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Shear Testing of Steel Fiber-Reinforced Lightweight Concrete Beams without Web Reinforcement

by Thomas H.-K. Kang, Woosuk Kim, Yoon-Keun Kwak, and Sung-Gul Hong

To investigate the effect of steel fibers on the shear strength of lightweight concrete beams without web reinforcement, a total of 12 beams were tested under four-point loads, including six steel fiber-reinforced lightweight concrete (SFRLC) beams and three normalweight steel fiber-reinforced concrete (SFRC) beams. The variables include the shear span-depth ratio (a/d) (2, 3, and 4), steel-fiber volume fraction (V_f = 0, 0.5, and 0.75%) and type of concrete (lightweight versus normalweight). The addition of steel fibers with V_f of 0.75% was found to increase the shear capacity by 30% and promote a ductility of 5.3 or higher. The test results also indicate that the a/d adversely affects the shear capacity. Models for SFRC beam shear strength were reexamined using the current and prior test data to assess the shear strength of the SFRLC beams. Finally, a design shear strength equation for SFRLC beams without web reinforcement has been proposed based on the review.

Keywords: fiber-reinforced concrete; lightweight; shear span-depth ratio; steel-fiber volume fraction.

INTRODUCTION

The use of steel fiber-reinforced concrete (SFRC) is increasing in popularity in the U.S. and other countries due to its improved material and structural behavior relative to plain concrete and even to conventionally reinforced concrete (with the same steel volume fraction). The addition of steel fibers to a reinforced concrete (RC) beam is known to increase shear and flexural strengths and promote ductile behavior when the fiber volume fraction is at least 1%.[1,2] The increased strength and ductility of SFRC members are associated with the postcracking tensile strength of SFRC[3]; thus, the use of SFRC helps to reduce the extent and width of cracking. The use of steel fibers also tends to increase the compressive ductility of brittle high-strength concrete.[4,5] Along with these advantages, one of the most useful applications of SFRC would be to relieve reinforcing steel congestion by reducing the amount of shear or confining transverse reinforcement without sacrificing structural performance.[6]

A similar improvement may be anticipated in steel fiber-reinforced lightweight concrete (SFRLC); however, the application of the ACI 318-08[6] minimum steel-fiber volume fraction (0.75%) to lightweight concrete is questionable due to the lack of data. Also, the shear strength of SFRLC beams has not been well examined. The purpose of this study is to address this knowledge gap. First, the material properties of SFRLC are investigated, and then structural performance is determined by large-scale experimental testing. Finally, a database is compiled and examined to develop a shear strength model.

Although a large number of studies on SFRC structural members with normalweight concrete have been conducted by many investigators[1,2,7-9] over the past decades, studies on the structural behavior of large-scale SFRLC are limited. In this research, an experimental study on the shear behavior of SFRLC beams without web reinforcement is carried out. In general, the shear strength of RC beams without web reinforcement is dependent on the compressive strength of the concrete, longitudinal reinforcing ratio, shear span-depth ratio (*a*/*d*), and member dimension (ACI 318-08,[6] Section 11.2). The main parameters selected in this experimental study include the *a*/*d*, steel-fiber volume fraction, and concrete density (normalweight versus lightweight). A relatively high-strength concrete is used, as there is a growing demand for the use of high-strength concrete.

RESEARCH SIGNIFICANCE

While a number of large-scale experimental studies have been conducted to examine the shear behavior of SFRC beams, similar experimental studies addressing SFRLC beams have been limited. Particularly, steel-fiber volume fractions of 0.5 and 0.75% were not previously tested with lightweight concrete beams; however, the fraction of 0.75% (the new ACI 318-08[6] minimum requirement) or less is demanded for practical applications in the industry. An experimental study and data analysis were performed to address these issues.

PRIOR EXPERIMENTAL RESEARCH

Most previous tests of SFRLC materials were performed using approximately 100 x 100 x 360 mm (4 x 4 x 14 in.) prisms, 150 x 300 mm (6 x 12 in.) cylinders, and/or small-scale shear specimens (for example, 80 x 80 x 155 mm [3 x 3 x 6 in.]).[10-17] To the authors' knowledge, only two large-scale structural testing programs of SFRLC members were previously undertaken by Swamy's research team.[8,18]

Large-scale structural testing

Swamy et al.[8] tested nine large-scale specimens of SFRLC I-section beams with a span length of 3 m (118 in.). The main variables studied were the shear span-depth ratio (*a*/*d* = 2, 3.4, and 4.9), steel-fiber volume fraction (V_f = 0 and 1%), and reinforcing ratio of the bottom bars (ρ = 1.6, 2.8, and 4.3%). The test results indicated that the ultimate shear strength was dependent on *a*/*d* and ρ, and that SFRLC with V_f = 1% showed significantly greater load capacity (by 60 to 210%) and greater ductility (by 20 to 150%) than equivalent beams without steel fibers. The coarse lightweight aggregate used contained a sintered, pulverized fuel ash (PFA) with a maximum size of 14 mm (0.55 in.), and the steel fibers

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Table 1—Test results and failure modes of specimens

Specimen ID	Displacement at failure, mm	Shear load, kN: At first diagonal cracking, V_{cr}	Shear load, kN: At peak, V_u	μ^*	Failure mode
LB-0-2	11.1	33.5	70.2	1.7	Shear
FLB-0.5-2	28.2	39.9	81.7	3.2	Flexure-shear
FLB-0.75-2	41.7	45.6	83.1	5.3	Flexure
LB-0-3	11.0	25.4	45.1	1	Shear
FLB-0.5-3	47.4	34.5	45.4	4.6	Flexure
FLB-0.75-3	50.0	36.3	48.2	5.8	Flexure
LB-0-4	7.5	22.6	25.8	1	Shear
FLB-0.5-4	7.8	30.9	35.4	1	Shear
FLB-0.75-4	49.3	31.8	42.1	6.3	Flexure
FNB-0.5-2	34.3	40.7	77.2	2.9	Flexure-shear
FNB-0.5-3	43.2	37.3	47.2	5.4	Flexure
FNB-0.5-4	44.8	34.5	39.5	5.7	Flexure

$^*\mu = (\Delta_u/\Delta_y)$; Δ_u = displacement at failure; Δ_y = yield displacement.
Notes: 1 mm = 0.039 in.; 1 kN = 0.2248 kips.

used were crimped, 0.5 x 50 mm (0.02 x 2 in.) fibers with an ultimate strength of 1560 N/mm^2 (226 ksi).

Theodorakopoulos and Swamy[18] investigated the punching shear behavior and strength of SFRLC slab-column connections with PFAs. Twenty connection specimens were tested for variables of steel-fiber shapes (crimped, rectangular sectional, hooked, and paddle types), V_f (0.5 and 1%), reinforcing ratios of tension and compression slab steel (0.3 and 0.6%), column size (100, 150, and 200 mm [4, 6, and 8 in.]), and concrete compressive strength (f_c' = 17.8 to 58.6 MPa [2.6 to 8.5 ksi]). Overall, the addition of steel fibers in the SFRLC slab-column connections with a V_f of 1% increased the gravity loads at first cracking, at yielding, and at punching by approximately 30 to 45%. The types of hooked, crimped, and paddle (dumbbell-shaped) steel fibers performed similarly well.

Small-scale materials testing

Experimental studies were conducted by Balaguru and Dipsia[11] and Balaguru and Foden[12] to assess the applicability of discrete steel fibers for improving the mechanical properties of normal-strength (42 MPa [6 ksi]) and high-strength (62 MPa [9 ksi]) lightweight concrete with expanded shale aggregates. The experimental programs consisted of third-point loading tests of prisms per ASTM C1018-97,[19] splitting tensile and compressive strength tests of cylinders per ASTM C496/496M-04,[20] and direct shear tests. In their experimental studies, it was found that the addition of steel fibers to lightweight concrete increased the compressive strength f_c' by 7 to 70%, splitting tensile strength f_{ct} by 10 to 170%, and modulus of elasticity E_c by up to 65%. These improved mechanical properties were observed for all combinations of the fiber aspect ratios (60, 75, and 100) and steel-fiber volume fractions (0.55, 0.75, 0.9, and 1.1%).

Higashiyama and Banthia[17] evaluated relations between shear and flexural toughness for both SFRC and SFRLC, where the shear and flexural toughness are defined as the area under the load-versus-deflection relation curves up to a certain deflection point, which are obtained from direct shear tests and third-point loading tests in accordance with ASTM C1609/C1609M-10,[21] respectively. The materials used for their research consisted of two types of lightweight coarse aggregates (pumice and expansive shale), two different lengths (38 and 63.5 mm [1.5 and 9.2 in.]) of crimped steel fibers with 1 mm (0.039 in.) diameter, and two fiber volume fractions (V_f = 0.5 and 1%). The test results indicated that there was a linear relationship between shear and flexural strength for both SFRC and SFRLC, and that for a given fiber type and volume fraction, SFRC exhibited better shear and flexural toughness properties than SFRLC.

Swamy and Jojagha[13] performed a variety of workability tests for both SFRC and SFRLC in the fresh state, including inverted slump cone tests, standard slump and flow table tests, and vibrator-based remolding (VB) tests. Four different types of steel fibers (plain, paddle, hooked, and crimped) and four (length-to-diameter) aspect ratios of steel fibers ranging between 50 and 100 were tested. Both SFRC and SFRLC with a V_f = 1% showed relatively poor workability, and it was concluded that water-reducing-plasticizing admixtures, along with PFAs, should be added to release interlocking friction between fibers and aggregates.

Swamy and Jojagha[14] experimentally assessed the material characteristics of SFRC and SFRLC with PFAs under impact loads by means of a drop hammer test and a drop ball test in accordance with ACI 544R-78.[22] Three and four mixtures were tested for normalweight and lightweight concrete, respectively. Both SFRC and SFRLC with a V_f = 1% had greater impact resistance in terms of strength and energy absorption than those without steel fibers by a substantial degree up to a factor of 10. The effects of steel-fiber shape and geometry were evident by the fact that the number of shocks needed to fail was 793 and 536 for hooked and paddle shapes, respectively, but much less (192 and 124) for plain and crimped shapes, respectively.

EXPERIMENTAL PROGRAM

Twelve concrete beams (six SFRLC [FLB-series; refer to Table 1 for Specimen ID], three SFRC [FNB-series], and three lightweight RC beams [LB-series]) without stirrups were simply supported and loaded with two equal concentrated loads using a spreader steel beam (Fig. 1). The callout FLB-0.5-2 in Table 1 refers to the following variables: fiber-reinforced, lightweight concrete, and beam, with a V_f of 0.5% and *a*/*d* of 2. All beams were tested until failure to evaluate the influence of *a*/*d*, steel-fiber volume fraction V_f, and concrete density on the shear behavior of SFRLC and SFRC.

Design of test specimens

All beams were singly reinforced. The beams had the same cross-sectional dimension (125 x 250 mm [4.9 x 9.8 in.]) with an effective depth of 210 mm (8.3 in.); however, different total beam lengths L of 1.6, 2, and 2.4 m (61, 78, and 95 in.) were used for a/d = 2, 3, and 4, respectively (Fig. 1). The a/d was varied by varying the length of the beams, keeping the cross-sectional area constant. When the a/d is less than 2, the beam end outside the loading points is considered to be a D-region (Appendix A of ACI 318-08[6]). For this reason, all specimens had an a/d greater than 2; however, when the a/d is typically less than approximately 3.4, the arching effect is likely to affect the behavior.[2]

To investigate the effect of the dosage rate of steel fibers on shear behavior, three kinds of steel-fiber volume fractions (0, 0.5, and 0.75%) were selected. According to the new provision of ACI 318-08,[6] Section 5.6.6.2(a), SFRC should be considered acceptable for shear resistance when the dosage rate of deformed steel fibers is not less than 60 kg/m^3 (100 lb/yd^3). This rate is equivalent to a mixture with V_f = 0.75%. Of the 12 specimens, four (FLB-0.75-series and FNB-0.75-2 with V_f = 0.75%) satisfied this minimum requirement (60 kg/m^3 [100 lb/yd^3]).

All specimens were designed such that shear failure would occur in the absence of steel fibers and web reinforcement between the support and loading point. Two D16 (d_b = 16 mm [0.6 in.]) bottom reinforcing bars were placed, resulting in a reinforcing ratio of 1.5%. Concrete shear capacity (average V_c = 23.3 kN [5.2 kips] for lightweight and 31.8 kN [7.1 kips] for normalweight) at any point between the support and loading point was designed to be smaller than the shear applied (V at M_n = 31.5 to 54.9 kN [7.1 to 12.3 kips]) when the moment capacity of the beam would be reached. Here, the concrete shear capacity of V_c and the nominal moment strength of M_n are calculated as per Eq. (11-5) and Chapter 10 of ACI 318-08,[6] respectively. Simple supports were located at positions 150 mm (6 in.) from the beam ends (that is, span length = 1.25, 1.7, and 2.1 m [50, 65, and 83 in.], respectively, for a/d = 2, 3, and 4). Shear stirrups were provided at the supports (A_v = 2 x 71 = 143 mm^2 [0.22 in.^{2}]) at a spacing of 40 mm (1.6 in.). This was to avoid any potential local failure near the supports. To provide the development length l_p of the bottom bars, a 90-degree standard hook sufficiently satisfied the requirements in ACI 318-08.[6] For example, FNB-0.5-2 had an l_p of 21.4 mm (0.85 in.), greater than an l_{dh} of 8.7 mm (0.34 in.), where l_{dh} is the development length for a hooked bar required by ACI 318-08,[6] Section 12.5.2. Two D10 (d_b = 10 mm [0.4 in.]) top reinforcing bars were placed to engage the stirrups at the beam ends. The concrete clear cover used was 30 mm (1.2 in.) for both the top and bottom reinforcement.

Material tests

The concrete was made of Type I portland cement. The coarse aggregates used for the lightweight concrete beams were expanded clay aggregates with a maximum size of 19 mm (0.75 in.), and those used for the normalweight concrete beams were crushed gravels. The physical properties of expanded clay aggregates are shown in Table 2. The fine aggregates used were natural river sands with a fineness modulus of 2.17. The unit weights of lightweight and normalweight concrete were 112.5 and 137 lb/ft^3 (1800 and 2194 kg/m^3), respectively, on average. A polynaphthalene sulfonate high-range water reducer (HRWR) containing 41% solids by mass and with a specific gravity of 2.6 was used to obtain relatively high-strength concrete properties (f'_c = 39.6 to 57.2 MPa [5.7 to 8.3 ksi]). The HRWR dosages and other detailed mixture proportions are provided in Table 3. The water-cement ratio (w/c) was 0.33 for all beams. The measured properties of concrete mixtures are summarized in Table 4.

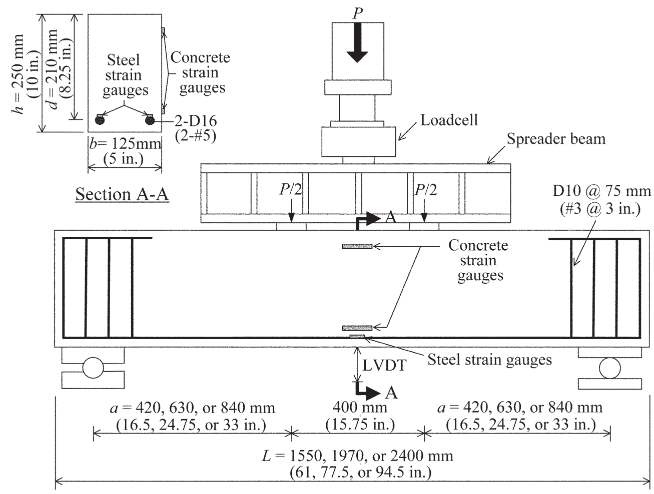

Fig. 1—Test setup and tested beam (not scaled).

Table 2—Physical properties of expanded clay and crushed gravel aggregates

	FM	SG	ARA, %	Maximum diameter, mm (in.)
Expanded clay aggregate	6.6	1.34	9	19 (0.75)
Crushed gravel aggregate	6.77	2.57	1.1	19 (0.75)

Notes: FM is fineness modulus, measurement of coarseness or fineness of given aggregate; SG is specific gravity, ratio of density of given solid to density of water; and ARA is absorption rate of aggregate, rate of moisture absorption into lightweight aggregate (per ASTM C127-07).[23]

The compressive and splitting tensile strengths f'_c and f_{ct} and modulus of elasticity E_c of concrete were obtained using 100 x 200 mm (4 x 8 in.) cylinders in accordance with ASTM C39/C39M-10[24] and ASTM C496/C496M-04.[20] The modulus of rupture f_r was evaluated from third-point bending tests of 150 x 150 x 530 mm (6 x 6 x 20 in.) concrete prisms in accordance with ASTM C78/C78M-10[25]; however, the midspan deflection δ_{mid} could not be measured as newly required by ACI 318-08,[6] Section 5.6.6.2, which refers to ASTM C1609/C1609M-10[21] during the modulus of rupture testing. The forms for all concrete specimens were stripped after 48 hours of curing, followed by moisture curing with burlap. All cylinder, prism, and beam specimens were tested at 28 days after casting.

Steel fibers with hooked ends were used (Fig. 2). The nominal ultimate strength of the steel fibers was 1100 MPa (160 ksi) (provided by the manufacturer). The fiber factor F of the steel fibers used is 0.23 for V_f = 0.5% and 0.35 for V_f = 0.75%, where F is equal to $(L_f/D_f)V_f d_f$ (refer to Fig. 2 for notations) and d_f is the bond factor (= 0.5 for circular section plain fiber, 0.75 for crimped fibers or hooked fibers, and 1 for indented fibers).[1,6] The flexural reinforcing bars

Table 3—Physical characteristics of SFRLC and SFRC mixtures

Specimens	Quantity or fraction of elements								
	Cement, kg/m³	Sand, kg/m³	Lightweight aggregate, kg/m³	Normalweight aggregate, kg/m³	Silica fume, kg/m³	HRWR, ℓ/m³	Steel-fiber volume fraction, %	w/c, %	Slump, mm
LB-0-series	480	560	480	NA	57.6	9.4	0	33	105
FLB-0.5-series	480	560	480	NA	57.6	10.0	0.5	33	100
FLB-0.75-series	480	560	480	NA	57.6	10.7	0.75	33	70
FNB-0.5-series	477	602	NA	909	NA	3.3	0.5	33	110

Notes: NA is not available; HRWR is high-range water reducer; 1 kg/m^3 = 1.667 lb/yd^3; 1 ℓ = 0.001 m^3 = 0.0353 ft^3; 1 mm = 0.039 in.

Table 4—Measured material properties of SFRLC and SFRC

Specimens	V_f, %	f_c', MPa	f_{ct}, MPa	f_r, MPa	E_c, GPa
LB-0-series	0	39.6	2.64	5.3	26.8
FLB-0.5-series	0.5	44.6	3.63	7.4	28.4
FLB-0.75-series	0.75	47.7	4.43	10.8	38.7
FNB-0.5-series	0.5	57.2	4.86	8.4	34.2

Notes: V_f is steel-fiber volume fraction; f_c' is concrete compressive strength; f_{ct} is splitting tensile strength; f_r is modulus of rupture; E_c is modulus of elasticity; 1 MPa = 0.145 ksi; 1 GPa = 145 ksi.

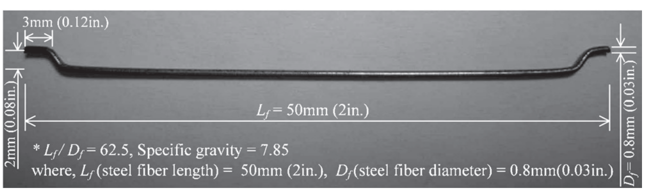

Fig. 2—Dimension for hooked steel fiber.

had an average yield strength of 442 MPa (64 ksi) and an ultimate strength of 638 MPa (92.5 ksi), measured from three steel coupons.

Testing and instrumentation

In shear testing, the beams were simply supported and subjected to two-point gravity loads. Ideally, no shear forces were applied inside the two loading points (Fig. 1). Steel plates were installed to transfer the load from the spreader beam to the top surface of the beam to avoid local stress concentration. The vertical load was measured using a 50 ton (112 kips) capacity compression-tension load cell. Linear variable displacement transducers (LVDTs) were mounted on the floor at the midspan of the beam to measure beam deflections during testing. In each loading step, crack width was measured using an eye gauge with a minimum resolution of 0.01 mm (0.0004 in.). To observe if the steel yielded, two strain gauges were affixed on two reinforcing bars near the midspan of each specimen. Also at the midspan, two concrete strain gauges were mounted on the side surface of each beam at 13 mm (0.5 in.) from the top and bottom surfaces.

OBSERVATIONS AND TEST RESULTS

In this section, the observed shear behavior and damage and experimental data obtained from lightweight RC, SFRC, and SFRLC specimens are described, with an emphasis on the effects of lightweight concrete, steel-fiber volume fraction, and a/d on the shear load capacity.

Material test results

Table 4 reports the average of the test results from three specimens for each property of each series. As the steel-fiber volume fraction increased, the compressive strength and stiffness of the SFRLC increased. The compressive strengths of f_c' were increased by 13% and 20% for the addition of steel fibers for a V_f of 0.5% and 0.75%, respectively. These relatively higher increases for SFRLC mixtures are noteworthy, as the compressive strength increase has been negligible to slight for SFRC with normalweight aggregates.[26] This was also clear from the results of the previous material tests[11,12] of SFRLC, where a 17 to 70% increase in f_c' was reported when a V_f of at least 0.75% was used. Thus, the new data reinforce the finding that the use of steel fibers is more effective in lightweight concrete than in normalweight concrete. Similarly, the average E_c values were increased by 6% and 44% with a V_f of 0.5% and 0.75%, respectively. The tensile properties of f_{ct} and f_r were increased by 68% and 104%, respectively, when a V_f of 0.75% was used, and by approximately 40% when a V_f of 0.5% was used.

If lightweight and normalweight concrete mixtures were being compared, higher compressive and tensile properties would be produced for normalweight concrete. The ratios of SFRLC to SFRC properties are as follows: 1) f_c' ratio = 0.78; 2) f_{ct} ratio = 0.75; 3) f_r ratio = 0.88; and 4) E_c ratio = 0.83. These results are consistent with those obtained from the material tests by Balaguru and Ramakrishman[10] and Higashiyama and Banthia.[17]

Difference in shear capacity of SFRC and SFRLC beams

Figure 3(a) depicts the shear load-deflection relationships for the FLB-0.5-series and FNB-0.5-series and compares the behavior between SFRC and SFRLC beams. For an a/d of 2 and 3, the shear load-deflection behavior was similar; however, the behavior of FNB-0.5-4 and FLB-0.5-4 was completely different when an a/d of 4 was used. FLB-0.5-4 failed in shear mode at the measured peak shear load V_u of 157.5 kN (35.4 kips) or v_u of $0.2\sqrt{f_c'}$ MPa ($2.4\sqrt{f_c'}$ psi), where v_u is taken as $V_u/(b_w d)$; b_w is the web width; d is the effective depth; and f_c' is the measured concrete compressive strength. The more ductile behavior of the FNB-0.5-series versus the FLB-0.5-series indicates different shear capacities between SFRC and SFRLC beams at a given deflection. Such different shear capacities signal that the use of the lightweight concrete multiplier λ should be applied to SFRC just as it was applied to conventionally reinforced concrete members (for example, ACI 318-08,[6] Section 11.2).

Little research has been done on the specific case of SFRC members with lightweight aggregates. The previous material tests[17] reported that the shear and flexural strength of SFRLC with expansive shale aggregates, which were linear, was approximately 65% of that of SFRC, whereas the other previous material tests[10] indicated that the flexural strength of SFRLC with expanded shale was approximately 80% of that of SFRC and that the compressive strength and toughness were approximately the same for SFRC and SFRLC. The current material tests noted that the tensile strength of SFRLC was reduced to the level of 75 to 88% of SFRC's. In this study, a lightweight concrete modifier λ is proposed as a function of [$(M/V)(1/d)$; in this study, a/d], f_c', ρ, and the type of lightweight aggregates based on the research by Hanson[27] (refer to Table 5 and Fig. 4). Here, M/V is the moment-shear ratio at the section considered. It is noted that the research by Hanson[27] was used as a basis for the lightweight concrete provision of ACI 318-08[6] (for example, Section 8.6.1).

Effect of steel-fiber volume fraction

The failure mode for each specimen was defined based on the ductility μ, where the ductility was taken as the midspan deflection at failure Δ_u normalized by the yield deflection Δ_y (refer to Table 1). The yield deflection Δ_y was determined as a point where the slope of the load-displacement relationship began to drop. If the value of μ is within the range of 2 to 3.5, the failure mode is denoted as flexure-shear. Outside of this range, the failure mode was denoted as either shear ($\mu < 2$) or flexure failure ($\mu > 3.5$). Three lightweight concrete beams without steel fibers (LB-0-series) and one SFRLC beam (FLB-0.5-4) failed in a brittle shear mode, whereas most SFRC and SFRLC beams failed in a ductile flexure mode, exhibiting μ values of at least 3.2 (Table 1). The observed damage is consistent with the failure mode definition. The extent of cracking and the crack width were reduced as the steel-fiber volume fraction increased (Appendix B*).

The failure mode and ductility of SFRLC specimens were also generally improved when the steel-fiber volume fraction was increased. In addition, the measured shear loads V_{cr} at the first diagonal tension cracking yielded consistent results (Table 1). The V_{cr} values for SFRLC beams (FLB-series) were larger than those for comparable lightweight concrete beams without steel fibers (LB-0-series) by 35% on average (refer to Table 1), likely due to increased tensile strength of the SFRLC. Similar results were found for the shear loads at peak V_u (Table 1). Given the same a/d, the peak shear load V_u was increased by 18% on average when a V_f of 0.5% was used and by 30% on average when a V_f of 0.75% was used. These results indicate that the shear load capacity of SFRLC beams was increased with an increasing steel-fiber volume fraction. Only the increased load capacity trend was not evident for FLB-0.5-3 when compared to LB-0-3; however, given that the failure mode and ductility are quite different, the effect of steel fibers on the SFRLC shear strength was substantial. The shear capacity of LB-0-3 was almost the same as the shear load at which the moment strength was reached of FLB-0.5-3.

Shear load versus deflection relations for beams with an a/d of 2 are compared in Fig. 3(b). The linear behavior of all specimens was represented up to diagonal tensile cracking, and subsequently nonlinear behavior was observed (Fig. 3(b)). Lightweight concrete beam LB-0-2 exhibited brittle behavior, whereas FLB-0.5-2 and FLB-0.75-2 were characterized by more ductile behavior. The deflections of FLB-0.5-2 and FLB-0.75-2 at failure were 510% and 1250% of the deflections at first bar yielding (as monitored by strain gauges), respectively. Similar behavior was noted for beams

*The Appendixes are available at **www.concrete.org** in PDF format as an addendum to the published paper. They are also available in hard copy from ACI headquarters for a fee equal to the cost of reproduction plus handling at the time of the request.

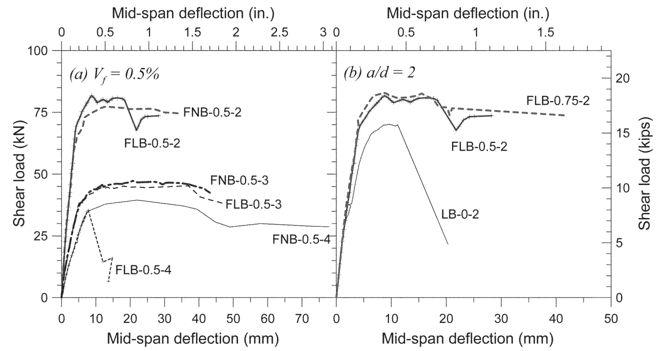

Fig. 3—Shear load-deflection relationship.

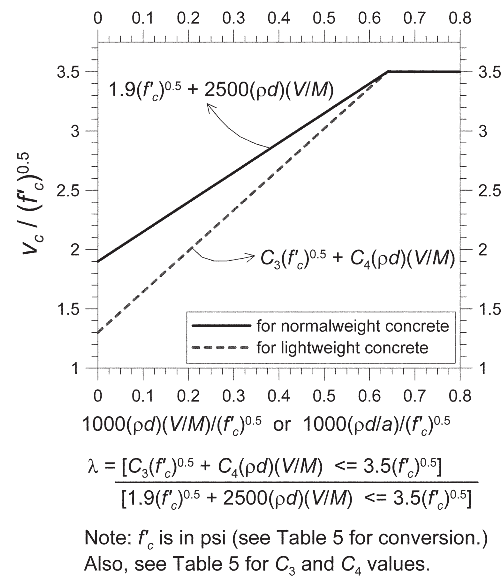

Fig. 4—Design consideration of lightweight concrete modification factor λ based on research by Hanson.[27]

Table 5—Design constants needed for lightweight concrete modification factor λ, proposed based on research by Hanson[27]

Coarse aggregates	C_3, U.S. units (SI units)	C_4, U.S. units (SI units)
Expanded shale (both rounded and angular)	1.1 (0.092)	3750 (25.82)
Expanded slag	1.3 (0.108)	3440 (23.72)
Expanded clay	1.5 (0.125)	3120 (21.52)
Sintered PFA	1.7 (0.142)	2810 (19.38)
Expanded slate or normalweight concrete aggregate	1.9 (0.158)	2500 (17.24)

Notes: PFA is pulverized fuel ash or fly ash;

$$\lambda = \frac{C_3\sqrt{f_c'} + C_4\dfrac{\rho V d}{M}\left(\le 3.5\sqrt{f_c'}\right)}{1.9\sqrt{f_c'} + 2500\dfrac{\rho V d}{M}\left(\le 3.5\sqrt{f_c'}\right)} \text{ in U.S. units;}$$

$$\lambda = \frac{C_3\sqrt{f_c'} + C_4\dfrac{\rho V d}{M}\left(\le 0.292\sqrt{f_c'}\right)}{0.158\sqrt{f_c'} + 17.24\dfrac{\rho V d}{M}\left(\le 0.292\sqrt{f_c'}\right)} \text{ in SI units.}$$

with an a/d of 3. These results, along with the compressive cylinder test results, indicate that steel fibers increased the shear load capacity (that is, the compressive strength of a diagonal strut in the arching region) so that crushing failure of

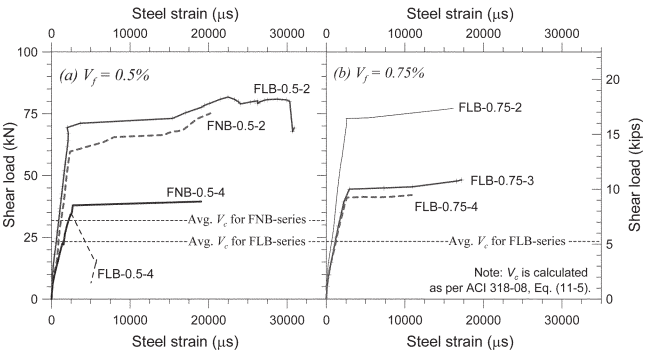

Fig. 5—Shear load-bar strain relationship.

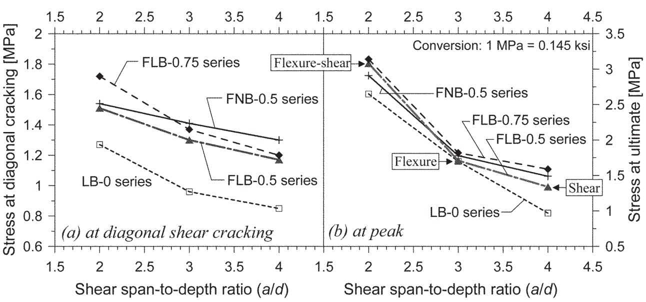

Fig. 6—Measured average shear stresses versus a/d.

the strut did not occur prior to flexural yielding for the SFRC and SFRLC beams with an a/d of 2 to 3. The larger deflections for the beams with V_f = 0.75% (versus 0.5%) were likely due to the increased compressive toughness. The concrete beam shear capacity (V_c = average 23.3 kN [5.2 kips]) without the effect of steel fibers predicted by ACI 318-08,[6] Eq. (11-5) was 28 to 66% of the measured peak.

The average of two strain gauge readings at the midspan was examined. The presence of flexural yielding and strain hardening of reinforcing bars can be identified from shear load versus reinforcing bar strain relations, as shown in Fig. 5. For FLB-0.5-4, yielding of the flexural reinforcing bars did not occur (Fig. 5(a)). This was also noted from the observation that shear failure occurred with little ductility. No brittle shear failure, however, was observed from FLB-0.75-4 and FNB-0.5-4 with similar configurations (Fig. 5(b) and Appendix B). This result indicates that although the effect of steel fibers on the tensile strength was reported to be greater in lightweight concrete than normalweight concrete,[10] the lightweight concrete member has a much lower shear strength (diagonal tensile strength) than the normalweight concrete member. The increase in tensile strength of SFRC due to the addition of steel fibers has been reported by many researchers (ACI 544.1R-96,[28] 544.3R-08,[29] and 544.4R-88[26]), and that of SFRLC was reported from the material tests conducted as part of this study. The observation gives a quantitative indication of the reasonable combination of an a/d and steel-fiber volume fraction that produce a ductile design of SFRLC beams without web reinforcement. Based on the results from this test program, a minimum V_f of 0.75% is recommended for SFRLC beams without web reinforcement and a V_f of 0.5% may be acceptable for an a/d of 2 to 3.

The V_f values of 0.5% and 0.75% are equivalent to the steel-fiber dosage rates of 75 and 100 lb/yd^3 (45 and 60 kg/m^3), respectively. The ACI 318-08[6] minimum requirement of 100 lb/yd^3 (60 kg/m^3) for shear resistance (Section 5.6.6.2(a)) also appears to be appropriate for steel fiber-reinforced lightweight concrete beams. The use of V_f = 0.75% would somewhat improve the concrete workability compared to V_f = 1%.

Increased shear capacity due to steel fibers

Based on the difference in the measured peak shear load between the FLB-series and LB-series, a minimum increase in the shear capacity due to steel fibers was assessed. When a 0.75% volume fraction of steel fibers (ACI 318-08[6] minimum requirement) was added, the shear capacity of at least 13.8 to 57.4 kN (3.1 to 12.9 kips) was provided only by the steel fibers. These values are higher than the ACI 318-08[6] calculated shear capacity $V_{s,min}$ = 11.1 kN (2.5 kips) of a minimum amount $A_{v,min}$ of stirrups. The amount $A_{v,min}$ was determined in accordance with ACI 318-08,[6] Section 11.4.6.3, and its steel shear capacity $V_{s,min}$ was calculated using the equation of [$V_{s,min} = (A_{v,min} f_{yt}d)/s$] as per Section 11.4.7.2 of ACI 318-08.[6] This result signals that the minimum stirrup amount requirement may also be waived for lightweight concrete—that is, Section 11.4.6.1(f) may also be applicable to lightweight concrete. The justification can be further strengthened by making a direct comparison between an SFRLC beam without web reinforcement and an RC beam with $A_{v,min}$ of web reinforcement, which was not included in this testing.

When a V_f of 0.5% was used, the shear capacities were increased by at least 51.2 and 42.7 kN (11.5 and 9.6 kips) for FLB-0.5-2 and FLB-0.5-4, respectively, which are substantially larger than $V_{s,min}$ (11.1 kN [2.5 kips]). The increased shear capacity, however, was negligible for FLB-0.5-3. Because the used fraction of 0.5% is lower than the ACI 318-08[6] required minimum fraction of 0.75%, these data are not used for the evaluation of applicability of Section 11.4.6.1(f).

Effect of *a/d*

Figure 6(a) shows the relations of the measured shear load at first diagonal cracking V_{cr} normalized by the cross-sectional area $b_w d$ versus a/d. The average shear stress ($v_{cr} = V_{cr} / b_w d$) at diagonal cracking was decreased as the a/d was increased due to the larger bending moment and associated principal stresses and also because the arching effect appeared to diminish as the a/d increased. The a/d eventually affected the ultimate shear behavior of the SFRLC beams. Similar trends were observed for the relations of the measured peak shear load normalized by the area (that is, the average shear stress at peak); $v_u = V_u/(b_w d)$) versus a/d (refer to Fig. 6(b)). Furthermore, the FLB-0.5-2 and FLB-0.5-3 specimens exhibited quite ductile behavior, whereas FLB-0.5-4 experienced a brittle shear failure (Fig. 3(a) and Appendix B). These results indicate that the effect of a/d on the SFRLC shear strength inherently exists, and thus a term associated with the a/d needs to be included in the shear strength equation of SFRLC beams, as also seen in the shear strength Eq. (11-5) in ACI 318-08[6] for RC beams. Figure 6 indicates that the a/d adversely affects the shear behavior of the SFRLC to a greater degree than it does that of the SFRC. In the following section, a design shear strength model for SFRLC beams is proposed.

DESIGN SHEAR STRENGTH MODEL

The shear strength equations for SFRC beams without web reinforcement were evaluated as to whether or not they

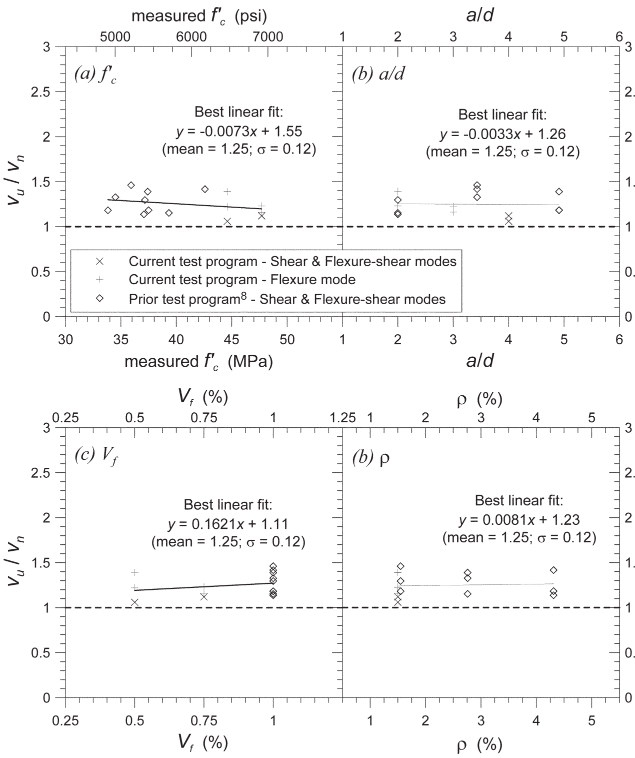

Fig. 7—Ratio of measured average shear stress at peak to shear stress v_u/v_n calculated based on design shear strength model developed for SFRLC beams (Eq. (1) and (2)).

are also applicable to SFRLC beams in consideration with lightweight concrete effects. Results from the current tests and the SFRLC beam tests conducted by Swamy et al.[8] were used for this evaluation to draw the best conclusions. The lack of available data underscores the importance of the current test program and warrants additional large-scale experimental studies.

The following three steps of the calibration approach were used. First, the most available shear strength models for SFRC beams (none lightweight) were extracted from the literature. Detailed equations are provided in Appendix A. Second, the lightweight concrete modification factor of λ, which was proposed as a function of $M/(Vd)$ or a/d, f'_c, ρ, and the type of lightweight aggregates earlier in this paper (refer to Table 5 and Fig. 4), was accounted for by replacing f'_c with $\lambda^2 f'_c$ and τ with λτ for SFRLC beams. Here, τ is the average fiber matrix interfacial bond stress. Note that the current test program used the expanded clay lightweight aggregates and the prior test programs[8] used the sintered PFA. The process is analogous to the application of the λ factor in ACI 318-08.[6] Finally, the ratio v_u/v_n of measured average shear stress at peak to shear strength calculated based on the existing model, except the consideration of lightweight concrete effects, was determined for each specimen to make a comparison between the models. Here, as-measured material properties were used for the calculation of v_n. The values of the measured average shear stress at peak v_u for the beams failing in flexure also give some indication of shear strength (the actual shear strength level is at least v_u); thus, the data of FLB-0.5-3 and FLB-0.75-series (refer

Table 6—Comparisons of measured average shear stresses at peak and shear strengths calculated based on available SFRC shear strength models except replacement of f'_c by $\lambda^2 f'_c$ and τ by λτ

ID	V_f, %	a/d	Failure mode	Measured average shear stress at peak/calculated shear strength, v_u/v_n									
				Narayanan and Darwish[1]	Ashour et al.[7] A	Ashour et al.[7] B	Kwak et al.[2]	Khuntia et al.[3]	Sharma[29]	Imam et al.[5]	Shin et al.[4]	Li et al.[30]	Choi et al.[9]
FLB-0.5-2	0.5	2	Flexure-shear	1.77	1.39	1.06	1.66	2.06	1.24	1.24	1.19	1.05	0.77
FLB-0.75-2	0.75	2	Flexure	1.57*	1.23*	0.93*	1.49*	1.83*	1.22*	1.08*	1.13*	1.04*	0.74*
FLB-0.5-3	0.5	3	Flexure	1.18*	1.22*	0.90*	1.04*	1.38*	0.78*	1.18*	1.30*	0.95*	0.54*
FLB-0.75-3	0.75	3	Flexure	1.07*	1.16*	0.82*	0.97*	1.26*	0.80*	1.10*	1.22*	0.99*	0.54*
FLB-0.5-4	0.5	4	Shear	1.21	1.06	0.94	1.01	1.10	0.66	1.17	1.13	0.77	0.47
FLB-0.75-4	0.75	4	Flexure	1.18*	1.12*	0.96*	1.03*	1.11*	0.76*	1.23*	1.16*	0.90*	0.52*
ITLF-1[8]	1.0	2	Shear	1.40	1.14	1.09	1.54	2.38	2.13	0.73	0.84	1.24	2.04
ITLF-2[8]	1.0	3.4	Shear	1.38	1.42	1.35	1.24	1.87	1.72	1.31	1.37	1.93	1.62
ITLF-3[8]	1.0	4.9	Shear	1.32	1.18	1.44	1.25	1.45	1.46	1.42	1.15	1.52	1.40
2TLF-1[8]	1.0	2	Shear	1.47	1.15	1.00	1.51	2.11	1.89	0.90	1.01	1.28	1.55
2TLF-2[8]	1.0	3.4	Shear	1.24	1.33	1.13	1.12	1.63	1.49	1.38	1.29	1.68	1.31
2TLF-3[8]	1.0	4.9	Shear	1.42	1.39	1.50	1.37	1.48	1.48	1.79	1.32	1.62	1.28
3TLF-1[8]	1.0	2	Shear	1.64	1.30	0.98	1.64	2.09	1.87	1.29	1.32	1.51	1.25
3TLF-2[8]	1.0	4.9	Flexure-shear	1.23	1.46	1.05	1.14	1.47	1.35	1.73	1.35	1.62	1.00
3TLF-3[8]	1.0	4.9	Flexure-shear	1.05	1.18	1.08	1.05	1.08	1.08	1.66	1.03	1.20	0.82
Mean	NA	NA	NA	1.34	1.25	1.08	1.27	1.62	1.33	1.28	1.19	1.29	1.06
Standard deviation	NA	NA	NA	0.21	0.12	0.20	0.24	0.42	0.46	0.29	0.15	0.34	0.48
Minimum	NA	NA	NA	1.05	1.06	0.82	0.97	1.08	0.66	0.73	0.84	0.77	0.47
Maximum	NA	NA	NA	1.77	1.46	1.50	1.66	2.38	2.13	1.79	1.37	1.93	2.04

* is minimum.

Notes: NA is not available; failure mode definition: shear (μ < 2); flexure-shear (2 ≤ μ ≤ 3.5); flexure (3.5 < μ); 1 MPa = 0.145 ksi.

to the + marks in Fig. 7 or the * marks in Table 6) were conservatively included in the analysis.

The mean, standard deviation, and minimum and maximum values of the ratios, as well as the slope of the linear regression lines, are compared (Table 6 and Appendix C). The standard deviation or coefficient of variation is a good statistical indicator of consistent accuracy. The models by Narayanan and Darwish,[1] Ashour et al.,[7] and Shin et al.[4] showed lower standard deviations (average = 0.18) relative to other models (Table 6). The mean values of v_u/v_n indicate that the models by Ashour et al.[7] (Model A) and Kwak et al.[2] have reasonable safety margins (approximately 25%), whereas the models by Ashour et al.[7] (Model B) and Shin et al.[4] have small safety margins of 9% on average. Particularly for the model by Shin et al.,[4] only one data point of v_u/v_n is much lower (0.84). Given the small number of data points, it is important that all 11 data points failing in shear or flexure-shear mode fall above the unity line. On the other hand, the models by Khuntia et al.,[3] Sharma,[30] and Choi et al.[9] are overly conservative or exhibit substantial scatter (Table 6).

The slope (steepness) of the linear regression line for v_u/v_n ratios is one statistical indicator to evaluate the sensitivity of the dependable variable v_u/v_n to each independent variable. The models by Narayanan and Darwish,[1] Ashour et al.[7] (Model A), Kwak et al.,[2] and Shin et al.[4] are overall satisfactory in terms of sensitivity, and the model by Ashour et al.[7] (Model A) gives the best results (Appendix C). Based on this review, the following design shear strength equation is proposed for SFRLC beams, which is the modified version of the SFRC equations developed by Ashour et al.[7] (Model A).

$$v_n = \left(2.11\sqrt[3]{\lambda^2 f_c'} + 7F\right)\sqrt[3]{\left(\rho\frac{d}{a}\right)} \quad \text{(MPa) for } a/d \geq 2.5 \tag{1}$$

$$v_n = \left(58.2\sqrt[3]{\lambda^2 f_c'} + 1015F\right)\sqrt[3]{\left(\rho\frac{d}{a}\right)} \quad \text{(psi) for } a/d \geq 2.5$$

$$v_n = [\text{Eq. (1)}]\left(\frac{2.5}{a/d}\right) + v_b\left(2.5 - \frac{a}{d}\right) \quad \text{(MPa; psi) for } a/d < 2.5 \tag{2}$$

where f_c' is the cylinder concrete strength of SFRLC; ρ is the flexural reinforcement ratio; a/d is the shear span-depth ratio, which is also expressed as $M_u/(V_u d)$; V_u is the factored shear force; M_u is the factored moment occurring simultaneously with V_u at the section considered; v_b is the fiber pullout stress (= $0.41\tau F$); and τ is taken as 4.15λ MPa (0.6λ ksi). The constant value of 4.15 was based on the recommendations by Li et al.,[31] Swamy et al.,[32] and Kwak et al.[2] The fiber factor F was defined earlier in the paper as equal to $(L_f/D_f)V_f d_f$ (refer to Fig. 2 for notations), where d_f is the bond factor (= 0.75 for both crimped and hooked fibers that were used for the prior[8] and current specimens, respectively). Note that a/d in Eq. (2) is taken as 1 when a/d or $V_u d/M_u$ is less than 1. The model of Eq. (1) and (2) empirically considers the arch action, which tends to occur when a/d is less than approximately 2.5 to 3.4 for SFRC beams.[1,2,4,31] For more details on the original SFRC model, the reader is referred to the paper by Ashour et al.[7] The models by Narayanan and Darwish[1] and Kwak et al.,[2] with a consideration of lightweight concrete effects, are also reasonable and acceptable.

Figure 7 illustrates the distributions of the v_u/v_n ratios against four different independent variables, showing that the proposed model is not overly sensitive to the variation of these four main variables. Furthermore, Fig. 7 depicts that the proposed model corresponds well to the current and prior data[8] of SFRLC beams in terms of the prediction (mean = 1.25), consistency (standard deviation = 0.12), safety (minimum = 1.06), and structural efficiency (maximum = 1.46).

SUMMARY AND CONCLUSIONS

The research undertaken comprised four-point bending tests, material tests, and design model studies. Based on the test results and analysis for the design shear strength, the following conclusions were drawn:

1. The compressive strength of SFRLC was increased by 13% for V_f = 0.5% and 20% for V_f = 0.75%. This indicates that the effectiveness of steel fibers is greater in lightweight concrete than in normalweight concrete. The tensile strength of SFRLC was also increased by 40% and approximately 70% to 100% for a V_f of 0.5% and 0.75%, respectively.

2. The measured material properties of SFRC were larger than those of SFRLC: f_c' by 28%, f_{ct} by 33%, f_r by 14%, and E_c by 20% on average. Due to these increased properties, the shear capacity of the SFRC beam was greater than that of the SFRLC beam at a given deflection. Therefore, a lightweight concrete modification factor λ is also recommended to be applied to a steel fiber-reinforced beam.

3. The addition of steel fibers significantly improved the resistance to structural damage, ductility, and measured shear load V_{cr} at diagonal cracking (by 35% on average). Most importantly, the increased steel-fiber volume fraction resulted in a change in the failure mode from brittle to ductile. Also, the steel-fiber volume fraction V_f of 0.5 % and 0.75% increased the shear capacities by 13% and 30%, respectively. The increased shear capacity for V_f = 0.75% demonstrated that the provision of ACI 318-08,[6] Section 11.4.6.1(f), could also be applicable to steel fiber-reinforced beams with lightweight aggregates.

4. The SFRLC beams with V_f = 0.75% or the combination of V_f = 0.5% and a/d = 2 to 3 performed well without any signs of brittle failure. Based on the results, the ACI 318-08[6] minimum requirement of 0.75% (Section 5.6.6.2(a)) appears to also be reasonable for steel fiber-reinforced lightweight concrete beams.

5. The a/d adversely affects the shear behavior of the lightweight fiber-reinforced beam (to a greater degree than it does that of the SFRC beam). The average shear stresses at diagonal cracking v_{cr} and peak v_u were reduced as the a/d increased. Thus, a term associated with the a/d needs to be included in the shear strength equation of SFRLC beams.

6. A design shear strength model for SFRLC beams without web reinforcement has been proposed based on the available research regarding the SFRC shear strength model and the lightweight concrete modification factor λ. The proposed model gives a satisfactory comparison of the test results and models.

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The views expressed herein are those of the authors and do not necessarily represent those of the sponsors.

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APPENDIX A

The following are the design shear strength equations for steel fiber-reinforced concrete (SFRC) beams.

I. Narayanan and Darwish[1]

$$v_n = e\left[0.24 f_{sp} + 80\rho\frac{d}{a}\right] + v_b \text{ (MPa)} \tag{A1}$$

$$v_n = e\left[0.24 f_{sp} + 11600\rho\frac{d}{a}\right] + v_b \text{ (psi)}$$

where f_{sp} = estimated using the splitting tensile strength of SFRC as shown in Eq. (A2);

$$f_{sp} = f_{cuf} / \left(20 - \sqrt{F}\right) + 0.7 + 1.0\sqrt{F} \text{ (MPa)} \tag{A2}$$

$$f_{sp} = f_{cuf} / \left(20 - \sqrt{F}\right) + 101.5 + 145\sqrt{F} \text{ (psi)}$$

e = arch action factor = 1 for $a/d > 2.8$, and $e = 2.8d/a$ for $a/d \leq 2.8$;

a/d = shear span-to-depth ratio;

ρ = flexural reinforcement ratio;

F = fiber factor = $(L_f/D_f)V_f d_f$;

f_{cuf} = cube strength of fiber concrete = 1.2 f_c', MPa (psi);

f_c' = concrete compressive strength, MPa (psi);

L_f = fiber length, mm (in.);

D_f = fiber diameter, mm (in.);

V_f = volume fraction of steel fibers;

d_f = bond factor = 0.5 for round fibers, 0.75 for crimped fibers, and 1 for indented fibers;

v_b = fiber pullout stress = $0.41\tau F$, MPa (psi); and

τ = average fiber matrix interface bond stress, taken as 4.15 MPa (600 psi), based on the recommendations of Swamy et al.[28]

II. Ashour et al.[7]

i) Model A (modified Zsutty equation)

For $a/d \geq 2.5$,

$$v_n = \left(2.11\sqrt[3]{f_c'} + 7F\right)\left(\rho\frac{d}{a}\right)^{0.333} \text{ (MPa)} \tag{A3}$$

$$v_n = \left(58.2\sqrt[3]{f_c'} + 1015F\right)\left(\rho\frac{d}{a}\right)^{0.333} \text{ (psi)}$$

For $a/d < 2.5$,

$$v_n = [\text{Eq. (A3)}]\left(\frac{2.5}{a/d}\right) + v_b\left(2.5 - \frac{a}{d}\right) \text{ (MPa; psi)} \tag{A4}$$

ii) Modified ACI equation

$$v_n = \left(0.7\sqrt{f_c'} + 7F\right)\frac{d}{a} + 17.2\rho\frac{d}{a} \text{ (MPa)} \tag{A5}$$

$$v_n = \left(8.4\sqrt{f_c'} + 1015F\right)\frac{d}{a} + 2494\rho\frac{d}{a} \text{ (psi)}$$

where f_c' = concrete compressive strength, MPa (psi);

a/d = shear span-to-depth ratio;

ρ = flexural reinforcement ratio;

F = fiber factor = $(L_f/D_f)V_f d_f$;

L_f = fiber length, mm (in.);

D_f = fiber diameter, mm (in.);

V_f = volume fraction of steel fibers;

d_f = bond factor = 0.5 for round fibers, 0.75 for crimped fibers, and 1 for indented fibers;

v_b = fiber pullout stress = $0.41\tau F$, MPa (psi); and

τ = average fiber matrix interface bond stress, taken as 4.15 MPa (600 psi), based on the recommendations of Swamy et al.[28]

III. Kwak et al.[2]

$$v_n = 3.7ef_{sp}^{2/3}\left(\rho\frac{d}{a}\right)^{1/3} + 0.8v_b \text{ (MPa)} \tag{A6}$$

$$v_n = 19.5ef_{sp}^{2/3}\left(\rho\frac{d}{a}\right)^{1/3} + 0.8v_b \text{ (psi)}$$

where e = arch action factor = 1 for $a/d > 3.4$; $e = 3.4d/a$ for $a/d \leq 3.4$;

f_{sp} = estimated using the splitting tensile strength of SFRC as shown in Eq. (A7);

$$f_{sp} = f_{cuf} / \left(20 - \sqrt{F}\right) + 0.7 + 1.0\sqrt{F} \text{ (MPa)} \tag{A7}$$

$$f_{sp} = f_{cuf} / \left(20 - \sqrt{F}\right) + 101.5 + 145\sqrt{F} \text{ (psi)}$$

a/d = shear span-to-depth ratio;

ρ = flexural reinforcement ratio;

F = fiber factor = $(L_f/D_f)V_f d_f$;

f_{cuf} = cube strength of fiber concrete = $1.2 f_c'$, MPa (psi);

L_f = fiber length, mm (in.);

D_f = fiber diameter, mm (in.);

V_f = volume fraction of steel fibers;

d_f = bond factor = 0.5 for round fibers, 0.75 for crimped fibers, and 1 for indented fibers;

v_b = fiber pullout stress = $0.41\tau F$, MPa (psi); and

τ = average fiber matrix interface bond stress, taken as 4.15 MPa (600 psi), based on the recommendations of Swamy et al.[28]

IV. Khuntia et al.[3]

$$v_n = (0.167\alpha + 0.25F_1)\sqrt{f_c'} \text{ (MPa)} \tag{A8}$$

$$v_n = (2\alpha + 3F_1)\sqrt{f_c'} \text{ (psi)}$$

where f_c' = concrete compressive strength, MPa (psi);

α = arch action factor = 1 for $a/d \geq 2.5$, and $\alpha = 2.5d/a \leq 3$ for $a/d < 2.5$;

F_1 = fiber factor = $\beta V_f (l_f/d_f)$;

l_f = fiber length, mm (in.);

d_f = fiber diameter, mm (in.);

V_f = volume fraction of steel fibers; and

β = factor for fiber shape and concrete type = 1 for hooked or crimped steel fibers, 2/3 for plain or round steel fibers with normal concrete, 3/4 for hooked or crimped steel fibers with lightweight concrete.

V. Sharma[26]

$$v_n = kf_t'(d/a)^{0.25} \text{ (MPa, psi)} \tag{A9}$$

where a/d = shear span-to-depth ratio;

f_t' = tensile strength of concrete, MPa (psi);

k = 1 if f_t' is obtained by direct tension test;

k = 2/3 if f_t' is obtained by indirect tension test;

k = 4/9 if f_t' is obtained using modulus of rupture; or

$f_t' = 0.79(f_c')^{0.5}$, MPa ($f_t' = 9.5(f_c')^{0.5}$, psi); and

f_c' = concrete compressive strength, MPa (psi).

VI. Imam et al.[5]

$$v_n = 0.6\Psi\sqrt[3]{\omega}\left[(f_c')^{0.44} + 275\sqrt{\frac{\omega}{(a/d)^5}}\right] \text{ (MPa)} \tag{A10}$$

$$v_n = 0.6\Psi\sqrt[3]{\omega}\left[16.2(f_c')^{0.44} + 39875\sqrt{\frac{\omega}{(a/d)^5}}\right] \text{ (psi)}$$

where f_c' = concrete compressive strength, MPa (psi);

a/d = shear span-to-depth ratio;

Ψ = size effect factor $= \dfrac{1+\sqrt{(5.08/d_a)}}{\sqrt{1+d/(25d_a)}}$, for mm $\left(\dfrac{1+0.2\sqrt{(5.08/d_a)}}{\sqrt{1+d/(25d_a)}}, \text{for in.}\right)$;

ω = reinforcing factor = $\rho(1+4F)$;

F = fiber factor = $(L_f/D_f)V_f d_f$;

L_f = fiber length, mm (in.);

D_f = fiber diameter, mm (in.);

V_f = volume fraction of steel fibers; and

d_f = bond factor = 0.5 for smooth fibers, 0.9 for deformed fibers, and 1.0 for hooked fibers.

VII. Shin et al.[4]

For $a/d \geq 3.0$,

$$v_n = 0.19 f_{sp} + 93\rho\left(\frac{d}{a}\right) + 0.834 v_b \text{ (MPa)} \tag{A11}$$

$$v_n = 0.19 f_{sp} + 13485\rho\left(\frac{d}{a}\right) + 0.834 v_b \text{ (psi)}$$

For $a/d < 3.0$,

$$v_n = 0.22 f_{sp} + 217\rho\left(\frac{d}{a}\right) + 0.834 v_b \text{ (MPa)} \tag{A12}$$

$$v_n = 0.22 f_{sp} + 31465\rho\left(\frac{d}{a}\right) + 0.834 v_b \text{ (psi)}$$

where f_{sp} = estimated using the splitting tensile strength of SFRC as shown in Eq. (A13);

$$f_{sp} = f_{cuf} / \left(20 - \sqrt{F}\right) + 0.7 + 1.0\sqrt{F} \text{ (MPa)} \tag{A13}$$

$$f_{sp} = f_{cuf} / \left(20 - \sqrt{F}\right) + 101.5 + 145\sqrt{F} \text{ (psi)}$$

a/d = shear span-to-depth ratio;

ρ = flexural reinforcement ratio;

F = fiber factor = $(L_f/D_f)V_f d_f$;

f_{cuf} = cube strength of fiber concrete = $1.2 f_c'$, MPa (psi);

f_c' = concrete compressive strength, MPa (psi);

L_f = fiber length, mm (in.);

D_f = fiber diameter, mm (in.);

V_f = volume fraction of steel fibers;

d_f = bond factor = 0.5 for round fibers, 0.75 for crimped fibers, and 1 for indented fibers;

v_b = fiber pullout stress = $0.41\tau F$, MPa (psi); and

τ = average fiber matrix interface bond stress, taken as 4.15 MPa (600 psi), based on the recommendations of Swamy et al.[28]

VIII. Li et al.[27]

For $a/d \geq 2.5$,

$$v_n = 1.25 + 4.68\left[\left(f_f f_t\right)^{3/4}\left(\rho\frac{d}{a}\right)^{1/3} d^{-(1/3)}\right] \text{ (MPa)} \tag{A14}$$

$$v_n = 181 + 0.134\left[\left(f_f f_t\right)^{3/4}\left(\rho\frac{d}{a}\right)^{1/3} d^{-(1/3)}\right] \text{ (psi)}$$

For $a/d < 2.5$,

$$v_n = 9.16\left[\left(f_f\right)^{2/3}\left(\rho\right)^{1/3}\left(d/a\right)\right] \text{ (MPa)} \tag{A15}$$

$$v_n = 48.3\left[\left(f_f\right)^{2/3}\left(\rho\right)^{1/3}\left(d/a\right)\right] \text{ (psi)}$$

where f_f = modulus of rupture $= 2.5 f_{cc}$, MPa (psi);

f_{cc} = tensile strength = $f_t\left(1 - V_f\right) + \alpha_1\alpha_2\tau V_f\left(L_f / D_f\right)$, MPa (psi);

f_t = tensile strength of concrete $= 0.292\sqrt{f_c'}$, MPa ($3.5\sqrt{f_c'}$, psi) proposed by MacGregor et al. (1960);

f_c' = concrete compressive strength, MPa (psi);

V_f = volume fraction of steel fibers;

L_f / D_f = steel fiber aspect ratio;

α_1 = coefficient representing the fraction of bond mobilized at first matrix cracking, taken as 0.5, based on the recommendation by Naaman and Reinhardt (2003);

α_2 = efficient factor of fiber orientation in the uncracked state of the composite, taken as 0.1, based on the recommendation by Naaman and Reinhardt (2003);

τ = average fiber matrix interface bond stress, taken as 4.15 MPa (600 psi), based on the recommendations of Swamy et al.[28];

a/d = shear span-to-depth ratio;

ρ = flexural reinforcement ratio; and

d = beam depth, mm (in.).

IX. Choi et al.[9]

$$v_n = \lambda_s\sqrt{f_{cc}[f_{cc} + \alpha_{x1}\varepsilon_{of}E_c/2]}\,bc_{x1} / \left(bd\right) \text{ (MPa)} \tag{A16}$$

$$v_n = 0.085\lambda_s \sqrt{f_{cc}[f_{cc} + \alpha_{x1}\varepsilon_{of} E_c/2]} bc_{x1} / (bd) \text{ (psi)}$$

where λ_s = size effect factor $= 1.2 - 0.3(a/d)d \geq 0.65$, d in m (d in yd);

f_{cc} = tensile strength = $f_t(1 - V_f) + \alpha_1\alpha_2\tau V_f(L_f / D_f)\beta$, MPa (psi);

f_t = tensile strength of concrete $= 0.292\sqrt{f_c'}$, MPa ($3.5\sqrt{f_c'}$, psi) proposed by MacGregor et al. (1960);

f_c' = concrete compressive strength, MPa (psi);

V_f = volume fraction of steel fibers;

L_f / D_f = steel fiber aspect ratio;

α_1 = coefficient representing the fraction of bond mobilized at first matrix cracking, taken as 0.5, based on the recommendation by Naaman and Reinhardt (2003);

α_2 = efficient factor of fiber orientation in the uncracked state of the composite, taken as 0.1, based on the recommendation by Naaman and Reinhardt (2003);

τ = average fiber matrix interface bond stress, taken as 4.15 MPa (600 psi), based on the recommendations of Swamy et al.[28];

$\alpha_{x1}\varepsilon_{of}$ = compressive normal strain at the extreme compression fiber of the cross section at location x_1 =

$$\frac{2[f_f h^2/(6x_0) + 0.05\sqrt{f_{cf}'}d](x_0 + h - c_{x1})}{E_c c_{x1}(jd_{x1})}, \text{ mm}$$

$$(x_1 = \frac{2[f_f h^2/(6x_0) + 0.6\sqrt{f_{cf}'}d](x_0 + h - c_{x1})}{E_c c_{x1}(jd_{x1})}, \text{ in.});$$

c_{x1} = depth of the compressive zone in a newly cracked section at location x_1, mm (in.);

x_0 = the location of crack initiation: $0.6a - h + c_{x1}$ for $2 \leq a/d \leq 5$, and $a - 2d - h + c_{x1}$ for $a/d > 5$, mm (in.);

f_{cf}' = compressive strength $= 1.9V_f(L_f / D_f)\beta + f_c'$, MPa ($275.5V_f(L_f / D_f)\beta + f_c'$, psi);

β = 1 for hooked or crimped steel fibers;

β = 2/3 for plain or round steel fibers with normal concrete;

β = 3/4 for hooked or crimped steel fibers with lightweight concrete;

jd_{x1} = length of moment arm at location $x_1 = jd_{c1} + jd_{t1}$, mm (in.);

jd_{c1} = distance from the neutral axis to the centroid of the resultant compressive force at location x_1 = $(2/3)c_{x1}$, mm (in.);

jd_{t1} = distance from the neutral axis to the centroid of the resultant tensile force at location x_1, mm (in.);

a/d = shear span-to-depth ratio;

b = beam width, mm (in.);

d = beam depth, mm (in.);

h = beam height, mm (in.); and

E_c = modulus of elasticity, MPa (psi).

References

MacGregor, J. G.; Sozen, M. A.; and Siess, C. P., "Strength and Behavior of Prestressed Concrete Beams with Web Reinforcement," University of Illinois Civil Engineering Studies, Structural Research Series 210, Urbana, IL, Aug. 1960.

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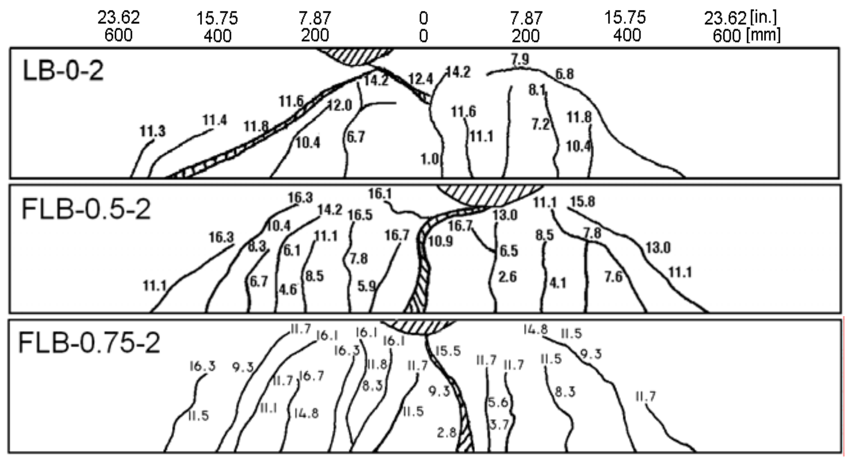
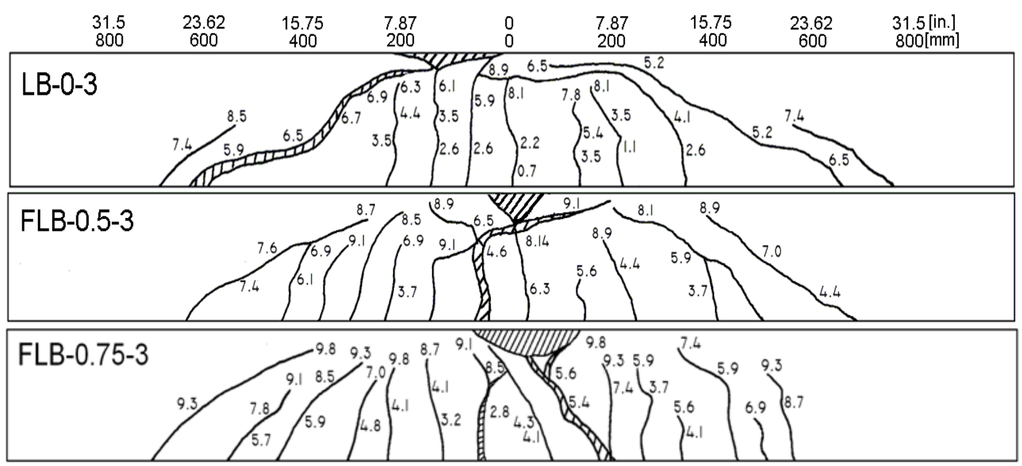
APPENDIX B

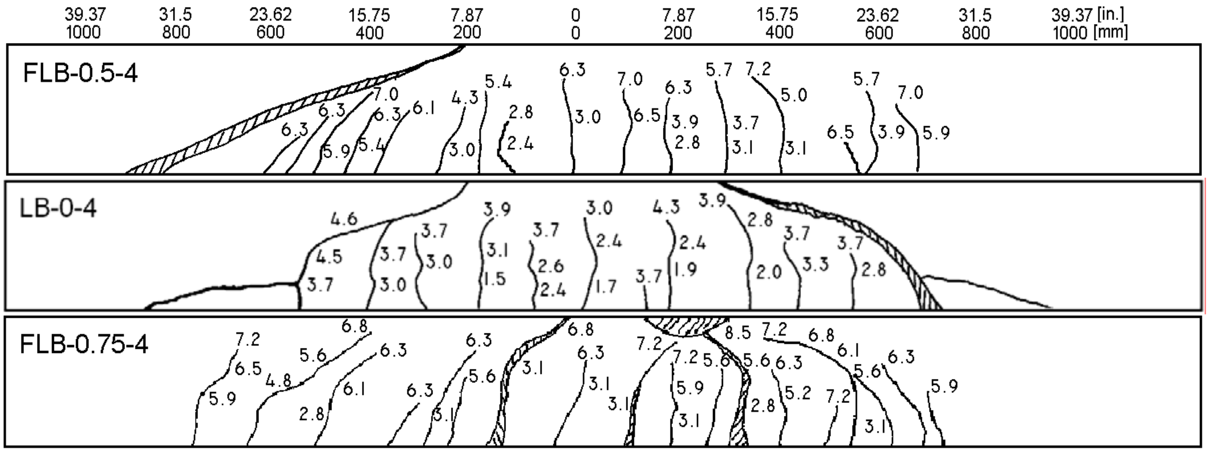
The following shows the sequential crack patterns for tested SFRLC beams.



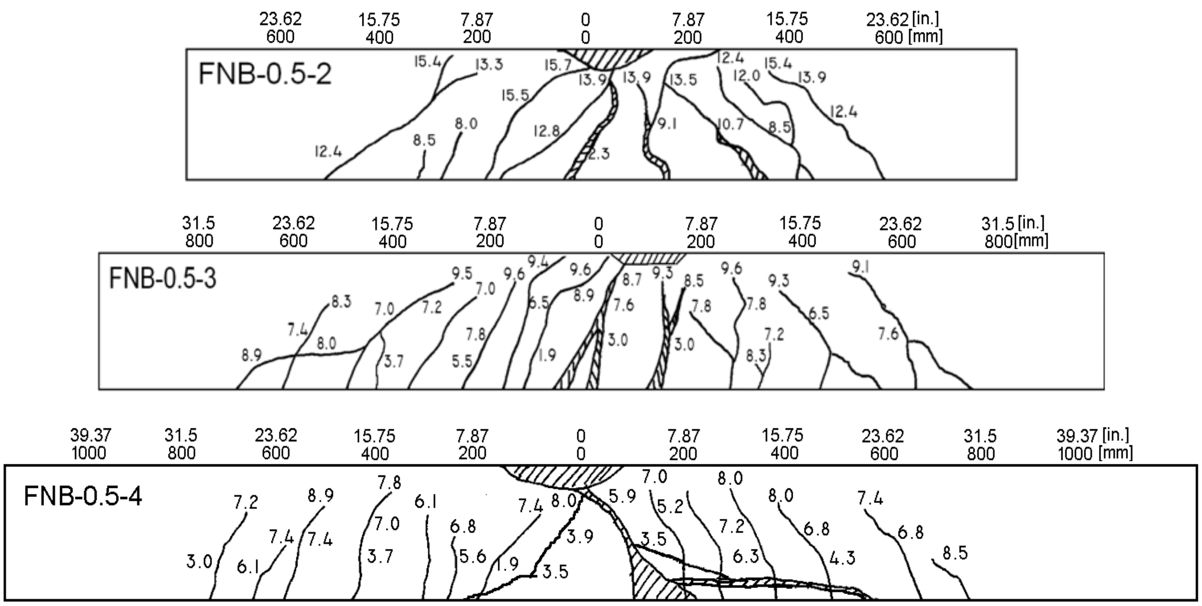
(a) SFRLC beams with (*a*/*d*) = 2



(b) SFRLC beams with (*a*/*d*) = 3



(c) SFRLC beams with (*a*/*d*) = 4



(d) SFRC beams with $V_f = 0.5\%$

Fig. B1 – Sequential crack patterns (numbers on the beam: load [tons]; Conversion: 1 ton = 9.8 kN = 2.24 kips)

APPENDIX C

The following indicates the steepness (slope) of the linear regression line for the ratio of measured average shear stress at peak (v_u) to calculated shear strength (v_n), with a consideration of lightweight concrete effects.

Table C1 – Steepness (slope) of the linear regression line for each (v_u/v_n) data set

Independent variable	Absolute slope of linear regression line for each (v_u/v_n) data set									
	Narayanan and Darwish[1]	Ashour et al.[7] A	Ashour et al.[7] B	Kwak et al.[2]	Khuntia et al.[3]	Sharma[26]	Imam et al.[5]	Shin et al.[4]	Li et al.[27]	Choi et al.[9]
f'_c (MPa)	0.0019	0.0073	0.0192	0.0072	0.0144	0.0529	0.0257	0.0017	0.0437	0.0575
a/d	0.1129	0.0033	0.0954	0.138	0.2859	0.1512	0.1857	0.0244	0.0511	0.0705
V_f (%)	0.0266	0.1621	0.5105	0.2228	0.5407	1.5893	0.397	0.0318	1.2977	1.7219
ρ (%)	0.0188	0.0081	0.1245	0.0475	0.1542	0.2641	0.0403	0.3568	0.1791	0.3568

Conversion: 1 MPa = 0.145 ksi