Low Complexity Phase-Unaware Detectors Based on Estimator-Correlator Concept

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1. Introduction

Our goal is to present an overview of a class of low complexity detectors working in linear fading multipath channels. In addition, we present briefly a unified theory based on the optimal maximum a posteriori probability (MAP) receiver concept (Woodward & Davies, 1952), which in additive Gaussian noise leads to the estimator-correlator receiver (Price, 1956; Middleton, 1957; Kailath, 1960; Kailath, 1969). The terms receiver and detector are interchangeable. Detectors are estimators where the parameter or symbol set to be estimated is discrete (Kay, 1993; Kay, 1998).

We consider phase-unaware detectors (PUDs) such as differentially coherent detector (DD), noncoherent detector (ND), and energy detector (ED). The term PUD is used to emphasize that the receiver does not have any knowledge of the absolute phase of the received signal although it may have some knowledge of the internal phase structure. We use the term noncoherent to represent a special case of PUD system, and this will be clarified later. PUD detectors are more robust than coherent detectors in a fading multipath channel since the carrier phase of a signal with a wide bandwidth or high carrier frequency may be difficult to estimate with a low complexity. Earlier extensive reviews include (Schwarz et al., 1966; Van Trees, 1971) and more recently (Garth & Poor, 1994; McDonough & Whalen, 1995; Proakis, 2001; Mämmelä et al., 2002; Simon & Alouini, 2005; Witrisal et al., 2009). A summary of the estimator-correlator receiver is presented in (Kay, 1998).

Unless stated otherwise, we exclude equalizers which increase the complexity of the receiver significantly (Lodge & Moher, 1990; Colavolpe & Raheli, 1999). Thus we avoid intersymbol interference (ISI) by signal design and concentrate on the reception of a single symbol, which may include several bits in M-ary communications. It is, however, conceptually straightforward to generalize the single symbol or “one-shot” detectors to symbol sequence detection by replacing the symbols by symbol sequences. The noise is assumed to be additive white Gaussian noise (AWGN). The frequency offset caused by the channel is assumed to be known and compensated. We also assume that the receiver is synchronous in the sense that the start of each symbol interval is known. Estimation of frequency and timing is a highly nonlinear problem, which is studied in (Mengali & D’Andrea, 1997; Meyr et al., 1998), see also (Turin, 1980). Also because of complexity reasons in general we exclude coherent detectors which are such that they assume that the alternative received
symbol waveforms are known including the absolute phase. Obviously, there are also other interesting physical and higher layer aspects we are not able to include due to space limitation.

In our review we emphasize that PUD systems can be derived from the optimal estimator-correlator receiver with suitable simplifying assumptions. In addition, our purpose is to emphasize recent ultra-wideband (UWB) M-ary communications and multiple-input multiple-output (MIMO) diversity systems which enable increase of date rates. One interesting modulation method to consider is the pulse-amplitude modulation (PAM), which has been recently selected for short-range wireless standards such as ECMA-387 and IEEE802.15.3c in which the carrier phase recovery can be a major problem. We also present a historical review of PUDs and summarize the problems in the performance analysis of such systems.

2. Conceptual analysis

General theoretical background is given for example in (Papoulis, 2002; Ziemer & Tranter, 2002; Kay, 1993; Kay, 1998; Proakis, 2001). To make our presentation as compact as possible, we use the complex envelope concept to define the signals as explained in (Franks, 1969). Furthermore, we use some matrix equations, which are explained in (Marple, 1987).

2.1 Coherency

Signal coherency is an important concept that leads to several orthogonality concepts, each of which refers to a certain idealized detector structure. The channel is assumed to be a wide-sense stationary uncorrelated scattering (WSSUS) channel with a time-variant impulse response $c(\tau, t)$ and time-variant transfer function $C(f, t) = a(f, t)e^{j\theta(f, t)} = \int_{-\infty}^{\infty} c(\tau, t)e^{-j2\pi ft}\; d\tau$ (Bello, 1963; Proakis, 2001). If the transmitted signal is $s(t)$, the received signal without noise is $h(t) = \int_{-\infty}^{\infty} c(\tau, t)s(t - \tau)d\tau$.

If we transmit an unmodulated carrier or complex exponential $s(t) = e^{j2\pi f_1 t}$ with a unit amplitude and frequency $f_1$ through the channel, we receive a fading carrier $v(t) = C(f_1, t)e^{j2\pi f_1 t}$ whose amplitude $a(f_1, t)$ and phase $\theta(f_1, t)$ are time-variant. We compare the received signal at two time instants $t_1$ and $t_2$ where $\Delta t = t_1 - t_2$. In general, the magnitude of the correlation $E[h(t_1)h^*(t_2)]$ between $h(t_1)$ and $h(t_2)$ is reduced when $|\Delta t|$ is increased. In a WSSUS channel, the normalized correlation $E[h(t_1)h^*(t_2)]/E[h(t_1)^2] = |E[C(f_1, t_1)C^*(f_1, t_2)]/E[C(f_1, t_1)^2]|$ does not depend on $f_1$ or $t_1$, only on $\Delta t$. The minimum positive interval $\Delta t$ where the normalized correlation is $E[C(t_1)C^*(t_2)]/E[C(t_1)^2] = \epsilon$, where $\epsilon$ is a real constant ($0 \leq \epsilon < 1$), is defined to be the coherence time $(\Delta t)_c$. If $|\Delta t| \ll (\Delta t)_c$, the complex samples are correlated in such a way that in general $h(t_1) \approx h(t_2)$. We say that the two samples at $t_1$ and $t_2$ are coherent with each other, and the fading channel is coherent over the time interval $|\Delta t| \ll (\Delta t)_c$.

If the transmitted signal is modulated and the symbol interval $T$ is small so that $T \ll (\Delta t)_c$, the channel is slowly fading and the channel is essentially constant within the symbol interval, otherwise the channel is fast fading. In practice symbol waveforms are often band-limited, for example Nyquist pulses (Proakis, 2001), and their duration may be several symbol intervals. In a slowly fading channel the channel is assumed to be approximately constant during the whole length of the symbol waveform.

In a similar way, if we transmit either $s_1(t) = e^{j2\pi f_1 t}$ or $s_2(t) = e^{j2\pi f_2 t}$, the normalized correlation at time $t_1$ is $|E[h_1(t_1)h_2^*(t_1)]/E[h_1(t_1)^2]|$. If $|\Delta t| \ll (\Delta t)_c$, we say that the received signal is coherent for $t_1$ and $t_2$.
which does not depend in a WSSUS channel on \( v_1 \) or \( v_2 \), only on \( \Delta v = v_1 - v_2 \). The minimum positive frequency interval \( \Delta f_c \) where the normalized correlation is

\[
|E[C(f_1, t_1)C^*(f_2, t_1)]|/E[C(f_1, t_1)C^*(f_1, t_1)] = \varepsilon,
\]

where \( \varepsilon \) is a real constant \((0 \leq \varepsilon < 1)\), is defined to be the coherence bandwidth \( (\Delta f)_c \). If \( |\Delta f| \ll (\Delta f)_c \), the complex samples are correlated in such a way that in general \( h_1(t_1) \approx h_2(t_1) \). If this happens over the frequency band \( B \) of the modulated signal so that \( B \ll (\Delta f)_c \), the channel is frequency-nonselective or flat fading, otherwise it is frequency-selective.

### 2.2 Classification of detectors

As discussed in (Kay, 1993, p. 12), we must separate optimal detectors, their approximations, and suboptimal detectors. In optimal detectors some parameters related to the channel are assumed to be known. In practice they must be estimated, which leads to an approximation of the optimal detector. A suboptimal detector is not an approximation of any of the known optimal detectors. An example is the discriminator detector when used in a frequency-shift keying (FSK) receiver (Shaft, 1963).

A **coherent detector** is such a detector where \( h(t, a) \) is assumed to be known for each \( a \), and the problem is to estimate \( a \) when \( r(t) \) is known. Knowledge of \( h(t, a) \) implies that we know \( c(t, t) \). A **partially coherent** or **pseudocoherent detector** is an approximation which estimates \( c(t, t) \), and there is some error in the estimate. All practical detectors that are called coherent are only partially coherent since \( c(t, t) \) must be estimated since it is unknown a priori.

A **differentially coherent detector** or **differential detector** is a partially coherent detector, which is based on the assumption of a known pilot symbol in the beginning of the transmission and differential coding in modulation, which observes the received signal over two symbol intervals, and which uses the earlier symbol as a phase reference. The idea can be generalized to several symbol intervals (Leib & Pasupathy, 1988; Divsalar & Simon, 1990). We classify DNs among PUDs since no absolute phase reference is needed. In fact, the equivalence of binary differential phase shift keying (DPSK) detection and noncoherent detection was shown in (Schwartz et al., 1966, pp. 307-308, 522-523) when the observation interval is two symbol intervals. In this case the phase of the channel must remain constant over two symbol intervals.

A **noncoherent detector** is such a detector where the received symbol waveform is assumed to have the form \( h(t, a) = v(t, a) e^{j\theta} \) where the waveform \( v(t, a) \) is assumed to be known and the absolute phase \( \theta \) is an unknown constant over the reception of the symbol waveform. Thus the received symbol waveforms are known except for the phase term. If the phase \( \theta \) would change during the reception of the waveform \( v(t, a) \), it would be distorted, and the noncoherent detector could not be implemented. The term noncoherent is usually used in this meaning in wireless communications. The term incoherent is usually used in optical communications. Some authors do not want to use the terms noncoherent or incoherent at all because the detector uses the internal phase structure of the signal although an absolute phase reference is missing (Van Trees, 1968, p. 326). The terms are still widely used. Noncoherent detectors have been considered for continuous phase wideband and narrowband signals in (Hirt & Pasupathy, 1981; Pandey et al., 1992).
A generalized noncoherent detector is a detector where the received symbol waveform has the form \( h(t, a) = ae^{i\beta} v(t, a) \) where \( v(t, a) \) is assumed to be known and \( ae^{i\beta} \) is an unknown complex gain, which is constant over the duration of the symbol interval. The term “generalized” is used to emphasize that the amplitude gain \( a \) is unknown but in a noncoherent detector it is known and for simplicity set to unity.

### 2.3 Orthogonality of modulated signals

Orthogonality is an important concept since we must avoid as much as possible any crosstalk between signals. In a diversity system crosstalk or interference may appear between diversity channels. An example is multipath diversity where crosstalk is equivalent to interpath interference (Turin, 1980). ISI is another form of crosstalk (Van Etten, 1976). Crosstalk is different from correlation, which is measured by the covariance matrix. There may be correlation although crosstalk is avoided and vice versa. There are different orthogonality concepts for different detectors, including coherent, noncoherent, and energy detectors.

#### 2.3.1 Coherently orthogonal signals

We define the inner product of two deterministic signals \( h_1(t) \) and \( h_2(t) \) as \( <h_1, h_2> = \int_{-\infty}^{\infty} h_1(t)h_2^*(t)dt \). The signals are orthogonal or coherently orthogonal (Pasupathy, 1979; Madhow, 2008) if \( \text{Re}(<h_1, h_2>) = 0 \). This form of orthogonality is used in coherent detection. As an example we give two complex exponential pulses \( h_1(t) = A_1 \exp(j2\pi f_1 t), 0 \leq t < T \) and \( h_2(t) = A_2 \exp[j2\pi (f_1 + \Delta f)t + \varphi], 0 \leq t < T \) with an arbitrary amplitude \( A_1 \) or \( A_2 \), frequency offset \( \Delta f \) and phase offset \( \varphi \). The pulses are coherently orthogonal if either 1) \( A_1 = 0 \) or \( A_2 = 0 \) or 2) \( \Delta f = n/T \) or 3) \( \varphi = \pi \Delta f T + (n + 1/2)\pi \) where \( n \) is an integer, \( n \neq 0 \). Signals with \( A_1 = 0 \) or \( A_2 = 0 \) are used in on-off keying (OOK) systems. When \( \Delta f = n/T, n \neq 0 \), the pulses are always orthogonal irrespective of the value of \( \varphi \). However, for an arbitrary \( \Delta f \) we can always find a phase offset \( \varphi \) for which the pulses are orthogonal. If we set \( \varphi = 0 \), the pulses are orthogonal if \( \Delta f = n/2T \) where \( n \neq 0 \) is an integer. Such signals are used in coherent FSK systems. If we alternatively set \( \Delta f = 0 \), the pulses are orthogonal if \( \varphi = \pi/2 + n\pi, n \neq 0 \). Such signals are used in quadrature phase-shift keying (QPSK) systems. The examples were about orthogonality in the frequency domain. Time-frequency duality can be used to find similar orthogonal signals in the time domain, for example by using sinc pulses (Ziemer & Tranter, 2002). Furthermore, some codes are also orthogonal, for example Hadamard codes (Proakis, 2001).

#### 2.3.2 Noncoherently orthogonal signals

The signals \( h_1(t) \) and \( h_2(t) \) are noncoherently orthogonal or envelope-orthogonal (Pasupathy, 1979; Madhow, 2008; Turin, 1960) if \( <h_1, h_2> = 0 \). This form of orthogonality is used in noncoherent detection. In the previous example, the two complex exponential pulses are noncoherently orthogonal if 1) \( A_1 = 0 \) or \( A_2 = 0 \) or 2) \( \Delta f = n/T, n \neq 0 \). Such signals are used in noncoherent ASK and FSK systems, respectively. In these cases there is no requirement for the phase \( \varphi \), i.e., it can be arbitrary, but it must be constant during the interval \( 0 \leq t < T \). Noncoherently orthogonal signals are also coherently orthogonal signals.

#### 2.3.3 Disjointly orthogonal signals

Coherently and noncoherently orthogonal signals can be overlapping in time or frequency. To define disjointly orthogonal signals \( h_1(t) \) and \( h_2(t) \), we must first select a window function
\( w(t) \) and define the short-time Fourier transform (Yilmaz & Rickard, 2004) \( S_k(\tau, f) = \int_{-\infty}^{\infty} w(t - \tau) h_k(t) e^{-j2\pi f \tau} dt, k = 1, 2 \) which can be interpreted as the convolution of a frequency-shifted version of the signal \( h_k(t) \) with a frequency shift \(-f\) and the time-reversed window function \( w(-t) \). The signals are \( w \)-disjoint orthogonal if \( S_1(\tau, f)S_2(\tau, f) = 0, \forall \tau, f \). If \( w(t) = \delta(t) \), the \( w \)-disjoint orthogonal signals are time disjoint, which can be implemented in an FSK system. If \( w(t) \) = \( \delta(t) \), the \( w \)-disjoint orthogonal signals are frequency disjoint, which can be implemented in a pulse-position modulation (PPM) system. If two signals are frequency disjoint, they do not need to be time disjoint and vice versa. Time and frequency disjoint signals are called disjointly orthogonal. Our main interest is in the time and frequency disjoint signals. A special case of both of them is OOK. Disjointly orthogonal signals are used in energy detection. Disjointly orthogonal signals are also coherently and noncoherently orthogonal signals.

2.4 Optimal MAP receiver
When defining an optimal receiver, we must carefully define both the assumptions and the optimization criterion. We use the MAP receiver, which minimizes the symbol error probability. A maximum likelihood (ML) receiver is a MAP receiver based on the assumption that the transmitted symbols have identical a priori probabilities. The easiest way to derive the optimal receiver is to use the time-discrete model of the received signal. The received signal \( r(t) = h(t, a) + n(t) \) is filtered by an ideal low-pass filter, whose two-sided bandwidth \( B \) is wide enough so that it does not distort \( h(t, a) \). The output of the filter is sampled at a rate \( f_s = B \) that is defined by the sampling theorem. In this case the noise samples are uncorrelated and the time-discrete noise is white. The sampling interval is normalized to unity.

2.4.1 Optimal MAP receiver
The covariance matrix of a column vector \( x \) is defined as \( R_{xx} = E[(x - E(x))(x - E(x))^H] \) where \( E(x) \) refers to the statistical mean or expectation of \( x \) and the superscript \( H \) refers to conjugate transposition. The received signal \( r(a) \) depends on the transmitted symbol \( a \) and may be presented as the \( N \times 1 \) vector (Kailath, 1961) \( r(a) = h(a) + n \). The vectors \( h(a) \) and \( n \) are assumed to be mutually uncorrelated. The received signal \( r \) has the \( N \times N \) covariance matrix \( R_{rr}(a) = R_{hh}(a) + R_{nn} \) where \( R_{hh}(a) \) is the covariance matrix of \( h(a) \) and \( R_{nn} = N_0I \) is the covariance matrix of \( n, N_0 > 0 \) is the noise variance, and \( I \) is a unit matrix.

In the MAP detector, the decision \( s(\tilde{a}) \) is based on the rule (Proakis, 2001)

\[
  s(\tilde{a}) = \text{arg}_s(\tilde{a}) \max P(\tilde{a} | r) \tag{1}
\]

where

\[
  P(\tilde{a} | r) = \frac{p(r | \tilde{a})p(\tilde{a})}{p(r)} \tag{2}
\]

is the a posteriori probability that \( s(\tilde{a}) \) was transmitted given \( r \), \( p(r | \tilde{a}) \) is the a priori probability density function of \( r \) given \( s(\tilde{a}) \) was transmitted, \( P(\tilde{a}) \) denotes the a priori probability for the symbol \( \tilde{a} \), and \( p(r) \) denotes the probability density function of \( r \) averaged over all \( a \). The symbol \( \tilde{a} \) refers to the symbol under test and \( \tilde{a} \) to the final decision. We
assume that the a priori probabilities $P(\alpha)$ are equal, and $p(r)$ does not have any effect on the maximization in (2). An equivalent decision variable is the a priori probability density function or the likelihood function $p(r|\alpha)$. To proceed, we need some knowledge of the statistics of $r$ to compute $p(r|\alpha)$. By far the simplest case is to assume that for each $\alpha$, $r$ is Gaussian. The decision variables to be defined can be used also in diversity systems by using simple addition when there is no crosstalk or correlation between the diversity channels, see for example (Turin, 1980).

**Coherent receiver:** In the coherent receiver, $h(a)$ is assumed to be known for each $a$. Since $n$ is Gaussian, also $h$ is Gaussian, and (Barrett, 1987; Papoulis, 1991)

$$p(r|a) = \frac{1}{\pi N_0 \det[R_{rr}(\alpha)]} \exp\{-[r - E(r)]^H[R_{rr}(\alpha)]^{-1}[r - E(r)]\}$$

viewed as a function of $\alpha$. The right-hand side of (3) represents the probability density function of a random vector whose elements are complex Gaussian random variables. Since the noise is assumed to be white with $N_0 > 0$, the matrix $R_{rr}(\alpha)$ is always positive definite (Marple, 1987) and nonsingular. In the coherent receiver the $R_{rr}(\alpha) = R_{nn} = N_0 I$. We take the natural logarithm and the MAP criterion leads to the decision variable

$$y(\alpha) = \frac{1}{N_0} \text{Re}[r^H h(\alpha)] + B(\alpha), B(\alpha) = -\frac{1}{2N_0} h^H(\alpha) h(\alpha)$$

where $B(\alpha)$ is the bias term, which depends on the energy of $h(\alpha)$. The term $\text{Re}[r^H h(\alpha)]$ corresponds to the correlator which can be implemented also by using a matched filter, which knows the absolute phase of the received signal. In a diversity system the receiver can be generalized to **maximal ratio combining**.

### 2.4.2 Noncoherent receiver

In a noncoherent receiver $h(a)$ has the form $h(a) = v(a) e^{j\theta}$ where $\theta$ is a random variable uniformly distributed in the interval $[0, 2\pi]$ and is $v(a)$ assumed to be known for each $a$. Now for a given $\theta$ the received signal is Gaussian and

$$p(r|\alpha, \theta) = \frac{1}{\pi N_0 \det[R_{rr}(\alpha, \theta)]} \exp\{-[r - E(r)]^H[R_{rr}(\alpha, \theta)]^{-1}[r - E(r)]\}.$$  

The MAP criterion is obtained from $p(r|\alpha, \theta)$ by removing $\theta$ by averaging (Meyr et al., 1998), i.e., $p(r|\alpha) = \int_0^{2\pi} p(r|\alpha, \theta) p(\theta) d\theta$. The conditional probability density function $p(r|\alpha)$ is not Gaussian although $p(r|\alpha, \theta)$ is Gaussian and therefore the receiver includes a nonlinearity. When $p(r|\alpha)$ is maximized, the decision variable is

$$y(\alpha) = \ln I_0 \left( \frac{1}{N_0} |r v^H(\alpha)| \right) + B(\alpha), B(\alpha) = -\frac{1}{2N_0} v^H(\alpha) v(\alpha)$$

where $I_0(\cdot)$ is the zeroth order modified Bessel function and $B(\alpha)$ is the bias term that depends on the energy of $v(\alpha)$. The term $|r v^H(\alpha)|$ corresponds to noncoherent correlation and can be implemented with a noncoherent matched filter, which includes a matched filter and a linear envelope detector. The envelope detector is needed because the absolute phase of the received signal is unknown.

For large arguments, an approximation is (Turin, 1980)

$$y(\alpha) \approx \frac{1}{N_0} |r v^H(\alpha)| + B(\alpha).$$
In a diversity system the decision variable (6) leads to a nonlinear combining rule and the approximation (7) to a linear combining rule. It can be shown that the performance of the linear envelope detector is almost identical to that of quadratic or square-law envelope detector, but performance analysis is easier for square-law envelope detector although in practical systems the dynamic range requirements are larger (Proakis, 2001, p. 710; Skolnik, 2001, p. 40; McDonough & Whalen, 1995). If the energies of $v(\bar{a})$ for all $\bar{a}$ are identical, no bias terms are needed and the decision variable (6) is simplified to the form $y'(\bar{a}) = |r v^H(\bar{a})|^2$. In a diversity system the receiver can be generalized to square-law combining. The use of these simplifications is an approximation only since the signals coming from different diversity channels do not in general have identical energies, and ideally the nonlinearity in (6) is needed (Turin, 1980).

2.4.3 Estimator-correlator receiver

Now the signal part $h(a)$ for a given $a$ is random and complex Gaussian and it has zero mean, i.e., $E[h(a)] = 0$ where $0$ is a zero vector. This implies that the channel is a Rayleigh fading multipath channel. As in the noncoherent receiver, the effect of the channel can be removed by averaging (Kailath, 1963). The MAP criterion (2) corresponds to the decision variable (Kailath, 1960)

$$y(\bar{a}) = -r^H[R_{rr}(\bar{a})]^{-1}r + B(\bar{a}), B(\bar{a}) = -\ln[\det[R_{rr}(\bar{a})]].$$

The bias term $B(\bar{a})$ can be ignored if the determinant of $R_{rr}(\bar{a})$ does not depend on $\bar{a}$. The conditions where the bias terms are identical are considered in (Mämmelä & Taylor, 1998). The inverse of the covariance matrix can be expressed in the form $[R_{rr}(\bar{a})]^{-1} = R_{nn}^{-1} - R_{nn}^{-1}G(\bar{a})$ where the matrix

$$G(\bar{a}) = R_{hh}(\bar{a})[R_{rr}(\bar{a})]^{-1} = I - R_{nn}[R_{rr}(\bar{a})]^{-1}$$

is a linear minimum-mean square error (MMSE) estimator of the received signal. The optimal estimator is an MMSE estimator although the whole receiver is a MAP detector (Kailath, 1969). Since the noise covariance matrix in (9) does not depend on the transmitted signal, and the noise is white, the decision variable

$$y'(\bar{a}) = \frac{1}{n_0}r^H G(\bar{a})r + B(\bar{a})$$

(10)
can be maximized where $G(\bar{a})$ is a Hermitian matrix since it is a difference of two Hermitian matrices. Thus the decision variables (10) are real. Since the expression $r^H G(\bar{a}) r$ has a Hermitian quadratic form, it is nonnegative and almost always positive.

In (10) the receiver estimates the received signal, and the estimate is $s(\bar{a}) = G(\bar{a})r$. However, the estimate is the actual signal estimate only in the receiver branch where $\bar{a} = a$ (Kailath, 1961). The receiver based on the decision variables (10) is called the estimator-correlator receiver (Kailath, 1960) and the quadratic receiver (Schwartz et al., 1966; Barrett, 1987), see Fig. 1. It does not use any knowledge of the absolute phase of the received signal. Thus for phase-modulated signals there is a phase ambiguity problem, which can be solved by using known pilot signals. The structure is similar to that of the DPSK detector when two consecutive symbols are observed and only the earlier symbol is used in the estimator. The detector (6) can be also interpreted as an estimator-correlator receiver, but the estimator is nonlinear because $p(r|\bar{a})$ is not a Gaussian probability density function (Kailath, 1969). In
fact, any MAP receiver used in a fading channel with AWGN has an estimator-correlator interpretation having an MMSE estimator, possibly nonlinear.

Fig. 1. Estimator-correlator. Asterisk (*) refers to complex conjugation. For each $\hat{a}$ there is a similar receiver branch and the maximum of the outputs corresponds to the MAP decision.

We now assume that $h(\hat{a})$ can be expressed in the form $h(\hat{a}) = S(\hat{a})c$ where $S(\hat{a})$ is a suitably defined signal matrix (Kailath, 1961) and $c$ is the channel vector. As shown in (Kailath, 1961), the decision variable can be alternatively expressed in the form

$$y'(\hat{a}) = \frac{1}{N_o} r^H S(\hat{a}) F(\hat{a}) S^H(\hat{a}) r + B(\hat{a})$$

(11)

where

$$F(\hat{a}) = \frac{1}{N_o} (R^{-1} + \frac{1}{N_o} S^H(\hat{a}) S(\hat{a}))^{-1}$$

(12)

and the inverses can be shown to exist. We now assume that the channel is flat fading and the variance of the fading gain is $\sigma_c^2 = E(|c(n)|^2)$. The matrix $F(\hat{a})$ reduces to the scalar

$$F(\hat{a}) = \frac{\sigma_c^2}{E(\hat{a}) \sigma_n^2 + N_o}$$

(13)

and the signal matrix $S(\hat{a})$ reduces to a vector $s(\hat{a})$ whose energy is denoted by $E(\hat{a})$. The decision variable has now the form

$$y'(\hat{a}) = \frac{F(\hat{a})}{N_o} |s^H(\hat{a}) r|^2 + B(\hat{a}).$$

(14)

This receiver represents the generalized noncoherent receiver where the amplitude of the received signal is an unknown random variable. The detector includes a square-law envelope detector. In a diversity system the receiver corresponds to generalized square-law combining. Compared to the ordinary noncoherent detectors the generalized noncoherent receiver (14) must know the second order statistics of the channel and noise. The instantaneous amplitude is assumed to be unknown.

The effect of weighting with $F(\hat{a})$ is discussed in channel estimators in (Li et al., 1998). An important special case is equal gain combining (EGC), which has some loss in performance but the robustness is increased and the complexity is reduced partially because the noise variance and the mean-square strengths of the diversity branches are not needed to estimate. It is important not to include weak paths in EGC combining.

As a positive definite matrix, $G(\hat{a})$ can be factored in the form $G(q) = [G_i(\hat{a})]^H G_i(\hat{a})$ where $G_i(\hat{a})$ is a lower-triangular matrix (Kailath, 1961). Therefore
$y'(\bar{a}) = \frac{1}{N_t} [G_1(\bar{a})r]^H G_1(\bar{a})r + B(\bar{a}). \quad (15)$

This receiver is called the filter-squarer-integrator (FSI) receiver (Van Trees, 1971).

If the knowledge about the received signal is at the minimum, we may assume that $G_1(\bar{a})$ corresponds to an ideal band-pass filter, and the receiver corresponds to the energy detector (ED). If the signals share the same frequency band and time interval, the ED can only discriminate signals that have different energies. If the received symbols have similar energies, they must be time disjoint or frequency disjoint.

**Joint data and channel estimation.** In joint estimation both the data and channel are assumed to be unknown as in the estimator-correlator but they are estimated jointly (Mämmelä et al., 2002). In a Rayleigh fading channel the MAP joint estimator is identical to the estimator-correlator (Meyr et al., 1998). Due to symmetry reasons the MAP estimator for this channel is identical to the MMSE estimator. This is not true in more general channels and joint estimation differs from the optimal MAP detector whose aim is to detect the data with a minimum error probability.

### 3. Historical development of phase-unaware detection methods

Optimal MAP receivers were first analyzed by Woodward and Davis (1952). They showed that the a posteriori probabilities form a set of sufficient statistics for symbol decisions. Price (1956) and Middleton (1957) derived the estimator-correlator receiver for the time-continuous case. In addition, Middleton presented an equivalent receiver structure that has been later called the FSI receiver (Van Trees, 1971). Kailath (1960) presented the estimator-correlator for the time-discrete case and generalized the results to a multipath channel where the fading is Gaussian. If the channel includes a known deterministic part in addition to the random part, the receiver includes a correlator and the estimator-correlator in parallel (Kailath, 1961). Later Kailath (1962) extended the result to a multi-channel case. Kailath (1969) also showed that the estimator-correlator structure is optimum for arbitrary fading statistics if the noise is additive and Gaussian. If the noise is not white, a noise whitening filter can be used (Kailath, 1960).

According to Turin (1960) the noncoherent matched filter was first defined by Reich and Swerling and Woodward in 1953. Noncoherent receivers were studied by (Peterson et al., 1954; Turin, 1958). Noncoherent diversity systems based on square-law combining were considered in (Price, 1958; Hahn, 1962).

Helström (1955) demonstrated the optimality of orthogonal signals in binary noncoherent systems. Jacobs (1963) and Grettenberg (1968) showed that energy-detected disjointly orthogonal and noncoherent orthogonal $M$-ary systems approach the Shannon limit and capacity in an AWGN channel. Scholtz and Weber (1966) showed that in $M$-ary noncoherent systems noncoherently orthogonal signals are at least locally optimal. They could not show the global optimality. Pierce (1966) showed that the performance of a noncoherent $M$-ary system with $L$ diversity branches approaches the Shannon limit just as that of a coherent system when $M$ and $L$ approach infinity. However, in a binary system ($M = 2$) there is a finite optimal $L$ dependent on the received signal-to-noise ratio (SNR) per bit for which the bit error probability performance is optimized (Pierce, 1961). In this case there is always a certain loss compared to the corresponding binary coherent orthogonal system.

One of the earliest papers on differential phase-shift keying (DPSK) includes (Doelz, 1957). Cahn (1959) analyzed the performance of the DPSK detector. DPSK was extended to multiple
symbols in (Leib & Pasupathy, 1988; Divsalar & Simon, 1990; Leib & Pasupathy, 1991). An extension to differential quadrature amplitude modulation (QAM) is described in a voiceband modem standard (Koukourlis, 1997). The estimator-correlator principle was used in a DPSK system in (Dam & Taylor, 1994).

EDs are sometimes called radiometers. Postdetection or noncoherent integration similar to energy detection has been originally considered in radar systems by North in 1943 (North, 1963) and Marcum in 1947 (Marcum, 1960). The authors analyze the noncoherent combining loss. Peterson et al. (1954) showed the optimality of energy detection when the signal is unknown. A general analysis of EDs was presented in (Urkowitz, 1967; Urkowitz, 1969).

Energy detection was studied for digital communications in (Helström, 1955; Middleton, 1957; Harris, 1962; Glenn, 1963). Dillard (1967) presented an ED for pulse-position modulation (PPM), and Hauptschein & Knapp (1979) for M-ary orthogonal signals. A general result from these studies was the fact that the performance of the system is decreased when the time-bandwidth product is increased.

4. Recent trends in designing phase-unaware detectors

In this section, a more detailed view on selected signal design and data estimation methods, suitable for PUDs is given. Specifically, we first focus on basic signal design principles, followed by a discussion on the data estimation and generation of the decision variable for the subsequent symbol decision approaches at the receiver. Advanced signal processing approaches, which represent more recent trends, are considered next. Finally, we discuss specific analysis problems arising with the PUD. With a PUD system, any information on the absolute signal phase is not recovered, thus demodulation methods based on absolute phase information are useless unless pilot symbols are used.

4.1 Basic signal design principles

We start from a transmission model for single-input single-output (SISO) time-division-multiplexed (TDM) signals given as

\[ s(t) = \sum_{k=-\infty}^{\infty} a_k g_k(t - kT - \tau_k) \]

where \( T \) is the symbol interval, \( a_k \) is the \( k \)th amplitude selected from the symbol set with OOK, \( g_k(t) \) is the \( k \)th pulse shape selected the symbol set for binary frequency-shift keying (BFSK), and \( \tau_k \) is the \( k \)th delay selected the symbol set for binary pulse position modulation (BPPM). In general, the overall pulse modulation method for the selected PUD method can be based on one of these approaches or a combination of them.

Alternatively, we can use the frequency domain to multiplex signals by using appropriate frequency-division-multiplexed (FDM) signals. In this case, (16) becomes

\[ s(t) = \sum_{k=-\infty}^{\infty} \sum_{j=1}^{N} a_{k,j} g_{k,j}(t - kT - \tau_{k,j}) \]

where now \( a_{k,j} \) and \( \tau_{k,j} \) are, respectively, the amplitude and delay at \( k \)th time and \( j \)th (\( j = 1, 2, \ldots, N \)) frequency. Typically, in PUD-based systems, the pulses \( g_{k,j}(t) \) are nonoverlapping in frequency. An example of nonoverlapping FDM with OOK is given for UWB systems in (Paquet, 2004). Overlapping FDM signals were analyzed in (Al-Dweik, 2003) using the PUD approach. Nonoverlapping FDM is also called as a multiband modulation system (Anttonen & Mämmelä, 2009).
Some PUD structures may require additional reference, pilot, or training signals in order to be able to recover the transmitted information. For instance, an unmodulated reference symbol and modulated information symbol are required to be sent in pairs or a known training signal is needed to acquire some knowledge of the instantaneous state of the channel. This former system is sometimes called as a transmitted-reference system (Franz & Mitra, 2006). It is also possible to use the previous symbol as a local reference template given rise to differential modulation approach. In this case, variants of DPSK become possible (Ma et al., 2005).

The comparison of different modulation methods depends on the target system specification and selected receiver structure but some general conclusions can be drawn (cf. Proakis, 2001; Guvenc, 2003; Simon & Alouini, 2005). For instance, the OOK can be preferred for its simple transceiver structure. Orthogonal BFSK and BPPM result in improved energy efficiency per information bit at the cost of occupying larger bandwidth.

### 4.2 Symbol-by-symbol data estimation without interference

As described in the previous section, data estimation is in general based on the estimator-correlator structure. In the optimal receiver the aim is to develop a symbol detector which is somehow matched to the transmitted signal and the channel. On the other hand, in a suboptimal symbol detection approach, the aim is to match the combination of the channel and receiver front-end to a simpler detector by using suitable signal pre-processing. Several important pre-processing tasks include an out-of-band noise filtering, solving the phase ambiguity problem, and a multipath energy combiner. In case of a PUD system, these pre-processing tasks have some special features and will be discussed in more detail.

Figure 2 illustrates some important pre-processing parts for the given received signal \( r(t) \). We have excluded parts such as amplifiers and down-converters which may be needed in some PUD systems. The order of the blocks is naturally not fixed and can be changed resulting in different trade-offs. As an example, the sampling operation can take place at any stage after limiting the bandwidth of the noise. If the signal bandwidth is very high, as it is typical for UWB systems, it is desired to locate the sampling unit as late as possible to avoid the use of extremely high sampling rates. In an ideal case, the noise filtering can follow two principal approaches, namely sinc filtering and matched filtering. In the former case, the frequency response is a rectangular function in frequency domain for removing all frequency components outside a given two-sided bandwidth \( B \). On the other hand, in the latter case the aim is to match the impulse response of the receiver filter to the transmitted pulse \( g(t) \). In practice, some approximations of these approaches are usually used. After the noise filtering, the phase ambiguity between resolvable multipaths must be removed by an appropriate co-phasing scheme in order to combine the energy from different multipaths.

Fig. 2. Pre-processing of received signal for different PUD systems (linear modulation assumed).
The difference between DD and ED is clear, i.e., for ED the correlation delay is zero whereas for the DD it is nonzero. The DD method is sometimes called as an autocorrelation detector (Franz & Mitra, 2006).

The main consequence is that the noise characteristics become different at the output of the correlator. However, the difference between the ED and ND is often not so clear when used together with a multipath combiner. In fact, with certain approximate assumptions they become equivalent. The main differences lie in the assumptions on noise filtering and time-variability. Typically, ND is defined as a pulse matched filter structure followed by a quadratic envelope detector. Thus, ND must assume that the channel phase is constant over at least the symbol interval. On the other hand, such assumptions are not, by definition, made with ED indicating that ED is a more robust concept.

In a PUD-based system, the multipath combiner can be based on similar approaches as with the systems which have an access to the phase information. The integration interval determines the amount of multipath and noise energy accumulation. The aim is to collect the energy optimally from different resolvable multipaths separated by a delay so that the SNR at the output of the combiner is maximized. The most convenient approach is to use an EGC where the weighting signal $w(t)$ is one. The EGC approach with different PUD systems has been analysed in (Proakis, 2001; Simon & Alouini, 2005, Anttonen et al., 2011a). A more complicated approach is to use a weighted gain combiner where $w(t)$ is now changing in time based on a selected criterion (Romme & Witrisal, 2006; Wang et al., 2011). If weighting is done with sampled signals, the weighting signal can be presented as a weight vector $\mathbf{w}$. Typically, weight optimization based on the minimization of symbol error probability of a PUD system is difficult for non-Gaussian statistics and other criteria such as minimum mean square error or maximum SNR are used. Weighting changes also the distribution of the decision variable and this needs to be taken into account in symbol detection (cf. Kotz, 1967). Consequently, the maximum available gain from the weighting of the diversity paths with respect to the EGC depends especially on the fading statistics. Using the results form (Kotz, 1967) for a weighted chi-square distribution, the effect of different normalized weighting vectors $\mathbf{w}$ is illustrated in Fig. 3.

After the pre-processing blocks, a discrete decision variable $y(kT)$ is provided for the subsequent symbol detector. The PUD systems can be also classified based on the used symbol decision approach. Typically the symbols are equally probable and ML criterion is used in all cases. However, the ML criterion may or may not need instantaneous channel energy information depending on the used modulation method and the corresponding decision variable. In case of un-coded nonconstant envelope OOK signals, the receiver must know the noise level and the instantaneous SNR in order to recover the transmitted information. However, with constant envelope BPPM and BFSK, the transmitted information is detected by comparing the decision variables at each candidate time-frequency intervals. If the channel does not change relatively within these intervals, the symbols can be detected without instantaneous channel information.

### 4.3 Advanced signal processing approaches

In this section, we overview some recent trends to improve the performance of the basic PUD-based systems described in the previous section. The selected techniques we tend to highlight include multilevel modulation, multiantenna modulation, multiple-symbol sequence detection, multiuser communication techniques, and ISI avoidance methods.
Fig. 3. Illustration of the effect of weighting on the pdf of the decision variable $y$.

4.3.1 Multilevel modulation and demodulation

Multilevel modulation is a powerful way to increase the spectral efficiency or the energy efficiency to transmit the information bits. The binary modulation methods are extended to $M$-PAM, $M$-PPM, or $M$-FSK with $M$-ary constellations. Each of the methods has its own specific advantages and challenges. For instance, the optimum delay parameter of $M$-PPM depends on the selected pulse shape and receiver structure (Jayaweera, 2005). In case of $M$-PAM signals, it is often necessary to use nonnegative symbol constellations when applied with a PUD system as shown in (Anttonen et al., 2009; Anttonen et al., 2011a). When combining nonnegative $M$-PAM signals with ED, the receiver must know the noise level and the instantaneous SNR to calculate $M$-1 symbol decision thresholds. However, it has been shown recently that the decision thresholds can be found blindly without using a known training signal (Anttonen et al., 2010; Anttonen et al., 2011b). The main advantages of the $M$-PAM approach are that the complexity of the generation of the decision variable for the symbol detector is independent of the number of modulation levels $M$, and the bandwidth is decreased when $M$ is increased for a given bit rate. On the other hand, $M$-PPM and $M$-FSK result in an improved bit error probability with $M$ as, unlike with $M$-PAM, the symbol distances do not change for a given average signal energy. Naturally, various hybrids of the modulation methods presented above are possible. Combinations of $M$-PAM and $M$-PPM schemes can be applied to provide compromises between the spectral and energy efficiency. It is also possible to use multilevel differential phase shift keying and combine it with amplitude modulation, resulting in a differential QAM approach (Koukourlis, 1997) without a need to know the absolute phase information of the received signal.

4.3.2 Multiantenna modulation and demodulation

Using multiple-input-multiple-output (MIMO) methods with coherent systems have become a standard approach to improve the performance of the system. The use of MIMO methods with
PUD systems lags behind but has become more popular recently. Spatial diversity methods have been most popular with ND and DD. The capacity of multiple antenna systems without having access to the instantaneous channel state was studied in (Hochwald & Marzetta, 2000). Under the AWGN and Rayleigh channel assumptions, it has been shown that at a high SNR, or when the coherence time in symbols intervals is much larger than the number of transmitter antennas, the capacity can be achieved by using constellations of unitary matrices as codebooks. Differential space-time encoding and decoding principles are developed in (Hughes, 2000) and analysed for UWB short-range systems in (Zhang & Ng, 2008). Beamforming with an ND-based PUD receiver is studied in (Naguib & Paulraj, 1996). While MIMO-extended PUD systems typically rely on orthogonal space-time matrices, some techniques have been proposed that allow the transmission of independent space-time signals (Song et al., 2007). This kind of differential spatial multiplexing approach can achieve maximum bandwidth efficiency without the need of any channel state information.

4.3.3 Multiple symbol sequence detection
So far we have assumed that the symbol decision is made using a symbol-by-symbol detector. In order to improve the performance, an increasing trend for PUD systems is to use a sequence of symbols at each time instant and apply a ML estimator for the used symbol sequence (Leib & Pasupathy, 1988; Divsalar & Simon, 1990; Lodge & Moher, 1990; Leib & Pasupathy, 1991; Leib & Pasupathy, 1992; Colavolpe & Raheli, 1999; Guo & Qiu, 2006; Tian & Yang, 2008). The performance of noncoherent sequence detectors, which have no access to the absolute signal phase information, has been shown to approach that of the corresponding coherent sequence detectors if the phase ambiguity problem is solved but at the cost of increasing the decoder complexity (Raphaeli, 1996).

4.3.4 ISI avoidance by signal design
In systems using a PUD approach, the receiver often includes some nonlinear operation which makes it difficult to post-equalize the ISI. Furthermore, phase information is required to completely remove the ISI. Consequently, it would be more reasonable to aim at avoiding the interference using appropriate signal design methods. In principle, the avoidance is possible via pre-distortion or pre-equalization of the transmitted signal according to the instantaneous ISI (Harashima & Miyakawa, 1972), spread-spectrum signalling with interference-rejecting autocorrelation characteristics of the pseudo-noise codes (Peterson et al., 1995), increasing the symbol interval for a given bit rate by using $M$-ary modulation, or frequency or spatial multiplexing, using gaps longer than the delay spread of the channel between symbols (Proakis, 2001), delay spread reduction of the channel with signal beamforming (Hansen & Loughlin, 1981), or commutating the signal, e.g., with frequency-hopping code according to the delay spread of the channel (Turin, 1984). A rough comparison of these fundamental approaches is presented in Table 1.

4.3.5 Multiuser communications
Multiuser detection involves the study of methods for the demodulation of simultaneously transmitted information from different user terminals. In general, the user information can be detected in a serial or parallel fashion. Multiuser and multiantenna detection methods pose similar type of problems but from different viewpoints, and the techniques that are used for data recovery have many commonalities. There has been a considerable research on
the coherent multiuser detection problem in the past, see a good summary in (Verdu, 1998).
A pioneering work in studying multiuser detection techniques for DD-based PUD systems in
found from (Varanasi, 1993). Other recent work on studying and analysing multiuser
techniques with DD can be found from (Dang & van der Veen, 2007). Although multiuser
DD schemes have been a more popular research topic, recently multiuser approaches have
also been applied to ED (Xu & Brandt-Pearce, 2006). The most significant challenge in
designing PUD transceivers with multiple users is to compensate the nonlinear interference
generated by the nonlinear operations at the receiver front-end. Another challenge is
naturally the evaluation of the analytical error probability of these systems.

5. On analysis of phase-unaware detectors

In this section, we outline some important analysis challenges and available solutions to
evaluate the error probability of PUD-based systems. Our purpose is not to explicitly
compare the error probability performance of different PUD approaches as this has been
done in many contributions, see good overviews from (Proakis, 2001; Simon & Alouini,
2005). We discuss selected approximation approaches which significantly ease the analysis
of PUD systems.

5.1 Idealization of the receiver filter
The receiver filter reduces the noise by convolving the received signal with a selected
impulse response which is sometimes matched to the pulse shape of the transmitted signal.
The receiver filter can be a bandpass or lowpass filter depending on the location of the filter
with respect to the possible down-converter. The consequences of the nonideal filtering are
that the received signal is distorted and the output noise samples become correlated.
Typically, these effects are difficult to include in the performance analysis of PUD systems.
To avoid such a situation, the filter bandwidth should be larger than the bandwidth of the
received signal without noise (Choi & Stark, 2002). In the bandpass case, this kind of filter is
called as ideal bandpass zonal filter in (Quek & Win, 2005). At the output of the idealized
filter with a sufficiently large bandwidth $B$, no ISI is introduced and the noise samples,
which are separated by $1/B$, can be approximated to be statistically independent.

5.2 Integration model and the sampling theorem
In order to come up with a proper probability density function for the decision variable of a
PUD system, we need to approximate the continuous integration operation involved with
the multipath combiner in Fig. 2. A natural approximation is obtained from the sampling
theorem as follows (cf. Urkowitz, 1967). In a general case, with an appropriate choice of time origin, we may express a real signal $r(t)$ in the form

$$r(t) = \sum_{k=-\infty}^{\infty} r_k \text{sinc}(Bt - k) \approx \sum_{k=1}^{K} r_k \text{sinc}(Bt - k)$$  \hspace{1cm} (18)

where $r_k = r(k/B)$ is a sampled version of $r(t)$, $K$ is the finite number of signal components, and $B$ is the bandwidth of the receiver filter. It is obvious that the approximation improves as $K$ increases. It is shown in (Urkowitz, 1967) that for a given integration interval $T$, it is sufficient to select $K = BT$ discrete terms to obtain

$$\int_0^T r(t) dt \approx y = \frac{1}{B} \sum_{k=1}^{BT} r_k.$$  \hspace{1cm} (19)

An example to use the result is presented as follows by applying the ED principle. If $r(t) = [s(t) + n(t)]^2$, where $s(t)$ is the information signal with nonzero energy and $n(t)$ is the zero mean Gaussian random variable, $y$ can readily be shown to follow the noncentral chi-square distribution with $2BT$ degrees of freedom since each complex sample includes two real samples.

5.3 Gaussian quadratures

Important approaches to solve analytical problems of PUD systems arise from the application of Gaussian quadratures. Gaussian quadratures approximate the integrals of the form (Abramowitz & Stegun, 1972)

$$\int_a^b \omega(x) f(x) dx \approx \sum_{i=1}^{G} \omega_i f(x_i)$$  \hspace{1cm} (20)

where $a$ and $b$ set the integration interval, $\omega(x)$ is a positive weight function, $f(x)$ is an arbitrary function, $\omega_i$ and $x_i$ are, respectively, the weighting factors and abscissas of the

Fig. 4. SER approximation of binary ED-PAM system with different orders of Gaussian quadratures.
selected $G$th order Gaussian quadrature. The selection of the quadrature depends on the form of $f(x)$. An important example arises from the analysis of ED-based PAM systems (Anttonen et al., 2011a). The optimal ML symbol decision thresholds of ED-PAM systems cannot be represented in a closed form. Consequently, the evaluation of the analytical error probability in multipath fading channels becomes difficult, if not impossible. Following the results of (Anttonen et al., 2011a), the symbol error rate is given as Gaussian quadratures enable a convenient framework to select only few discrete points in which the continuous time integral is evaluated, and the threshold values are calculated. Figure 4 illustrates the effect of the order of the Gauss quadrature for the symbol error rate (SER) of the ED-PAM system with $M = 2$ in a flat lognormal fading channel, see further details of the analysis in (Anttonen et al., 2011a). It is seen that already when $G \geq 6$, the approximation accuracy is not significantly improved anymore. In addition to the Gaussian quadratures, other types of quadratures exist as well if a suitable weighting factor is not available for the function $f(x)$ at hand or the function involves multiple random variables, see an excellent summary from (Cools, 2002).

5.4 Probability density function of the decision variable

Essential information for ML symbol detection is to know the probability density function (pdf) of the decision variable after sufficient pre-processing has been performed to allow the maximum possible signal-to-noise ratio for the decision variable and a simple symbol-by-symbol detector structure. The noncentral chi-square distribution, which was inferred from the sampling theorem in the previous subsection, is used extensively to model the distribution of the decision variable of PUD-based systems (Quek & Stark, 2005; Anttonen et al., 2011a). In case of a weighted PUD system, we must use a weighted chi-square distribution which has alternate series forms as shown in (Kotz et al., 1967). Since the chi-square distribution (both weighted and nonweighted) involves complicated functions with series forms, Gaussian approximation approach can be used to approximate the pdf of the decision variable, provided that the number of degrees of freedom is large enough. This is justified by the Berry-Esseen theorem given in (Feller, 1972). The Gaussian approximation approach enables also a nice framework to compare the noise statistics which has a major role in determining the error probability of the system. Let $N(\mu, \sigma)$ denote the Gaussian or normal distribution where $\mu$ and $\sigma$ are, respectively, the mean and standard deviation of the of the decision variable $y$. Following the work from (Guvenc et al., 2006; Wang et al., 2011), the noise statistics of $y$ with nonzero signal energy for different PUD systems are approximated as

$$
y \sim \begin{cases} 
N(E_s, BT \sigma^4 + 2E_s \sigma^2) & \text{for DD} \\
N(2BT \sigma^2 + E_s, 4BT \sigma^4 + 4E_s \sigma^2) & \text{for ED} \\
N(2BT \sigma^2 w^T 1 + w^T z, 4BT \sigma^4 w^T w + 4\sigma^2 w^T Z w) & \text{for WED} 
\end{cases}
$$

(21)

where $E_s$ is the signal energy, $w$ is the weighting column vector of the weighted energy detection (WED), $1$ is the column vector of ones, $z$ is the column vector including the energies from different diversity paths, $Z = \text{diag}(z)$, and diag($a$) is the diagonal matrix where vector $a$ is on the diagonal.

5.5 Nonlinear filtering models for analysis and equalization of interference

PUD systems introduce inherent nonlinearity in the signal processing in the process of co-phasing the signal. Consequently, the compensation of various types of interferences becomes
difficult with linear filters. In case the interference cannot be avoided by signal design, optimal receiver principles, nonlinear filtering models including linearization represent systematic procedures for reducing the interference or distortion caused by a nonlinear device. Linearization techniques have been traditionally used to ease the design of power amplifiers which are typically nonlinear (Katz, 2001). Linearization techniques as well as simplified nonlinear models are also very useful with the design and analysis of nonlinear PUD systems in the presence of interference. A nice framework is proposed in (Witrisal et al., 2005) for DD-type PUD systems. A second-order Volterra model (cf. Sicuranza, 1992) is proposed to describe the data dependency in the presence of ISI, whereby the nonlinearity is caused by the multiplication in the pulse-pair correlators. This Volterra model divides the problem into the linear and nonlinear counterparts to find more efficient ways to equalize the interference. Consequently, nonlinear structures can be more effectively handled. Furthermore, if the interference is not severe, the nonlinear parts may be found to have an insignificant effect on the performance. The approach can readily be extended to other type of PUD systems as well.

6. Concluding remarks

In this chapter, we have presented an overview of low complexity PUD systems which do not need carrier phase recovery at the receiver. This is an important advantage for the systems using a very high bandwidth or centre frequency. We started from the optimal MAP receiver which lead to the estimator-correlator concept. We then provided a selected snapshot of historical landmark papers of PUD systems. Furthermore, some recent trends in designing advanced PUD systems were discussed. Finally, we provided some insight for the approximation approaches to ease the analysis of PUD systems using some specific examples. The design and analysis of advanced PUD systems lags behind the corresponding coherent receivers and there is a clear need for better understanding of these systems, especially in more complicated environments. To help to achieve this goal we emphasize the importance of the estimator-correlator concept which nicely connects the approaches under the same theoretical framework. The main motivation of using the PUD systems is to lower the complexity of the transceivers not being able to recover the carrier phase information inexpensively. However, in more complicated environments with significant ISI or multiuser interference, the nonlinear operations involved with the PUD receivers may also increase the complexity of some parts of the symbol detection with respect to the coherent detection. To this end, we also emphasize the significance of interference avoidance with PUD systems by signal design. It is obvious that the PUD systems inherently tend to amplify the noise energy at the receiver. Consequently, a remaining open question is to reveal the overall trade-offs including the required transmitter signal energy, signal processing energy, and the related hardware complexity. The PUD systems have traditionally been used with low data rate applications. Due to the recent advances in both algorithm and implementation designs, the PUD systems have created much attention among academic and industrial research communities to apply PUD-based transceivers also for high data rate applications. This is true especially in case of UWB wireless short-range systems operating at 3-10 GHz as well as 60 GHz frequency ranges.

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8. References


