Analysis of UMTS radio channel access delay

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Abstract

We present an analysis of delay encountered in successfully accessing the Random Access Channel (RACH) of Universal Mobile Telecommunication System (UMTS) Radio Interface by User Equipments (UE) that want to initiate data transfer. The process of random channel access is described and the MS state modeled as a DTMC in order to derive the delay. We evaluate the variation of the channel access delay with the preamble power, preamble detection threshold, maximum attempts, inter-attempt time interval, number of mobile users, number of slots, persistence level, rate of incoming data and rate of retransmissions. We also derive the capture probability of preambles sent in the same slot by multiple UEs in the presence of Rayleigh fading.

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1. Introduction

The radio interface of wireless networks poses a challenge for data transfer. Radio Interface delays affect the overall network performance to a great extent. Salkintzis et al. [11] have reported that in case of handoff, the delays involved in GPRS and UMTS networks form the bottleneck. In [2,13] the handoff delay is estimated using a break-up of all the delays caused during handoff, including both wireless and wireline delays of the networks involved. The wireless or radio interface delay would typically involve the channel access delay, resource allocation delay and data transfer delay. The channel access delay is a variable delay over the radio channel faced by every UE trying to access the network and has a direct impact on the response time of the UE. Therefore, we analyse the UMTS channel access delay.

The Medium Access Control (MAC) layer at the UMTS radio interface is based on slotted ALOHA mechanism. UEs that have data to transfer, send their resource request to the network on a common channel called Random Access Channel (RACH). A number of parameters like the dynamic persistence level, persistence scaling factor, maximum preamble retransmission cycles, maximum preamble ramp-up steps, the backoff range and inter-attempt spreading factor, broadcast as System Information messages, enable centralised, prioritized and fast access.

In the literature [6] shows that RACH access delay and error probability increases with increase in preamble arrival rate and decreases with increase in the number of RACH message part processing units. It shows by simulation that the overlapping Access Service Class (ASC) allocation method has better performance than the non-overlapping method. Ref. [9] shows the impact of preamble target Signal to Interference Ratio (SIR) value and power ramp-up step on the random access delay, success probability and uplink interference by means of simulation. The higher the SIR target, higher is the access success ratio and lesser the access delay. A high power ramp-up step leads to lesser delay but a lower access success ratio as it increases interference. When traffic load increases, the mean SIR of received preambles decreases and gaussian deviation increases due to increased interference level. Ref. [5] uses OpNet simulation for generating traffic on
the RACH and shows that with small packet sizes the average access delay corresponds to the persistence value but with increased load and large packet size, delay is lower for smaller persistence values. Ref. [3] shows different mechanisms to introduce access priority, among which Random Backoff based access priority scheme is best in terms of access delay and throughput. Ref. [14] shows that linear and geometric ramping schemes are more effective than the fixed ramping scheme in increasing the success probability at the expense of high interference to existing traffic.

This paper presents an analysis of the variation of the UEs radio channel access delay with parameters like preamble power, preamble detection threshold, maximum attempts, inter-attempt time interval, number of mobile users, number of slots, persistence level, rate of incoming data and rate of retransmissions. The effect of one user and that of multiple users selecting the same slot on the delay is evaluated. The expression for capture probability of preambles being sent in the same slot by multiple UEs in the presence of Rayleigh fading is derived.

2. Delay modeling

In the access procedure the Physical RACH (PRACH) channel is used for sending a preamble, and the Acquisition Indicator Channel (AICH) carries acquisition indicators responding quickly to PRACH preambles. The PRACH access slots have 5120 chips per slot. Fifteen access slots occupy two radio frames of 20 ms total duration. A RACH preamble is of length 4096 chips, with spreading factor 256 and consists of 256 repetitions of a signature of length 16 chips. The RACH message part is like any other uplink 10 ms or 20 ms transmission consisting of a data part and a control part. With available spreading factors 32–256, the data part can support a data rate of 60 kbps to 7.5 kbps. The control part carries spreading factors 32–256, the data part can support a data rate of 60 kbps to 7.5 kbps. The control part carries a combination of both is considered as a slot here.

Centralised probabilistic access control is performed individually for each PRACH through signalling of dynamic persistence levels that can be translated into class specific persistence probability values, $p_i$ where $i = 0, 1, \ldots, 7$ and $i$ is the Access Service Class (ASC).

$$p_b = 1$$

$$p_i = 2^{(N-1)}, \quad N = 1, \ldots, 8$$

$$p_i = s_i p_1, \quad i = 2, \ldots, 7$$

where $N$ is the dynamic persistence level and $\{s_i = 0.2, \ldots, 0.9\}$ is the persistence scaling factor.

A cell may have up to 16 different PRACHs. PRACH resources are access slots and signatures. The access slots of a PRACH are split into 12 sub-channels. Every 12th access slot belongs to a specific PRACH sub-channel. More than one ASC or all ASCs can be assigned to the same access slot or signature space.

The access procedure can be broken into functionalities performed by the different layers. The Radio Resource Control (RRC) layer reads relevant information from BCCH and configures various parameters related to RACH access in the MAC and PHY layers. It also selects the PRACH, Transmission Time Interval (TTI) and Transport format (TF). The Medium Access Control (MAC) layer controls the timing of RACH transmissions on TTI level and Physical (PHY) layer controls the timing of RACH transmissions on access slot level.

2.1. Channel access delay

The model assumes that all UEs belong to the same Access Service Class and that all UEs’ incoming data rates and retransmission rates are the same.

1. Data arrives at the MAC layer of the UE at rate $\lambda_d$.
2. MAC selects an ASC number $i$.
3. It selects a random variable $r$ between 0 and 1.
4. If $r \leq p_i$, MAC sends the Transport Block (TB) to PHY layer for transmission on the PRACH.
5. If $r > p_i$, MAC waits for a TTI before starting the persistency check again.
6. PHY layer selects a RACH sub-channel and a signature. A combination of both is considered as a slot here.
7. It determines the transmission power [16] and transmits the preamble on the PRACH.
8. PHY layer checks the acknowledgement on the corresponding DL AICH signal.
9. If positive acknowledgement, it informs MAC, gets the message part and transmits it on the PRACH after 3–4 slots depending on the AICH transmission timing parameter after setting the power correctly [16].
10. If negative acknowledgement, it informs MAC. MAC starts a backoff timer and retries again, if its maximum retries are not exceeded.
11. If no acknowledgement, it selects another slot, another signature, steps up the Preamble Transmission Power $T_P^{\text{pr}}$ by a power ramp-up step $\Delta_p$ and retransmits the preamble. If power limit or the number of retries is exceeded it stops and informs MAC.
12. MAC retries in the next TTI till the maximum number of attempts are over. After that it stops the access and informs the higher layer.

The UE MAC layer functions during random access are modeled as a finite Discrete Time Markov Chain (DTMC) with two absorbing states, as shown in Fig. 1. $D$ is the start state where UE has data to transmit. The UE starts channel access procedure with probability $p_b$ and enters the first PHY layer action state $PH_0$. With probability $1 - p_b$, it defers access and remains in state $D$. If the PHY layer returns success it forwards the message to PHY layer for transmission and goes to the absorbing state $S_a$ denoting...
message transmission success with probability $P_m^1$. In case message transmission is unsuccessful it goes to the absorbing state $F_m$ denoting message transmission failure with probability $P_{mf}$. If the PHY layer indicates receipt of a negative acknowledgement, it goes to the transient state $B_1$ with probability $P_g$. Backoff state $B_1$ represents a random wait time distributed uniformly between $N_{b0\text{max}}$ and $N_{b0\text{min}}$. If the PHY layer indicates that maximum retries are over or maximum power has been used, UE goes to the transient state $M_1$ with probability $P_1$. From backoff state $B_1$ it goes to state $M_1$ with probability 1 and waits for next TTI before making an attempt. $M_1$ is the retry count state of MAC layer. MAC waits in this state till the next TTI. From the state $M_1$, UE moves to next PHY layer action state $PHY_1$ with probability $p_b$ to once again start preamble transmission. It remains in $M_1$ with probability $1-p_b$. This procedure is repeated for $M_{\text{max}}$, the maximum number of attempts at MAC layer (here 4). States $D$, $B$, $M$ represent backlogged states and states $F_m$, $S_m$ represent failure and success states, respectively. The $M_j$ and $PHY_j$ states, here $j = 1, \ldots, 4$ represent retransmission states of MAC and PHY layers, respectively. $P_x^j$ represents the corresponding transition probability, $x \in \{pb,pf,ms,mf\}$, in the $j$th trial.

The state of the UEs PHY layer during random access procedure is also modeled as a finite Discrete Time Markov Chain (DTMC) with four absorbing states, as given in Fig. 2. Control from $PHY_j$ states of MAC goes to state $R_0$ of PHY layer. $R_0$ is the state where the first preamble transmission is done by the UE. The $R_i$ states, $\{i = 1, \ldots, 5\}$, denote the PHY layer retransmission state. The UE goes to these states with probability $P_{r_i}$ if there was no acknowledgement to the preamble transmission of the previous state. $P_{r_i}$, $\{i = 1, \ldots, 5\}$ denotes that the probability of transition at each step may be different depending on the network parameters at the time of access. The power level increases at each retry state in the PHY layer, increasing the probability of successful access. If the outcome of the preamble transmission is unsuccessful, the UE goes to the absorbing state $Ack$ with probability $P_{a}$ from the $R_i$ states. In the Ack state the UE transmits the message and waits for its acknowledgement. In case of positive acknowledgement, it goes to state $S_m$ with probability $P_{ma}$. In case of negative acknowledgement, it goes to state $W$ with probability $P_{mf}$. $W$ is a wait state corresponding to the additional wait encountered till the timeout for acknowledgement and from there it goes to the failure state $F_m$. If the BSS sends a negative acknowledgement to the UE request, it enters the absorbing state $Nack$ with probability $P_{bf}$ from the $R_i$ states. In case the UEs maximum power is reached in the $R_i$ states, it goes to the absorbing state $F_p$ with probability $P_{pf}$. When the number of retries $PRM$ at PHY layer are over (five, here) and still no acknowledgement is received, the UE enters the absorbing state $F_p$ with probability $P_{pf}$.

Consider $N_a$ UEs, out of which $N_a$ have active sessions, $N_i$ are idle and $N_b$ are backlogged. Let $t$ be the number of UEs that select the same preamble, $n$ the number of UEs that transmit either a different preamble or any message constituting the noise for the $t$ UEs, $\lambda_d$ the Poisson distributed data arrival rate at the idle UE in packets/ms, $X = 0, 1, 2, 3, \ldots$ is the number of packets, $I(n)$ the probability that $n$ users transmit in an access slot when there are $N_b$ backlogged users, $N_i$ the number of slots available for access, where slot refers to a signature and RACH sub-channel, $P_{\text{slot}}(t,n)$ the probability that $t$ users out of $n$ select the same slot, $\lambda_s$ the Poisson distributed maximum data arrival rate at the backlogged UE in packets/ms [8], $N_{\text{ac}}$ the average number of access slots between two preamble transmissions, $T_{\text{ac}}$ the duration of one access slot.
in ms and $p_i$ is the persistence probability value given by Eq. (3).

$$\lambda_p = \frac{1}{T_{acc} \cdot (N_{acc}) + 1} \text{packets/ms} \quad (4)$$

$$I(t) = \frac{[p_i T_{acc} (N_{acc} \lambda_d + N_{b} \lambda_e)]^t}{t!} e^{-p_i T_{acc} (N_{acc} \lambda_d + N_{b} \lambda_e)} \quad (5)$$

$$P_{slo}(t, n) = \left( \frac{n}{t} \right) \left( \frac{1}{N_s} \right)^{n-t} \quad (6)$$

If more than one UE selects the same slot (same signature, same RACH sub-channel), the total received preamble power at the Node B is the sum of Rayleigh faded signal powers of all transmissions using the same preamble received within a small time interval. Let $u_o$ be the initial preamble power received from one source, $u_0[t]$ the initial preamble power received from $t$ sources, $u_{max}$ the maximum UE power, $z_o$ the preamble power threshold at the Node B and $z_{max}$ the maximum preamble power threshold. Assuming that $N_s$ is constant in one access slot duration and all preamble transmissions are independent and identically distributed and undergo Rayleigh fading, the distribution of the capture probability of preambles has been derived in the Appendix [1]. Some of the results are presented here. The probability $P_{p-access}(1)$ that a preamble is received successfully when only one UE selects a preamble and $P_{p-access}(t)$ the probability of preamble reception when $t$ UEs select the same preamble is given by

$$P_{p-access}(1) = 1 - \sum_{n=1}^{\infty} I(n)P_{slo}(1, n) \frac{(n + N_s)\sqrt{z_o}}{(n + N_s)\sqrt{z_o} + 1} \quad (7)$$

$$P_{p-access}(t) = 1 - \sum_{n=1}^{\infty} I(n) \sum_{i=1}^{n} P_{slo}(1, n) \times \frac{(n - t + N_s)\sqrt{z_o}}{(n - t + N_s)\sqrt{z_o} + t} \quad (8)$$

At every retry of the PHY layer, the signal power $u_o$ is increased by the power ramp-up step $\Delta_{u-p}$. If $u_o$ is also considered then the above probabilities may be obtained as shown below [1].

$$P_{p-access}(1) = 1 - \sum_{n=1}^{\infty} I(n)P_{slo}(1, n) \frac{2(n + N_s)\pi}{(n + 4u_o)^{3/2}} \times \sqrt{\frac{z_o}{(n + N_s)^2 \pi z_o + 4u_o}} \quad (9)$$

$$P_{p-access}(t) = 1 - \sum_{n=1}^{\infty} I(n) \sum_{i=1}^{n} P_{slo}(1, n) \times \frac{2(n - t + N_s)\pi}{(t \pi + 4u_o)^{3/2}} \times \sqrt{\frac{z_o}{(n - t + N_s)^2 \pi z_o + 4u_o}} \quad (10)$$

Let $M$ be the total number of message part processing resources at Node B, $T_{msg}$ the time taken for message transmission, $acc$ the number of previous access slots in which messages are received and $T_{p-m}$ the time between a preamble acknowledgement and actual message transmission. Assuming that previous message transmissions are of 10 ms duration each and that a newly acknowledged preamble request results in a message transmission after 3 access slots, we find the number of messages being received by the Node B when it gets a preamble request. These message receptions will lead to message part processing resources’ occupancy [6]. The probability $P_{res}$ of resource availability at Node B is then the probability that the number of resources $M$ is greater than the number of messages received by the Node B in the previous $acc$ access slots.

$$T_{msg} = T_{acc}(acc + 1) + T_{p-m} \quad (11)$$

$$P_{res} = M \sum_{j=0}^{\infty} I(y)|_{y} = 1 \quad (12)$$

Let $P_{acc}$ be the probability of correct preamble acknowledgement detection on AICH given that the acknowledgement was sent, $P_{fa}$ the probability of false alarm of preamble acknowledgement detection on AICH given that the acknowledgement was not sent [15], $P_t$ the probability of getting a positive acknowledgement for a preamble transmission, $P_b$ the probability of getting a negative acknowledgement, $P_n$ the probability of getting no acknowledgement, $P_f$ the probability that the UE power limit is exceeded, $u_{max}$ the maximum UE output power and $T_{p}^{WR}$ the instantaneous preamble signal power.

$$P_t = P_{p-access}(t)P_{res}P_{det} + P_{p-access}(t)(1 - P_{res}) \times (1 - P_{det})/2 + (1 - P_{p-access}(t))P_{fa}/2 \quad (13)$$

$$P_b = P_{p-access}(t)(1 - P_{res})P_{det} + P_{p-access}(t)P_{res} \times (1 - P_{det})/2 + (1 - P_{p-access}(t))P_{fa}/2 \quad (14)$$

$$P_n = (1 - P_{p-access}(t))(1 - P_{fa}) + P_{p-access}(t) \times (1 - P_{det})/2 \quad (15)$$

$$P_f = \begin{cases} 
0 & \text{when } T_{p}^{WR} < u_{max} \\
1 & \text{when } T_{p}^{WR} \geq u_{max} 
\end{cases} \quad (16)$$

Eventually, when the message is sent (on getting a positive acknowledgement) then only one out of $t$ UEs will succeed in transmitting the message and $t-1$ UEs’ messages will fail as governed by the capture probability for messages, $P_{cm}$ [7,10]. The message transmission power $T_{p}^{WR}$ is given below in terms of the preamble-to-message power step-up $\Delta_{p-m}$, $T_{p}^{WR}$ the transmission power of a successfully received preamble, $p_{d}$ the message data part gain factor and $p_{c}$ the message control part gain factor. The detection threshold for messages is higher and is given by $E_o/N_o$. Therefore, $P_{ms}$, the probability of a single message reception success when $n$ UEs transmit simultaneously and the
detection power threshold for messages is $M_o$ and $P_{af}$, the probability of message failure are given by

$$
P^{\text{PHY}}_{\text{msg}} = P^{\text{PHY}}_\theta + \Delta_{p-m} + 10 \log \left(1 + \frac{\beta_2}{\beta_1}\right)^2$$

$$P_{cm} = 1 - \frac{1}{\sum_{n=1}^{\infty} \lambda(n) \frac{\alpha}{\beta^2} + 1}$$

$$P_{ms} = (1 - P_{\text{bler}})P_{cm}$$

$$P_{nt} = 1 - P_{ms}$$

The state transitions of the PHY layer as shown in Fig. 2 can be captured in the state transition probability matrix $P_{\text{PHY}}$ [12],

$$P_{\text{PHY}} = \begin{bmatrix}
0 & P^1_n & 0 & 0 & 0 & 0 & P^1_i & P^1_h & P^1_p & 0 & 0 \\
0 & 0 & P^2_n & 0 & 0 & 0 & P^2_i & P^2_h & P^2_p & 0 & 0 \\
0 & 0 & 0 & P^3_n & 0 & 0 & 0 & P^3_i & P^3_h & P^3_p & 0 \\
0 & 0 & 0 & 0 & P^4_n & 0 & 0 & 0 & P^4_i & P^4_h & P^4_p \\
0 & 0 & 0 & 0 & 0 & P^5_n & 0 & 0 & 0 & P^5_i & P^5_h & P^5_p \\
0 & 0 & 0 & 0 & 0 & 0 & P^6_n & 0 & 0 & 0 & P^6_i & P^6_h \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{ms} & P_{nt} & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}$$

where $j$ is the trial number of the MAC layer, $Q_{\text{PHY}}$ is a $7 \times 7$ substochastic matrix capturing the transient state transitions only, $C_{\text{PHY}}$ is a $7 \times 4$ column vector, $O_{\text{PHY}}$ is a $4 \times 7$ vector of zeros and $I_{\text{PHY}}$ is a $4 \times 4$ identity matrix. $J_{\text{PHY}}$ is the solution matrix whose first row $J_{\text{PHY}}(1,:)$ gives the steady state probabilities of MS for being in a particular state. $A_{\text{PHY}}(1,1)$ gives the probability of going to \textit{Nack} state, $A_{\text{PHY}}(1,2)$ gives the probability of going to $F_j$ state, $A_{\text{PHY}}(1,3)$ gives the probability of going to $S_m$ state and $A_{\text{PHY}}(1,4)$ gives the probability of going to $F_m$ state.

Using the probabilities obtained in Eqs. (26)–(29) and denoting $p^*_i = 1 - p_i$, the state transition matrix, $P_{\text{MAC}}$ for four trials by the MAC layer is obtained from Fig. 1 and can be written as

$$P_{\text{MAC}} = \begin{bmatrix}
P^1_{nt} & P^1_{ms} & P^1_{af} & P^1_{mf} \\
P^2_{nt} & P^2_{ms} & P^2_{af} & P^2_{mf} \\
P^3_{nt} & P^3_{ms} & P^3_{af} & P^3_{mf} \\
P^4_{nt} & P^4_{ms} & P^4_{af} & P^4_{mf} \\
P^5_{nt} & P^5_{ms} & P^5_{af} & P^5_{mf} \\
P^6_{nt} & P^6_{ms} & P^6_{af} & P^6_{mf} \\
P_{nt} & P_{ms} & P_{af} & P_{mf} \\
\end{bmatrix}$$

and

$$P_{\text{MAC}} = \begin{bmatrix}
Q_{\text{MAC}} & C_{\text{MAC}} \\
O_{\text{MAC}} & I_{\text{MAC}} \\
\end{bmatrix}$$

$$J_{\text{MAC}} = (I - Q_{\text{MAC}})^{-1}$$

$$A_{\text{MAC}} = J_{\text{MAC}} C_{\text{MAC}}$$

where $Q_{\text{MAC}}$ is a $13 \times 13$ substochastic matrix capturing the transient state transitions only, $C_{\text{MAC}}$ is a $13 \times 2$ column vector, $O_{\text{MAC}}$ is a $2 \times 13$ vector of zeros and $I_{\text{MAC}}$ is a $2 \times 2$ identity matrix. $J_{\text{MAC}}$ is the solution matrix whose first row $J_{\text{MAC}}(1,:)$ gives the steady state probabilities of MS for being in a particular state. $A_{\text{MAC}}(1,1)$ gives the probability of going to $S_m$ state, $A_{\text{MAC}}(1,2)$ gives the probability of going to $F_m$ state.

Given the average time spent in each state, and the probability of being in each state the total channel access delay in the PHY and MAC layer can be obtained. The time spent in different states of PHY layer state transition may be categorised as

- $T_{\text{start}}$, time to start access in next access slot set
- $T_{\text{acc}}$, time spent in access
- $T_{\text{ack}}$, wait time for preamble acknowledgement
- $T_{\text{msg}}$, time for transmitting message
- $T_{\text{ack}}$, time for receiving message acknowledgement
- $T_{\text{int}}$, total timeout for message acknowledgement

The total time in one PHY layer execution $T_{\text{gap}}$, the time spent in the PHY layer in case of message success, $T_{\text{phy}}(s)$ and the time spent in the PHY layer in case of message failure, $T_{\text{phy}}(f)$ are given by
select the same slot as compared to one user using one slot due to Rayleigh fading effects for the same number of active users $N_u$.

As $t$ increases, the $(n\sqrt{z_0})/(n\sqrt{z_0} + t)$ decreases and $P_{p\rightarrow acc}(t)$ decreases. Thus, the increase in the second term of Eq. (8) although present is low and leads to a lower $P_{p\rightarrow acc}(t)$. As $N_u$ decreases, the probability of successful access decreases as attempts by idle and backlogged users increase. At very low $z_0$ it is almost the same for 1 user or $t$ users. In this case, the value of $(n\sqrt{z_0})/(n\sqrt{z_0} + 1)$ is higher for 1 user than for $t$ users. However, the $I(n)$ value for 1 user is lesser for higher $n$ as compared to $t$ users. Thus, the values for low $z_0$ are same. For further increase in $z_0$, the increase in $(n\sqrt{z_0})/(n\sqrt{z_0} + 1)$ is much higher than that for $t$ users and thus the $P_{p\rightarrow acc}(t)$ value is lesser.

The probability of successful access for one user and $t$ users selecting one slot considering $u$, the preamble transmission power is given by Eqs. (9) and (10), respectively. Its variation is shown in Fig. 4 for $z_0 = 0.114$ W or $-9.4$ dB.

As $u_0$, the preamble transmission power increases, the probability of successful access increases. The main contributor is the term $\frac{2(u_0 + N_u)t}{(x + 4u_0)^2}$ and it decreases with increase in $u_0$, leading to an increase in $P_{p\rightarrow acc}(1)$. $P_{p\rightarrow acc}(t)$ is lesser when $t$ users select the same slot as compared to one user using one slot due to Rayleigh fading effects. The $P_{p\rightarrow acc}(t)$ term in Eq. (10) decreases rapidly with increase in $t$ with the effect that only the first few $t$ values contribute to a slight increase in the value of $\frac{2(u_0 + N_u)t}{(x + 4u_0)^2}$ leading to a lower $P_{p\rightarrow acc}(t)$. With increase in $N_u$, the value of $I(n)$ decreases as the number of UEs generating messages falls, leading to an increase in $P_{p\rightarrow acc}(t)$.

The probability of resource availability, $P_{res}$ is given by Eq. (12) and shown in Fig. 5. $P_{res}$ increases initially as $M$, the number of resources increases but becomes constant for high $M$ as it depicts a probability distribution function of resource availability. It is higher initially for lower incoming data rate $\lambda_d = 0.01$.

The probability that a preamble is received successfully, $P_s$ is given by Eq. (13) and depicted in Fig. 6. The effect of $P_{res}$ is higher on $P_s$ as compared to $P_{p\rightarrow acc}(t)$. $P_s$ increases.

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**T phy** detection threshold for 1 user and

**Fig. 3.** Variation of the probability of successful access with preamble detection threshold for 1 user and $t$ users.

**Fig. 4.** Variation of the probability of successful access with preamble transmission power for 1 user and $t$ users.
as $P_{res}$ increases and as $P_{p-access}(t)$ increases. The probability that a preamble negative acknowledgement is received, $P_b$, is given by Eq. (14) and depicted in Fig. 7. The effect of $P_{res}$ is higher on $P_b$ as compared to $P_{p-access}(t)$. $P_b$ decreases as $P_{res}$ increases. The probability that no acknowledgement is received for a preamble, $P_n$, is given by Eq. (15) and depicted in Fig. 8. The effect of $P_{res}$ is lower on $P_n$ as compared to $P_{p-access}(t)$. $P_n$ decreases as $P_{res}$ increases.

The delay at PHY layer with $z_o$, the preamble detection threshold is shown in Fig. 10. As $z_o$ increases, delay increases. The delay is more when more than one user selects the same slot, as the $P_{p-access}(t)$ decreases.

The delay at PHY layer with $u_o$, the preamble transmission power is shown in Fig. 11. The delay decreases with increase in $u_o$ as $P_{p-access}(t)$ increases. The delay is more when more than one user selects the same slot, again because $P_{p-access}(t)$ decreases.

As the number of UEs $N_m$ increases the total access delay increases as shown in Fig. 12. However, for less
number of message part processing resources \(M = 10\), the backoff probability \(P_b\) is higher. Thus, delay is more as backoff duration is added. The delay is higher as \(N_m\) increases for lesser number of resources. If the number of slots is increased to 64, the delay reduces slightly as the value of \(P_{p-\text{access}}(t)\) increases and leads to \(P_t\) increase. For more number of resources, the probability of success, \(P_t\) is higher and message transmission succeeds when \(N_m\) is low. As \(N_m\) increases, \(P_t\) increases and the delay again increases. For very large \(N_m\), the delay becomes the same irrespective of the number of resources. The increase in number of resources is more effective in reducing delay as compared to increase in the number of slots. The variation of delay with persistence probability \(p_i\) is captured in Fig. 13. A decrease in \(p_i\) reduces the number of accesses and increases the \(P_{\text{res}}\) or resource availability probability and also the \(P_{p-\text{access}}(t)\) or access success probability. Thus, delay reduces as \(p_i\) reduces. For the case depicted here, as the \(p_i\) reduces, the delay due to backoff is slowly converted to delay in successful outcome. Thus, delay decreases gradually as the \(p_t\) decreases. However, a reduction in \(p_i\) also increases the time that is spent in the wait states \(M_i\), leading to an increase in delay as \(p_t\) reduces beyond 0.4.

The variation of total delay with incoming data rate \(\lambda_d\) is shown in Fig. 14. For large number of idle users \(N_i\) and low data rate \(\lambda_d\), the probability of success \(P_t\) is high as both \(P_{p-\text{access}}(t)\) and \(P_{\text{res}}\) are high. Thus, outcome is successful and delay is low. As \(\lambda_d\) increases, both \(P_{p-\text{access}}(t)\) and \(P_{\text{res}}\) fall, leading to an increase in \(P_b\) and delay due to backoff. Further increase in \(\lambda_d\), increases \(P_{n}\) or no acknowledgement probability causing more attempts at PHY layer. Thus, the delay increases but not as much as it would in a backoff case. For less number of idle users \(N_i\) or more number of backlogged users \(N_b\), even at low data rates \(P_b\) is high and therefore delay is high as the chances of success of backlogged users are higher. As \(\lambda_d\) increases, both
$P_{\text{access}}(t)$ and $P_{\text{res}}$ fall much more rapidly, leading to an increase in delay due to more increase in $P_b$ and less increase in $P_n$. As $\lambda_d$ increases further, $P_n$ increase is greater and delay still rises.

The variation of total delay with retransmission rate $\lambda_r$ is shown in Fig. 15 for $\lambda_d = 0.01$. For retransmission rates below 0.03, the probability of successful access $P_t$ is high and delay is low irrespective of the number of users. For lesser number of backlogged users as the retransmission rate increases, the fall in $P_{\text{access}}(t)$ and $P_{\text{res}}$ is low, the outcome is success and though the delay increases, it does so gradually. However, for large number of backlogged users, the fall in $P_{\text{access}}(t)$ and $P_{\text{res}}$ is high and the delay increases steeply due to backoff delay and increase in retransmission attempts.

Fig. 16 shows that as the backoff interval increases the overall delay increases.

Fig. 17 shows for a successful outcome, the increase in access delay with an increase in the number of retransmission attempts at the PHY layer.

Fig. 18(a) shows for a failed outcome that as the number of retransmission attempts at PHY layer increase, the PHY layer delay becomes same irrespective of the $N_m$ value. The reason is that the large retransmission delay predominates the other delays. Fig. 18(b) shows for a failed outcome that as the number of retransmission attempts at PHY layer increase, the total delay increases gradually for $N_m = 25$, as the delay is the sum of attempt delay and backoff delay.

However, for $N_m = 100$, the delay decreases as the backoff delay component gradually decreases due to $P_b$ decrease and $P_t$ or success probability increase.

For a successful outcome, where retries took place due to no acknowledgement, Fig. 19(a) shows that the lower $p_t$, the higher the delay. The increase in delay with every MAC retry is higher for a low $p_t$. Fig. 19(b) shows the delay variation with different reasons of MAC retry. In case a negative acknowledgement is received, the delay is more due to backoff as shown in the ‘Backoff’ option. In case the retries at PHY layer are exceeded, the delay is lesser in comparison, shown by the ‘Wait’ option.

The value of minimum delay is 35.5 ms and it occurs due to instant success at PHY and MAC layers. The maximum
As number of UEs increases, the delay increases. For very large number of UEs, the delay is high and the same irrespective of the number of resources. The increase in number of resources is more effective in reducing delay as compared to increase in the number of slots.

For very low persistence probability value, delay is high and reduces as persistence probability increases. On further persistence probability increase delay once again increases.

For large number of idle users and low incoming data rate, the outcome of message transmission is successful and the delay is low. As incoming data rate increases, delay increases due to backoff. A further increase in incoming data rate increases delay as it causes more attempts at PHY layer. For less number of idle users delay is high even at low data rates and the outcome unsuccessful.

For large number of backlogged users and low retransmission rate, the outcome of message transmission is successful and the delay is low. As the retransmission rate increases, delay increases due to backoff. For less backlogged users the outcome of message transmission is successful and the delay is low even as retransmission rate increases.

The backoff interval increase and the increase in the number of retransmission attempts at the PHY layer increases the access delay significantly. With less number of UEs, PHY layer delay is less. As the number of retries at PHY layer increase, it increases for both more and less number of UEs and ultimately becomes equal for both. The total delay increases with number of retries at PHY layer for less number of UEs. However for more UEs, it decreases as the number of retries increases, although it is much higher compared to less UEs case.

The increase in delay with every MAC retry is higher for a low persistence probability. The delay varies with the cause of MAC retry. In case a negative acknowledgement is received, the total delay is more due to backoff. In case the retries at PHY layer are exceeded, the total delay is lesser in comparison.

The value of minimum delay is 35.5 ms and that of maximum delay obtained is 2121.8 ms.

4. Conclusion

As the preamble detection threshold increases, the probability of successful access decreases. It is lesser when more than one user selects the same slot as compared to one user using one slot due to Rayleigh fading effects for the same number of active users. As the number of active users decreases, the probability of successful access decreases. At very low values of preamble detection threshold, it is almost the same for 1 user or more than one user. As preamble detection threshold increases, delay increases.

As the preamble transmission power increases, the probability of successful access increases. It is lesser when more than one user selects the same slot as compared to one user using one slot due to Rayleigh fading effects. The delay decreases with increase in preamble transmission power.

As the number of UEs increases, the total access delay increases. The delay is higher with increase in number of UEs for less number of resources. If the number of slots is increased, the delay reduces slightly, the reduction is more for greater number of UEs. For more number of resources, delay is less when number of UEs is less.

Appendix

We derive the capture probability in the presence of interfering signals and Rayleigh fading of all signals based on the work presented in [1]. Let \( P_s \) be the signal power of a packet, \( P_n \) the noise power of \( n \) interfering signals, \( f_{P_n}(p_n) \) the p.d.f of the mean received packet power, \( f_{Z_n}(p_n) \) the p.d.f of the mean received interference power of \( n \) packets, \( f_{P_n}(p_n) \) the p.d.f of the packet power, \( f_{P_s}(p_s) \) the p.d.f of the interference power of \( n \) packets and \( Z_n \) the preamble detection threshold power. If \( P_s = U_s \) and \( P_n = U_s/Z_n \) then the threshold at Node B can be exceeded. The probability that the packet power exceeds the threshold when transmitted with a particular power \( u_s \), in the presence of Rayleigh fading can be derived as follows [1]:

\[
\text{probability} = f_{P_n}(p_n) \cdot f_{Z_n}(p_n) \cdot f_{P_s}(p_s)
\]
For the case where \( t \) UEs transmit the same preamble, we continue from Eq. (53) to derive the probability that \( P_s \leq u_o \) as shown below

\[
F_{U_s}(u[t]) = \int_0^\infty \frac{\exp(-u/p_s)}{2} \frac{t}{2} p_s^{-3/2} \exp\left(-\frac{4u + t^2 \pi}{4p_s}\right) \, dp_s
\]

\[
= t \sqrt{\pi} \left[ \frac{1}{\sqrt{t^2\pi + 4u_o}} - \frac{1}{\sqrt{t^2\pi + 4u_o}} \right]
\]

\[
= 1 - t \sqrt{\frac{\pi}{t^2\pi + 4u_o}}
\]

The pdf of \( z \) is given by [1]

\[
f_{z_s}(z) = \int_0^\infty f_{s_s}(u) f_{r_s}\left(\frac{u}{z}\right) \frac{u}{z^2} \, du
\]

\[
= \int_0^\infty \frac{\exp(-u/p_s)}{p_s} \frac{1}{2} p_s^{-3/2} \exp\left(-\frac{n^2\pi}{4p_s}\right) \, dp_s
\]

\[
= \int_0^\infty \frac{1}{2} p_s^{-3/2} \exp\left(-\frac{4u + \pi}{4p_s}\right) \, dp_s
\]

\[
= \frac{2\sqrt{\pi}}{\pi + 4u_o)^{3/2}}
\]

The probability that \( P_s \leq u_o \) is given by

\[
F_{U_s}(u_o) = \int_0^{u_o} du \int_0^\infty f_{s_s}(u) f_{r_s}\left(\frac{u}{z}\right) \frac{u}{z^2} \, dz
\]

\[
= \int_0^{u_o} \frac{2\sqrt{\pi}}{(\pi + 4u_o)^{3/2}} \, du
\]
\[ = \int_{-\infty}^{\infty} f(u,t)dt = P(P_s/P_n \leq z \mid U_n = u) f_{U_n}(u) \] (78)

Using Eqs. (70) and (76) the probability \( P_{p-\text{access}}(1) \) that a preamble is received successfully when 1 UE selects a preamble and \( P_{p-\text{access}}(t) \) the probability of preamble reception when \( t \) UEs select the same preamble is given by

\[ P_{p-\text{access}}(1) = 1 - \sum_{n=1}^{\infty} I(n)P_{\text{slot}}(1,n)P(P_s/P_n \leq z_o) \]

\[ P_{p-\text{access}}(t) = 1 - \sum_{n=1}^{\infty} I(n)P_{\text{slot}}(t,n)P(P_s/P_n \leq z_o) \]

If \( u_o \) is also considered then the above probabilities may be obtained using Eqs. (85) and (88) as shown below

\[ P_{p-\text{access}}(1) = 1 - \sum_{n=1}^{\infty} I(n)P_{\text{slot}}(1,n) \times P(P_s/P_n \leq z_o \mid U_n = u_o)f_{U_n}(u_o) \]

\[ P_{p-\text{access}}(t) = 1 - \sum_{n=1}^{\infty} I(n) \sum_{t=1}^{n} P_{\text{slot}}(t,n) \times P(P_s/P_n \leq z_o \mid U_n(t) = u_o) \]

\[ P_{p-\text{access}}(1) = 1 - \sum_{n=1}^{\infty} I(n) \sum_{t=1}^{n} P_{\text{slot}}(1,n) \times \left( \frac{2\pi}{(t^2 + 4u_o)^{3/2}} \right) \times \left( \frac{z_o}{n^2\pi z_o + 4u_o} \right) \]

\[ P_{p-\text{access}}(t) = 1 - \sum_{n=1}^{\infty} I(n) \sum_{t=1}^{n} P_{\text{slot}}(t,n) \times \left( \frac{2(n-t)t\pi}{(t^2 + 4u_o)^{3/2}} \right) \times \left( \frac{z_o}{n^2\pi z_o + 4u_o} \right) \]

References


