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Anatomy of a stimulus domain: The relation between multidimensional and unidimensional scaling of noise bands

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Theoretically, there are many possible relationships between multidimensional scaling and unidimensional scalings of the same stimulus domain. In particular, it is uncertain what will happen if the number of psychological "dimensions" exceeds the number of physical variables. The multidimensional scaling of noise bands, unidimensionally a relatively well-understood domain, was done to explore these problems. In correspondence with the number of physical variables, a two-dimensional configuration was found to give a satisfactory account of the judgments of magnitude of stimulus difference. Axes of loudness, volume, and density were found to fit the configuration with a high degree of precision, lending support to the metric value of numbers produced in magnitude estimation. Pitch, or frequency, also had a simple relationship to the configuration, but was not an axis or dimension. Therefore, the usual conceptualization of judgments of overall similarity as the result of combining difference on separate dimensions is questioned. It is suggested that multidimensional configurations may sometimes correspond to internal representations of general importance.

Nonmetric scaling is thought to be a method for discovering the important psychological dimensions underlying measures of similarity between stimuli or other objects of interest. Measures of stimulus difference are summarized in a spatial configuration of two or more dimensions which is then examined for evidence of the influence of known physical or psychological variables. Clearly, it is assumed that differences in overall similarity are some simple function of psychological differences along single dimensions. The multidimensional scaling model realized in Kruskal's (1964) program permits consideration of several combination rules. For example, the city block metric assumes that differences on each dimension are added to give overall stimulus difference. Hyman and Well (1967) suggested that this is the way subjects ought to deal with multidimensional stimuli. The Euclidean metric assumes that overall difference is the square root of the sum of the individual scale differences squared, Minkowski R-metrics do the same with other exponents, and the dominance metric uses the largest of the individual scale differences as the measure of overall difference. Selection of the correct combination rule should result in directions in the configuration being identifiable with the underlying psychological dimensions. Furthermore, the position of the axes is fixed and given in the solution for every dimensional combination rule except the Euclidean. However, Tversky and Krantz (1970) have pointed out that the mathematical assumptions underlying multidimensional scaling, while less restrictive than those of factor analysis, permit consideration of only a limited range of combination rules, excluding some plausible alternatives. For example, if overall stimulus difference were the product of differences on the separate dimensions, this rule would be incompatible with the multidimensional scaling assumptions, and the underlying psychological dimensions would not be evident as directions in the configuration. Therefore, the use of multidimensional scaling as a discovery method requires the assumption that overall stimulus similarity is related to differences on unidimensional scales in one of these simple ways.

What might be the relationship between the dimensions underlying measures of similarity and the dimensions that can be scaled by unidimensional scaling techniques in the same domain? The concept of "dimension" in unidimensional psychophysical scaling has changed over the past 50 years. At first, psychological dimensions were conceived of as distorted versions—through a logarithmic or power law—of the physical dimensions of stimulus variation. Thus, Stevens (1934) suggested that the psychological dimensions of a stimulus domain should be independent of each other, in the same way that a good set of physical dimensions are independent.

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However, it became evident that sometimes subjects could produce reliable psychophysical scales of more attributes than there are dimensions of physical variation; Stevens and Davis (1938) suggested that subjects might be equipped to compute an indefinite number of functions of the physical variables, and they proposed new criteria for psychological dimensions: that they be reliable and continuous. While we can see that the conversion of a few physical variables into a large number of psychological variables might have advantages for the perceiver, since physical multidimensionality is known to increase the perceiver's ability to handle information (Garner, 1962) and added psychological dimensions might have the same effect, one wonders what this suggests about the mapping of a physical stimulus space into psychological similarity space. What will happen to these "extra" dimensions? How will distance in similarity space be related to differences along all of these psychological dimensions? It is plausible to assume that the subject in the multidimensional task of judging overall similarity would judge difference in every respect, or along every psychological dimension available to him, and then apply his combination rule to these results. This would suggest that the multidimensional solution should have the same number of dimensions as the number of readily accessible single psychological dimensions, unless there are linear relations among the psychological scales.[1] Furthermore, since psychological scales are never pure representations of variation in a single physical variable (for example, both frequency and intensity influence loudness), the dimensions of the configuration might not be closely identified with the physical variables. Finally, it should be recognized that the similarity configuration might not be related to unidimensional scales in as simple a way as has been generally assumed.

Usually, multidimensional scaling is done with the assumption, or the hope, that it will be possible to identify the physical dimensions of stimulus variation that are affecting overall psychological similarity. Of course, it is recognized that the relationship between the physical variable and psychological similarity is likely to be nonlinear, so that the dimension will be streteched or contracted to give a psychological scale. However, an ordinal relationship with the physical variable is still expected. In addition, one might expect a linear relationship between the dimension in the similarity space and the psychological dimension most closely associated with the physical variable. The scaling of a well-understood domain offers the opportunity to test the validity of these implicit assumptions. In particular, it seems important to determine what happens when there are more psychological dimensions than physical variables because, ordinarily, the nature and number of psychological attributes which could be scaled unidimensionally within a stimulus domain are unknown. Therefore, a result in multidimensional scaling which is not closely related to the physical dimensions of stimulus variation is likely to be uninterpretable.

The Stimulus Domain

Both pure tones and narrow bands of filtered white noise present the opportunity to address all of the issues raised above. The underlying physical variables, frequency and intensity, are known. Four psychological scales, loudness, pitch, volume, and density, have been defined and measured. Thus, either of these stimulus domains permits the exploration of the relationship between overall similarity judgments and both physical and psychological dimensions of stimulus variation. In particular, the effect of extra psychological dimensions, beyond the number of physical dimensions can be explored.

Narrow bands of filtered white noise were selected as the stimulus domain to be studied for two possibly related reasons. Stevens, Guirao, and Slawson (1965) reported that subjects find it easier to make judgments of noise bands. Since judgments of stimulus difference can be quite difficult, this was an important consideration. In addition, it is generally believed that "unanalyzable" stimuli, which do not give the impression of distinct and separate attributes, are more suitable for multidimensional scaling. Noise bands are certainly less analyzable, in this sense, than pure tones.

The Unidimensional Studies

The results of multidimensional scaling were to be compared with the extensive background of unidimensional studies of these stimuli. Because of its practical significance, the loudness of noise bands has been very thoroughly studied. Stevens (1961) was the source of the loudness values used in this experiment. In contrast, there appear to be no extensive studies of the pitch of narrow bands of noise. It is generally assumed that the pitch of narrow bands of noise behaves in the same way as the pitch of pure tones (Stevens, personal communication, and, e.g., Rainbolt & Shubert, 1968). Nábělek and Krútel (1968) demonstrated that subjects match the pitch of a narrow noise band with a tone approximately equal to the central frequency of the noise band. Therefore, Stevens and Volkman (1940) was used as the source of the pitch values for the stimuli in this experiment. Guirao and Stevens (1964) measured the density of noise bands with exactly the same characteristics as those used in the experiment reported here; density values were estimated from the graph in that paper. The only measurements of the volume of noise bands were done by Stevens et al. (1965); these were not extensive enough to provide a complete picture of the

Table 1
The Stimuli

Stimulus Number	Intensity (Decibels)	Frequency	Sones	Mels	Volume	Density
1	80	330	12.0	440	8.0	2.8
2	100	510	57.0	611	10.0	12.5
3	65	2040	7.8	1563	.7	5.8
4	90	2520	45.0	1780	1.3	20.0
5	55	5000	5.5	2478	.016	7.0
6	82	1200	19.0	1125	3.5	9.5
7	93	3600	66.0	2146	.8	25.0
8	60	390	3.8	499	5.3	.95
9	72	720	9.0	775	4.0	3.6

function but were found to be consistent with the measurements of the volume of pure tones done by Terrace and Stevens (1962). Estimates of volume were therefore taken from Terrace and Stevens (1962); it is possible that these estimates are somewhat in error, since the density of noise bands was found to differ somewhat from the density of pure tones.

The domain of noise bands offers one additional advantage for a study of this sort. Stevens et al. (1965) claimed that loudness = volume x density. If this nonlinear relationship is true, then we will not find a two-dimensional solution for the similarity judgments on which can be fit loudness, volume, and density as axes. If the best solution is two-dimensional, and its axes are closely related to two of the four psychological dimensions uncovered in unidimensional scaling, then they should be *either* pitch and loudness *or* volume and density. If they are volume and density, then these results will cast serious doubts on the use of multidimensional scaling as a discovery method for underlying psychological dimensions. For, without the unidimensional psychophysics which established these dimensions, one would never look for them as axes in the similarity space.

METHOD

Stimuli

Bands of noise were generated by filtering a white noise with the aid of a continuously variable filter (Allison 23). For a given stimulus, the high- and low-pass sections of the filter were set to the same frequency (as calibrated with a tone oscillator) which produced a band about a quarter octave wide at the half-power level. The filtered signal was then amplified, monitored for intensity by a voltmeter to a power level previously established as resulting in a sound pressure level of 100 dB re 0.0002 microbar, and passed through a decibel attenuator set to the appropriate value. There were two such arrangements of apparatus, one for each member of the stimulus pair. A switch controlled by the subject had one position for each member of the stimulus pair as well as a neutral position. Stimuli were presented binaurally through calibrated earphones known to give a reasonably flat response in the relevant range.

Nine stimuli, well distributed in both frequency and intensity, were selected for this experiment. Table 1 presents the values of these stimuli on the two physical variables and the four psychological scales.

Subjects

The subjects were four graduate students in psychology and one undergraduate psychology major; all had some previous experience in magnitude estimation experiments.

Procedure

Judgments of stimulus difference were obtained for each of the 36 possible pairs of the nine stimuli. The procedure was magnitude estimation of stimulus difference without a standard. After a few initial practice trials, the 36 pairs were presented twice to each of the subjects, always in a different random order. The instructions to the subjects were: "You are being asked to judge the size of the overall difference between the members of a pair of noises. Assign an arbitrary number to the difference between the first pair. If the second pair sounds ten times as different, assign it a number ten times as large; if it seems only half as different, assign it a number half as large. In the same way, compare the third pair to the second pair, and so on. Feel free to use decimals and fractions if they are necessary to express your judgments. You will be given five practice pairs to familiarize yourself with the judgments. Remember that a large difference between the two members of a pair should be assigned a large number." The subjects were shown three pairs of visual forms, varying in similarity, to illustrate the nature of the task. Because the range of stimuli exceeded the dynamic range of any of the available recording devices, stimuli were manually set up for each presentation. In order to minimize the time between judgments, two experimenters participated in the running of the experiment, one for each member of the stimulus pair. Once the stimuli had been set up, the subject was free to switch from one stimulus to the other, or to the neutral position, as often as desired while making his judgment. During the experiment, the subject was seated in a sound-isolation booth and communicated with the experimenters through an intercom system.

RESULTS

Reduction of Data

In the multidimensional scaling procedure, distances in a spatial configuration are iteratively adjusted so that the rank-ordering of distance between points approximates the rank ordering of judgments of stimulus similarity as closely as possible. Because nonmetric scaling requires only ordinal data and because little is known about the statistical properties of magnitude estimations of distance, each set of judgments was ranked individually. These 10 ranks were then averaged to give an overall ranking which was used as input to the Kruskal (1964) MDSCAL program. The reliability of the rankings,

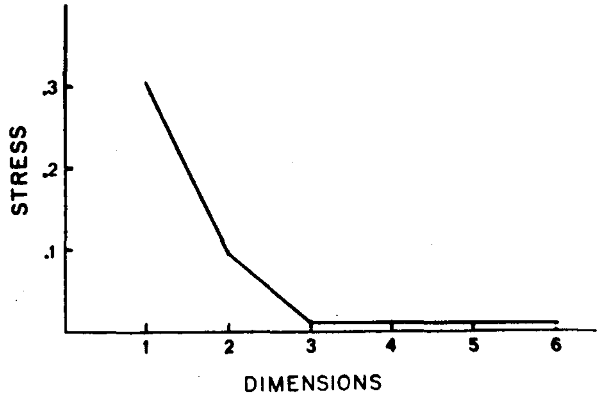

Figure 1. Stress as a function of the number of dimensions. Each point represents the best of four Euclidean solutions.

estimated by computing average ranks for two groups of five judgments, was .92. According to the Spearman-Brown formula (Gulliksen, 1950), this gives an estimated reliability of .96 for the total group. Since the correlation between these average ranks and the geometric means of the original judgments was .96, this ranking procedure made little difference; however, the estimated reliability of the geometric means was somewhat lower, .86 corrected to .92.

Dimensionality of the Solution

Generally, two criteria have been used for deciding what dimensionality is appropriate as an explanation of the data. One is the reduction of stress, a measure of goodness of fit to the input rank-ordering, with increasing dimensionality; a graph of this function can be examined for indications of diminishing returns. Figure 1 presents the lowest stress value (Kruskal's Formula 2) obtained in four Euclidean solutions at each level of dimensionality. Because these scaling procedures are plagued with local minima, several solutions from different starting configurations are necessary in order to be reasonably certain that the best solution has been achieved. Figure 1 indicates that the appropriate number of dimensions is either two or three. According to the empirically derived criteria reported by Wagenaar and Padmos (1971), the stress values for either of these solutions can be considered statistically significant at the .05 level. However, it is unclear whether the improvement obtained in going from two to three dimensions is significant.

The other established criterion is the interpretability of the solutions in the various dimensions. As described below, the two-dimensional solution was highly interpretable. The three-dimensional solution appeared to be very much the same; in particular, the one axis which did not fit the two-dimensional solution very well, frequency or pitch, also could not be fitted to the three-dimensional solution. Therefore, the addition of another dimension did not improve the explanatory value.

Finally, when the original data can be expected to have metric quality, distances in the solution can be compared with the data. In this case, the correlation between the distances in the best two-dimensional solution and the geometric means of the magnitude estimations was .96. Since this was approximately the same as the estimated reliability, this solution was considered to be an entirely adequate description of the data.

The Combination Rule

Several different metrics, which represent the way in which values on the several dimensions combine to yield overall perceptual differences, can be used in obtaining the multidimensional solution. The most commonly considered are the Euclidean metric, ordinary distance, and the city-block metric, simple addition of distances along each dimension. Superiority of the city-block metric is considered to indicate that the subject analyzes the stimuli into the dimensions given as axes in the solution. Typically, stress values for the metrics are compared to determine which combination rule applies. In this case, the stress values for the best Euclidean (stress = .093) and the best city-block (stress = .10) solutions were nearly identical; the configurations were nearly identical in appearance, and the distances in the two solutions have a correlation of .997. Thus, the customary criteria provided no basis for deciding between the metrics.

The results of several previous experiments indicate that this situation is not unusual. A recent analysis of pure tones (Carvellas & Schneider, 1971) also found very little difference between the stress and configurations of Euclidean and city-block solutions.

The results of Donald Gaudino's unpublished scaling of geometric forms varying in size and color indicate that stress values may not be very effective in differentiating between metrics, even when it is possible to do so by other means. Repeated Euclidean and city-block solutions yielded comparable stress values, although the corelation between Euclidean and city-block distances was only .54. However, city-block distances gave a correlation of .98 with ratings of stimulus difference, while Euclidean distances gave a correlation of only .54. It seems likely that this clear-cut differentiation between metrics can be obtained only with a large number of stimuli, well beyond the number required to outline the basic structure of the configuration, since Gaudino's study employed 25 stimuli, in contrast to 9 in this study of noise bands and 7 in the study of pure tones.

While the criteria of goodness-of-fit could not decide between the two metrics, there were other reasons for preferring the Euclidean solution. The axes of a city-block solution are fixed and defined in the solution. In this experiment, one of these axes corresponded to loudness or intensity (r = .976 and r = .94); the other did not correspond to any known attribute. It was clearly related to pitch or frequency (r = -.66 and r = -.58), but because there was an obvious interaction between frequency and intensity as determinants of stimulus similarity, these were not acceptable interpretations. In addition, it is not as reasonable to fit additional axes into a city-block configuration as to do so for a Euclidean configuration; yet, it was possible to fit two additional axes.

The Best Euclidean Solution

Figure 2 presents the best two-dimensional Euclidean solution together with the best-fitting axes for the two physical and four psychological variables. These axes were obtained by maximizing the linear correlation between the projections on the axis and the values given in Table 1 for each variable. Of course, there is considerable uncertainty about the location of these axes; for instance, the sones axis gave correlations greater than .90 in any location between +10 and -55 deg as measured from the right horizontal. The best location was -23 deg with a correlation of .986. In addition, of the values for the psychological variables, only those for loudness are very well established.

The roughly horizontal axis is the most obvious in the configuration and corresponds to either sound intensity or loudness in sones. The ordering of the stimuli corresponds perfectly to their intensity in decibels; however, the expanded spacing between 80 and 100 dB is characteristic of the sone scale. Thus, as expected, the psychological scale has a closer relationship to the axis in the similarity space than does the physical scale. The importance of loudness was also revealed in a correlation of .83 between the differences in sone values and the geometric means of judgments of overall differences between the noises.

In addition, it is evident that the low-frequency tones are arrayed above this axis and the high-frequency tones below. However, the best-fitting axes for frequency (r = .62) and its psychological counterpart, pitch (r = .72), do not fit very well. There seems to be an interaction between frequency and intensity as determinants of overall psychological difference: at low loudness levels, frequency differences have less effect upon perceived similarity than at high loudness levels. The projection along the radii of a circle centered at the point where the field of stimuli converges gives a much better approximation to the pitch scale along the arc of the circle. While the effect of frequency is very clearly revealed in this configuration, the interaction violates the assumption of the spatial model, and neither frequency nor pitch can be regarded as a "dimension" in this space unless intensity proves to have a large effect upon the pitch of noise bands.

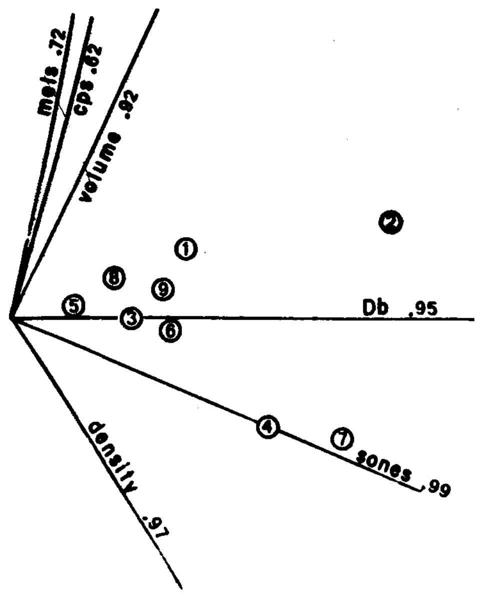

Figure 2. The two-dimensional Euclidean configuration of stimuli with best-fitting axes corresponding to the two physical variables, intensity and frequency, and the four psychological dimensions of this domain: loudness, pitch, volume, and density. The correlation between projections on each axis and the values of the corresponding variable is also indicated.

Axes for volume and density also fit this configuration very well, (r = .92) and (r = .97). Since volume was measured for pure tones rather than noise bands, the actual relation between volume and spacing in the multidimensional configuration could be even closer.

DISCUSSION

The results of the multidimensional scaling of this domain show that to a considerable degree the expectation of a close relationship between unidimensional and multidimensional scaling was supported. Three of the known unidimensional scales fit very well into the multidimensional configuration. In fact, they fit so well that we can regard the reproduction of spacing along the sones axis, for instance, as providing independent evidence for the metric meaningfulness of the use of numbers in the magnitude estimation task. In addition, fitting the three axes of loudness, volume, and density with such a high degree of precision implies a linear relationship among them. This, of course, is in direct conflict with Stevens et al.'s (1965) claim that loudness equals volume times density. Their claim is based on unidimensional scaling results. Figure 3 presents an exploration of the somewhat arbitrarily chosen hypothesis that volume and density are perpendicular axes and that loudness is another axis midway between the two. Similar graphs were drawn with the axes of volume and density at 122 deg separation, as

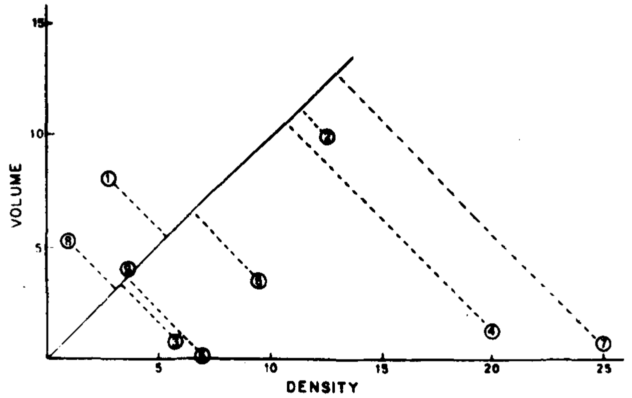

Figure 3. The stimuli plotted by volume and density values and projected onto a hypothetical sones axis.

they are in Figure 2, one with the sones axis at the position shown in Figure 2 and another with the sones axis midway between the volume and density axes. Both configurations resemble the multidimensional solution, showing somewhat less contraction of the frequency effect at low loudness levels. When the points are projected onto these sones axes, positions along these axes have correlations of .99, .97, and .99 with the sones value of the stimuli. In contrast, for these stimuli, the product of volume and density has a correlation of .61 with loudness. Stimuli 4 and 7 deviate drastically from a linear relationship between these two quantities and the less intense high-frequency stimuli show smaller deviations. Stevens, Guirao, and Slawson obtained relatively higher estimates of volume for comparable stimuli and reported difficulties with high-frequency stimuli. In the graph of their results, there are considerable deviations from the multiplicative relationship, especially for high-frequency stimuli. Thus, there is little question that the hypothesis suggested by the spatial analysis obtains greater support from the available data, and multidimensional scaling proved its usefulness in suggesting a relationship among the unidimensional scales which was not at all obvious without it.

Nevertheless, some difficulties do arise in relating the configuration to the unidimensional results. Since the pitch scale does not correspond to an axis in the configuration, volume and density, or perhaps one of these in combination with loudness, must be regarded as the best explanation in terms of dimensions if there are such things. In terms of the general application of multidimensional scaling, this outcome is awkward because these are not the psychophysical scales which correspond most closely to physical variables in the stimulus domain; these axes are improbable hypotheses which would not be recognized as meaningful if it were not for the extensiveness of the unidimensional explorations of this domain. Nor do they emerge as the axes of a city-block analysis, which might encourage further exploration of them. In the Euclidean solution, there is no a priori reason for selecting those particular directions as worthy of further exploration.

In addition, the failure to find pitch as an axis of either the two- or the three-dimensional solution (where it should have appeared even if not linearly related to the other dimensions) calls into question the straightforward notions, implicit in the use of axis fitting, of the relationship which should exist between the two scaling methods. Admittedly, there is an orderly relationship between pitch or frequency and location in the configuration; at any given intensity level, it appears likely that the spacing of stimuli would correspond to their pitches. The configuration has utility because it displays a relationship between pitch differences, loudness differences, and overall psychological difference which might otherwise be quite difficult to discover. However, the relationship between pitch and the configuration must be regarded as disappointing in one of two ways. Either the notion that it should be possible to project onto an axis across the whole configuration must be abandoned or it must be concluded that the rules for combining differences on the psychological scales sometimes do not meet the conditions imposed by the multidimensional scaling model. To some extent, of course, the conclusion is contingent upon further study of the pitch continuum. Stevens (1935) reported that intensity affects pitch in the same way that is suggested by the multidimensional configuration, but the effect was not nearly so large. Perhaps it will prove to be larger for noises than for tones.

A general issue emerges from this discussion: what relation can be expected to exist between the multidimensional scaling configuration and performance in unidimensional scaling or other experimental tasks? This question was implicit in some previous discussions of multidimensional scaling. For instance, Cliff and Young (1968) asked whether the configuration represents everything the subjects can do with the stimuli, but they did not specify what was meant by representation, what rule should be applied to the configuration to generate predictions for other experimental tasks. Similarly, Hyman and Well (1967) suggested that the Euclidean spatial model carried with it a large excess baggage of unjustified assumptions and implications. Presumably, they meant that the spatial configuration implied certain results for other experimental situations, but these implications were not specified. Essentially, these authors were asking whether the configuration had psychological reality, whether it corresponded to an internal representation which was the basis for performance in a variety of tasks.

At the very least, the results of the present experiment call into question the generality of the relationship which has generally been presumed to exist between unidimensional scales and the multidimensional configuration. In addition, the presence of an interaction between the effects of the two physical variables casts doubt upon Hyman and Well's claim that the subject should be assessing difference on each dimension separately. Nevertheless, the results of this experiment can be seen as conforming to our general way of thinking about psychophysical scaling; that is, the similarity space is a reasonably regular distortion, a conformal mapping of the physical stimulus space just as the usual unidimensional scales are reasonably regular distortions of single physical dimensions. Furthermore, the stimuli could be projected onto a pitch axis if the rules of projection were altered in accordance with the distortion of the similarity space. This, too, would be a projection rule of some generality, even if it presents practical difficulties in application.

Perhaps there should be a minor revolution in our way of thinking about multidimensional scaling to consider the possibility that the configuration is psychologically real. Nearly everyone has considered that multidimensional scaling involves separate and distinct dimensions, combining according to some rule in order to give overall similarity. Clearly, this assumption lies behind Tversky and Krantz's analysis of the mathematical foundations of multidimensional scaling. Yet, there is another line of thinking in the multidimensional scaling literature that is incompatible with this assumption: the distinction between unanalyzable and analyzable stimulus domains. Analyzable domains, which have perceptually obvious and distinct dimensions, are thought to be unsuitable for multidimensional scaling. Furthermore, some of the results accumulating in the literature are consistent with the psychological reality of the configuration. Shepard (Note 1) described other results, like the results of this experiment, in which regular contour lines can be fitted into the configuration but axes cannot. Presumably, the possibility of obtaining unidimensional scaling along multiple directions, as in the present experiment, is something to be expected if there were an internal representation of the configuration. It is quite striking, too, that many intuitively derived arrangements of stimulus domains are now emerging from multidimensional scaling: the Henning taste tetrahedron (Yoshida, 1963) and the color circle (Shepard, 1962). Similarly, the results of Rummelhart and Abrahamson's (1973) animal analogy experiment seem consistent with the notion that the configuration itself is meaningful, apart from any axes.

It is still unclear what the alternative conceptualization of multidimensional scaling would be if the notion of separate dimensions and combination rules were abandoned. Possibly, judgments of overall similarity must be considered as primary and immediate. The configuration would then be a cognitive representation of these relations, perhaps even a mapping onto two- or three-dimensional space. This mapping would be comparable to the mapping onto a general magnitude scale which is so familiar a concept in unidimensional scaling.

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NOTE

1. David Krantz pointed out that this would imply a manifold of the dimensionality of the physical stimulus space embedded in the psychological space of higher dimensionality.

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