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A Comparative Study of Mohand and Aboodh Transforms

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Abstract: Mohand and Aboodh transforms are very useful integral transforms for solving many advanced problems of engineering and sciences like heat conduction problems, vibrating beams problems, population growth and decay problems, electric circuit problems etc. In this article, we present a comparative study of two integral transforms namely Mohand and Aboodh transforms. In application section, we solve some systems of differential equations using both the transforms. Results show that Mohand and Aboodh transforms are closely connected.

Keywords: Mohand transform, Aboodh transform, System of differential equations.

1. INTRODUCTION

In modern time, integral transforms (Laplace transform [1, 7-11], Fourier transform [1], Hankel transform [1], Mellin transform [1], Z-transform [1], Wavelet transform [1], Mahgoub transform [2], Kamal transform [3], Elzaki transform [4], Aboodh transform [5], Mohand transform [6], Sumudu transform [12], Hermite transform [1] etc.) have very useful role in mathematics, physics, chemistry, social science, biology, radio physics, astronomy, nuclear science, electrical and mechanical engineering for solving the advanced problems of these fields.

Many scholars [13-30] used these transforms and solve the problems of differential equations, partial differential equations, integral equations, integro-differential equations, partial integro-differential equations, delay differential equations and population growth and decay problems. Aggarwal et al. [31] used Mohand transform and solved population growth and decay problems. Aggarwal et al. [32] defined Mohand transform of Bessel's functions. Kumar et al. [33] solved linear Volterra integral equations of first kind using Mohand transform.

Kumar et al. [34] used Mohand transform and solved the mechanics and electrical circuit problems. Solution of linear Volterra integral equations of second kind using Mohand transform was given by Aggarwal et al. [35]. Aboodh [36] used a new integral transform "Aboodh transform" for solving partial differential equations. Aboodh et al. [37] solved delay differential equations using Aboodh transformation method. Solution of partial integro-differential equations using Aboodh and double Aboodh transforms methods were given by Aboodh et al. [38]. Aggarwal et al. [39] applied Aboodh transform for solving linear Volterra integro-differential equations of second kind. A new application of Aboodh transform for solving linear Volterra integral equations was given by Aggarwal et al. [40]. Mohand et al. [41] used Aboodh transform and solved ordinary differential equation with variable coefficients. Aboodh transform of Bessel's functions was given by Aggarwal et al. [42]. Aggarwal et al. [43] gave the solution of population growth and decay problems using Aboodh transform method. Aggarwal and Chaudhary [44] gave a comparative study of Mohand and Laplace transforms. A comparative study of Mohand and Kamal transforms was given by Aggarwal et al. [45]

In this paper, we concentrate mainly on the comparative study of Mohand and Aboodh transforms and we solve some systems of differential equations using these transforms.

2. DEFINITION OF MOHAND AND ABOODH TRANSFORMS:

2.1 Definition of Mohand transforms:

In year 2017, Mohand and Mahgoub [6] defined "Mohand transform'' of the function $F(t)$ for $t \geq 0$ as

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt = R(v), k_1 \le v \le k_2$$

where the operator M is called the Mohand transform operator.

2.2 Definition of Aboodh transforms:

The Aboodh transform of the function $F(t)$ for all $t \ge 0$ is defined as [5]:

$$A\{F(t)\} = \frac{1}{v}\int_0^\infty F(t)e^{-vt}dt$$

$= K(v), 0 < k_1 \le v \le k_2,$ where the operator A is called the Aboodh transform operator.

The Mohand and Aboodh transforms of the function $F(t)$ for $t \ge 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mohand and Aboodh transforms of the function $F(t)$.

3. PROPERTIES OF MOHAND AND ABOODH TRANSFORMS: In this section, we present the linearity property, change of scale property, first shifting theorem, convolution theorem of Mohand and Aboodh transforms.

3.1 Linearity property of Mohand and Aboodh transforms:

a. Linearity property of Mohand transforms [31-32, 35]: If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of $[aF_1(t) + bF_2(t)]$ is given by $[aR_1(v) + bR_2(v)]$, where a, b are arbitrary constants.

b. Linearity property of Aboodh transforms [39-40]: If Aboodh transform of functions $F_1(t)$ and $F_2(t)$ are $K_1(v)$ and $K_2(v)$ respectively then Aboodh transform of $[aF_1(t) + bF_2(t)]$ is given by $[aK_1(v) + bK_2(v)]$, where a, b are arbitrary constants.

3.2 Change of scale property of Mohand and Aboodh transforms:

a. Change of scale property of Mohand transforms [32, 35]: If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $F(at)$ is given by $aR\left(\frac{v}{a}\right)$.

b. Change of scale property of Aboodh transforms: If Aboodh transform of function $F(t)$ is $K(v)$ then Aboodh transform of function $F(at)$ is given by $\frac{1}{a^2}K\left(\frac{v}{a}\right)$.

Proof: By the definition of Aboodh transform, we have

$$A\{F(at)\} = \frac{1}{v}\int_0^\infty F(at)e^{-vt}dt$$

Put $at = p \Rightarrow adt = dp$ in above equation, we have

$$A\{F(at)\} = \frac{1}{av}\int_0^\infty F(p)e^{\frac{-vp}{a}}dp$$

$$\Rightarrow A\{F(at)\} = \frac{1}{a^2}\left[\frac{1}{v/a}\int_0^\infty F(p)e^{\frac{-vp}{a}}dp\right]$$

$$\Rightarrow A\{F(at)\} = \frac{1}{a^2}K\left(\frac{v}{a}\right)$$

3.3 Shifting property of Mohand and Aboodh transforms:

a. Shifting property of Mohand transforms [35]: If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $e^{at}F(t)$ is given by $\left[\frac{v^2}{(v-a)^2}\right]R(v-a)$.

b. Shifting property of Aboodh transforms: If Aboodh transform of function $F(t)$ is $K(v)$ then Aboodh transform of function $e^{at}F(t)$ is given by $\frac{(v-a)}{v}K(v-a)$.

Proof: By the definition of Aboodh transform, we have

$$A\{e^{at}F(t)\} = \frac{1}{v}\int_0^\infty e^{at}F(t)e^{-vt}dt$$

$$= \frac{1}{v}\int_0^\infty F(t)e^{-(v-a)t}dt$$

$$= \frac{(v-a)}{v}.\frac{1}{(v-a)}\int_0^\infty F(t)e^{-(v-a)t}dt$$

$$= \frac{(v-a)}{v}K(v-a)$$

3.4 Convolution theorem for Mohand and Aboodh transforms:

a. Convolution theorem for Mohand transforms [33, 35]: If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of their convolution $F_1(t) * F_2(t)$ is given by

$$M\{F_1(t) * F_2(t)\} = \left(\frac{1}{v^2}\right)M\{F_1(t)\}M\{F_2(t)\}$$

$$\Rightarrow M\{F_1(t) * F_2(t)\} = \left(\frac{1}{v^2}\right)R_1(v)R_2(v),$$

where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\, F_2(x) dx$$
$$= \int_0^t F_1(x)\, F_2(t-x) dx$$

b. ***Convolution theorem for Aboodh transforms [38-40]:*** If Aboodh transform of functions $F_1(t)$ and $F_2(t)$ are $K_1(v)$ and $K_2(v)$ respectively then Aboodh transform of their convolution $F_1(t) * F_2(t)$ is given by

$A\{F_1(t) * F_2(t)\} = vA\{F_1(t)\}A\{F_2(t)\}$
$\Rightarrow A\{F_1(t) * F_2(t)\} = v\, K_1(v)K_2(v)$,
where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\, F_2(x) dx$$
$$= \int_0^t F_1(x)\, F_2(t-x) dx$$

4. MOHAND AND ABOODH TRANSFORMS OF THE DERIVATIVES OF THE FUNCTION $F(t)$:

4.1 Mohand transforms of the derivatives of the function $F(t)$ [33-35]:

If $M\{F(t)\} = R(v)$ then

a) $M\{F'(t)\} = vR(v) - v^2F(0)$
b) $M\{F''(t)\} = v^2R(v) - v^3F(0) - v^2F'(0)$
c) $M\{F^{(n)}(t)\} = v^nR(v) - v^{n+1}F(0) - v^nF'(0) - \cdots \ldots - v^2F^{(n-1)}(0)$

4.2 Aboodh transforms of the derivatives of the function $F(t)$ [39-40, 42-43]:

If $A\{F(t)\} = K(v)$then

a) $A\{F'(t)\} = vK(v) - \frac{F(0)}{v}$
b) $A\{F''(t)\} = v^2K(v) - F(0) - \frac{F'(0)}{v}$
c) $A\{F^{(n)}(t)\} = v^nK(v) - \frac{F(0)}{v^{2-n}} - \frac{F'(0)}{v^{3-n}} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v}$

5. MOHAND AND ABOODH TRANSFORMS OF INTEGRAL OF A FUNCTION $F(t)$:

5.1 Mohand transforms of integral of a function $F(t)$:

If $M\{F(t)\} = R(v)$ then $M\left\{\int_0^t F(t)dt\right\} = \frac{1}{v}R(v)$

Proof: Let $G(t) = \int_0^t F(t)dt$.
Then $G'(t) = F(t)$and $G(0) = 0$.
Now by the property of Mohand transform of the derivative of function, we have

$$M\{G'(t)\} = vM\{G(t)\} - v^2G(0) = vM\{G(t)\}$$
$$\Rightarrow M\{G(t)\} = \frac{1}{v}M\{G'(t)\} = \frac{1}{v}\, M\{F(t)\}$$
$$\Rightarrow M\{G(t)\} = \frac{1}{v}R(v)$$
$$\Rightarrow M\left\{\int_0^t F(t)dt\right\} = \frac{1}{v}R(v)$$

5.2 Aboodh transforms of integral of a function $F(t)$:

If $A\{F(t)\} = K(v)$ then $A\left\{\int_0^t F(t)dt\right\} = \frac{1}{v}K(v)$

Proof: Let $G(t) = \int_0^t F(t)dt$.
Then $G'(t) = F(t)$and $G(0) = 0$.
Now by the property of Aboodh transform of the derivative of function, we have

$$A\{G'(t)\} = vA\{G(t)\} - \frac{G(0)}{v} = vA\{G(t)\}$$
$$\Rightarrow A\{G(t)\} = \frac{1}{v}A\{G'(t)\} = \frac{1}{v}\, A\{F(t)\}$$
$$\Rightarrow A\{G(t)\} = \frac{1}{v}K(v)$$
$$\Rightarrow A\left\{\int_0^t F(t)dt\right\} = \frac{1}{v}K(v)$$

6. MOHAND AND ABOODH TRANSFORMS OF FUNCTION $tF(t)$:

6.1 Mohand transforms of function $tF(t)$:

If $M\{F(t)\} = R(v)$ then $M\{tF(t)\} = \left[\frac{2}{v} - \frac{d}{dv}\right]R(v)$

Proof: By the definition of Mohand transform, we have

$$M\{F(t)\} = v^2\int_0^\infty F(t)e^{-vt}dt = R(v)$$
$$\Rightarrow \frac{d}{dv}R(v) = 2v\int_0^\infty F(t)e^{-vt}dt + v^2\int_0^\infty (-t)F(t)e^{-vt}dt$$
$$\Rightarrow \frac{d}{dv}R(v) = \frac{2}{v}.v^2\int_0^\infty F(t)e^{-vt}dt - v^2\int_0^\infty tF(t)e^{-vt}dt$$
$$\Rightarrow \frac{d}{dv}R(v) = \frac{2}{v}R(v) - M\{tF(t)\}$$
$$\Rightarrow M\{tF(t)\} = \left[\frac{2}{v} - \frac{d}{dv}\right]R(v)$$

6.2 Aboodh transforms of function $tF(t)$:

If $A\{F(t)\} = K(v)$ then $A\{tF(t)\} = \left[\frac{-1}{v} - \frac{d}{dv}\right]K(v)$

Proof: By the definition of Aboodh transform, we have

$$A\{F(t)\} = \frac{1}{v}\int_0^\infty F(t)e^{-vt}dt = K(v)$$

$$\Rightarrow \frac{d}{dv}K(v) = \frac{-1}{v^2}\int_0^\infty F(t)e^{-vt}dt + \frac{1}{v}\int_0^\infty (-t)F(t)e^{-vt}dt$$

$$\Rightarrow \frac{d}{dv}K(v) = \frac{-1}{v^2}\int_0^\infty F(t)e^{-vt}dt - \frac{1}{v}\int_0^\infty tF(t)e^{-vt}dt$$

$$\Rightarrow \frac{d}{dv}K(v) = \frac{-1}{v}K(v) - A\{tF(t)\}$$

$$\Rightarrow A\{tF(t)\} = \left[\frac{-1}{v} - \frac{d}{dv}\right]K(v)$$

7. MOHAND AND ABOODH TRANSFORMS OF FREQUENTLY USED FUNCTIONS [31-35, 39-43]:

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = R(v)$	$A\{F(t)\} = K(v)$
1.	1	v	$\frac{1}{v^2}$
2.	t	1	$\frac{1}{v^3}$
3.	t^2	$\frac{2!}{v}$	$\frac{2!}{v^4}$
4.	$t^n, n \in N$	$\frac{n!}{v^{n-1}}$	$\frac{n!}{v^{n+2}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$	$\frac{\Gamma(n+1)}{v^{n+2}}$
6.	e^{at}	$\frac{v^2}{v-a}$	$\frac{1}{v^2-av}$
7.	$sinat$	$\frac{av^2}{(v^2+a^2)}$	$\frac{a}{v(v^2+a^2)}$
8.	$cosat$	$\frac{v^3}{(v^2+a^2)}$	$\frac{1}{v^2+a^2}$
9.	$sinhat$	$\frac{av^2}{(v^2-a^2)}$	$\frac{a}{v(v^2-a^2)}$
10.	$coshat$	$\frac{v^3}{(v^2-a^2)}$	$\frac{1}{v^2-a^2}$
11.	$J_0(t)$	$\frac{v^2}{\sqrt{(1+v^2)}}$	$\frac{1}{v\sqrt{(1+v^2)}}$
12.	$J_1(t)$	$v^2 - \frac{v^3}{\sqrt{(1+v^2)}}$	$\frac{1}{v} - \frac{1}{\sqrt{(1+v^2)}}$

8. INVERSE MOHAND AND ABOODH TRANSFORMS:

8.1 Inverse Mohand transforms [31, 35]: If $R(v)$is the Mohand transform of $F(t)$then $F(t)$is called the inverse Mohand transform of $R(v)$ and in mathematical terms, it can be expressed as
$F(t) = M^{-1}\{R(v)\}$, where M^{-1} is an operator and it is called as inverse Mohand transform operator.

8.2 Inverse Aboodh transforms [39-40, 43]: If $K(v)$ is the Aboodh transforms of $F(t)$ then $F(t)$ is called the inverse Aboodh transform of $K(v)$and in mathematical terms, it can be expressed as
$F(t) = A^{-1}\{K(v)\}$, where A^{-1} is an operator and it is called as inverse Aboodh transform operator.

9. INVERSE MOHAND AND ABOODH TRANSFORMS OF FREQUENTLY USED FUNCTIONS [31-32, 39-40, 42-43]:

Table: 2

S.N.	$R(v)$	$F(t)=M^{-1}\{R(v)\}=A^{-1}\{K(v)\}$	$K(v)$
1.	v	1	$\frac{1}{v^2}$
2.	1	t	$\frac{1}{v^3}$
3.	$\frac{1}{v}$	$\frac{t^2}{2}$	$\frac{1}{v^4}$
4.	$\frac{1}{v^{n-1}}, n\epsilon N$	$\frac{t^n}{n!}$	$\frac{1}{v^{n+2}}, n\epsilon N$
5.	$\frac{1}{v^{n-1}}, n>-1$	$\frac{t^n}{\Gamma(n+1)}$	$\frac{1}{v^{n+2}}, n>-1$
6.	$\frac{v^2}{v-a}$	e^{at}	$\frac{1}{v^2-av}$
7.	$\frac{v^2}{(v^2+a^2)}$	$\frac{sinat}{a}$	$\frac{1}{v(v^2+a^2)}$
8.	$\frac{v^3}{(v^2+a^2)}$	$cosat$	$\frac{1}{v^2+a^2}$
9.	$\frac{v^2}{(v^2-a^2)}$	$\frac{sinhat}{a}$	$\frac{1}{v(v^2-a^2)}$
10.	$\frac{v^3}{(v^2-a^2)}$	$coshat$	$\frac{1}{v^2-a^2}$
11.	$\frac{v^2}{\sqrt{(1+v^2)}}$	$J_0(t)$	$\frac{1}{v\sqrt{(1+v^2)}}$
12.	$v^2-\frac{v^3}{\sqrt{(1+v^2)}}$	$J_1(t)$	$\frac{1}{v}-\frac{1}{\sqrt{(1+v^2)}}$

10. APPLICATIONS OF MOHAND AND ABOODH TRANSFORMS FOR SOLVING SYSTEM OF DIFFERENTIAL EQUATIONS:

In this section some numerical applications are give to solve the systems of differential equations using Mohand and Aboodh transforms.

10.1 Consider a system of linear ordinary differential equations

$$\left.\begin{aligned}\frac{d^2x}{dt^2}+3x-2y=0\\ \frac{d^2x}{dt^2}+\frac{d^2y}{dt^2}-3x+5y=0\end{aligned}\right\}\dots..(1)$$

with $x(0)=0, y(0)=0, x'(0)=3, y'(0)=2 \dots\dots..(2)$

Solution using Mohand transforms:

Taking Mohand transform of “Eq. (1)”, we have

$$\left.\begin{aligned}M\left\{\frac{d^2x}{dt^2}\right\}+3M\{x\}-2M\{y\}=0\\ M\left\{\frac{d^2x}{dt^2}\right\}+M\left\{\frac{d^2y}{dt^2}\right\}-3M\{x\}+5M\{y\}=0\end{aligned}\right\}\dots.(3)$$

Now using the property, Mohand transform of the derivatives of the function, in “Eq. (3)”, we have

$$\left.\begin{aligned}v^2M\{x\}-v^3x(0)-v^2x'(0)+3M\{x\}-2M\{y\}=0\\ v^2M\{x\}-v^3x(0)-v^2x'(0)+v^2M\{y\}-v^3y(0)-v^2y'(0)-3M\{x\}+5M\{y\}=0\end{aligned}\right\}\dots\dots(4)$$

Using “Eq. (2)” in “Eq. (4)”, we have

$$\left.\begin{aligned}(v^2+3)M\{x\}-2M\{y\}=3v^2\\(v^2-3)M\{x\}+(v^2+5)M\{y\}=5v^2\end{aligned}\right\}\ldots\ldots\ldots\ldots(5)$$

Solving the "Eq. (5)" for $M\{x\}$and $M\{y\}$, we have

$$\left.\begin{aligned}M\{x\}=\frac{11}{4}\left[\frac{v^2}{(v^2+1)}\right]+\frac{1}{4}\left[\frac{v^2}{(v^2+9)}\right]\\M\{y\}=\frac{11}{4}\left[\frac{v^2}{(v^2+1)}\right]-\frac{3}{4}\left[\frac{v^2}{(v^2+9)}\right]\end{aligned}\right\}\ldots\ldots\ldots..(6)$$

Now taking inverse Mohand transform of "Eq. (6)", we have

$$\left.\begin{aligned}x=\frac{11}{4}sint+\frac{1}{12}sin3t\\y=\frac{11}{4}sint-\frac{1}{4}sin3t\end{aligned}\right\}\ldots\ldots\ldots\ldots.(7)$$

which is the required solution of "Eq. (1)" with "Eq. (2)".

Solution using Aboodh transforms:

Taking Aboodh transform of "Eq. (1)", we have

$$\left.\begin{aligned}A\left\{\frac{d^2x}{dt^2}\right\}+3A\{x\}-2A\{y\}=0\\A\left\{\frac{d^2x}{dt^2}\right\}+A\left\{\frac{d^2y}{dt^2}\right\}-3A\{x\}+5A\{y\}=0\end{aligned}\right\}\ldots\ldots\ldots..(8)$$

Now using the property, Aboodh transform of the derivatives of the function, in "Eq. (8)", we have

$$\left.\begin{aligned}v^2A\{x\}-x(0)-\frac{x'(0)}{v}+3A\{x\}-2A\{y\}=0\\v^2A\{x\}-x(0)-\frac{x'(0)}{v}+v^2A\{y\}-y(0)-\frac{y'(0)}{v}-3A\{x\}+5A\{y\}=0\end{aligned}\right\}\ldots\ldots\ldots(9)$$

Using "Eq. (2)" in "Eq. (9)", we have

$$\left.\begin{aligned}(v^2+3)A\{x\}-2A\{y\}=\frac{3}{v}\\(v^2-3)A\{x\}+(v^2+5)A\{y\}=\frac{5}{v}\end{aligned}\right\}\ldots\ldots\ldots\ldots(10)$$

Solving the "Eq. (10)" for $A\{x\}$ and $A\{y\}$, we have

$$\left.\begin{aligned}A\{x\}=\frac{11}{4}\left[\frac{1}{v(v^2+1)}\right]+\frac{1}{4}\left[\frac{1}{v(v^2+9)}\right]\\A\{y\}=\frac{11}{4}\left[\frac{1}{v(v^2+1)}\right]-\frac{3}{4}\left[\frac{1}{v(v^2+9)}\right]\end{aligned}\right\}\ldots\ldots\ldots..(11)$$

Now taking inverse Aboodh transform of "Eq. (11)", we have

$$\left.\begin{aligned}x=\frac{11}{4}sint+\frac{1}{12}sin3t\\y=\frac{11}{4}sint-\frac{1}{4}sin3t\end{aligned}\right\}\ldots\ldots\ldots..(12)$$

which is the required solution of "Eq. (1)" with "Eq. (2)".

10.2 Consider a system of linear ordinary differential equations

$$\left.\begin{aligned}\frac{dx}{dt}+y=2cost\\x+\frac{dy}{dt}=0\end{aligned}\right\}\ldots\ldots\ldots\ldots..(13)$$

with $x(0)=0, y(0)=1 \ldots\ldots\ldots\ldots\ldots$ (14)

Solution using Mohand transforms:

Taking Mohand transform of "Eq. (13)", we have

$$\left.\begin{array}{r} M\left\{\frac{dx}{dt}\right\}+M\{y\}=2M\{cost\} \\ M\{x\}+M\left\{\frac{dy}{dt}\right\}=0 \end{array}\right\} \ldots\ldots\ldots\ldots(15)$$

Now using the property, Mohand transform of the derivatives of the function, in "Eq. (15)", we have

$$\left.\begin{array}{r} vM\{x\}-v^2x(0)+M\{y\}=\frac{2v^3}{(v^2+1)} \\ M\{x\}+vM\{y\}-v^2y(0)=0 \end{array}\right\} \ldots\ldots\ldots\ldots(16)$$

Using "Eq. (14)" in "Eq. (16)", we have

$$\left.\begin{array}{r} vM\{x\}+M\{y\}=\frac{2v^3}{(v^2+1)} \\ M\{x\}+vM\{y\}=v^2 \end{array}\right\} \ldots\ldots\ldots\ldots(17)$$

Solving the "Eq. (17)" for $M\{x\}$and $M\{y\}$, we have

$$\left.\begin{array}{r} M\{x\}=\left[\frac{v^2}{(v^2+1)}\right] \\ M\{y\}=\left[\frac{v^3}{(v^2+1)}\right] \end{array}\right\} \ldots\ldots\ldots\ldots(18)$$

Now taking inverse Mohand transform of "Eq. (18)", we have

$$\left.\begin{array}{r} x=sint \\ y=cost \end{array}\right\} \ldots\ldots\ldots\ldots\ldots(19)$$

which is the required solution of "Eq. (13)" with "Eq. (14)".

Solution using Aboodh transforms:

Taking Aboodh transform of "Eq. (13)", we have

$$\left.\begin{array}{r} A\left\{\frac{dx}{dt}\right\}+A\{y\}=2A\{cost\} \\ A\{x\}+A\left\{\frac{dy}{dt}\right\}=0 \end{array}\right\} \ldots\ldots\ldots\ldots(20)$$

Now using the property, Aboodh transform of the derivatives of the function, in "Eq. (20)", we have

$$\left.\begin{array}{r} vA\{x\}-\frac{x(0)}{v}+A\{y\}=\frac{2}{(v^2+1)} \\ A\{x\}+vA\{y\}-\frac{y(0)}{v}=0 \end{array}\right\} \ldots\ldots\ldots\ldots(21)$$

Using "Eq. (14)" in "Eq. (21)", we have

$$\left.\begin{array}{r} vA\{x\}+A\{y\}=\frac{2}{(v^2+1)} \\ A\{x\}+sA\{y\}=\frac{1}{v} \end{array}\right\} \ldots\ldots\ldots\ldots(22)$$

Solving the "Eq. (22)" for $A\{x\}$and $A\{y\}$, we have

$$\left.\begin{array}{r} A\{x\}=\left[\frac{1}{v(v^2+1)}\right] \\ A\{y\}=\left[\frac{1}{(v^2+1)}\right] \end{array}\right\} \ldots\ldots\ldots\ldots(23)$$

Now taking inverse Aboodh transform of "Eq. (23)", we have

$$\left.\begin{array}{r} x=sint \\ y=cost \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

which is the required solution of "Eq. (13)" with "Eq. (14)".

11. CONCLUSIONS

In this paper, we have successfully discussed the comparative study of Mohand and Aboodh transforms. In application section, we solve systems of differential equations comparatively using both Mohand and Aboodh transforms. The given numerical applications in application section show that both the transforms (Mohand and Aboodh transforms) are closely connected to each other.

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