Sovereign Borrowing, Sanctions and Yield Spreads*

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Abstract

We model sovereign debt in the absence of a bankruptcy code. Threat of trade sanctions and seizure of exports by the lenders are the drivers of enforcement of sovereign debt contracts. The borrower takes these potential actions into account when choosing optimal voluntary default and debt capacity. We obtain a closedform solution for the sovereign yield spreads that depend on the costs of sanctions and seizures of export collateral. Our model predicts that sovereign debt will be renegociated more often and yield higher credit spreads than otherwise identical corporate debt.

JEL Classification: G12, F34

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I Introduction

The study of sovereign loans and bonds has tended to focus properly on the question of why there is sovereign lending in the first place. Corporate debt and sovereign debt differ in many important respects. A corporate debt contract is a legally enforceable claim on the assets of the corporate borrower. In the event of default (defined as non-payment or a non-timely payment of contractual obligations), the lender has the right to initiate actions against the borrower under the framework of a bankruptcy code. The essence of the bankruptcy code is to enable the assets of the borrower to be accessible to the lender under the provisions of the bankruptcy code. The nature of the bankruptcy code and the jurisdiction under which the borrower operates are important factors in determining the relative attractiveness of a corporate debt (or loan) contract to the lenders. The sovereign debt on the other hand may not necessarily be a legally enforceable contract. The borrowing country may default on its obligations without the lender having any recourse to a bankruptcy code that enables the lender to access the assets of the sovereign borrower. The recent debt crisis (in the summer of 1998) in the Russian GKO market is clearly illustrative of this problem. As a result of this, much of the sovereign debt literature has naturally tended to focus on reasons as to why sovereign lending takes place at all. The seminal contributions in the theory of sovereign debt of Eaton and Gersovitz (1981a), Fernandez and Rosenthal (1990) and Bulow and Rogoff (1989a, 1989b) provide explanations for rational sovereign lending. These explanations are based on the costs of access in the future to global credit markets (sometimes referred to as reputation costs) and on the potential for retaliatory actions by the lender by way of sanctions and trade barriers. Empirical work on sovereign spreads appears to suggest that sovereign credit spreads are much higher and more volatile than investment grade domestic (U.S.) corporate debt. Papers by Cantor and Packer (1996) and Eichengreen and Mody (1995) provide some evidence in this regard. The theory of sovereign debt however, has not yet been used to shed light on the behavior of sovereign credit spreads.

In this paper, we make several contributions to the literature on sovereign borrowing and strategic default. We construct a continuous-time bargaining model in which the lender has no recourse to a bankruptcy code but can threaten to enforce trade sanctions and to possibly seize some of the borrower's exports. In such a framework, we explore the optimal amount of debt that the sovereign borrower will issue and the manner in which the borrower will default. We explicitly characterize the default trigger: the point at which the sovereign borrower decides to default and faces the possibility of sanctions and expropriation. The value of the sovereign borrower and of the sovereign debt under the optimal default strategy are also characterized. An important contribution of our paper is that we study the behavior of the credit spread between a sovereign bond and an otherwise identical Treasury issue which is risk-free. We show how the spreads differences capture the underlying potential for sanctions and seizures as well as the presence or absence of a bankruptcy code. Using data on sovereign debt credit spreads from Argentina, Brazil, Mexico and Russia over the period October 1999 to June 2001, we also show some stylized facts about the relationship between sovereign debt credit spreads and the structure of a country's foreign trade policy that are consistent with our model's predictions. Indeed, we observe that the degree of a country's trade policy concentration influences the effectiveness of the sovereign lenders' trade sanctions and thus the level of the credit spreads.

The paper is organized as follows: In section 2 we provide a very brief review of the literature on sovereign borrowing. Rather than attempting an exhaustive survey of the literature we focus on some of the major contributions that have a direct bearing on our paper. We also set forth in section 2 some unresolved questions and some empirical regularities on sovereign debt that motivate our paper. In section 3, we use credit spreads and trade data on four emerging market countries, namely Brazil, Argentina, Mexico and Russia, to provide some stylized facts about the relationship between a

country's trade policy, its asset seizure capacity, its cost of sanctions and its sovereign debt credit spreads. In section 4, we present our model of sovereign debt and analytically determine the value of the sovereign country, of the sovereign debt and the main features of the sovereign debt contract. In section 5, we derive and examine the sovereign debt credit spreads. We conclude in section 6 by pointing out the limitations of our model and the directions in which it may be extended.

II Review of Sovereign Debt Literature

We review briefly models of sanctions that have a direct bearing on our paper. Since our model only considers a single issue of debt we do not survey in detail the literature on repeated borrowing and the role of reputational costs. Eaton and Gersowitz (1981) develop an intertemporal competitive equilibrium model in which they show that there exists an equilibrium in which a borrower who needs to borrow repeatedly optimizes by repaying in periods of high income and borrowing new one-period debt in periods of low output. Sanctions can be an effective tool to enforce the right incentives. In a model of trade sanctions, Bulow and Rogoff (1989b) consider a small repeated borrower who produces a single good while subject to random shocks to output. They show that, in any sequential equilibrium with perfect information, a lender will not lend under a pure reputation contract. The critical assumption in their model is that the sovereign can smooth its output shocks with cash-in-advance contracts. Thus, in a reputation based contract, the sovereign will inevitably choose to default in the future because increased savings (with the help of cash-in-advance contracts) allow him to hedge the country's future stochastic output as well as future lending. Lenders will thus refuse to lend in equilibrium. The introduction of additional penalties, e.g. economic and political sanctions, are thus necessary to achieve a positive-lending equilibrium. Bulow and Rogoff (1989b) equilibrium result suggests that the sovereign should bargain for debt forgiveness schemes since forgiven debt is indeed forgotten. The failure of reputation

as the sole monitoring device, on the one hand, and the empirical reality of frequently renegotiated partial defaults on sovereign loans, on the other hand, are also at the core of the constant recontracting model developed in Bulow and Rogoff (1989a). These authors show that it is the threat of economic and political sanctions which actually enforce a debt contract with positive lending and constant recontracting of the debt servicing.

Bulow and Rogoff leave an important issue unexplored. Their model does not provide guidelines about the credit spreads at which sovereign loans should trade. While all the previously analyzed models were cast in discrete- time, Kulatilaka and Marcus (1987) develop a model of strategic default which analyzes the timing of debt repudiation in a continuous-time setting. They do not endogeneize the debt servicing, or the sovereign credit spreads which is the focus of our study. Claessens and Pennachi (1996) propose a continuous-time pricing model for the *relative valuation* of Brady bonds. They use the market prices of Brady bonds to extract the repayment capacity [a latent variable] of the country. Their model takes into account the non-linear relationship between the debt prices, the repayment functions and the contractual features such as the underlying collateral. The implied repayment function is then used to value other sovereign debt issues. More recently, Duffie, Pedersen and Singleton (2003) estimate the Russian bond yields using a reduced form model of default. Our model differs from theirs in the sense that we look at the economic determinants of sovereign debt credit spreads. Since World War II, international legal changes have contributed to strengthen the role played by trade sanctions to enforce sovereign debt contracts according to Bulow and Rogoff (1989a). Furthermore, they show that the disruptive cost of imposing those sanctions can be quite substantial. Given the large fraction of exports in Mexico (90% of their trade is with industrialized countries) or Brazil (around 60 %), for instance, the World Bank estimates that trade disruptions costing less than 3% of these countries GNP would be more disruptive than making payments of 5% of their total external debt. Even more extreme, is the example of Rhodesia, since the estimated cost of trade sanctions represented 15% of its annual GNP. Thus, Bulow and Rogoff claim that trade sanctions do support the observed sovereign debt levels and ac tual debt renegotiations frequencies in the late 80's. The implicit costs of sanctions are a key parameter of our structural model of sovereign debt pricing under optimal default.

We summarize below some stylized facts that emerge from the empirical and historical studies which we believe motivate the choice of the assumptions underpinning the sovereign debt model presented in this study. The data appears to support the following broad regularities:

- 1. The history of sovereign debt highlights that frequent partial debt renegotiations are the rule rather than the exception.
- 2. Yield premia on sovereign debt issued by a country with significant exports are lower [see Cantor and Packer (1996)].
- 3. Corporate yield spreads tend to be lower than "otherwise identical" sovereign debt issues spreads[see Cantor and Packer (1996)].

III Credit Spreads and Trade Policy: Stylized Facts

In this section, we use trade and credit spreads data on four emerging markets countries, that is on Argentina, Brazil, Mexico and Russia, in order to emphasize some stylized facts about the relationship between a country's credit spreads and the structure of its foreign trade.

First, as noted by Boot and Kanatas (1995)," the more "open" is the debtor's country economy, the more assets it has to loose to its creditors". The authors argue that it is a credible threat to confiscate the assets owned by the sovereign. This threat is reinforced for countries with a higher exports capacity. Cantor and Parker (1996) support this conjecture when they note that sovereign yield spreads tend to be lower for countries

with higher exports levels. The role played by the level of exports seizure on credit spreads will be examined for the four countries under review.

Second, we assume that the degree of diversification of a country's foreign trade policy is also relevant in explaining sovereign credit spreads. More precisely, we relate the level of a country's trade policy diversification to the credibility of the sovereign creditors' trade sanctions mechanism. In other words, we conjecture that if a sovereign country has a very diversified trade policy, it will be more difficult and more costly for its creditors to credibly enforce trade embargoes or other forms of trade sanctions. Indeed, such actions are much harder to implement and to coordinate at the international than at the national level. If this conjecture is fulfilled than we should expect, even after controlling for the exports level, that countries with a very concentrated trade policy have tighter credit spreads than borrowers with a more diversified trade policy.

Thus, in the remainder of this section we will relate the strength of the lenders' assets seizure capacity and of their trade sanctions effectiveness to the levels of each sovereign country's exports capacity and to its foreign trade diversification respectively. This should thus enable us to explain the ranking of the credit spreads observed in those four emerging markets countries.

We obtained sovereign spreads on representative bonds for Argentina, Brazil, Mexico and Russia. We use daily yield spread series collected over the period ranging from the 10/18/1999 to the 06/05/2001. The yield spreads data was collected for individual bonds, that is, an Argentinian bond with a coupon of 11 3/4 % maturing in 2009, a Brazilian bond with a coupon rate of 14.5 % maturing in 2009, a Mexican bond with a coupon rate of 10 3/8 % maturing in 2009 and a Russian bond with a coupon rate of 10 % maturing in 2007. The bonds are non-callable bullet issues¹.

In tables 1 and 2 below, we provide the mean and the standard deviations of the four sets of credit spreads as well as their correlation over the sample period. We observe that

 $^{^{1}}$ We did not have access to the market prices or to other details concerning the covenants of these bond issues.

Mexico displayed the lowest and least volatile credit spreads whereas Russia displayed the highest and most volatile credit spreads over the sample period. Furthermore, while the credit spreads of Brazil and Argentina have a similar mean value, the latter is much more volatile over the sample period.

[Insert Table 1]

[Insert Table 2]

In addition to the spreads data, we also collected information about the volume of trade, patterns of trade, Debt/GDP ratios of these countries during the same sample period from various sources such as IMF, UNCTAD and Datastream. This data is used to interpret the costs of sanctions and the seizure costs of collateral from a trade policy's perspective. In figure 1, we summarize the GDP in US dollars and in figure 2 we summarize the external debt information for each country in the sample over the last five years. Using the 2000 figures, we find that the ratio of debt to GDP is 0.479 for Argentina, 0.268 for Brazil, 0.259 for Mexico and 0.335 for Russia. We note that all countries bear a debt to GDP ratio that is equal to or above the sustainability level of 0.25 as defined by the Bretton Woods Institutions. The leverage measure is highest for Argentina and lowest for Mexico. The Debt to GDP ratio is not however the sole determinant of sovereign credit spreads and cannot explain why the credit spread of Mexico is so much lower than the one observed in Brazil or why Argentina has a lower credit spread than Russia. We next examine the trade pattern of these countries through the importance of their exports to GDP levels and through their foreign trade diversification policy.

[Insert Figure 1]

[Insert Figure 2]

In figures 3 and 4 we report the top three trading partners for exports and for imports for these four countries. Several interesting patterns emerge. First, the total Exports - to -GDP ratios in 2000 reach 9.3 %, 8.9%, 28.9% and 42 % for Argentina, Brazil, Mexico and Russia, respectively. According to our conjecture, this would imply that the largest potential for asset seizure is with Russia while the lowest is with Brazil and Argentina. Looking at a country's exports to GDP ratio still does not explain why Argentina has such a lower credit spread than Russia and would point to the opposite conclusion. We next examine the degree of foreign trade concentration of each of these four countries. We note that Mexico's trade is lopsidedly linked with the United States both from the viewpoint of imports and exports. Implicitly, this means that the threat of a sanction is potentially very costly to Mexico. Thus, it is not at all surprising to observe that Mexico has the lowest credit spreads since it has both a strong Exports to GDP ratio and a strong concentration of its trade policy. For the other countries and especially for Russia, foreign trade appears to be much more diversified and evenly distributed among the three top partners. Thus, according to our conjecture, Russia is the country where trade sanctions would be the least effective and this phenomenon could thus explain why its credit spreads are higher than the ones observed in Argentina and in Brazil even though it has a higher exports to GDP ratio than the latter countries.

[Insert Figure 3]

[Insert Figure 4]

Building on these stylized facts, the threat of trade sanctions and its positive relationship to the degree of foreign trade concentration seem to play a crucial role in explaining emerging markets credit spreads. In the next section, we develop a theoretical model which integrates both the threats of trade sanctions and the degree of asset seizure while recognizing the dominant role played by the former in explaining sovereign debt credit spreads.

IV A Model of Sovereign Borrowing

We develop a continuous-time bargaining model that explicitly accounts for the role played by trade sanctions and by asset seizure on the market value of sovereign debt, on its debt capacity, and on the sovereign debt credit spreads. The approach that we follow is a continuous-time version of Anderson and Sundaresan (1996). In the absence of borrowing, the country has an initial wealth level of V_0 . Let us assume that the capitalized value of all goods and services produced by the country at time t in the absence of any borrowings follows the stochastic process shown below in equation (1). In particular, we assume that there are no exports in the absence of borrowings. The value is measured in units of domestic currency which is assumed to be the numeraire. In autarky, the country's wealth evolves as shown in equation (1).

(1)
$$dV_t = V_t \left[\mu - \beta\right] dt + V_t \sigma dz.$$

By assumption, in autarky, the country's wealth is distributed lognormally. The parameter μ defines the expected gross rate of growth of the country's wealth. The parameter β denotes the rate of outflow of the country's domestic wealth. When it is high, the economy is depleting its wealth and is growing slowly. When β is low, the economy is growing at a faster rate. This interpretation is crucial in order to better understand some of the following results. In order to generate exports and to create higher value, the country needs foreign resources. We assume that the country needs to import an amount of I_0 of the foreign currency to be in a position to generate exports of $g(I_0)V_t$ per unit time and to add to its capitalized value of goods and services. The whole amount borrowed is allocated to the exports technology ². The country issues a debt contract whose value is denoted by $D(V_0, c)$ whereby it agrees to pay at each instant a constant flow rate of interest of c per unit time on a par value which is set equal to

²We could also assume that a fraction q of the amount borrowed is allocated to exports and that the remaining fraction (1-q) is allocated to domestic investments. In this paper, we assume that q = 1. Results for the intermediate cases are available from the authors upon request.

1 without loss of generality. There is no exchange rate uncertainty in the model. We assume that the function g(I) is such that g(0) = 0, g' > 0, and g'' < 0. Thus, with borrowing the country's total value at time 0 is $V_0 + PV_0 [g(I_0)V_t] - I_0$ in the absence of default, where $PV_0 [g(I_0)V_t]$ stands for the present value at date 0 of the constant flow of exports.

More precisely:

$$PV_0\left[g(I_0)V_t\right] = E_0\left[\int_0^\infty V(s)g\exp(-\rho(s))ds\right],$$

where ρ^{3} denotes the country's discount rate. This expression can be simplified as follows:

$$PV_0[g(I_0)V_t] = \frac{g(I_0)V_0}{\rho - (\mu - \beta)}$$

The denominator of this expression is simply a scalar. In order to simplify the presentation, we will from now on assume that $\rho = \mu$, and thus, the above expression reduces to:

(2)
$$PV_0[g(I_0)V_t] = \frac{g(I_0)V_0}{\beta}$$

We assume that there are incentives to trade in the absence of default so that $PV_0[g(I_0)V_t] - I_0 > 0$. We assume that the export earnings are constant to keep the exposition simple. Since there are no bankruptcy codes that govern sovereign borrowing, it is assumed that the parties will negotiate when the contractual payments are not made. Therefore, it is in the interest of the lender to impose sanctions and appropriate export earnings if the borrower fails to perform his contractual obligations. This is the only way in which the lender can credibly bargain with the sovereign borrower. An important point of departure in our model as compared to the work of Bulow and Rogoff (1989a) is that

 $^{{}^{3}\}rho$ denotes the country's discount rate in the actual economy. Later on, we will work under risk -neutrality and thus assume that $\rho = r$, the default-free interest rate.

we require

$$D(V_0, c) = I_0.$$

This is the feasibility constraint by which we only consider the set of debt contracts that can finance the investment I_0 .

The lenders have the ability to seize a fraction $(1 - \gamma)$ of the export earnings per unit time if the borrower is not able to provide the contractual debt service. In the absence of default, the value process evolves as shown in (1). The first time the borrower defaults, trade sanctions are imposed so that the flow rate of output at time $s > \hat{t}$ reduces to $V_s\beta e^{-\delta(s-\hat{t})}$ per unit time, where δ represents the constant sanction rate per unit of time. The basic idea that we capture here is the following: the sanctions have the greatest impact on the flow rates of output of the borrower immediately following default and decays exponentially over time. Let, r denote the default-free interest rate, then assuming that the sovereign country is risk-neutral, the capitalized value of his output at time \hat{t} can be written as:

$$(3) \quad E\left[\int_{\widehat{t}}^{\infty}\beta V_{s}e^{-\delta(s-\widehat{t})}ds\right] = \int_{\widehat{t}}^{\infty}\beta V_{\widehat{t}}e^{-r(s-\widehat{t})}e^{(r-\delta)(s-\widehat{t})}ds = \frac{\beta V_{\widehat{t}}}{\beta+\delta} \equiv \alpha V_{\widehat{t}}.$$

Note that if there is default, the trade sanction against the borrowing country amounts to a penalty such that the capitalized value of the goods and services at the time of default \hat{t} becomes equal to $\alpha V_{\hat{t}}$ where $\alpha \in (0, 1)$. The borrower decides at each point whether to pay the contractual debt service or not. A critical level of the capitalized value of the goods and services denoted by V_S defines the point of departure: above this default trigger, the borrower will pay the contractual obligations. Below this trigger, the borrower defaults. To characterize the trigger level we now formulate the borrower's problem.

IV.A Borrowing Versus Autarky

The decision whether to borrow or not is based strictly on economic grounds in our model. At time t = 0, the country will compare the wealth associated with autarky with the wealth associated with the strategy of borrowing. In essence, it will compare V_0 with the value under borrowing which we denote by $S(V_0 + g(I_0)V_0 - I_0)$. If $V_0 > S(V_0 + g(I_0)V_0 - I_0)$, then the country will prefer autarky, else it will engage in borrowing. When the borrower has some debt outstanding, the value of the borrower is less than the capitalized value of the goods and services plus the capitalized value of exports. In other words, the value of the sovereign borrower denoted by S(V) is bounded from above by the quantity $\overline{S}(V) = V + PV_t[g(I_0)V]$. At the trigger point V_S the value of the sovereign borrower becomes equal to $\underline{S}(V) = \alpha V_S + PV_0[\gamma g(I_0)V].$ Under normal circumstances when the borrower is current in his debt obligations, the value of the sovereign borrower lies between these bounds. The sovereign borrower will compare the value under borrowing with the value under autarky at time 0. This difference, $S(V_0 + g(I_0)V_0 - I_0) - V_0$ is the difference between the sum of the benefits and the sum of the costs and the value under autarky. The benefits are the capitalized value V_0 , and the net present value of the exports $PV_0(Vg(I_0) - I_0)$. The costs are: (a) the value of the debt, $D(V_0, c^*)$, issued (which is a liability to be paid off) where c^* is the flow rate of coupon promised by the sovereign borrower that will yield at time 0 an amount I_0 . It thus consists of the present value of the coupon if the borrower stays solvent plus the present value of the costs of sanctions if the borrower does not honor his debt service. Formally, gains from trade, defined by $G(I_0)$, satisfy the following identity:

(4)
$$G(I_0) \equiv S(V_0 + Vg(I_0) - I_0) - V_0 =$$

 $PV_0(V_tg(I_0) - I_0 - (1 - p)PV(coupon) - pPV(Sanctions),$

where $PV_0(V_tg(I_0))$ is defined in equation 2 and where p is the probability of default. Equation 4 helps us to articulate the benefits and costs associated with sovereign borrowing. If there are no exports, there is no incentive in our model to engage in borrowing. Likewise, if the costs of sanctions are "too high" then there is no incentive for the country to borrow. The present value of sanctions is however endogenously determined in the model and it depends on the optimal voluntary default strategy of the sovereign borrower. Our goal is to determine endogenously the trigger point V_S and the borrower's value in the absence of default so that we can characterize the conditions of sovereign borrowing more sharply. From (4) we can say that there is an optimal level of borrowing denoted by I_0^* which is based on a trade-off between the marginal increase in the present value of sanctions due to a marginal increase in borrowing. In the absence of default risk, relying on equation (4) without the sanctions term and on equation (2), the optimal level of borrowing will be determined by the first order condition shown below:

(4a)
$$\frac{dG(I_0)}{dI_0} = 0 \Longrightarrow V_0 g'(I_0) = \beta.$$

This is the "first-best" outcome for the borrower⁴. In the absence of any sanctions, it is in the borrower's interest to stick to this policy. The presence of sanctions will cause the borrower to borrow less than the amount implied by equation (4a).

IV.B Optimal Default Strategy

Under the assumption than t the borrower is risk-neutral, the value of his country satisfies the partial differential equation shown next when $V \ge V_S$. The most intuitive way to see this is to recognize that under risk-neutrality, the local expected return must be

⁴For the numerical examples, we rely on the function $g(I_0) = I_0^{\zeta}$ which guarantees that equation (4a) has an interior optimum.

the default-free rate of interest. Or,

$$E_t\left[\frac{dS}{S}\right] = rdt.$$

By application of Itô's lemma, we then obtain the following valuation equation:

(5)
$$S_V(r-\beta)V + \frac{1}{2}S_{VV}\sigma^2V^2 - rS + Vg(I_0) + (\beta V - c) = 0.$$

The treatment of what the borrower obtains depends on the relative bargaining positions of the two counterparties. In order to tackle this problem, we will rely on the Nash solution to the bargaining problem as stated in Fan and Sundaresan (2000).

[Insert Table 3]

We provide in table 3, the possible outcomes to the sovereign lender and the borrower. The share of the borrower if he chooses to bargain is θ and that of the lender is $(1 - \theta)$. The share will be found endogenously. Let us assume that the endogenous bargaining boundary is V_S .

Define $\Psi = PV(gV_s)$. Note that given equation (2), we have $\Psi = \frac{g(I_0)V_s}{\beta}$. Then, the Nash bargaining formulation, at time zero, can now be stated as follows:

$$\theta^* = \arg \max \left[\theta [V_S + \Psi] - [\alpha V_S + \gamma \Psi] \right] \left[(1 - \theta) \left[V_S + \Psi \right] - (1 - \gamma) \Psi \right].$$

The unconstrained solution to the Nash Bargaining problem is:

$$\theta^* = \frac{(1+\alpha)V_S + 2\gamma\Psi}{2(V_S + \Psi)} = \frac{(1+\alpha) + 2\gamma\frac{g}{\beta}}{2(1+\frac{g}{\beta})}.$$

The share of the payoffs to the borrower is $\theta^*(V_S + \Psi) = \frac{(1+\alpha)V_S}{2} + \gamma \Psi$. It is useful to contrast the share that an otherwise identical corporate borrower (operating under a

bankruptcy code) might have received. Note that in the presence of a bankruptcy code, the lender has an outside option of liquidating. This would have given the lender access to the borrower's assets. In such a setting, the lender would have received in the absence of bargaining (under liquidation) an amount given by $(1 - \gamma) PV(gV_s) + (1 - \alpha) V_s$. The borrower would have only received an amount equal to $\gamma PV(gV_s)$. This would have significantly improved the lender's share in the bargaining. In fact, we can show that the borrower's share for an otherwise identical corporate contract where the lender has access to the collateral (under Nash bargaining) would be:

$$\widehat{\theta} = \frac{\alpha V_S + \gamma \Psi}{(V_S + \Psi)} = \frac{\alpha + \gamma \frac{g}{\beta}}{(1 + \frac{g}{\beta})}$$

It is easy to verify that $\hat{\theta} < \theta^*$. This shows that the outside option provided by the bankruptcy code helps the lender in the bargaining process. This also foreshadows the outcome that a sovereign debt contract is likely to be less valuable than an otherwise identical corporate debt contract. It is easy to show that the difference between the share of a corporate and that of a sovereign debt is

$$\theta^* - \widehat{\theta} = \frac{(1-\alpha)}{2(1+\frac{g}{\beta})}.$$

We observe that if α is strictly between zero and one, i.e. if there are sanctions, we find that the share of the corporate borrower is always less than that of the sovereign borrower. The borrower is better off in a sovereign contracting situation. Consequently, the lender is better off with a bankruptcy code. These results have strong implications for the spreads at which the debt contracts will be traded in the market: corporate debt will trade at a tighter spread than an otherwise identical sovereign debt. It is also reasonable to suppose that the costs of coordination and renegotiations are significantly higher in a sovereign setting than in a corporate setting where a formal bankruptcy code helps to coordinate the resolution of the process of financial distress. If the economy is

growing at a slow rate, then the sovereign borrower is able to take a higher share: this is due to the fact that the present value of the exports is much smaller and hence that the collateral value is lower.

The valuation equation (5) is valid when the value of the assets of the borrower is above V_S (in the continuation region) When the value of the assets falls below this threshold (in the stopping region), the valuation equation becomes:

(5')
$$\widehat{S}_{V}(r-\beta)V + \frac{1}{2}\widehat{S}_{VV}\sigma^{2}V^{2} - r\widehat{S} + \gamma Vg(I_{0}) + (\beta V - s(V)) = 0.$$

In the above equation, we need to determine the recontracting debt service payments at each instant. This payment is denoted by s(V). This payment will be such that the borrower and the lender will be able to implement the Nash outcome. The debt servicing strategy consists of two regions: In the top region when the borrower is in good financial health, contractual payments are made. In the lower region, the debt is recontracted at each instant. The regions are accessible from both sides in the following sense. A healthy borrower can fall into the recontracting region. Subsequently, the borrower can recover and resume contractual debt service. The mathematical requirements are the value matching and smooth pasting conditions. These are stated below:

(6)
$$\widehat{S}(V_S) = S(V_S) = \theta^* [V_S + PV(gV_s)] = \theta^* [1 + \frac{g}{\beta}] V_S$$

and

(7)
$$\widehat{S}_V(V_S) = S_V(V_S) = \theta^* [1 + \frac{g}{\beta}].$$

We now present our first major results in proposition 1. In this proposition, we explicitly define the recontracting trigger point for sovereign debt and the recontracting debt service. It can be shown that the trigger point is higher than the reorganization trigger point for an otherwise identical corporate debt contract. This result implies that in a sovereign debt contracting, there will be more renegotiations and rescheduling than in corporate debt contracting.

Proposition 1

1. The trigger point for default that maximizes the value to the sovereign borrower is:

(8)
$$V_S = \frac{\frac{\lambda}{\lambda - 1} \frac{c}{r}}{(1 + \frac{g}{\beta})(1 - \theta^*)} = \frac{\frac{\lambda}{\lambda - 1} \frac{c}{r}}{\frac{1 - \alpha}{2} + \frac{g}{\beta}(1 - \gamma)}$$

where λ is the negative root of the quadratic equation shown below.

$$\lambda(r-\beta) + \frac{1}{2}\lambda(\lambda-1)\sigma_1^2 - r = 0.$$

It is easy to check that λ satisfies the equation below.

(9)
$$\lambda = \left[\frac{1}{2} - \frac{(r-\beta)}{\sigma_1^2}\right] - \sqrt{\left[\frac{1}{2} - \frac{(r-\beta)}{\sigma_1^2}\right]^2 + \frac{2r}{\sigma_1^2}} < 0.$$

2. The recontracting debt service in the region $V \leq V_S$ is given by

(10)
$$s(V) = [\gamma g + \beta - \beta \theta^* (1 + \frac{g}{\beta})]V = \beta \frac{1 - \alpha}{2}V < c.$$

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Proposition 1 says that the trigger point at which a sovereign borrower recontracts is based on a number of factors: the cost of sanctions parameter denoted by $(1 - \alpha)$, the fraction of exports that may be seized, $(1 - \gamma)$, and the present value of promised interest payments. In addition, the fundamentals of the economy such as the volatility of the underlying country's wealth generating process, and the outflow rate β affect the recontracting boundary. The recontracting boundary for sovereign debt is strictly higher than the corresponding boundary for an otherwise identical corporate debt contract. Note that V_S is increasing in α . This implies that the threat of a direct action (a decrease in α) serves to increase the region of contractual debt service. To see this clearly recall from equation (3) that $\frac{\beta}{\beta+\delta} \equiv \alpha$. It follows from equation (9) that $\frac{\partial V_S}{\partial \delta} < 0$, and $\frac{\partial^2 V_S}{\partial \delta^2} > 0$.

Next, the trigger point V_S at which the sovereign chooses optimally to default is compared to the trigger point V_s^c of an otherwise identical corporate borrower. For illustrative purposes, in what follows, we use the following base parameters: $\beta = 0.06$, r = 0.05, $V_0 = 50$, $\sigma = 0.20$, $I_0 = 50$ and $g(I_0) = I_0^{\zeta}$, with $\zeta = 0.25$. The default trigger point is always higher for the sovereign than for an otherwise corporate borrower and it is for both borrower types a convex decreasing function of the retaliatory action parameter as can be seen from figure 5.

[Insert figure 5]

It can further be shown that $\frac{\partial V_S}{\partial (1-\gamma)} < 0$. The ability to seize exports causes the sovereign to stay current on its debt payments for a much longer period of time. The threat of seizing exports and the threat of a direct action on the terms of trade complement each other in ensuring that the sovereign borrower has a greater incentive to stay current on its debt obligations. As the cost of sanctions increases, the difference between the two recontracting regions widens in favour of the sovereign thereby illustrating its improving recontracting position.

The recontracting payments also reflect these trade-offs: the costliness of sanctions as reflected by $(1 - \alpha)$ improves the flow rate of recontracting payments. The seizure threat affects it through the recontracting boundary. As the promised coupon rate increases, the recontracting boundary increases. This suggests an upper bound for the amount of risky debt (and hence for the promised coupon) that the borrowing country may issue, a topic which we will further examine in section 4.E.

IV.C The Sovereign Country's Value Under Borrowing

We now turn our attention to the value of the sovereign country with sovereign debt. Define $p = \left[\frac{V}{V_S}\right]^{\lambda} > 0$. It turns out that p has an intuitive interpretation as the probability of default by the sovereign borrower. In the next proposition, we characterize the value of the sovereign country under borrowing.

Proposition 2

The value of the sovereign country under borrowing is equal to

(11)
$$S(V, I_0) = \left(1 + \frac{g}{\beta}\right)V + \frac{1}{1 - \lambda} \frac{c}{r} \left[\frac{V}{V_S}\right]^{\lambda} - \frac{c}{r}.$$

We can write the value of the sovereign country under borrowing as:

(11a)
$$S(V, I_0) = (1 + \frac{g}{\beta})V - (1 - p)\frac{c}{r} - p\left[(1 - \theta^*)(1 + \frac{g}{\beta})V_S\right].$$

With a probability p the sovereign borrower will default and he pays an amount given by $\left[(1-\theta^*)(1+\frac{q}{\beta})V_S\right]$. With a probability of [1-p] the borrower will not default in which case he pays $PV_t(c)$. The borrower's total value is the sum of the capitalized value of the domestic output and of the capitalized value of export earnings minus the present value of payments to debtholders in all states of nature. The value of the sovereign country $S(V, I_0)$ is a convex decreasing function of the fraction of exports $(1-\gamma)$ that can be seized by the lender and its contingent claim-like payoff structure is an increasing function of the country's wealth volatility (σ) . The penalizing role of an increasing cost of sanctions $(1-\alpha)$ is less severe for the sovereign than for an otherwise identical corporate borrower whose value is denoted by S_c . Finally, the value of the sovereign borrower is a convex decreasing function of beta, the output flow rate of the sovereign country⁵.

IV.D Value of the Sovereign Debt

It follows directly from our analysis that the value of sovereign debt in our setting is given by equation (12) in Proposition 3.

Proposition 3

The value of sovereign debt is given by the default probability-weighted debtholders payoffs as shown below.

(12)
$$D(V, P, c) = (1-p)\frac{c}{r} + p\left[(1-\theta^*)(1+\frac{g}{\beta})V_S\right] = \frac{c}{r}\left[1-p\frac{1}{1-\lambda}\right]$$

The value of the sovereign debt is sensitive to many of the deep parameters of the model. The pay-out ratio β affects the value of sovereign debt in two ways. It affects both the parameter λ and the probability of default p. The probability of default tends to increase as β increases: this is due to the fact that the future growth potential is lower when β is higher. This has the effect of reducing the value of the sovereign borrower. On the other hand, an increase in beta also tends to increase λ . This reduces the default trigger as well. Both effects will decrease the value of the sovereign debt. Since high β implies a low growth rate, it follows that sovereign borrowers with falling growth rates are adversely affected in this model. Finally, it can also be shown that the value of sovereign debt is a monotonically increasing function of the cost of sanctions parameter $(1 - \alpha)^6$.

⁵Illustrative graphs depicting those relationships are available from the authors upon request.

⁶Illustrative graphs depicting those relationships are available from the authors upon request.

IV.E Promised Coupon Rate and Debt Capacity

The borrowing country would like to set the promised coupon level at a rate such that the proceeds of the debt issue support the amount I_0 needed to finance the exporting capacity. Using (12) we can then set,

(12a)
$$I_0 = D(V, c^*) = (1-p)\frac{c}{r} + p\left[(1-\theta^*)(1+\frac{g}{\beta})V_S\right].$$

We can solve (12a) to determine the relationship between the promised level of coupons $c^*(I_0)$ for any investment level I_0 . We substitute for the probability of default in (12a) and use the requirement that $V_0 = I_0 + W_0$, where W_0 denotes the country's initial wealth before borrowing. This leads to the following relationship.

(12b)
$$I_0 = \frac{c^*}{r} + V_0^{\lambda} \left[\frac{\frac{\lambda}{\lambda - 1} \frac{c}{r}}{\frac{1 - \alpha}{2} + \frac{g}{\beta} (1 - \gamma)} \right]^{-\lambda} \left[\frac{c^*}{r} \frac{1}{\lambda - 1} \right]$$

Equation (12b) allows us to characterize the promised coupon levels associated with any feasible investment level.

Proposition 4

There exists a coupon $c^*(I_0)$ satisfying equation (12b) that is an increasing convex function of I_0 . \Box

The coupon rate is a decreasing convex function of the fraction of exports that serve as collateral and the latter function is especially important for countries with high wealth volatility levels. The promised coupon rate tends to be insensitive to the cost of sanctions $(1 - \alpha)$. Indeed, the cost of sanctions will essentially affect the default trigger point at which renegotiation occurs for a given coupon rate. Finally, the promised coupon rate is a decreasing convex function of the value of the country V_0 and will be higher for countries with lower productivity rates (ζ) of their imports⁷.

⁷The graphs illustrating those relationships are available from the authors upon request.

[Insert figure 6]

Obviously, there will be a maximal debt capacity that can be supported by the sovereign. Indeed, as the coupon increases, so does the recontracting trigger level and thus there must be a limit to the amount of debt that the sovereign can initially raise without provoking immediate renegotiation. This maximal coupon rate c^{\max} is obtained by solving for c^{\max} in equation (8), in the limiting case when the ratio (V_S/V_0) tends to one. This knife-edge coupon rate is then used in conjunction with equation (12a) to determine the maximal debt capacity of the sovereign. The relationship between the coupon level and the amount borrowed by the sovereign is illustrated in figure 6.

We can now determine the borrower's decision to enter the sovereign debt market. Applying the results of our paper to equation (4) we obtain:

(4a)
$$G(I_0) \equiv S(V_0 + Vg(I_0) - I_0) - V_0 =$$

$$\frac{g(I_0)}{\beta}V_0 - (1-p)c/r - p(1-\theta^*)(1+\frac{g(I_0)}{\beta})V_S - I_0.$$

The optimal level of investment is then found by solving the following equation:

(4b)
$$\frac{dG(I_0)}{dI_0} = 0.$$

Proposition 5

The optimal level of investment (I_0) is strictly lower than the first-best level. Furthermore, the gains from trade are increasing in the level of exports, decreasing with the cost of sanctions $(1 - \alpha)$ and decreasing in the fraction $(1 - \gamma)$ of the exports that can be seized by the lender. \Box

This last relationship can be explained by the fact that an increase in the retained exports proceeds raises the gross value of the sovereign country which in turn dominates their reduced impact as collateral in raising the recontracting default trigger ⁸.

[Insert figure 7]

The model can be used to examine the impact of debt overhang and debt relief programs even though we do not consider repeated borrowing. In our case, the Sovereign borrower may be facing debt overhang if the capitalized value of its goods and services deteriorates subsequently to the debt issuance date. For instance, let us suppose that the sovereign initially (at date t = 0) raised an amount $I_0 = 100$, when the value of the country's goods and services was set at $V_0 = 50$ with the other parameters set at their base case levels. The debt contract promised coupon rate was accordingly set at $c^* = 5.57\%$ in order to fulfill equation 12a. For that coupon rate, as illustrated on the Debt-Laffer curve in figure 7, the market value of the debt initially satisfies $D^* = I_0 = 100$. Subsequently, the economic situation of the country may deteriorate which translates into lower values of V and obviously of the debt contract $D(V, I_0)$ as illustrated in the same figure. If the country ever reaches the renegotiation trigger point $V_s = 10.28$, the resulting debt servicing renegotiation process can be viewed as an implicit debt relief obtained by the sovereign. For instance, if V drops to 10, the sovereign borrower will only have to service a coupon rate of s(V = 10) = 2.7%(instead of $c^* = 5.57\%$). This corresponds to an implicit debt relief resulting from the bargaining process. Thus, debt overhang situations can arise not only from excessive repeated borrowing but also from a deterioration of the total wealth of a "static" Sovereign borrower whose debt servicing gradually becomes less sustainable.

⁸Illustrative graphs depicting those relationships are available from the authors upon request.

V Sovereign Debt Credit Spreads

We can express the sovereign credit spread π , in closed form in our model:

(13)
$$\pi = \frac{rp}{1 - \lambda - p}$$

We can formally show that the credit premium of sovereign debt is strictly greater than the credit spread π_c of an otherwise identical corporate debt contract. Indeed, since the derivative of $\frac{\partial \pi}{\partial p} = r(1-\lambda)/(1-\lambda-p)^2$ is strictly positive and p is strictly greater than p^c , the probability of default for an otherwise identical corporate bond contract, the conclusion follows.

The default spreads for the sovereign and the corporate borrower are increasing slightly convex functions of the promised coupon rate. The sovereign always commands a higher spread than his corporate counterpart. For instance, in our base case example, for a 6.5% coupon rate issue, the sovereign and the corporate borrowers will have to yield spreads respectively equal to 38 and 37 basis points and in the case of 8.5% coupon rate issues, their spreads will amount to 49 and 47 basis points respectively.

The distance between both credit spreads also widens due to the fact that the absolute difference between both bargaining shares $(\theta^* - \hat{\theta})$ increases as the output flow rate β increases. The sovereign credit spread is an increasing convex function of the volatility parameter σ with a steeper slope for higher promised coupon rates. The sovereign credit spreads are inversely related to the level of the default free interest rate r. Thus, sovereign spreads will tighten (widen) in periods of high (low) interest rates. This in turn portrays the business cycle attribute of sovereign credit spreads which will tend to rise (fall) in periods of recession (expansion). Again, spreads associated with high coupon rate sovereign issues are more severely affected by an economic slowdown ⁹. Table 4 illustrates the joint impact of the cost of sanctions $(1 - \alpha)$ and of the fraction

⁹Illustrative graphs depicting those relationships are available from the authors upon request.

of seizable exports $(1 - \gamma)$ on the sovereign and the corporate debt spreads.

[Insert Table 4]

[Insert Table 5]

[Insert Table 6]

We illustrate the impact of both parameters in three panels: the first two refer to debt issues with a volatility of 20% and a coupon rate of 6.5% and 9% respectively and the last panel displays the spreads for 9% coupon debt issues with a volatility level of 30%. It appears immediately, that both the sovereign and the corporate spreads are higher when the coupon rate and-even more so-when the volatility level of the borrower increase. If we examine table 6, we see that both the sovereign and the corporate spreads decrease with the cost of sanctions. Indeed, a sovereign (corporate) 9% coupon debt issue with a 5% recovery rate will command a spread of 213.83 (209.83) basis points when the cost of sanctions amounts to 0.1 whereas its spread will drop to 186.24 (163.89) basis points when the cost of sanctions reaches a value of 0.9. We further observe that the difference between sovereign and corporate spreads widens substantially as the cost of sanctions increases. Indeed, when the recovery rate reaches 5% of the exports and the cost of sanctions is equal to 0.1, the difference between both spreads is negligible and amounts to 4 basis points. The difference increases however to 22.35 basis points when the cost of sanctions reaches 0.9. A similar although less pronounced effect of the cost of sanctions on the difference between both spreads is observed for lower volatility/and or coupon levels in the first two panels of table 2. Indeed, keeping a recovery rate of 5%, a cost of sanctions of 0.9 and lowering the volatility to 20%, the maximal difference between both spreads drops to 17.14 and is reduced further to 11.20 basis points when the coupon is lowered to 6.5%. This illustrates that the protection provided by the bankruptcy code to the corporate lender becomes relatively more valuable as the cost of sanctions increases. Indeed, in section III, we already proved that the difference between both bargaining shares is increasing in $(1 - \alpha)$, which in turn leads the sovereign lenders to demand relatively higher spreads as their relative position weakens. However, this phenomenon is mitigated and becomes almost negligible in the presence of higher exports recovery rates. We observe in table 6 that the maximal difference between both spreads for a cost of sanctions equal to 0.9, reduces to 7.34 (1.62) basis points for a recovery rate of 10% (25%). Let us further observe that both the corporate and the sovereign debt spreads are decreasing convex functions of $(1 - \gamma)$. For instance, we see in table 6, that if we double the recovery rate of exports from 5% to 10%, for a cost of sanctions level of 0.5, the sovereign credit spread will drop from 198.87 to 119.41 basis points.

When combined, the above results suggest that the absence of a bankruptcy code leads to relatively higher sovereign spreads when the cost of sanctions is high and recovery rates are low. The difference between sovereign and corporate spreads also widens for highly volatile sovereign countries which issue higher coupon rate debt contracts. However, the absence of a bankruptcy code can vastly be compensated for and thus sovereign debt will command smaller credit spread premia with respect to corporates whenever lenders can impose higher trade sanctions and/or have access to a large fraction of the sovereign country's exports. In the stylized facts section III, we related those two attributes to the degree of foreign trade concentration and to the level of a country's exports respectively. We showed that in this context, Mexico qualifies as a country which bears both a high export capacity (higher access to collateral for the lenders) and a high potential for trade sanctions due to the concentration of its trade policy whereas the latter statement did not apply to Russia which had a very diversified foreign trade policy. As a result, the average sovereign spread (10.98%) observed in Russia is almost three times higher than the one observed in Mexico (3.25%). These stylized facts can be reconciled with our model's predictions if we assume that the retaliatory parameter δ —and thus the cost of sanctions parameter $(1 - \alpha)$ —is an increasing function of the degree of a country's trade concentration.

VI Conclusion

We presented a model of sovereign borrowing in which the borrower optimally chooses when to default. We showed that the ability to punish the sovereign borrower with terms of trade and to collateralize the debt obligations with exports significantly improves the value of sovereign debt. The value of sovereign debt and the spreads over Treasuries were also explicitly characterized. The credit spreads obtained in our model are consistent with the stylized fact that sovereign debt spreads over Treasuries are systematically higher than for identically rated corporate debt issues especially in the presence of low recovery rates.

Using foreign trade and indebtedness data, we showed that the access to collateral and the effectiveness of trade sanctions in our model can be related to the level and the degree of concentration of exports respectively observed in a given country. For instance, Mexico has a much more concentrated trade policy than Russia and, as a result, commands lower credit spreads than the latter country.

There are several ways in which our model could be extended. We assumed that the exports were fixed in foreign currency units. Introducing uncertainty in exports is a fruitful avenue for further research. We did not explore alternative debt instruments such as floating rate sovereign debt. Such an analysis would require the explicit modeling of stochastic interest rates. Our model could also be extended to a repeated borrowing problem in which the reputation of the borrower is also likely to play a role. We also ignored the role of multiple lenders and of loan syndicates which, as Chowdry (1991) shows, provide strong incentives for the sovereign borrower not to repudiate its debt voluntarily. Finally, our model could be extended to account for asymmetric information considerations that would even further reinforce the borrower's strategic

renegotiation position.

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	1995	1996	1997	1998	1999	2000	2001(e)
Argentina	258.1	272.2	293.0	299.1	283.4	285.2	282.8
Brazil	704.1	774.9	807.7	787.4	529.5	624.3	529.0
Mexico	287.0	333.0	401.0	421.0	480.0	575.0	602.0
Russia	337.9	419.0	436.0	282.4	193.2	251.1	305.0

Nominal Gross Domestic Product (in bn US\$)

Source: Datastream

Figure 1: Nominal Gross Domestic Product (in bn US Dollars).

	Argentina	Brasil	Mexico	Russia
Mean	0.0717	0.0712	0.0325	0.1098
StDev	0.0157	0.0060	0.0038	0.0255

Table 1: Mean and standard deviations of the sovereign credit spreads series for the four countries over the period 10/18/1999 to 06/05/2000.

	Argentina	Brasil	Mexico	Russia
Argentina	1	0.7379	0.2919	-0.3519
Brasil	0.7379	1	0.5944	0.1633
Mexico	0.2919	0.5944	1	0.4221
Russia	-0.3519	0.1633	0.4221	1

Table 2: Correlations between the sovereign credit spreads across the four countries computed over the period 10/18/1999 to 06/05/2000.

	Borrower's Perspective	Lender's Perspective
Don't negotiate	$\alpha V_S + \gamma P V_S(gV)$	$(1-\gamma)PV_S(gV)$
Negotiate	$\theta[V_S + PV_S(gV)]$	$(1-\theta)[V_S + PV_S(gV)]$

Table 3: Description of the bargaining game between the sovereign and the lenders.

Statistics on external debt (end of June 2000)

	Total external	Debt	issued abroad ⁽²⁾	Brady	bonds	Short term ⁽³⁾	liabilities
Country	debt (T)mn US\$ (1)	mn US\$	% of (T)	mn US\$	% of (T)	mn US\$	to banks
							% of (T)
Argentina	136,799	69,630	50.9%	11,597	8.5%	36,016	26.3%
Brazil	167,747	48,793	29.1%	29,793	17.8%	35,957	21.4%
Mexico	149,178	57,562	38.6%	20,990	14.1%	22,606	15.2%
Russia	84,112	17,381	20.7%	0	0.0%	11,509	13.7%

⁽¹⁾ Total external debt includes:

- Bank loans: loans from banks resident in 20 major developed countries and 6 offshore centres.

⁽²⁾ - Debt securities issued abroad: money market instruments, bonds and notes issued in international markets by both public and private sector borrowers.

- Brady Bonds: bonds issued to restructure commercial bank debt under the 1989 Brady Plan.

- Multilateral claims: loans from the African Development Bank, Asian Development Bank, and Inter-American Development Bank, use of IMF credit, and IBRD loans and IDA credits from the World Bank.

- Official bilateral loans: Concessional (aid) and other loans provided mainly for developmental

purposes by the 21 member countries of the OECD Development Assistance Committee. - Non-bank trade credits: official and officially guaranteed non-bank export credits from 25 OECD countries.

(3) Liabilities with an original maturity of one year or less, plus repayments due within the next 12 months on liabilities with an original maturity of over a year, plus arrears.

(Source: Joint BIS-IMF-OECD-World Bank statistics on external debt from 30.11.2000)

Figure 2: External Debt Statistics.

		Imports (bn US\$)	Argentina	Asia	Belarus	Brazil	European Union	Germany	Japan	Ukraine	USA				
Argentina	1996	23.8				22.3 ²⁾	29.3 ¹⁾				19.9 ³⁾				
	1998	31.4				22.5^{2}	27.4 ¹⁾				19.8 ³⁾				
	1999	25.5				21.9 ²⁾	27.9 ¹⁾				19.5 ³⁾				
	2000	25.2				25.7 ¹⁾	22.9 ²⁾				18.9 ³⁾				
Brazil	1996	53.3	12.7 ³⁾				26.5 ¹⁾				22.3 ²⁾				
	1997	61.3	13.4 ³⁾				26.5 ¹⁾				23.2 ²⁾				
	1998	57.7	13.9 ³⁾				29.2 ¹⁾				23.7 ²⁾				
	1999	49.2	11.8 ³⁾				30.4 ¹⁾				24.1 ²⁾				
	2000	55.8	12.3 ³⁾				25.2 ¹⁾				23.3 ²⁾				
Mexico	1996	92.0					7.9 ²⁾		4.2 ³⁾		73.5 ¹⁾				
	1997	113.1					8.2 ²⁾		3.8 ³⁾		72.6 ¹⁾				
	1998	129.1					8.5 ²⁾		3.5 ³⁾		72.3 ¹⁾				
	1999	146.2					8.0 ²⁾		3.5 ³⁾		72.2 ¹⁾				
	2000	174.5					9.6 ²⁾		3.7 ³⁾		73.1 ¹⁾				
Russia	1996	68.8		9.5 ³⁾				11.6 ²⁾		14.1 ¹⁾					
	1997	72.0		9.3 ²⁾	8.8 ³⁾			12.7 ¹⁾							
	1998	58.0		10.0 ³⁾	10.5 ²⁾			12.6 ¹⁾							
	1999	39.5		9.2 ³⁾	10.6 ²⁾			13.9 ¹⁾							
	2000	44.9			10.0 ²⁾			11.0 ¹⁾		6.9 ³⁾					
Sources.	The Wo	dd EactE	Rook 1996	-2001 IN/	E ctaff r	Sources: The World EastBook 1996 2001 IME staff reports Detastroom									

Diversification of Imports and the Value of Implicit Collateral (in % of total imports) Listed developing countries (vertical order) for 1996-2000

*) not work in a clock is a clock in the comparing with other trading partners in the upper case indicate the rank of the selected country when comparing with other trading partners

Figure 3: Diversification of Imports.

		US\$				loir					
		h l	æ			L L					
		ts (tina		a D	ear	any	_		e	
		bot	den	azil	anae	rop	E La	par	ain	trair	A8
		ű	Ar	Ъ	ő	ш	Ğ	Ja	S	Ś	ŝ
Argentina	1996	23.8		27.8 ¹⁾		19 ²⁾					8.2 ³⁾
	1997	26.4		30.7 ¹⁾		15.1 ²⁾					8.3 ³⁾
	1998	26.4		30.0 ¹⁾		17.4 ²⁾					8.4 ³⁾
	1999	23.3		24.4 ¹⁾		20.3 ²⁾					11 ³⁾
	2000	26.4		26.5 ¹⁾		17.8 ²⁾					11.9 ³⁾
Brazil	1996	47.7	10.8 ³⁾			26.9 ¹⁾					19.5 ²⁾
	1997	53.0	12.8 ³⁾			27.4 ¹⁾					17.8 ²⁾
	1998	51.1	13.2 ³⁾			28.8 ¹⁾					19.3 ²⁾
	1999	48.0	11.2 ³⁾			28.6 ¹⁾					22.6 ²⁾
	2000	55.1	11.3 ³⁾			26.8 ¹⁾					24.3 ²⁾
Mexico	1996	95.9			2.3 ³⁾	3.7 ²⁾					84.0 ¹⁾
	1997	110.4			2.0 ³⁾	3.6 ²⁾					85.6 ¹⁾
	1998	117.5			1.3 ³⁾	3.3 ²⁾					87.9 ¹⁾
	1999	136.7			1.73)	3.9 ²⁾					88.4 ¹⁾
	2000	166.4			2.03)	3.9 ²⁾					88.7 ¹⁾
Russia	1996	88.6					8.0 ²⁾			9.0 ¹⁾	7.6 ³⁾
	1997	89.0					7.6 ²⁾			8.5 ¹⁾	5.8 ³⁾
	1998	74.9					8.0 ²⁾			7.7 ³⁾	8.4 ¹⁾
	1999	75.7					8.5 ²⁾			6.6 ³⁾	8.9 ¹⁾
1	2000	105.6					9.0 ²⁾			9.1 ¹⁾	7.3 ³⁾

Diversification of Exports and the Value of Implicit Collateral Listed developing countries (vertical order) for 1996-2000 (in % of total exports)

Sources: The World FactBook 1996-2001, IMF staff reports, Datastream

Figure 4: Diversification of Exports.



Figure 5: Recontracting trigger point as a function of δ . The figure below illustrates the recontracting trigger point, expressed as a fraction of the coupon rate and the retaliatory parameter. The dotted (bold) line refers to the sovereign (corporate) borrower.



Figure 6: **Promised coupon rate as a function of** I_0 . The figure below illustrates the promised coupon as a function of I_0 . The dotted (bold) line refers to the promised coupon rate for a sigma of 20% (40%). It also depicts the maximum promised coupon rate and its associated maximal level of imports I_0 .



Figure 7: The Debt Laffer curve as a function of V. This figure illustrates the evolution of the sovereign debt's market value $D_s(I_0 = 100, V)$ as the sovereign country's capitalized value V changes. Initially, the debt is issued for an amount $I_0 = D^* = 100$, yielding a promised coupon c* =5.57%. At date 0, the capitalized value of the country was equal to $V_0 = 50$. The straight line refers to the sovereign debt's market value $D^*(I_0, V_0)$ at the issuance date t=0.

	π_{s}	$\pi_{ m c}$	$\pi_{ m s}$	$\pi_{ m c}$	π_{s}	$\pi_{ m c}$
1-a	1-γ=0.05	1-γ=0.05	1-γ=0.1	1-γ=0.1	1-γ=0.25	1-γ=0.25
0.1	83.72	81.62	39.09	38.63	15.04	14.97
0.2	81.62	77.72	38.63	37.73	14.97	14.83
0.3	79.62	74.17	38.17	36.87	14.90	14.70
0.4	77.72	70.93	37.73	36.06	14.83	14.57
0.5	75.90	67.96	37.30	35.27	14.77	14.44
0.6	74.17	65.24	36.87	34.52	14.70	14.31
0.7	72.51	62.72	36.46	33.81	14.63	14.19
0.8	70.93	60.39	36.06	33.12	14.57	14.06
0.9	69.42	58.22	35.66	32.46	14.50	13.94

Table 4: Sovereign and corporate credit spreads as a function of the cost of sanctions and the fraction of seized exports. This table displays the sovereign(π_s) and corporate(π_c) credit spreads (in basis points) for a cost of sanctions parameter ranging from 0.10 to 0.90 and for values of the fraction of seized exports equal to 0.05, 0.10 and 0.25 respectively. The coupon rate of both debt contracts is set at 6.5% and the volatility is equal to 20%.

	π_{s}	$\pi_{ m c}$	π_{s}	$\pi_{ m c}$	π_{s}	π_{c}
1-a	1-γ=0.05	1-γ=0.05	1-γ=0.1	1-γ=0.1	1-γ=0.25	1-γ=0.25
0.1	123.90	120.58	55.80	55.12	21.07	20.97
0.2	120.58	114.45	55.12	53.80	20.97	20.78
0.3	117.43	108.91	54.45	52.55	20.87	20.59
0.4	114.45	103.88	53.80	51.35	20.78	20.40
0.5	111.61	99.30	53.17	50.20	20.68	20.22
0.6	108.91	95.10	52.55	49.11	20.59	20.04
0.7	106.33	91.24	51.94	48.06	20.49	19.86
0.8	103.88	87.69	51.35	47.05	20.40	19.69
0.9	101.54	84.40	50.77	46.09	20.31	19.51

Table 5: Sovereign and corporate credit spreads as a function of the cost of sanctions and the fraction of seized exports This table displays the sovereign(π_s) and corporate(π_c) credit spreads (in basis points) for a cost of sanctions parameter ranging from 0.10 to 0.90 and for values of the fraction of seized exports equal to 0.05, 0.10 and 0.25 respectively. The coupon rate of both debt contracts is set at 9% and the volatility is equal to 20%.

	π_{s}	π_{c}	π_{s}	π_{c}	π_{s}	π_{c}
1-a	1-γ=0.05	1 <i>-γ</i> =0.05	1-γ=0.1	1-γ=0.1	1-γ=0.25	1 <i>-γ</i> =0.25
0.1	213.83	209.83	123.41	122.38	64.30	64.10
0.2	209.83	202.36	122.38	120.38	64.10	63.71
0.3	206.01	195.51	121.37	118.46	63.90	63.33
0.4	202.36	189.21	120.38	116.62	63.71	62.95
0.5	198.87	183.38	119.41	114.84	63.52	62.58
0.6	195.51	177.98	118.46	113.14	63.33	62.21
0.7	192.30	172.96	117.53	111.49	63.14	61.85
0.8	189.21	168.27	116.62	109.91	62.95	61.49
0.9	186.24	163.89	115.72	108.38	62.76	61.14

Table 6: Sovereign and corporate credit spreads as a function of the cost of sanctions and the fraction of seized exports. This table displays the sovereign(π_s) and corporate(π_c) credit spreads (in basis points) for a cost of sanctions parameter ranging from 0.10 to 0.90 and for values of the fraction of seized exports equal to 0.05, 0.10 and 0.25 respectively. The coupon rate of both debt contracts is set at 9% and the volatility is equal to 30%.