Predictive Approaches to Rear Axle Regenerative Braking Control in Hybrid Vehicles

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Abstract—We consider the problem of rear axle regenerative braking maximization in hybrid vehicles. We focus on cornering maneuvers on low friction surfaces, where excessive braking at the rear axle might induce vehicle instability.

We present and compare two predictive control approaches, where the objective is maximizing the regenerative braking and distributing the friction braking at the four wheels, while (i) delivering the braking force requested by the driver, (ii) preserving vehicle stability and (iii) fulfilling system constraints, e.g., bounds on regenerative braking set by the hybrid powertrain.

We present simulation results in combined braking and cornering manoeuvres, showing that the proposed approaches are able to effectively balance regenerative and friction braking, while preserving vehicle stability.

I. INTRODUCTION

In hybrid vehicles, regenerative braking is used in order to recover energy when vehicle brakes. Energy is recovered by converting the vehicle kinetic energy into electric energy to be stored in electricity buffers, i.e., batteries or capacitors. The recovered energy can then be used for powering the vehicle, thus reducing the fuel consumption. In this paper, we consider hybrid drivelines where the regenerative braking is delivered at the rear axle and focus on the implications of the regenerative braking on the vehicle stability.

While regenerative braking maximization on high friction surfaces is not involving from a control point of view, maximizing the regenerative braking on slippery surfaces, while preserving the vehicle stability, is a challenging nonlinear control problem with constraints [6]. We first observe that the force, which can be delivered through the regenerative braking, is bounded by the hybrid driveline. Hence, a driver’s braking request might have to be delivered only through a combination of regenerative and friction braking. Secondly, excessive braking on slippery surfaces might (i) lead to wheels lock up and (ii) induce undesired yaw moment in cornering maneuvers [5]. In both cases, vehicle instability might occur and Anti-lock Braking Systems (ABS) or yaw stability controls would probably stop the regenerative braking in order to prevent wheels lock-up and vehicle spinning, respectively.

In [8] the authors focus on point (i) and propose a wheel slip-based approach for maximizing regenerative braking at the rear axle, while preventing wheels’ lock up. The controller consists of three main parts, a feedback term based on a modified PID and an optimizer consisting of an extremum seeking control. The first two components of the controller guarantee closed loop stability and the third component maximize the regenerative braking at the rear axle while maintaining a stable operating point.

In this paper, instead, we focus on the effects of regenerative braking at the rear axle on the yaw dynamics (i.e., point (ii)). We consider testing scenarios where the driver demands a braking force while the vehicle is cornering on slippery surfaces, e.g., ice. The control objective is to maximize the energy recovery (i.e., the regenerative braking), while (i) delivering the requested braking force by introducing front and rear friction braking, if necessary, and (ii) preventing the activation of vehicle yaw stability systems. As explained next in the paper, the constraint (ii) can be reformulated as a constraint on the vehicle state.

We show how the considered control problem can be effectively formulated as a Model Predictive Control (MPC) problem. In particular, we define a cost function and a set of design constraints in order to achieve our control objectives. Every time step, based on the demanded braking force and measurements of the vehicle yaw rate, longitudinal and lateral velocities, we repeatedly solve a constrained optimization problem in order to find the braking policy minimizing the cost function while fulfilling design and system constraints. As shown in [1] for a similar vehicle control application, such control approach can be highly computational demanding and even prevent real-time implementation. In order to implement our MPC algorithms in real-time, we resort to the low complexity MPC formulation used in [3], [4], [2] to solve autonomous path following problems.

We present simulation results showing how the coordination of the friction braking torques at the four wheels can be used to counteract the undesired effects of the regenerative braking at the rear axle. The paper is organized as follows: in Section II, we derive simplified vehicle models used next, in Section III, to formulate the MPC problem. In Section IV we present simulation results, while Section V closes the paper with concluding remarks.

II. VEHICLE MODEL

In this paper we use simplified vehicle models, derived from the nonlinear four wheels vehicle model presented in [2] by introducing small angles and linear tire characteristics simplifying assumptions.

The nomenclature, used next in vehicle modeling, is introduced in Figure 1. Moreover, we use two subscript

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symbols to denote variables related to the four wheels. In particular \( \star \in \{f, r\} \) denotes the front and rear axles, while \( \bullet \in \{l, r\} \) denotes the left and right sides of the vehicle. As example, the variable \( (\cdot)_{f,l} \) is referred to the front left wheel.

The lateral, longitudinal and yaw dynamics of the vehicle are described through the following set of nonlinear differential equations

\[
\begin{align*}
    m\ddot{y} &= -m\dot{x}\dot{\psi} + \sum_{\star \in \{f, r\}} \sum_{\bullet \in \{l, r\}} F_{y_{\star, \bullet}}, \quad (1a) \\
    m\ddot{x} &= m\dot{y}\dot{\psi} + \sum_{\star \in \{f, r\}} \sum_{\bullet \in \{l, r\}} F_{x_{\star, \bullet}}, \quad (1b) \\
    I_{\psi} &= a \sum_{\bullet \in \{l, r\}} F_{y_{f, \bullet}} - b \sum_{\bullet \in \{f, r\}} F_{y_{r, \bullet}} \\
    &\quad + c \left( \sum_{\star \in \{f, r\}} F_{x_{\star, -}} - \sum_{\star \in \{f, r\}} F_{x_{\star, +}} \right), \quad (1c)
\end{align*}
\]

where \( \dot{x} \) and \( \dot{y} \) are the vehicle velocities along the longitudinal and lateral vehicle axes, respectively, \( \psi \) is the yaw angle, \( \dot{\psi} \) is the yaw rate, \( m \) is the mass of the car, \( I \) is the inertia of the vehicle along the vertical axis, the constants \( a \) and \( b \) are the distances of the front and rear axles from the CoG, respectively, \( c \) is the distance of the left and right wheels from the vehicle’s longitudinal axis.

Indeed, in combined cornering and braking manoeuvres, the lateral tire force depends on the tire longitudinal force as well. Sample plots of the lateral force \( F_{l} \) versus the longitudinal tire force \( F_{l} \) for different values of the tire slip angle \( \alpha \) [7] are shown in Figure 2. In order to model the interaction between lateral and longitudinal tire forces in combined cornering and braking maneuvers (see Figure 2), we modify the (3) as

\[
F_{c_{\bullet, \star}} = C_{c_{\bullet, \star}} \alpha_{\bullet, \star} + D_{c_{\bullet, \star}} F_{l_{\bullet, \star}}, \quad (5)
\]

where

\[
D_{c_{\bullet, \star}} = \frac{\partial F_{c_{\bullet, \star}}}{\partial F_{l_{\bullet, \star}}}. \quad (6)
\]

The linear approximation (5) depends on the current operating point, i.e., \( F_{l_{\bullet, \star}} \) and \( F_{c_{\bullet, \star}} \). In order to further simplify the vehicle model, hereafter we will make use of a simpler linear approximation of the curves in Figure 2, according to the following

**Simplification 1:** \( D_{c_{\bullet, \star}} = \text{const} \) in (6).

By combining the equations (1), (4)-(6), the simplified, yet nonlinear, vehicle dynamics can be described by the following compact differential equation:

\[
\dot{\xi}(t) = f(\xi(t), u(t), d(t)), \quad (7)
\]

where the state, input and disturbance vectors are \( \xi = [\dot{y}, \dot{x}, \dot{\psi}], u = [F_{f_{bf, l}}, F_{f_{bf, r}}, F_{f_{br, l}}, F_{f_{br, r}}, F_{RB}] \) and \( d = \delta_{f} \), respectively. \( F_{f_{bf, \bullet}} \) and \( F_{RB} \) are the friction forces.
braking forces at the four wheels and the regenerative braking force at the rear axle, respectively. Moreover
\[
F_{l_{f,*}} = F_{fb_{f,*}} \quad (8a)
\]
\[
F_{l_{r,*}} = F_{fb_{r,*}} + \frac{1}{2} F_{RB} \quad (8b)
\]

A. Simplified single track model

Starting from the simplified vehicle model (7), next we derive a simplified single track (or bicycle) model, sketched in Figure (3), by introducing the following Simplification 2: At front and rear axles, the left and right wheels are identical and lumped together in a single wheel. By Simplification 2, the equations (1) can be rewritten as follows:
\[
m \ddot{y} = -m \dot{x} \dot{\psi} + 2 F_{yf} + 2 F_{yr}, \quad (9a)
\]
\[
m \ddot{x} = m \dot{y} \dot{\psi} + 2 F_{xf} + 2 F_{xr}, \quad (9b)
\]
\[
I \ddot{\psi} = 2a F_{yf} - 2b F_{yr}, \quad (9c)
\]
where (4)-(6) are written for a single axle (i.e., the second symbol is dropped). By combining the equations (2)-(6), (9),

under the Simplification 2, the simplified bicycle model can be described by the following compact differential equation:
\[
\dot{\xi}(t) = f_{2\omega}(\xi(t), u(t), d(t)), \quad (10)
\]
where the state, input and disturbance vectors are \( \xi = [\dot{y}, \dot{x}, \dot{\psi}] \), \( u = [F_{fb_{f}}, F_{fb_{r}}, F_{RB}] \) and \( d = \delta_f \), respectively. Similarly to (8) we define
\[
F_{l_{f}} = F_{fb_{f}} \quad (11a)
\]
\[
F_{l_{r}} = F_{fb_{r}} + F_{RB}. \quad (11b)
\]

**Remark 1:** We remark that the complex interaction between longitudinal and lateral tire forces, depicted in Figure 2 is described by the approximation (5) in the simplified models (7) and (10). In Figure 4, we compare the full nonlinear model in [2] against the simplified model (10). In particular, the yaw rates predicted by models in [2] and (10) are shown in Figure 4(a) with solid and dashed lines, respectively. By comparing the yaw rates between 10 s to 12 s, we observe that the simplified model (10) is able to well approximate the yaw dynamics in combined cornering and braking as well. A comparison between models in [2] and (7) leads to similar conclusions.
### III. CONTROL DESIGN

Next we present a predictive approach to regenerative braking control, based on the simplified vehicle models presented in Section II. We assume that every time step a braking force request \( F_D \) and a yaw rate reference are available over future time horizons. The control problem can be then stated as follows: maximize the regenerative braking force \( F_{RB} \) while (i) delivering the requested braking force \( F_D \), (ii) limiting the yaw rate tracking error with respect to a given reference and (iii) fulfilling constraints on the maximum regenerative braking set by the hybrid powertrain.

In order to solve the problem stated above as a MPC problem, in this work we adopt the control design methodology used in [2] and presented next for the sake of completeness.

In Section IV, we present simulation results of two predictive approaches based on the simplified vehicle models (7) and (10), respectively. For the sake of brevity, in this section we present the design procedure for the first controller only.

We discretize the vehicle model (7) with a sampling time \( T_s \):

\[
\xi(t+1) = f^d(\xi(t), u(t), d(t)),
\]

\[
u(t) = u(t-1) + \Delta u(t),
\]

where \( u(t) = [F_{f_{lb}(t)}, F_{f_{rb}(t)}, F_{r_{lb}(t)}, F_{r_{rb}(t)}, F_{RB}(t)] \), \( \Delta u(t) = [\Delta F_{f_{lb}(t)}, \Delta F_{f_{rb}(t)}, \Delta F_{r_{lb}(t)}, \Delta F_{r_{rb}(t)}, \Delta F_{RB}(t)] \).

The output variables to be tracked are defined through the following output map:

\[
\eta(t) = h(\xi(t)) = [0 \ 0 \ 1] \xi(t).
\]

Moreover we consider the following cost function

\[
J(\xi(t), \Delta U(t), \epsilon) = \sum_{i=0}^{H_p} \|\eta(t+i) - \eta_{ref}(t+i)\|^2_Q + \sum_{i=0}^{H_c-1} \|\Delta u(t+i)\|^2_R + \sum_{i=0}^{H_s-1} \|u(t+i)\|^2_S + \rho \epsilon^2,
\]

where \( H_p \) and \( H_c \) are the prediction and control horizons respectively, \( \Delta u(t) = [\Delta u(t), \ldots, \Delta u(t+H_c-1)] \), \( \eta_{ref} = \psi_{ref} \), the first term penalizes the vehicle deviation from the yaw rate reference, the second term penalizes the change in the control input, i.e., the braking rate, and the third summand penalizes the braking effort. \( Q, R \) and \( S \) are the weighting matrices of appropriate dimensions, while the parameter \( \rho \) penalizes the slack variable \( \epsilon \).

At time \( t \) we consider the current vehicle state \( \xi(t) \) and the previous input \( u(t-1) \) of the system (12)-(13) and solve the following optimization problem:

\[
\min_{\Delta U(t), \epsilon} J(\xi(t), \Delta U(t), \epsilon)
\]

subject to

\[
\xi_{k+1,t} = A_t \xi_{k,t} + B_t u_{k,t} + d_{k,t},
\]

\[
\eta_{k,t} = h(\xi_{k,t})
\]

\[
k = t, \ldots, t + H_c - 1
\]

where the equations (15a) are a linear approximation of (12) computed at the current state \( \xi(t) \) and the previous control input \( u(t-1) \). The term \( d_{k,t} \) in (15a) is required since, in general, the operating point \( \xi(t) \), \( u(t-1) \) is not an equilibrium point. Further details can be found in [4], [2].

The (15e) limits the braking forces, while the (15f) limits the braking forces variations. The (15e) includes the upper bound on the regenerative braking set by the hybrid powertrain. The (15g), where \( u_{k,t} \) is the \( i \)-th element of \( u_{k,t} \) and \( F_{RB} = [F_{R_{lb}(t)}, \ldots, F_{R_{rb}(t)+H_c-1}] \), constrains the total braking force to equal the requested braking force \( F_D \). Finally, the (15j), where \( H_u \) is the constraint horizon, soft constraints the yaw rate tracking error. We remark that the (15j) forces the yaw rate tracking error to lie within bounds \([\eta_{min}, \eta_{max}]\) where yaw stability systems are not enabled.

**Remark 2:** The constraint (15g) might be unfeasible, if high braking force \( F_{RB} \) is required on low friction surfaces. Hereafter we assume that the requested braking force can always be delivered on the considered surface.

The problem (15) is solved in receding horizon and the braking at time \( t \) is computed through the following feedback control law

\[
u(t, \xi(t)) = u(t-1) + \Delta u^*_t(t, \xi(t)).
\]

where \( \Delta U_t^* = [\Delta u_{t+1}^*, \ldots, \Delta u_{t+H_c-1}^*] \) is the sequence of optimal braking forces increments computed at time \( t \) by solving (15) for the current observed states \( \xi(t) \) and the previous input \( u(t-1) \).

**Remark 3:** The stability of the closed-loop system (12)-(13) with the state feedback control (15)-(16) is not guaranteed in general. In [4] the stability properties of the LTV MPC algorithm (15)-(16) are studied and a sufficient condition enforcing the local closed loop stability is provided.

### IV. SIMULATIONS

Simulation results of two regenerative braking controllers, based on the approach presented in Section III, are presented next. The first, next referred to as Controller A, is based on the simplified bicycle model (10) (i.e., the (15a) is a discrete time linear approximation of (10)), where the control inputs are the friction braking forces at the front and rear axle.
and the regenerative braking force. The second controller, referred to as Controller B, is based on the model (7) and has been described in details in Section III. We recall that in Controller B the control inputs are the friction braking forces at the four wheels and the regenerative braking force at the rear axle. The two controllers have been simulated in a Simulink environment using the ve-DYNA [9] vehicle model, with the following set of tuning parameters.

**Controller A**
- sampling time: $T = 0.05s$.
- horizons: $H_p = 15$, $H_c = 5$, $H_u = 2$.
- bounds:
  - $F_{fR_{\min}} = 0$ N, $F_{fR_{\max}} = 910$ N, $F_{RB_{\min}} = 0$ N, $F_{RB_{\max}} = 2 \cdot 10^3$ N,
  - $\Delta F_{fR_{\min}} = -455 \cdot 10^2$ N, $\Delta F_{fR_{\max}} = 455 \cdot 10^2$ N, $\Delta F_{RB_{\min}} = -2 \cdot 10^5$ N, $\Delta F_{RB_{\max}} = 2 \cdot 10^5$ N.
  - $\eta_{\min} = -2$ deg/s, $\eta_{\max} = 2$ deg/s.
- weighting matrices:
  - $Q = 0$.
  - $R \in \mathbb{R}^{3 \times 3}$ with $R_{11} = R_{22} = 10^4$, $R_{33} = 10^2$ for $i \neq j$.
  - $S \in \mathbb{S}^{3 \times 3}$ with $S_{11} = 10^2$, $S_{22} = 10^2$ $S_{33} = 0$ and $S_{ij} = 0$ for $i \neq j$.
  - $\rho = 10^3$.

**Controller B**
- sampling time: $T = 0.05s$.
- horizons: $H_p = 15$, $H_c = 2$, $H_u = 1$.
- bounds:
  - $F_{fR_{\min}} = 0$ N, $F_{fR_{\max}} = 455$ N, $F_{RB_{\min}} = 0$ N, $F_{RB_{\max}} = 2 \cdot 10^3$ N,
  - $\Delta F_{fR_{\min}} = -195 \cdot 10^2$ N, $\Delta F_{fR_{\max}} = 195 \cdot 10^2$ N, $\Delta F_{RB_{\min}} = -2 \cdot 10^5$ N, $\Delta F_{RB_{\max}} = 2 \cdot 10^5$ N.
  - $\eta_{\min} = -2$ deg/s, $\eta_{\max} = 2$ deg/s.
- weighting matrices:
  - $Q = 0$.
  - $R \in \mathbb{R}^{5 \times 5}$ with $R_{ij} = 10^4$ for $i = j$ and $R_{ij} = 0$ for $i \neq j$.
  - $S \in \mathbb{S}^{5 \times 5}$ with $S_{ij} = 10^2$ for $i = j$, $i = 1, 2, 3, 4$, $S_{55} = 0$ and $S_{ij} = 0$ for $i \neq j$.
  - $\rho = 10^3$.

**Remark 4:** We point out that the upper bounds to the braking forces has been found, through extensive simulations, as the maximum braking forces that can be applied without locking the wheels.

The testing manoeuvre we consider is similar to the one described in Remark 1. In particular, the braking force, showed in the upper plot of Figure 5(b) with dashed line, is requested, between 10 s and 12 s, while the vehicle is traveling at 40 Km/h along a curve with a steering angle of 2.5 deg. Simulations have shown that if the requested braking force is entirely delivered through regenerative braking in the considered scenario, the vehicle spins.

In Figures 5 and 6, simulation results of Controllers A and B are shown, respectively.

Simulation results show three remarkable results next discussed in details. Both controllers (i) keep the yaw rate within the specified bounds, (ii) deliver an amount of regenerative braking such that the yaw rate is above the reference, yet below the upper bound, and (iii) the Controller B perform slightly better than Controller A.

In order to comment points (i) and (ii), we recall that large braking at the rear axle (e.g., regenerative braking) causes a variation of the rear lateral tire forces (see Figure 2) inducing a yaw moment that increases the yaw rate. This phenomenon is usually referred to as oversteering. Similarly, a reduction of the lateral forces at the front tires (i.e., braking the wheels at the front axle) reduces the turning rate of the vehicle. This phenomenon is usually referred to as understeering. We observe that Controller A blends the front friction and rear regenerative braking in order to counteract the oversteering effect of the regenerative braking. Controller B, instead, counteracts the effects of the regenerative braking, i.e., a positive yaw moment, by braking the wheels at the right side of the vehicle (see lower plot in Figure 6(b)), thus generating a negative yaw moment.

We further observe that both controllers induce an oversteering behaviour. In particular, the possibility of inducing oversteering, given by the upper bound in constraint (15j), is smarterly exploited by the two controllers in order to increase the regenerative braking.

By comparing the regenerative braking delivered by the two controllers in Figures 5(b) and 6(b), we observe that Controller B is able to deliver slightly more regenerative braking than Controller A. This is due to the possibility, in Controller B, of compensating an higher oversteering yaw moment, induced by the regenerative braking, by single wheel braking. In the considered braking maneouvre, the Controller B delivers 11.17% more regenerative braking than Controller A. We finally observe a non smooth behavior of both controllers when approaching the upper bound on the yaw rate. This is due to the approximation introduced in (5).

**V. CONCLUSIONS AND FUTURE WORKS**

We have presented two predictive approaches for controlling the regenerative braking at the rear axle in hybrid vehicles. We have considered low friction scenarios, where excessive regenerative braking might induce yaw instability. Simulation results of the presented approaches show that both controllers smartly blend regenerative and friction braking in order to maximize the regenerative braking while preserving vehicle stability and exactly delivering the requested braking force. We point out that the proposed approach only focuses on the yaw stability issues induced by regenerative braking, while vehicle instability, caused by wheels lock up, is avoided by an ad hoc selection of the tuning parameters (see Remark 4). The next step of this work is the modification of the vehicle prediction model and the inclusion of additional constraints, in order to prevent the wheels’ lock up without an ad hoc selection of bounds on the braking forces.
(a) Yaw rate reference (solid line), actual (dashed line) and bounds (dash-dotted lines).

(b) Regenerative braking (solid line) and requested braking (dashed line) forces in the upper plot, friction braking forces at the front (solid line) and rear (dashed line) in the lower plot.

Fig. 5. Simulation results of Controller A.

References


