Modelling of contact deformation for a pinch gripper in automated material handling

Hua Lin\textsuperscript{a,\textdagger,1}, Paul M. Taylor\textsuperscript{a,2}, Steve J. Bull\textsuperscript{b}

\textsuperscript{a} School of Mechanical and Systems Engineering, The University of Newcastle, Newcastle upon Tyne, UK
\textsuperscript{b} School of Chemical Engineering and Advanced Materials, The University of Newcastle, Newcastle upon Tyne, UK

Received 10 March 2005; received in revised form 25 January 2007; accepted 7 February 2007

Abstract

Grasping analysis is concerned with relating the characteristics of the grasped object to the requirements of the gripper structure. When a gripper presses a flexible specimen with a size larger than that of the gripping surface, complex deformations are produced (compression, shearing, bending and tension). The boundary conditions are complex too because of the changing compressed region and, for many grippers, a changeable contact area. In this paper a mathematical predictive model is presented for simulating these complex deformations, relating a flat gripper’s performance to the properties of flexible gripped materials using elasticity theory. In particular, the distributions of stress and strain within the materials are derived when the gripper presses them. The main variables in the picking-up action have been identified. Moreover, a contact solution has been derived for calculating the contact length between a sample and a rigid support table. The change in the compressed region within a sample as a function of external load has been calculated. In order to match experimental behaviour the non-linear elastic response of the flexible material and the large deformations have to be incorporated in the model. The model predictions have been tested against experimental data and the results of finite element analysis and reasonably good agreement has been obtained.

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Keywords: Material handling; Gripper; Modelling; Complex deformations

1. Introduction

There are a number of industries that deal with the handling of flexible materials. The practical use of robotics for the automatic handling of flexible objects such as textiles has been limited given the variations in physical, mechanical and surface properties of the flexible materials [1]. Therefore, to perform the functional manipulations of the flexible materials during the whole manufacturing process, the robot must be intelligent, i.e. changing its manipulation parameters as the material properties change. This intelligence can be supported by the three components of “robotic intelligence”, sensing, actuation, and control. Intelligent control can significantly improve a robotic manipulator and it relies on a knowledge base and a suitable reasoning procedure for arriving at a control decision, which will in turn

\dagger Corresponding author.
E-mail address: hua.lin@nottingham.ac.uk (H. Lin).

1 Current address: School of Mechanical, Materials and Manufacturing Engineering, The University of Nottingham, NG7 2RD, UK.
2 Current address: R&T Mechatronics Ltd, South End, Goxhill, North Lincs, DN19 7NE, UK.

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doi:10.1016/j.mcm.2007.02.019
initiate a corresponding control action [2]. Controlling a robot to properly handle and place a limp object has been a major challenge for many years and tremendous efforts have been made towards achieving this target. Examples are:

(a) Determining the importance of the different fabric properties to the operation of a variety of grippers [3,4], and complete manufacturing systems [5];
(b) Developing on-line approximations to the measurements of the critical properties [6];
(c) Exploring new control strategies for robotic handling of flexible materials [7] and
(d) Studying the dependence of fabric properties on environmental conditions [8].

A variety of grippers has been proposed and developed for handling the flexible materials and these have been reviewed in detail [9]. A number of different sensing techniques have been introduced into automatic flexible material handling systems, such as vision sensors [10,11] and tactile sensors [12,13], the aim being to enhance the reliability, flexibility and accuracy of handling operations. Many integrated “unit handling” systems have been proposed and invented for composite manipulation and garment handling [14–18], but in practice they usually work satisfactorily for just a limited range of materials. It was realized that the physical deformation behaviour of the flexible materials when they are being handled and the complex interactions between the material properties and handling devices would play a very important role in the automatic handling [19–24]. Taylor mentioned that an understanding of the interaction between automation systems and material properties is fundamental for automated limp material handling [25]. Knowledge-based systems are required to control highly flexible automated devices for handling limp materials.

This study focuses on picking and placing of pieces of flexible material, one of the most frequently occurring processes in the automated flexible material handling line. This operation can be carried out using pinch grippers comprising two pegs that push down on the top of the material. The pegs are then brought together so that the material buckles up and is secured between them.

Clearly, such a grasping operation is dependent on the properties of the material being grasped and its interaction with the gripping surfaces. The interaction is controlled by numerous parameters—e.g. the gripper design, the properties of the materials being gripped, the external applied loads and environmental conditions [9]. Fig. 1 shows that picking and placing of flexible material depends on three factors.

This area has still not been numerically defined because of the complexity of the interactions between flexible material and gripper systems. The physical or mechanical properties of a flexible material tend to be highly non-linear and so, in general, the interaction between the flexible material and gripper also tends to be highly non-linear. Owing to an incomplete mathematical understanding of these non-linear interactions, it is unrealistic to perform intelligent picking and placing of flexible material at present.

### Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$E$</td>
<td>Young’s Modulus.</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio.</td>
</tr>
<tr>
<td>$a$</td>
<td>Half width of indenter.</td>
</tr>
<tr>
<td>$a_0$ to $a_{14}$</td>
<td>Integrating constants.</td>
</tr>
<tr>
<td>$b$</td>
<td>The contact half-length between a sample and a table</td>
</tr>
<tr>
<td>$G$</td>
<td>Modulus of elasticity in shear.</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of sample.</td>
</tr>
<tr>
<td>$L_y$</td>
<td>The half-length of sample subjected to the load at any layer.</td>
</tr>
<tr>
<td>$N$</td>
<td>External load.</td>
</tr>
<tr>
<td>$q$</td>
<td>Distributed load $\left(\frac{N}{a}\right)$ at the top surface of sample.</td>
</tr>
<tr>
<td>$\sigma_x, \sigma_y$</td>
<td>Normal components of stress parallel to $x, y$ axes respectively.</td>
</tr>
<tr>
<td>$\gamma_{xy}$</td>
<td>Shear stress in the $x y$ plane.</td>
</tr>
<tr>
<td>$\gamma_{xy}$</td>
<td>Shear strain in the $x y$ plane.</td>
</tr>
<tr>
<td>$\varepsilon_x, \varepsilon_y$</td>
<td>Normal strains in the $x$ and $y$ directions.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Stress function.</td>
</tr>
<tr>
<td>$u, v$</td>
<td>Displacement in $x$ and $y$ directions respectively.</td>
</tr>
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</table>
It is important to minimize the necessary gripping force to avoid unnecessary stress and deformation on both the object and the robotic gripper. It is therefore necessary to establish a relationship between the load, displacement and geometry of the gripper and the behaviour of the materials being handled in order to achieve an optimal gripping process.

In this study, the first step of the picking-up action, the peg pushing down on the top surface of materials, is investigated. The aim is to develop a model for predicting/simulating the interaction between the performance of the pinch grippers, the external load and the properties of material to be grasped employing mathematical analysis in order to yield a scientific understanding of the picking action. A potential application for this study is in the design of an intelligent gripping system for flexible materials. This would allow designers to gain knowledge of the influence of the interaction on the performance of flexible material picking devices and then assist them in the design of control/adjustment features through a knowledge-based control or intelligent gripping system.

2. Mathematical modelling

2.1. Statement of the problem

Consider a 2D section through a long rectangular indenter pressing on a rectangular foam block of much larger size as illustrated in Fig. 2, where a rectangular sample (foam) is under vertical uniform load on its top surface by a finite flat punch indenter. The uniformity of the load distribution is an approximation backed up by the results of the FE analysis, see Fig. 5(b). Away from the punch the two sides are free to move. However, this movement is limited by the rigid table beneath the foam. The bottom surface of the foam keeps in contact with the support over a distance which is greater than the punch width. The material under the indenter is compressed and the two sides can deflect as the area compressed increases from the top surface to the bottom. This figure also shows that the deformation in the sample does not only include compression but also bending, shearing and tension, making it different from a general compression problem. In fact, it should be considered as a planar deflection problem, which is main novel feature of our current analysis. The sample dimensions, coordinate axes and loading are shown in Fig. 3.

2.2. Approach and assumptions

The foams used here behave in an elastic and non-linear manner as demonstrated by previous research [26]. The model is therefore developed based on the theory of elasticity. Additionally, the deformation of the foam tested is up to 12 mm. The elasticity theory and the Hertz elastic contact solutions were derived for linear and small strains. Here, the non-linear elastic material and large strain behaviour is approximated by piecewise linear spans of measured data. In each span, we assume that the generalized Hooke’s law [27] is valid. From span to span, the elastic parameters of the material change as a function of the deformation history.

The foams are assumed to be elastic, isotropic, continuous and homogeneous, although experimental measurements indicate a small degree of inhomogeneity in the foams tested. To simplify the problem, the following three assumptions need to be made:

(1) \( x = 0 \) is a plane of symmetry for the indenter, the sample, the external load, the displacements and the relevant stress components in the sample.
(2) The contact region between the sample and the indenter is assumed to be frictionless and the body force is negligible so only the external load is therefore transmitted.
(3) There is a neutral section between the compressed area and the lifted part.

2.3. Boundary conditions

1. Along the top surface of the sample, \(-a \leq x \leq a, y = -h\), the intensity of the continuously distributed load is \(q\).
2. At any layer of the sample, \(-L_y \leq x \leq L_y\) \((L_y\) is the half-length of sample subjected to the load at any layer, \(a \leq L_y \leq b\), \(-h \leq y \leq 0\), the sample has negligible mass. Using force equilibrium conditions, we have
   \[
   \int_{-L_y}^{L_y} \sigma_y \, dx = 2qa. 
   \]
3. Along the bottom of the sample, \(-b \leq x \leq b, y = 0\), the displacements in the \(y\) direction are zero.
4. At the hypothetical zero plane loading section, i.e., \(x = \pm b, -h \leq y \leq 0\),
   \[
   \int_{-h}^{0} \sigma_y (b, y) \, dy = 0 \quad \int_{-h}^{0} \sigma_x (b, y) \, dy = 0 \quad \int_{-h}^{0} \sigma_x (b, y) \, y \, dy = 0. 
   \]
5. There is a symmetry about the \(y\) axis. Thus, along \(x = 0, -h \leq y \leq 0\),
   \[
   u = 0 \quad \text{and} \quad \frac{dv}{dx} = 0. 
   \]

2.4. Principle of virtual work

The conditions for equilibrium of a body require that for any compatible, small, virtual displacements which satisfy the essential boundary conditions imposed on the body, the total virtual work is equal to the total external virtual work with a number of conditions to be met: (a) displacements should be compatible and continuous, (b) they must satisfy the displacement boundary conditions.
2.5. Development of stress component functions

Consider the foam compressed by a pinch gripper shown in Figs. 2 and 3. In Fig. 2, several lines were drawn on the sample in order to observe the deformation phenomenon and trend. In Fig. 4, the sample is divided into 5 layers in order to measure the displacement at different heights of the sample and to compare the experimental data with the predicted results. The trend of the experimental displacements in the x-direction is found to be parabolic, and it goes up as y decreases. The shear deformations increase as x increases. The stress in the y-direction and the shear stress can therefore be described approximately by the following expressions

\[ \sigma_y = a_0 + a_1 y + a_2 x^2 + a_3 x^2 y \]  
\[ \tau_{xy} = b_3 x + a_4 x^3 + a_5 y \]  \hspace{1cm} (1)

where \(a_0-a_5\), \(b_3\) are integration constants which will be determined by the stress boundary conditions and the force equilibrium conditions. The constants \(a_0-a_5\), and \(b_3\) are the same in each layer, with continuity of displacement and traction enforced at the interface.

\(\phi\), a stress function, may be obtained from these stress components since they are functions of its derivatives, i.e. [28]

\[ \sigma_x = \frac{\partial^2 \phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 \phi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}. \]  \hspace{1cm} (3)

Integrating Eq. (1) once with respect to \(x\) gives

\[ \frac{\partial \phi}{\partial x} = \int \sigma_y \, dx = \frac{1}{3} a_2 x^3 + \frac{1}{3} a_3 x^3 y + f_1(y) \]  \hspace{1cm} (4)

in which \(f_1(y)\) is an as yet unknown function of \(y\) only. It can be obtained by the following derivation.

Differentiating Eq. (4) once with respect to \(y\) gives

\[ \frac{\partial^2 \phi}{\partial x \partial y} = a_1 x + \frac{1}{3} a_3 x^3 + \frac{d f_1(y)}{dy}. \]  \hspace{1cm} (5)

Equating Eqs. (2) and (5), produces

\[ -a_1 x - \frac{1}{3} a_3 x^3 - \frac{d f_1(y)}{dy} = b_3 x + a_4 x^3 + a_5 y \]  \hspace{1cm} (6)

and comparing the two sides of Eq. (6), produces

\[ b_3 = -a_1 \]
\[ a_4 = -\frac{1}{3} a_3 \]
\[ a_5 y = -\frac{d f_1(y)}{dy} \]  \hspace{1cm} (7)

where \(f_1(y)\) can be obtained by integrating Eq. (7) once with respect to \(y\)

\[ f_1(y) = -\int a_5 y \, dy = -\frac{1}{2} a_5 y^2 + a_6. \]  \hspace{1cm} (8)

Substituting Eq. (8) in Eq. (4), gives

\[ \frac{\partial \phi}{\partial x} = \int \sigma_y \, dx = \frac{1}{2} a_0 x^2 + \frac{1}{2} a_1 x^2 y + \frac{1}{12} a_2 x^4 + \frac{1}{12} a_3 x^4 y - \frac{1}{2} a_5 y^2 x + a_6 x + f_2(y) \]  \hspace{1cm} (9)

Integrating Eq. (9) once with respect to \(x\), the stress function \(\phi\) is given in the form

\[ \phi = \int \int \sigma_y \, dx = \frac{1}{2} a_0 x^2 + \frac{1}{2} a_1 x^2 y + \frac{1}{12} a_2 x^4 + \frac{1}{12} a_3 x^4 y - \frac{1}{2} a_5 y^2 x + a_6 x + f_2(y) \]  \hspace{1cm} (10)

where \(f_2(y)\) is also an as yet unknown function of \(y\) only.
According to the principle of virtual work the displacements should be compatible and continuous. Therefore, the stress function \( \phi \) must satisfy compatibility equation [28] for an isotropic, homogeneous material in the absence of body forces

\[
\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0.
\]

Using the given form for stress function \( \phi \), it means that

\[
\frac{\partial^4 \phi}{\partial x^4} = 2a_2 + 2a_3 y \\
2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} = 0 \\
\frac{\partial^4 \phi}{\partial y^4} = f_2^{(4)}(y).
\]

Substituting these into the compatibility equation of large deflections, gives

\[
\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 2a_2 + 2a_3 y + f_2^{(4)}(y) = 0 \tag{11}
\]

where \( f_2^{(4)}(y) \) can be obtained by integrating Eq. (11) four times with respect to \( y \)

\[
f_2(y) = -\frac{1}{60} a_3 y^5 - \frac{1}{12} a_2 y^4 + \frac{1}{6} a_7 y^3 + \frac{1}{2} a_8 y^2 + a_9 y + a_{10}. \tag{12}
\]

The stress function \( \phi \) can be finally obtained by substituting Eq. (12) in Eq. (10) in the form

\[
\phi = \frac{1}{2} a_0 x^2 + \frac{1}{2} a_1 x^2 y + \frac{1}{12} a_2 x^4 + \frac{1}{12} a_3 x^4 y - \frac{1}{2} a_5 x y^2 + a_6 x \\
- \frac{1}{60} a_3 y^5 - \frac{1}{12} a_2 y^4 + \frac{1}{6} a_7 y^3 + \frac{1}{2} a_8 y^2 + a_9 y + a_{10}. \tag{13}
\]

The corresponding stress system can be therefore deduced by using Eq. (3)

\[
\sigma_x = \frac{\partial^2 \phi}{\partial y^2} = -a_5 x - \frac{1}{3} a_3 y^3 - a_2 y^2 + a_7 y + a_8 \tag{14}
\]

\[
\sigma_y = \frac{\partial^2 \phi}{\partial x^2} = a_0 + a_1 y + a_2 x^2 + a_3 x^2 y \tag{15}
\]

\[
\tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = -a_1 x - \frac{1}{3} a_3 x^3 + a_5 y. \tag{16}
\]

The stress fields in the foam compression are now complete in these primitive forms; the integration constants \( a_0 \)–\( a_8 \) are determined from the stress boundary conditions. They are

\[
a_0 = \frac{3qa}{2b} \\
a_1 = \frac{3qa}{2bh} - \frac{q}{h} = \frac{a_0 - q}{h} \\
a_2 = -\frac{3qa}{2b^2} = -\frac{a_0}{b^2} \\
a_3 = -\frac{3qa}{2hb^3} = -\frac{a_0}{hb^2} \\
a_5 = -\frac{N}{h^2}
\]
\[ a_7 = \frac{7ha_0}{10b^2} \]
\[ a_8 = \frac{h^2a_0}{10b^2} - \frac{Nb}{h^2}. \]

Substituting these values of \( a_2, a_3, a_5, a_7, a_8 \) in Eq. (14), \( a_1, a_2, a_3 \) in Eq. (15), and \( a_1, a_3, a_5 \) in Eq. (16), respectively, the three stress components are finally given by the following expressions:

\[ \sigma_x = \frac{a_0}{3hb^2}y^3 + \frac{a_0}{b^2}y^2 + \frac{7a_0h}{10b^2}y + \frac{h^2a_0}{10b^2} - \frac{Nb}{h^2} + \frac{N}{h^2}x \]  \hspace{1cm} (17)

\[ \sigma_y = a_0 + \frac{(a_0 - q)}{h}y - \frac{a_0}{b^2}x^2 - \frac{a_0}{hb^2}x^2y \]  \hspace{1cm} (18)

\[ \tau_{xy} = \frac{a_0}{3hb^2} - \frac{(a_0 - q)}{h}x - \frac{N}{h^2}y \]  \hspace{1cm} (19)

where \( a_0 = \frac{3qa_a}{2b} \).

### 2.6. Development of displacement functions

The displacements in the \( x \)-direction and in the \( y \)-direction, \( u \) and \( v \), and in-plane strains are related by the following formulae [28]

\[ \varepsilon_x = \frac{\partial u}{\partial x} \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \]

and

\[ \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \quad \gamma_{xy} = \frac{1}{G}\tau_{xy}. \]

The functions of displacements, \( u \) and \( v \) are therefore derived as follows:

\[ \varepsilon_x = \frac{\partial u}{\partial x} \]
\[ = \frac{1}{E} \left( -a_2y^2 - \frac{1}{3}a_3y^3 - a_5x + a_7y + a_8 \right) \]
\[ \text{(Note: } \nu = 0 \text{ for the foam used here and using Eq. (14) for } \sigma_x,) \]
\[ = \frac{1}{E} \left( -a_2y^2 - \frac{1}{3}a_3y^3 - a_5x + a_7y + a_8 \right) \]
\[ \text{ (using Eq. (15) for } \sigma_y) \]  \hspace{1cm} (20)

and

\[ \varepsilon_y = \frac{\partial v}{\partial y} \]
\[ = \frac{1}{E} \left( a_0 + a_1y + a_2x^2 + a_3x^2y \right) \]
\[ \text{ (using Eq. (16) for } \sigma_y) \]  \hspace{1cm} (21)

Integrating Eq. (20) in the \( x \)-direction and Eq. (21) in the \( y \)-direction respectively, the displacement functions are given by

\[ u = \int \frac{\partial u}{\partial x} \, dx \]
\[ = \frac{1}{E} \left( -\frac{1}{3}a_3xy^3 - a_2xy^2 + a_7xy - \frac{a_5}{2}x^2 + a_8x \right) + u_1(y) \]  \hspace{1cm} (22)

\[ v = \int \frac{\partial v}{\partial y} \, dy \]
\[ = \frac{1}{E} \left( a_0y + \frac{1}{2}a_1y^2 + a_2x^2y + \frac{1}{2}a_3x^2y^2 \right) + v_1(x) \]  \hspace{1cm} (23)
in which \( u_1(y) \) and \( v_1(x) \) are as yet unknown functions of \( y \) only and \( x \) only. They can be obtained by the following method:

\[
\frac{\partial u}{\partial y} = \frac{1}{E} (-a_3 xy^2 - 2a_2 xy + a_7 x) + \frac{du_1(y)}{dy} \quad (24)
\]

\[
\frac{\partial v}{\partial x} = \frac{1}{E} (2a_2 xy + a_3 y^2) + \frac{dv_1(x)}{dx}. \quad (25)
\]

Adding Eqs. (24) and (25)

\[
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{1}{E} a_7 x + \frac{du_1(y)}{dy} + \frac{dv_1(x)}{dx}. \quad (26)
\]

Employing the relation \( \tau_{xy} / G = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \) and using Eq. (16) for \( \tau_{xy} \), we have

\[
\frac{1}{E} a_7 x + \frac{du_1(y)}{dy} + \frac{dv_1(x)}{dx} = \frac{1}{G} \left( -a_1 x - \frac{1}{3} a_3 x^3 + a_5 y \right) \quad (26)
\]

where \( G = \frac{E}{2(1+v)} = \frac{E}{2} (v = 0 \text{ for the foam}) \).

Eq. (26) can be rewritten as follows

\[
-\frac{du_1(y)}{dy} + \frac{2}{E} a_5 y = \frac{1}{E} a_7 x + \frac{dv_1(x)}{dx} + \frac{2}{E} \left( a_1 x + \frac{1}{3} a_3 x^3 \right). \quad (27)
\]

In Eq. (27), some terms are function of \( x \) only, some are functions of \( y \) only. There are no mixed terms. Denoting these groups by \( F(x), G(y) \), yields

\[
F(x) = -\frac{du_1(y)}{dy} + \frac{2}{E} a_5 y
\]

\[
G(y) = \frac{1}{E} a_7 x + \frac{dv_1(x)}{dx} + \frac{2}{E} \left( a_1 x + \frac{1}{3} a_3 x^3 \right).
\]

These equations imply that \( F(x) \) must be a constant and \( G(y) \) must also be a constant. Otherwise, \( F(x) \) and \( G(y) \) would vary with \( x \) and \( y \), respectively, and by varying \( x \) alone, or \( y \) alone, the equality would be violated. Thus

\[
-\frac{du_1(y)}{dy} + \frac{2}{E} a_5 y = \frac{1}{E} a_7 x + \frac{dv_1(x)}{dx} + \frac{2}{E} \left( a_1 x + \frac{1}{3} a_3 x^3 \right) = a_{12}.
\]

Hence, equations of \( x \)-only and \( y \)-only are obtained

\[
\frac{du_1(y)}{dy} = \frac{2}{E} a_5 y - a_{12} \quad (28)
\]

\[
\frac{dv_1(x)}{dx} = a_{12} - \frac{1}{E} a_7 x - \frac{2}{E} \left( a_1 x + \frac{1}{3} a_3 x^3 \right). \quad (29)
\]

Functions of \( u_1(y) \) and \( v_1(x) \) are then obtained by integrating Eqs. (28) and (29) with respect to \( y \) and \( x \), respectively,

\[
u_1(x) = \int \frac{dv_1(x)}{dx} dx = a_{12} x - \frac{1}{2E} a_7 x^2 - \frac{1}{E} a_1 x^2 - \frac{1}{6E} a_3 x^4 + a_{13}. \quad (31)\]

By substituting Eqs. (30) and (31) into Eqs. (22) and (23) respectively. The functions of displacement, \( u \) and \( v \), in the \( x \) and the \( y \) directions can be expressed in the forms:

\[
u = \frac{1}{E} \left( -\frac{1}{3} a_3 x^3 - a_2 xy^2 - \frac{a_5}{2} x^2 + a_5 y^2 + a_7 xy + a_8 x \right) - a_{12} y + a_{14} \quad (32)\]
\[
v = \frac{1}{E} \left( a_0 y + \frac{1}{2} a_1 x^2 + a_2 x^2 y + \frac{1}{2} a_3 x^2 y^2 \right) + a_{12} x - \frac{1}{2E} a_7 x^2 - \frac{1}{E} a_1 x^2 - \frac{1}{6E} a_3 x^4 + a_{13}.
\]

Coefficient \(a_{12}a_{13}a_{14}\) in these functions are determined from the displacement boundary conditions, they are
\[
a_{12} = 0 \quad a_{13} = 0 \quad a_{14} = -\frac{1}{E} a_5 y^2.
\]

Finally, the displacement functions, \(u\) and \(v\), can be obtained by eliminating \(a_{12}\) and \(a_{14}\) in Eq. (32), \(a_{12}\) and \(a_{13}\) in Eq. (33) in the forms
\[
u = \frac{1}{E} \left[ \frac{a_0}{3hb^2} x y^3 + \frac{a_0}{b^2} x y^2 + \frac{N}{2h^2} x^2 + \frac{7a_0 h}{10b^2} x y + \left( \frac{a_0 h}{10b^2} - \frac{Nh}{h^2} \right) x \right]
\]

\[
v = \frac{1}{E} \left[ \frac{a_0}{6hb^2} x^4 - \frac{a_0}{2hb^2} x^2 y^2 + \frac{1}{2} \left( \frac{a_0 - q}{h} \right) y^2 - \frac{a_0}{b^2} x^2 y - \left( \frac{7a_0 h}{20b^2} + \frac{a_0 - q}{h} \right) x^2 + a_0 y \right]
\]

where \(a_0 = \frac{3qa}{2b}\).

A conclusion drawn from Eqs. (34) and (35) is that \(u\) and \(v\) are both non-linear functions of \(x\) and \(y\), namely any section of the foam is no longer a vertical or horizontal section after it deforms, which is consistent with the image of the pictured in Fig. 2.

2.7. Determining the contact length (b) between sample and table

Eqs. (34) and (35) contain an important parameter \(b\), the contact half-length between the sample and the rigid support. This leads to a contact problem between an elastic body and a rigid body. The contact is maintained only by compressive stress. An equation can be derived to determine the contact length, based on Eq. (35).

According to boundary condition 3: \(v = 0, (y = 0, x = b)\), Eq. (35) becomes
\[
v = -\frac{(a_0 - q)b^2}{h} - \frac{7ha_0}{20b^2} b^2 + \frac{a_0}{6hb^2} b^4 = 0
\]
\[
21h^2a - 50ab^2 + 40b^3 = 0.
\]

2.8. Determining the length (\(L_y\)) of sample subjected to the load at any layer

The regime compressed under the indenter increases from the top surface to the bottom, namely, \(L_y\) increases from the width of the indenter ‘\(a\)’ to the contact length ‘\(b\)’. \(L_y\) is derived as follows:

According to the boundary condition 2, we have
\[
\int_0^{L_y} \sigma_y dx = qa
\]
\[
a_0 L_y + \frac{a_0 - q}{h} L_y y - \frac{a_0}{3b^2} L_y^3 - \frac{a_0}{3hb^2} L_y^3 y = qa
\]

\(L_y\) in Eq. (37) may be obtained by a numerical approximation method, such as Newton’s method:

\[
L_{yn} = L_{yn-1} - f(x)/f'(x)
\]

where \(f(x) = \left( \frac{a_0}{3hb^2} y + \frac{a_0}{3b^2} \right) L_y^3 - \left( a_0 + \frac{a_0 - q}{h} y \right) L_y + qa
\]

\[
f'(x) = \frac{df(x)}{dL_y} = \left( \frac{a_0}{hb^2} y + \frac{a_0}{b^2} \right) L_y^3 - a_0 - \frac{a_0 - q}{h} y.
\]
3. Experimental methods

Compression tests were carried out on a compression rig, which is pictured in Fig. 2. The rectangular indenter (60 × 15 × 10 mm), length, thickness and height respectively, is driven downwards by a stepper motor to apply pressure on the foam. The compressive displacement is increased at steps of 0.26 mm, and corresponding loads are recorded with a load cell after waiting 20 s from each increase in displacement, to minimize the effects of creep. Each test was repeated five times with a new sample. All the tests were carried out at 20 ± 1 °C temperature and 65 ± 3% relative humidity.

The polyurethane (PU) foam samples were cut to 120×60×25 (mm) length, thickness and height, respectively with a density of 33.51 × 10^3 g/m^3. Five lines were ruled on the sample horizontally in order to measure the displacements of different sections (Fig. 4). A digital video camera was used to record images of the deformation of the samples, and then the images were analysed to determine the layer displacements and strains.

4. Verification of proposed model via numerical modelling and testing

Numerical simulation has been undertaken using the ANSYS finite element package based on the test geometry of Fig. 1. In this numerical model, PLANE 82 2-D 8-node structure solid elements were used for the entire model. MELAS, i.e., the multilinear elasticity data table was employed to describe the stress–strain curve of the foam in compression. A static structure analysis was performed with geometric non-linearities and material non-linearities using the ANSYS 5.5.1 software [23]. The results are used here to verify the mathematical model.

4.1. The theoretical and FE results of stress components

The identification of the significant stresses and strains and their distributions developed within the sample subjected to compression is of interest as it would allow better understanding of the action of picking-up; the results of the stress analysis can be used to set limits to protect the sample from damage when it is being handled. Furthermore, the stress and strain analysis would also be of benefit in optimizing the gripper design.

4.1.1. Stress in y(σ_y)

Using Eq. (18), the distribution of σ_y at different layers is shown in Fig. 5(a). The FE results for σ_y are shown in Fig. 5(b). A number of interesting observations can be made:

- The distribution of σ_y is uniform at the top surface for the mathematical modelling and approximately uniform for the FE analysis.
- σ_y decreases as x increase at other layers. The reason for this is the fact that the shear stress component increases in the x direction.
- σ_y decreases through the thickness of the sample due to the increase of the length over which the force acts from the top surface to the bottom.
- The values of σ_y are zero at the free edges and also at the bottom (x = ±b); beyond this point the foam is no longer in contact with the substrate.

4.1.2. Stress in x(σ_x)

Using Eq. (17), the distribution of σ_x is shown in Fig. 6(a). The FE results for σ_x are given in Fig. 6(b). The values of σ_x are relatively small in both simulations and can be ignored in most cases. This is because the external load is only in the y direction.
4.1.3. Shear stress ($\tau_{xy}$)

Using Eq. (19), Fig. 7(a) shows that the trend of shear stress distribution agrees well with both the observed shear deformation (Fig. 2) and the FE simulation (Fig. 7(b)). In Fig. 2, the initial horizontal lines become curved in response to the sharp corners of the indenter shearing the specimen as they are pressed into the foam. The slopes of these concave curves increase in the $x$ direction and decrease from the top surface to the bottom. The modelled shear stress also increases from the central section to the edge of compressed region and decreases with the thickness of sample. The maximum shear stresses occur at the interface of the indenter corner and the sample, which can explain why those grids near the indenter corners are severely distorted in the experimental image (Fig. 2).

In summary, reasonable agreements have been obtained between the results of mathematical model, experimental observations and FE analysis for the distributions of stress $\sigma_y$, $\sigma_x$ and $\tau_{xy}$. The mathematical models tend to overestimate the stress components of $\sigma_y$ and $\sigma_x$ at every layer, typically by 35%, which could arise from the assumption of zero friction between the sample and the indenter. The analytically predicted shear stresses, $\tau_{xy}$, are quite close to those from the FE analysis, apart from the shear stress concentration area.

4.2. Calculated and measured contact length ($b$)

The contact width ‘$b$’ is determined by two factors in Eq. (36). They are the thickness of sample ‘$h$’ and the width of the indenter ‘$a$’.

Experiments were performed on foam with different thicknesses to examine the effect of sample thickness ‘$h$’ on the contact width ‘$b$’ in Eq. (36) using a rectangular indenter with a fixed width ‘$a$’. Fig. 8 compares the results.
from Eq. (36) to the experimental data. This comparison indicates that Eq. (36) underestimates the contact width ‘b’, typically by about 8%.

Experiments were also carried out on foam using different widths of rectangular indenter to look at the effect of indenter width ‘a’ on the contact width ‘b’ in Eq. (36) for a fixed sample thickness ‘h’. The comparison of the calculated ‘b’ using Eq. (36) with the measured data shows that the predicted values of ‘b’ are also smaller than in the test (Fig. 9), typically by about 23%.
4.3. Calculated and measured displacements

Fig. 10 compares the theoretical and experimental vertical displacement directly beneath the indenter for every layer. It can be seen that reasonable agreement between theory and experiment is achieved in all cases. The prediction accuracy for the first stage of compression is better than for the second stage. A possible explanation is that the foam behaves in a linear elastic manner in the first stage of compression, in the second stage, the relationship of stress and strain is still elastic, but is highly non-linear.

The results predicted from the upper layers are closer the test data than those from lower layers. The difference may be attributed to the errors of measurement since it is very difficult to measure a displacement as small as 0.1 mm or less by the technique used.
5. Conclusions

In order to model the relationship between the indentation response and the properties of materials, a mathematical model has been developed. The theoretical analyses reveal that when the peg of a pinch gripper pushes down on the top of a flexible material sample the pressure of the peg is divided into three stress components within the materials, i.e. the normal stress along the height of the sample, the normal stress along the length of the sample and the shear stress. These stress components produce complex deformations that depend on the width of indenter, the external load and the behaviour of the material.

The model can sufficiently characterize the experimental behaviour in the problem considered in that:

- It gives the distributions of the stresses and displacements, which could help us to develop a theoretical understanding of the deformation mechanisms of flexible materials being pressed by a flat indenter.
- It links the deformations observed to the stress–strain characteristic of the material being compressed.
- It describes the non-linear behaviour and non-uniformity of compression.
- It determines the length of contact between the sample and the table and the change of area being compressed from the top surface to the bottom under a rectangular indenter.

The model proposed in this paper provides a fundamental understanding for the first step of the flexible materials grasping process using a robotic gripper. The grasping force can now be adjusted systematically depending on the properties of the materials to be grasped based on this model. Further research on the other stages of the grasping operation are needed to fully characterize the process.

References


