Coordinate-space and Observation-space Filtering Methods for Sidereally Repeating Errors in GPS: Performance and Filter Lifetime

Ahmed E. Ragheb, Peter J. Clarke, Stuart J. Edwards,
School of Civil Engineering and Geosciences, Newcastle University, *UK*

BIOGRAPHY

Ahmed Ragheb received his MSc degree from Ain Shams University, Egypt in 2004, and is now a PhD candidate at Newcastle University. His current research is concerned with filtered single-epoch GPS positioning for deformation monitoring. Dr Peter Clarke is Reader in Geophysical Geodesy, with research interests in precise GPS positioning applied to ocean tide and hydrological loading, crustal deformation and tectonics, the earthquake cycle, and post-seismic deformation. Dr Stuart Edwards is Senior Lecturer in Geodesy, researching GPS for precise tropospheric delay estimation and the improvement of GPS positioning for the monitoring of engineering structures and geohazards.

ABSTRACT

Multipath is a major error in GPS observations because it is not removed using differencing techniques. One can benefit from the repetition of satellite-receiver geometry every near sidereal day and apply filtering to minimize this error. For 1 second GPS data, a 10-hour window leads to a consistent and steady value of the geometry-repeat lag, when compared to a window as short as 2 hours giving fluctuating lag values. We conclude that there is little advantage in using a satellite-specific or other time-varying lag in double-difference processing. GPS data is filtered either by stacking and applying number of days of processed coordinate residuals ("coordinate-space filtering"), or double-difference phase residuals ("observation-space filtering"), using the optimum near-sidereal lag (23h 55m 54s). Both methodologies result in a more homogeneous set of coordinates compared with unfiltered processing. Coordinate filtering gives higher precision than observation filtering, but with similar hour-to-hour consistency. While stacking three prior days' data in a high multipath environment, the 24 hour percentage variance reduction reaches 73% for coordinate filtering, where as for observation filtering the corresponding value is 71%. However, the latter technique is advantageous in the less processing time required to achieve filtered coordinates. Thus, the optimal filtering method to use will depend on whether precision or computational time is the over-riding criterion. When using a 3-day stack to form the filter, as the time gap between the days forming the filter and the applied day increases, the final precision worsens gradually. This trend continues until about 23-30 days from the applied day, at which point the precision is the same as the unfiltered case. In competition with this "filter lifetime" effect, stacking more data improves the filter. Stacking 7 days immediately before the applied day results in the best possible precision for both coordinate-space and observation-space filtering. The former methodology reduces the variance of the 24-hour data set by 61%, while the latter has a slightly poorer variance reduction of 52%.

INTRODUCTION

Most of the errors affecting short-baseline GPS are eliminated or minimized by differencing techniques (e.g. Leick, 2004). However, multipath error will remain due to the highly site-specific nature of the reflection of GPS signals from nearby surfaces. Accordingly, multipath is often considered the most limiting factor in precise GPS positioning (e.g. Lau and Mok, 1999; Axelrad et al., 1996). Multipath could be reduced at the point of measurement using choke ring antennas or receivers incorporating hardware or firmware multipath mitigation techniques. On the other hand, multipath error may be reduced at the post-processing stage using several techniques. These include multipath maps of the GPS antenna environment containing multipath corrections for each satellite signal dependent on elevation and azimuth (Cohen and Parkinson, 1991), wavelet analysis (Satirapod and Rizos, 2005), a Vondrak filter with cross-validation (Zheng et al., 2005), or weighting the data using the observed signal to noise ratio (Lau and Mok, 1999).

Due to the nearly exact repetition of satellite geometry in the sky above a site every sidereal day (nominally 23h 56m 04s), multipath error is highly correlated across subsequent days if the same antenna and reflector environment remains constant, and it is possible to apply "sidereal filtering" techniques to mitigate multipath error (Genrich and Bock, 1992; Nikolaidis et al., 2001). Essentially, these methods subtract a filter value from the results at each epoch. The filter at a given epoch is composed from the coordinate or phase residuals with respect to the long-term mean value, at an epoch separated by a multiple of the near-sidereal error repeat interval. To improve the precision and robustness of the filter, residuals may be stacked (averaged) over several sidereal days (Ragheb et al., 2006).

Recent investigations based on satellite orbit analysis (Choi et al., 2004) have shown that the actual satellite geometry repeat interval is slightly less than the nominal sidereal period used in earlier sidereal filtering studies. Using cross-correlation within the coordinate and phase residuals, Ragheb et al. (2006) and Larson et al. (2006, http://spot.colorado.edu/~kristine/publications.html) confirmed that the overall multipath-repeat lag matches this actual satellite geometry repeat interval. However, Agnew and Larson (2006) have shown that the geometry repeat interval of an individual satellite varies at the few-second level across the constellation, with polar observing stations showing greater inter-satellite variability than equatorial ones. This work also suggested that the multipath error is most similar only on adjacent sidereal days and tends to differ as the time separation increases.

The main objective of this paper is to determine the "lifetime" during which a near-sidereal filter, generated from prior GPS data, can be used to improve the precision of receiver coordinates. In addition, we first investigate the optimal number of days' data to combine when deriving the filter, to produce the best and most consistent precision. This will be useful in all high-precision GPS applications in near-static environments with high multipath, such as the monitoring of small or slow deformations in engineering structures. In context of these investigations, we revisit our earlier questions as to the stability and consistency of the lag value for the repetition of satellite geometry, and the benefit of filtering using either coordinate residuals (coordinate-space filtering) or carrier-phase residuals (observation-space filtering) in mitigating the multipath error.

METHODOLOGY

A single epoch ambiguity resolution software called GASP (**G**PS **A**mbiguity **S**earch **P**rogram) is used to process the GPS data (Corbett, 1994; Al-Haifi, 1996). The program operates in kinematic mode, treating each epoch as an entirely independent solution and conducting a search in ambiguity space based on the L1 and L2 phase observables, validating the final ambiguity set using the F-test statistic. Because each epoch is independent, there is no possibility for common parameters to affect the level of multipath error. GASP processes baselines, either in fully-kinematic mode in which the coordinates of the "fixed" site are determined by a code pseudorange solution at each epoch, or in fixed-base mode where the "fixed" site coordinates are specified *a priori*. The GASP technique has the advantage of not requiring long GPS sessions or initialization, as well as eliminating the effect of cycle slips and the need for continuous satellite lock during receiver motion.

The day-to-day autocorrelation of either the coordinate or phase residual time series is used to determine the optimum value of the "sidereal" lag. After optimal lag determination, we filter the GPS data at this lag over one or more days' observations by either (1) stacking epoch-by-epoch coordinate residuals ("coordinate-space filtering"), or (2) stacking double difference phase residuals of each satellite pair ("observation-space filtering"). The latter residuals are obtained with respect to a fully-fixed GASP solution. Filtering is applied at the corresponding epoch of the day in question, in (1) by subtracting the residuals from the processed coordinates, or in (2) by subtracting the phase residuals from the L1 and L2 phase double difference as they are formed during processing.

The main criterion used to assess the efficiency of the sidereal filter, the optimum number of stacked days, and the filter lifetime is the repeatability (precision) of station coordinates over a certain time interval. This is done either in terms of the absolute station coordinate standard deviation or the percentage reduction in coordinate variance, calculated from the following expression:

$$VR(\%) = (1 - \frac{\sigma^2_{filter}}{\sigma^2_{unfiltered}}) \times 100 , \qquad (1)$$

where VR is the percentage of variance reduction, σ^2_{filter} is the 24-hour variance of the applied day based on the filter in question, and $\sigma^2_{unfiltered}$ is the 24-hour unfiltered variance. In addition, we consider not only the improvement in short-term coordinate precision, but also the consistency of improvement and processing time necessary to achieve final filtered coordinates. These additional criteria are important in near-real-time applications (Ragheb et al., 2006).

DATA COLLECTION AND HANDLING

Four stations on the Newcastle University campus are used, as reported in Ragheb et al. (2006), two in a low multipath environment, named **DRMN** and **DRMS**, and two in a higher multipath environment, called **SN02** and **NEWC**. The latter is part of the "Active GPS Network" of the Ordnance Survey of Great Britain (OS). The baseline lengths range between few metres and hundreds of meters. Four different GPS data sets were collected at 1 second epoch interval with a 5º elevation mask angle, in order to include low elevation satellites which generally cause higher multipath error, as is desired to test the robustness of the sidereal filtering for multipath reduction. Dual-frequency GPS code and carrier phase data were collected in irregular bursts of 4-31 days over a 15-month period, to allow a range of filter combinations and latencies to be tested (Table 1). Three Leica receivers on stations DRMN, DRMS and SN02 were used, while an Ashtech receiver is mounted on NEWC station. For all data sets, observations have been converted to RINEX format, while final and rapid precise orbit files have been obtained from the IGS website (http://igscb.jpl.nasa.gov/components/prods.html), (Neilan et al., 1997).

Table 1 Observation criteria for all collected data sets. All times are given in GPS Time

Data set	Apr_05	Dec_05	Mar_06	Jul_06
Start Time	04/04/2005 14:07:50	13/12/2005 12:35:05	22/03/2006 14:21:20	02/07/2006 08:42:11
End Time	08/04/2005 14:25:05	17/12/2005 14:16:20	21/04/2006 14:52:17	06/07/2006 11:42:17
Sites	DRMN, DRMS, NEWC, SN02	DRMN, DRMS, NEWC, SN02	DRMN, DRMS	DRMN, DRMS

Baselines were processed radially from fixed site DRMN. As the baseline length is sufficiently short (up to few hundred metres), all atmospheric, orbital, and clock errors may assumed to be removed by double-differencing, and hence our results will depend on receiver noise and multipath error only. The environment at each of the four stations can be characterised using the MP1 and MP2 code multipath proxy values produced by the UNAVCO software TEQC (Estey and Meertens, 1999). This suggests that both DRMN and DRMS stations sustain high multipath for very low elevation satellites only, whereas NEWC and SN02 suffer from high multipath over much or all of the sky.

COORDINATE AND OBSERVATION SPACE FILTERING

At first, using Apr_05 data set we sought the optimum sidereal lag (maximum autocorrelation among station coordinates), via the investigation of different window sizes (number of considered epochs). We found that using a 10-h window tends to the steadiest value of the "sidereal" lag around 86154 s (23h 55m 54s), while using a 2-h window leads to similar results but with higher variability due to the small sampling size. Figs. 1 and 2 show the sidereal lags having an autocorrelation equal to or greater than 95%, 97% and 99% of the maximum autocorrelation value at each epoch for the 2-h window with 10-h window optimum lag overlaid and the 10-h window with 2-h optimum lag overlaid respectively. The optimal 10-h lag lies nearly always within the 99% threshold, while the 2-h lag lies within the 97% threshold. Consequently, the longer-term estimate of sidereal lag is never significantly worse than the short-term one and thus we can use a constant value of the lag, as short-term variations are relatively unimportant for "sidereal" filtering, especially when using a double-differencing processing strategy (Ragheb et al., 2006).

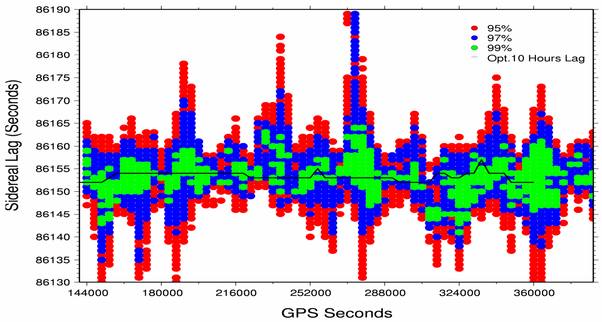

Fig. 1 2-hour window threshold as a percentage of the maximum autocorrelation. Solid line show the optimal lag for a 10-hour window (DRMS)

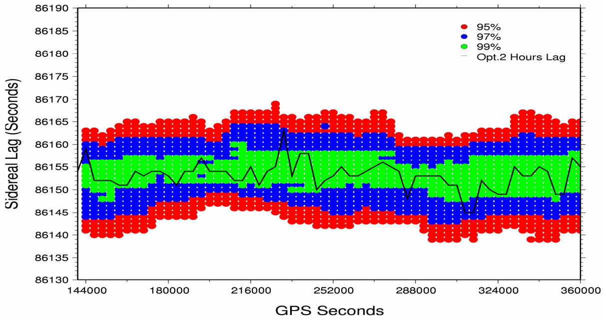

Fig. 2 10-hour window threshold as a percentage of the maximum autocorrelation. Solid line show the optimal lag for a 2-hour window (DRMS)

We formed three filters using the obtained optimum lag, by stacking one, two or three days together and then applying the stacked filter on the fourth day of the data set. In each case, the daily standard deviation dropped significantly after the filter, while stacking three days together produced the highest improvement in the coordinate precision, and thus we adopted the 3-day filter.

Table 2 gives the 3D coordinate standard deviation and percentage of variance reduction over the entire fourth day for both methodologies while stacking 3 days. Figs. 3, 4 and 5 represent the hourly 3D coordinate standard deviation of stations SN02, DRMS and NEWC respectively for unfiltered coordinates, coordinate- and observation-space filtering. The standard deviation after the filter becomes significantly smaller and more consistent at the 95% confidence level in almost every hourly window, decreasing to a roughly similar value for all three stations, regardless of the multipath environment surrounding each station. This confirms the efficiency of sidereal filtering. Statistically, coordinate filtering gives better precision than observation filtering, as evidenced by the 24-hour coordinate standard deviations, but with similar hour-to-hour precision consistency of the filtered coordinates.

Table 2 3D coordinate standard deviation over a 24-hour period using a 3-day filter

Station	Unfiltered	Coordinate Filter		Observation Filter	
	SD (mm)	SD (mm)	VR (%)	SD (mm)	VR (%)
SN02	6.9	4.9	49.6	5.1	45.4
DRMS	6.5	4.6	49.9	4.8	45.5
NEWC	9.5	4.9	73.4	5.1	71.2

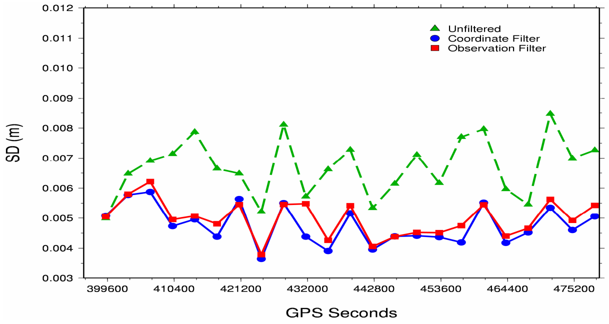

Fig. 3 SN02 hourly 3D coordinate standard deviation

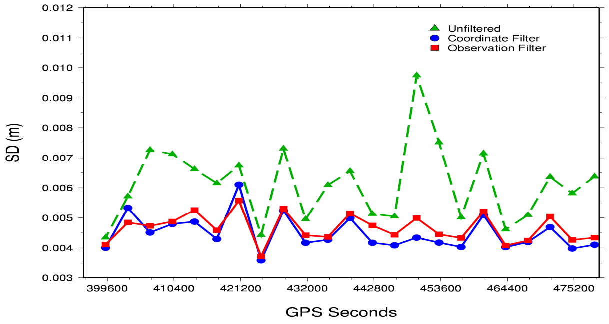

Fig. 4 DRMS hourly 3D coordinate standard deviation

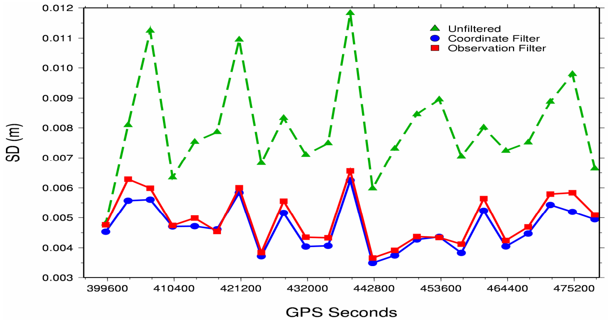

Fig. 5 NEWC hourly 3D coordinate standard deviation

On the other hand, observation-space filtering is clearly superior in the CPU time required for filter generation and application. The time required to produce coordinate residuals is much greater than that required to produce phase residuals when running GASP for the first "reference" days (about 444 s compared with 57 s for a 24-hour dataset). This is due to the fact that in the latter, GASP is run quickly for the "reference" days with fixed baseline coordinates, no least squares adjustment and without ambiguity search to output biased residuals. Fig. 6 illustrates the time required for each phase of both filtering methodologies until reaching final filtered coordinates. Another advantage of observation-space filtering arising from this figure is that the time required to produce final filtered coordinates is the same for all stations regardless of the multipath environment, which is not the case for coordinate-space filtering, as seen for NEWC station. We confirmed this analysis using the Dec_05 data set, in which almost identical results and conclusions were obtained.

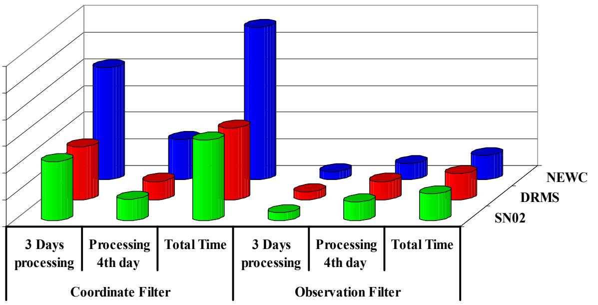

Fig. 6 GASP processing time for each step of both filtering methods on a Linux workstation

OPTIMUM FILTER STACKING

Our first objective in filter performance investigations is the determination of the optimum number of days to stack in the filter, after which stacking more days will produce similar or even worse precision. Using the Mar_06 data set only, increasing numbers of days' coordinate and phase residuals were stacked to form sidereal filters. The last day of the Mar_06 data set is chosen to be the day upon which all different stacked filters are applied, starting with one day immediately before the applied day, two days, and so on up to 21 days prior to the applied day. Fig. 7 shows the 24-hour 3D variance reduction for all 21 cases for both filtering methodologies. At first, as the number of stacked days increases, the variance reduction increases, in other words the overall precision improves. This continues until stacking 7 days together, after which the variance reduction starts to decrease gradually, i.e. the precision worsens. Table 3 shows the achieved precision for the optimal case of the 7-day stack.

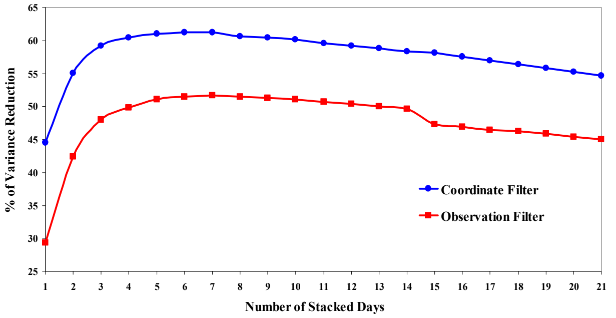

Fig. 7 Percentage of 24-h variance reduction (optimum stacked filter)

Table 3 24-h standard deviation and percentage variance reduction for the optimum (7-day) stack

Case	Unfiltered	Coordinate Filter		Observation Filter	
	SD (mm)	SD (mm)	VR (%)	SD (mm)	VR (%)
E	2.0	1.4	55.8	1.4	50.4
N	3.4	2.0	64.5	2.3	54.4
U	5.7	3.5	60.7	4.0	51.1
3D	**6.9**	**4.3**	**61.2**	**4.8**	**51.7**

The F-test statistic was again used to test the significance of the results. Except for the first four days where the filter precision increases rapidly, adding one day at a time to the stack does not improve the precision significantly (at the 95% confidence level). All stack sizes of 3 days or more are significantly better than the unfiltered case. Stacking between 5 and 8 days gives indistinguishable precision. Statistically, the optimum (highest precision) 7-day stack is significantly better than a 3- or 4-day stack, as well as the 10-day or longer stack.

OPTIMUM FILTER LIFETIME

We now turn to our main question: how long a filter established from a previous data set will last in order to be applied to a different data set and produce precision improvement. For this question, all four data sets were used. Satellites that were not common to all datasets were removed from the processing of the first three data sets, while Jul_06 data set was processed two months later. As before, the last day of the Mar_06 is the applied day.

The coordinates or carrier phase residuals of 3-day batches are stacked using the optimum lag, adopting the same method as before. 3-day batches represent a justifiable trade-off between better precision (shown above to result from 7-day stacks) and the best temporal sensitivity of the change in precision with filter age (which will result from 1-day stacks). More than one sidereal filter is formed, starting either 3 days prior to the applied day (that is the first day of the stack is three days away from the applied day), 5 days, 12 days, 19 days, 26 days, 30 days (end of Mar_06 data set), 128 days (Dec_05) and finally 381 days (Apr_05). We also formed a 72-days filter using the Jul_06 data set.

Fig. 8 illustrates the percentage of variance reduction for 3D coordinate component for all filter stacks based on coordinate and observation filtering. It can be seen that as the time gap between the produced stacked filter and applied day increases, the precision of the applied filter decreases gradually and even gets worse than the unfiltered case. Note that a negative value of variance reduction means the filtered case produced lower precision than the unfiltered one i.e. the filtering degraded the precision rather than improving it. In addition, as shown previously, coordinate filtering gives higher precision than observation filtering. For the 72-days filter, the variance reduction for coordinate filtering decreased even more than expected and was worse than the 128-days filter as well as the 72-days observation filter counterpart value. The reason for this unexpected behaviour was the processing of the Jul_06 data set – two months later - which had a different satellite constellation, with SV6 missing. This discrepancy shows another advantage of observation-space filtering other than the faster processing time over coordinate-space filtering; observation filtering was not affected by the change of constellation.

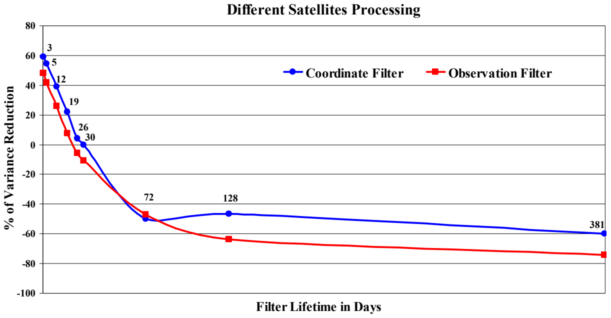

Fig. 8 Percentage of 24-h variance reduction (different constellation processing)

During phase residuals stacking of the Jul_06 data set, the filter did not include any double difference pairing involving SV6, and so when these filter values were applied to the last day of Mar_06, no carrier phase data with SV6 pairings were updated or corrected. This is then followed by a full least squares adjustment which absorbs these uncorrected errors. However, this is not the case in coordinate filtering: the coordinates of the Jul_06 data set were processed and stacked without SV6 being involved, and then applied directly – without least squares – to the coordinates of the applied day, which had SV6 included during processing. The application of coordinate residuals based on a different constellation and geometry causes a drop in precision not seen in observation filtering. Accordingly, all four data sets were re-processed using only the common satellites among them, forming the filters as before. Fig. 9 shows the filter lifetime plot using all reprocessed data sets, which follows the same path as Fig. 8, but with the expected trend in the region of the 72-day-old stack. In addition, Table 4 shows the minimum standard deviation achieved and percentage variance reduction in easting, northing, up and 3D components, for both filtering methods for the optimum (3-day-old) filter.

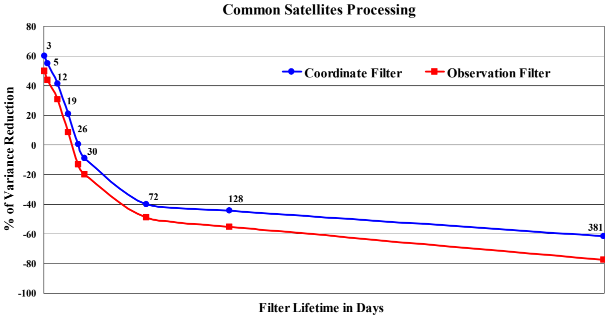

Fig. 9 Percentage of 24-h variance reduction (common constellation processing)

Table 4 24-h standard deviation and variance reduction of optimum 3-day lifetime filter

Case	Unfiltered	Coordinate Filter		Observation Filter	
	SD (mm)	SD (mm)	VR (%)	SD (mm)	VR (%)
E	2.2	1.5	55.2	1.6	47.8
N	3.5	2.2	62.6	2.4	52.4
U	6.0	3.8	59.6	4.3	49.1
3D	**7.3**	**4.6**	**59.9**	**5.2**	**49.8**

CONCLUSIONS AND RECOMMENDATIONS

In general, the "sidereal" lag tends to be steady and uniform around 23h 55m 54s using the maximum number of epochs (8-12 hours). However, variations within ±5 s of the optimal lag can be obtained using a 2-h window with 97% of the maximum autocorrelation. Such variations have slight effect on the filtering precision using double-difference processing. Coordinate filtering minimizes the multipath effect better than observation filtering, while both improving the overall precision of station coordinates. At the end, one has to decide whether to use the more accurate and homogeneous method of coordinate filtering which takes longer time, or the observation filtering with worse accuracy but less processing time and better robustness to constellation changes.

Stacking more days will increase the overall precision of final filtered station coordinates. A 7-day stack will result in the best precision, with 61% variance reduction for coordinate-space filtering and 52% for observation-space filtering, after which stacking more days will degrade the overall coordinate precision.

As the lifetime of any stacked "sidereal" filter increases, that is the increase in time difference between stacked and applied data, the efficiency of the filter decreases until about 30 days difference for coordinate filtering and 23 days for observation filtering, at which point the same precision as the unfiltered case is achieved. Any stacked filter of a larger time gap, formed after or before the applied day, will produce lower precision with respect to the case with no filter. During coordinate filtering, common satellite processing among the stacked and applied data sets is essential to achieve reliable results, while in observation filtering, any previously stacked filter can be directly applied to any data set without the need for re-processing. This can save major time-consuming processing effort.

ACKNOWLEDGEMENTS

We thank Martin Robertson (Newcastle University) for technical support, Mark Greaves and colleagues (Ordnance Survey) for timely provision of the 1 s data for NEWC. A. Ragheb is partially supported by an Overseas Research Scholarship funded by Universities UK.

REFERENCES

Agnew D, Larson K (2006). Finding the repeat times of the GPS constellation. GPS Solutions 10 (4).

Al-Haifi Y (1996). Short range GPS single epoch ambiguity resolution. PhD thesis, Department of Surveying, University of Newcastle upon Tyne, UK.

Axelrad P, Comp CJ, MacDoran PF (1996). SNR-based multipath error correction for GPS differential phase. IEEE Transactions on Aerospace and Electronic Systems 32 (2): 650-660.

Choi K, Bilich A, Larson K, Axelrad P (2004). Modified sidereal filtering: Implications for high-rate GPS positioning. Geophysical Research Letters 31(L22608).

Cohen C, Parkinson B (1991). Mitigating multipath error in GPS-based attitude determination. Guid Control Adv Astronaut Sci 74: 53-68.

Corbett S (1994). GPS single epoch ambiguity resolution for airborne positioning and orientation. PhD thesis, Department of Surveying, University of Newcastle upon Tyne, UK.

Estey L, Meertens C (1999). TEQC: The multi-purpose toolkit for GPS/GLONASS data. GPS Solutions 3 (1): 42-49.

Genrich J, Bock Y (1992). Rapid resolution of crustal motion at short ranges with the Global Positioning System. Geophysical Research Letters 97: 3261-3269.

Larson K, Bilich A, Axelrad P (2006). Improving the precision of high-rate GPS. Geophysical Research Letters (submitted).

Lau L, Mok E (1999). Improvement of GPS relative positioning accuracy by using SNR. Journal of Surveying Engineering-ASCE 125(4): 185-202.

Leick A (2004). GPS satellite surveying. New York, John Wiley & Sons Inc.

Neilan R, Zumberge J, Beutler G, Kouba J (1997). The International GPS Service: A global resource for GPS applications and research. Proc. 10th Technical meeting of the Satellite Division of the Institute of Navigation, ION GPS-97, Kansas City Convention Centre, Kansas City, USA.

Nikolaidis R, Bock Y, de Jonge P, Shearer P, Agnew D, Van Domselaar M (2001). Seismic wave observations with the Global Positioning System. Geophysical Research Letters 106 (B10): 21897-21916.

Ragheb A, Clarke P, Edwards S (2006). GPS sidereal filtering: coordinate- and carrier-phase-level-strategies. Journal of Geodesy (published online).

Satirapod C, Rizos C (2005). Multipath mitigation by wavelet analysis for GPS base station applications. Survey Review 38(295): 2-10.

Zheng D, Zhong P, Ding X, Chen W (2005). Filtering GPS time-series using a vondrak filter and cross-validation. Journal of Geodesy 79 (6-7): 363-369.