A Robust General Voronoi Graph based SLAM for a Hyper Symmetric Environment

Nakju Lett Doh† Wan Kyun Chung† SungOn Lee‡ SangRok Oh‡ BumJae You‡

† Robotics & Bio-Mechatronics Lab., Mechanical Engineering, Pohang University of Science & Technology (POSTECH), Pohang, KOREA
Tel : 82-54-279-2844 ; Fax : 82-54-279-5899; E-mail : {nakji,wkchung}@postech.ac.kr
‡ Intelligent System Control Lab., Korea Institute of Science and Technology (KIST), Seoul, KOREA
Tel : 82-2-958-5747 ; Fax : 82-2-958-5749; E-mail : {solee,sroh,ybj}@amadeus.kist.re.kr

Abstract—In this paper, an algorithm for hyper symmetric environment Simultaneous Localization And Mapping (SLAM) is developed based on the generalized Voronoi graph. The hyper symmetric environment is a challenging environment for the SLAM and the most difficult problem on this is a data association. To redeem this problem, we propose three techniques. The first one is a breadth first node navigation algorithm which highly reduces the complexity of the data association. The second scheme is an odometry calibration method which converts untrustable odometry into reliable one. The third method is a multi layered data association scheme. We integrate above three techniques into one framework and propose a SLAM algorithm for the hyper symmetric environment. The simulation result shows that the proposed algorithm can successfully cover a large hyper symmetric environment under 20% odometry uncertainty.

I. INTRODUCTION

A Hyper Symmetric Environment (HSE) is an environment where similar pattern repeats in many places. This environment has few features that can be used as a landmark and loops in the loop as shown in Fig. 1.

This kind of environment has not considered yet, as a target of the SLAM and thus previous SLAM algorithms are not robust against the complexity of the HSE. The main reason of the weakness is that the previous algorithms mostly rely on the current sensor data rather than inaccurate odometry. However, the sensor data of the HSE is similar with each other and this induces failure of the previous SLAM algorithms.

In this paper, we propose a framework for the Hyper Symmetric environment SLAM (HS-SLAM). Our approach is based on the Generalized Voronoi Graph (GVG) whose edge and node are equidistant to two and more than two objects, respectively [1]. We believe that the GVG is the best basis for the HS-SLAM because the GVG can use topology of the HSE as the landmarks and dramatically reduces the computation time.

But there is one major disadvantage on the GVG based SLAM. That is a data association which matches a newly met node to the one of the previous nodes. To overcome this drawback, we propose three techniques.

Firstly, we suggest a Robust Node Navigation Algorithm (RNNA). There are two kinds of node navigation strategies. One is a depth first and the other is a breadth first navigation. Upon the authors knowledge, all previous topology based SLAM algorithm implicitly used the depth first algorithm. We show that the depth first algorithm is apt to fail for the HS-SLAM. Then we use the breadth first navigation for each node to generate the RNNA.

Secondly, we discuss that the role of the odometry is important for the HS-SLAM and adopt the way of odometry error calibration. The odometry has known to be inaccurate and untrustable. Thus previous algorithms don’t pay attention to the role of the odometry. However, we claim that the odometry is very important for the HS-SLAM because the sensor scan of the HSE is similar with each other. We also re-interpret the physical meanings of the odometry error and adopt the way of odometry calibration from our previous result [2].

Finally, we propose a Multi Layered Data Association (MLDA) which uses as many trustable data as possible. For that purpose, we categorize the trustable data into a deterministic and a probabilistic group. Then we
propose the MLDA using both the deterministic and the probabilistic data in a multi layered manner.

For the HS-SLAM, we integrate these three techniques in one framework. This scheme utilizes the RNNA and relies on the calibrated odometry. It also uses the MLDA for the robust data association. We show that this framework can successfully cover a 3.7 km × 3.7 km HSE even under 20% odometry uncertainty.

II. THE ADVANTAGES OF THE GVG FOR THE HS-SLAM

In this section we describe the advantages of the GVG for the HS-SLAM. To explain the edge and the node of the GVG, we define $d_i(x)$ as a distance to the closest point on an object. Then the edge of the GVG can be defined as

$$E_{ij} = \{ x \in E_i : d_i(x) = d_j(x) \leq d_h(x) \text{ such that } \nabla d_i(x) \neq \nabla d_j(x) \}.$$  (1)

The node of the GVG edges is a point where $d_i(x) = d_j(x) = d_h(x)$ for at least one $h$ [1].

We think that there are four major advantages on the GVG based SLAM over other algorithms.

Firstly, it can use topologically meaningful place as a natural landmark. Thus it needs neither the artificial landmark nor easily detectable feature.

Secondly, we can easily embed the navigation strategy on the SLAM algorithm by using the GVG. Recently, this navigation embedded SLAM algorithm is denoted as an ‘integrated exploration’ [3].

Thirdly, the GVG based SLAM algorithms are computationally efficient because the GVG uses the graph structure of the environment. Thus the memory requirement and the computation time can be dramatically reduced compared to the Kalman filter SLAM or the probabilistic SLAM.

Fourthly, the GVG based SLAM ensures the complete mapping of the environment. Previous SLAM researches have not mentioned on this complete mapping. But we claim that the SLAM algorithm should completely map the environment for the practical use.

However, there are several drawbacks of the GVG based SLAM algorithm. The most critical one is that it is hard to perform a robust data association. The data association, for the GVG based SLAM algorithm, is a process of linking a current node to either one of the previous nodes or new node. If one of the data associations fails, then the SLAM algorithm is also broken. Thus the robust data association is the most important problem to solve in the GVG based SLAM algorithm.

Therefore, we propose three techniques for the robust data association of the GVG based HS-SLAM in the next 3 sections.

Fig. 2. Two graph searching algorithms for (a) a map: (b) depth first and (c) breadth first searching algorithm.

Fig. 3. The data association for a large loop(in (a)) by using the depth first navigation. The robot started its search on node $A$ and returned to $A$. However, all the shaded circles are possible matching candidates of the data association as in (b).

III. FIRST TECHNIQUE: A ROBUST NODE NAVIGATION ALGORITHM

The GVG maps the environment as a graph and it needs graph navigation scheme for the mapping. The graph navigation scheme can be classified into two groups. One is a depth first algorithm and the other is a breadth first algorithm. These two algorithms are shown in Fig. 2.

Upon the authors knowledge, all previous SLAM algorithms implicitly used the depth first searching scheme. However, the depth first navigation makes data association more ambiguous. For example, the data association after a large cyclic loop(Fig. 3) is not easy, because there are several possible matching candidates(shaded circles).

In this section, we propose a Robust Node Navigation Algorithm(RNNA) by using the breadth first navigation...
scheme. Firstly, we define two types of graph nodes as shown in Fig. 4. One is an open node whose emanating edges are not yet checked. The other is a closed node whose emanating edges are all examined.

The RNNA is constructed by applying the breadth first navigation algorithm to each node and the procedures are as follows:

---

The RNNA algorithm

for all the open nodes {

- apply the breadth first navigation algorithm to the current node.
- set current node as the closed node.
- go to the closet open node.

} Algorithm end

---

The RNNA has two advantages. Firstly, it completely maps the environment. Secondly, it dramatically reduces the complexity of the data association. The complexity is reduced because possible candidates of the data association is limited to the open nodes whose number is relative very small compared to the number of checked nodes.

One example is given in Fig. 5. It shows the mapping procedures of the RNNA for the map indicated in Fig. 3. Firstly, the robot starts at node A (Fig. 5(a)). Then it apply the RNNA to node A and close it as shown in (b). The robot keep using the BFA and expands its search in (c). Finally it comes back to the node that needs the data association in (d). However, all nodes but one are closed thus data association can be easily solved.

IV. SECOND TECHNIQUE: THE ROLE OF THE ODOMETRY AND THE WAY OF ODOMETRY CALIBRATION

The odometry has been known to be untrustable and really it is. Thus previous SLAM researches have not focused on the role of the odometry. But we claim that we should actively use the odometry for challenging environment SLAM such as the HS-SLAM.

In [4], Borentein firstly addressed on the systematic and the non-systematic sources of odometry errors. The systematic error sources are the system itself, such as wheel diameter difference, inaccurate wheel base length, etc. The non-systematic error sources are the sources that are independent from the robot, such as wheel slipping, floor cracking and so on.

Here we want to re-interpret these from the physical point of view. Assume a situation when the robot repeats navigations from a point A to a point B as shown in Fig. 6(a). Here the thick line is a real path and the dotted line is an odometry trajectory. The real end point is the point B, but the odometry will think it as one of the crosses above the point B. Also the covariance matrix,
The robot moves from A to B along the thick line, but the odometry path is the dotted line. (a) is before systematic error compensation and (b) is after compensation.

Fig. 7. An example where (a) the real point differs from the odometry but (b) belongs to the covariance matrix bound.

\[ P(X) = \Sigma \]

where \( X \) and \( \hat{X} \) are the real and the estimated location of the robot, which represents the uncertainty of the odometry is drawn as a banana shaped thick line.

Here, the final points of the odometry can be represented as a Gaussian distribution with its mean \( \mu \) and covariance \( \Sigma \) as \( \mathcal{N}(\mu, \Sigma^2) \). Our new interpretation on the systematic odometry error is that the amount of the systematic error can be defined as \( \mu \).

If the perfect compensation of the systematic error is conducted (\( \mu \approx 0 \)) then the final odometry points will show up as in Fig. 6(b). Once it is done, the covariance matrix is reduced to \( \Sigma \). Our new point of view on the non-systematic error is that the non-systematic error can be represented as \( \Sigma \) when the systematic errors are compensated.

Now we will explain three aspects of the role of the odometry on the HS-SLAM. Firstly, we claim that we can not trust odometry itself but we can trust the odometry whose covariance matrix is known. Fig. 7 shows an example where the real point differs from the odometry but belongs to the covariance matrix bound. Thus if we know the covariance matrix, we can acquire a reliable odometry and use it.

Secondly, if we compensate the systematic error then the magnitude of the covariance matrix is reduced as shown in Fig. 6(b). Furthermore, the probability that the mean \( \mu \) is the real point \( X \), i.e., \( P(\mu | X) \), also increases. These enhance the accuracy of the odometry and the robustness of the data association.

Thirdly, the compensation of the systematic error reduces the ambiguity of the node matching. An example is given in Fig. 8 which is a node matching procedure of the GVG for the synchro drive robot. The systematic error of the synchro drive robot varies according to the wheel angle [2] and the uncertainty could be spread as in Fig. 8(a). Here the real position is D but the robot could match it to B because there is a covariance overlapping. But if we compensate the systematic error, the uncertainty will show up as in Fig. 8(b). Then there is no covariance overlapping and the robot will match node D as a new node.

Based on these three aspects, we insist that it is important to compensate the systematic error and to know the covariance matrix for the HS-SLAM.

But it is not easy to compensate the systematic error because it depends on the type of the mobile base. Moreover, it is difficult to know the covariance matrix because we do not know the real position of the robot. But we already suggested the way of the systematic error compensation and the covariance matrix estimation method, the PC-method, in [2].

V. THIRD TECHNIQUE: A MULTI LAYERED DATA ASSOCIATION SCHEME

The most difficult problem of the GVG based HS-SLAM is the data association and the previously mentioned two
TABLE I
NOMENCLATURE

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{cur}}$</td>
<td>current node</td>
</tr>
<tr>
<td>$N_{\text{open},i}$</td>
<td>$i$-th open node</td>
</tr>
<tr>
<td>$\text{DD}(N)$</td>
<td>deterministic data of node $N$</td>
</tr>
<tr>
<td>$P_o(N)$</td>
<td>odometry probability of node $N$</td>
</tr>
<tr>
<td>$P_s(N)$</td>
<td>sensor scan probability of node $N$</td>
</tr>
<tr>
<td>$S_{\text{DT}}$</td>
<td>set of nodes that passed deterministic test</td>
</tr>
<tr>
<td>$S_{\text{PT}}$</td>
<td>set of nodes that passed probabilistic test</td>
</tr>
<tr>
<td>$N(S)$</td>
<td>number of nodes on set $S$</td>
</tr>
<tr>
<td>$\gamma_o$</td>
<td>threshold probability of odometry</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>threshold probability of sensor scan</td>
</tr>
</tbody>
</table>

techniques, the RNNA and the PC-method, dramatically reduce its complexity. In this section, we propose a Multi Layered Data Association scheme(MLDA) by using as many reliable data as possible. Firstly, we divide the reliable data into two categories. One is a deterministic data [5] and the other is a probabilistic data. The deterministic data is a data which is invariant and the probabilistic data is a data which can vary by uncertainties. These two data are listed as follows:

- **Deterministic Data**
  1) The number of edges of the nodes.
  2) The relative angles between the edges.
  3) The types of nodes. It could be 3 way node, boundary node and so on [5].

- **Probabilistic Data**
  1) The probability of the odometry.
  2) The probability of the sensor scan.

The MLDA can be described by using the nomenclatures in table I as follows:

```plaintext
The MLDA algorithm
//—— Deterministic Test ——————————
for all open node $N_{\text{open},i}$ {
  if $\text{DD}(N_{\text{cur}})$ is same with $\text{DD}(N_{\text{open},i})$
    then add $N_{\text{open},i}$ to $S_{\text{DT}}$
} if $N(S_{\text{DT}}) = 0$, $N_{\text{cur}}$ is new node. End.
else {
//—— Probabilistic Test
  for all $N_{\text{open},i}$ in $S_{\text{DT}}$ {
    if $P_o(N_{\text{open},i}) > \gamma_o$ and $P_s(N_{\text{open},i}) > \gamma_s$
      then add $N_{\text{open},i}$ to $S_{\text{PT}}$
  } if $N(S_{\text{PT}}) = 0$, $N_{\text{cur}}$ is new node. End.
  else if $N(S_{\text{PT}}) \geq 1$, data association failed.
} Algorithm end.
```

VI. HS-SLAM ALGORITHM AND SIMULATION RESULTS

In the previous 3 sections, we proposed three techniques for the robust data association. We can easily integrate these three methods into one framework and this integration gives synergy effect. The synergy effect is induced by the combination of the RNNA and the PC-method. The RNNA explores the same GVG path twice: one as forward and the other as backward. These two GVG paths can be used for the PC-method to achieve accurate odometry and its covariance matrix. The MLDA can be used whenever the robot meets a node during the GVG explorations. Now, we propose a HS-SLAM algorithm as follows:

```plaintext
The HS-SLAM Algorithm
for every open node $N_{\text{open},i}$ {
  for every unnavigated edges of $N_{\text{open},i}$ {
    • go through the edge to next node, $N_{\text{next}}$.
    • perform the data association to $N_{\text{next}}$ using the MLDA.
    • if current navigation constructs the longest forward and backward GVG paths, then recalibrate the odometry using the PC-method.
  }
} Algorithm end.
```

To validate the performance, simulations are conducted for the HSE in Fig. 9. This is a semi-unstructured, i.e. the surface is rough and arbitrary, HSE whose area is $3.7km \times 3.7km$ with 240 GVG nodes. In the simulation, we assumed that the robot does not miss a node during navigations. We also assumed that the systematic error is compensated and we know the covariance matrix.

We aimed to validate two things by the simulation. The first purpose is to prove that the proposed algorithm can successfully perform the SLAM for the HSE. This is

(a)  
(b)
Fig. 10. The percentage of successfully matched node versus odometry uncertainty by using (a) the proposed algorithm and (b) the depth first navigation algorithm with MLDA.

Fig. 11. Number of open nodes versus the number of mapped nodes.

proven in Fig. 10(a) which is a plot of the percentage of successfully matched node versus odometry uncertainty. We can see that the proposed algorithm completed its mission even under 20% odometry uncertainty.

The second objective is to show the superiority of the RNNA over the depth first algorithm. It can be shown by comparing Fig. 10(a) and Fig. 10(b). The only difference between these two results is the navigation algorithm. The RNNA is used for Fig. 10(a) and the depth first navigation algorithm is applied for Fig. 10(b). As we can see, the robustness against odometry uncertainty is increased by 20 times in the RNNA. The main reason is that the RNNA keeps the number of the open node below 10 during the whole navigation(Fig. 11).

However, the travel length of the complete SLAM of the RNNA is 19km and that of the depth first is 11km. This is both a drawback and a advantage of our method. It is a drawback because it should navigate longer distance for the complete SLAM. Also it is advantageous because the odometry can be calibrated during the navigation and increase the robustness of the data association. But in this simulation this is just a drawback because we assumed that the systematic error is already corrected and the covariance matrix is known.

VII. CONCLUSION AND REMARKS

In this paper, we proposed an algorithm for the Hyper Symmetric environment Simultaneous Localization and Mapping(HS-SLAM) which is based on the Generalized Voronoi Graph(GVG). Firstly, we analyzed the merits of the GVG based SLAM algorithms. Then we moved our attention to the data association which is the weakest point of the GVG based SLAM. To overcome this weakness, we proposed three techniques. Firstly the robust node navigation algorithm which highly reduced the complexity of the data association is suggested. Secondly, we analyzed the role of the odometry and adopted the way of odometry calibration. Finally, we proposed a multi layered data association scheme which uses both the deterministic and the probabilistic data.

Then we integrated these three into one framework and proposed the HS-SLAM algorithm. This algorithm mapped a hyper symmetric environment whose area is 3.7km × 3.7km with 240 nodes even under 20% odometry uncertainty in simulation.

VIII. ACKNOWLEDGEMENT

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IX. REFERENCES