A hybrid general lot-sizing and scheduling formulation for a production process with a two-stage product structure

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Sandra Transchel[1] ⋆ Josef Kallrath[2] ⋆ Stefan Minner[3] ⋆ Nils Löhndorf[3] ⋆ Ulrich Eberhard[4]

[1] *Department of Supply Chain and Information Systems, The Pennsylvania State University, University Park, PA, USA*

[2] *BASF SE, GVC/S Scientific Computing - B009, 67056 Ludwigshafen, Germany*

[3] *Faculty of Business, Economics and Statistics, University of Vienna, Austria*

[4] *BASF SE, GRS/PI - J660, 67056 Ludwigshafen, Germany*

Abstract

In this paper we describe the model of an actual problem from the chemical industry, for which we developed a planning and lot-sizing tool. The production process is characterized by a two-stage product structure as well as sequence-dependent setup cost and setup times. The develop a model which is a modified version of the General Lot-sizing and Scheduling Problem (GLSP) framework and combines discrete-time as well as continuous-time elements. We furthermore develop two alternative reformulations which are based on the transportation problem and use disaggregated production variables instead of standard production and inventory variables. We show that both alternative formulations yield significant improvements with respect to computational time and integrality gap.

Keywords: Lot-sizing, scheduling, mixed-integer programming, process industry

1 Introduction

Applications of quantitative models and computer-based planning systems have received considerable attention by the process industry over the past decades. Since changeover operations are complex and expensive in time and cost, spreadsheet-based planning and scheduling quickly becomes inadequate and more sophisticated solutions are required. The physical work associated with a changeover, i.e., setup and cleaning, causes significant opportunity cost due to the loss of productive time. Other factors which increase costs are complex product structures and production processes, e.g., multiple product stages, batch size constraints and sequence-dependent setup costs and times. Advances in information technology as well as progress in the development of quantitative methods enabled many successful implementations of advanced planning systems (APS) tailored to industry requirements which support decision-making on strategic, tactical, and operational levels (Günther and Van Beek, 2003). However, complex product and manufacturing structures are often not properly incorporated in standard APS and more customized planning models are required.

This paper is the result of a project with a chemical company. We developed a multi-product lot-sizing and planning tool which supports the mid-term decision-making process of a single manufacturing plant. The planning model contains a lot-sizing kernel where two subsequent product batches require sequence-dependent setup costs and setup times. Products are characterized by a two-stage product structure. While lot-sizing and sequencing is planned on the first level – the product level – contribution margin, holding cost, and penalty cost are determined on the second level – the article level. At the article level, each product is split up into one or more articles which are characterized by different packages. Articles differ in their respective contribution margin and holding cost. The demand data of each article is based on monthly sales forecasts for a planning horizon of 11 to 13 months. The majority of lot-sizing and planning literature treats the problem as one of minimizing holding and setup cost. In our model the objective is to maximize total profits by finding the optimal trade-off between penalty cost of lost sales against holding cost for products with different contribution margins and by taking sequence-dependent setup cost and setup times into account.

We present a hybrid mixed-binary optimization model based on the General Lot-sizing and Scheduling Problem (GLSP) of Fleischmann and Meyr (1997). The *hybrid GLSP* approach combines model structures of discrete-time and continuous-time approaches. The discrete time scale is predefined by monthly time buckets which represent the times of demand fulfillment, i.e., the times where revenue, holding cost, and penalty cost are generated. The continuous time-scale represents production events, i.e., beginning and ending of production and setup runs, or idle time. Moreover, several company-specific requirements were considered which are not found in many classic lot-sizing models. The hybrid GLSP formulation makes use of standard production and inventory variables. However, other approaches in the literature show that there are stronger equivalent formulations based on network structures. We propose two alternative reformulations based on the *Transportation problem* (TP). The first reformulation, called *Quantity-based transportation problem* (QTP), disaggregates production variables of each period into more detailed variables related to the size of its demand. The second formulation is called *Proportional transportation problem* (PTP). Like the QTP, the PTP approach disaggregates production variables. However, instead of defining demand quantities, the PTP disaggregates production variables by their proportion of total demand. An extensive computational experiment based on real industry data highlights the superiority of the TP reformulations compared to the classical formulation. We also describe how we integrated the model into the company's planning process.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. In Section 3, we describe the general *hybrid GLSP* model. The TP reformulations are explained in Section 4. Section 5 gives a brief impression about the complexity of the hybrid-GLSP formulation and shows the results of a computational experiment using real industry data. Summary and conclusion follow in Section 6.

2 Literature review

Simultaneous lot-sizing and planning problems with sequence-dependent setup cost and setup times have received a lot of attention in the literature. Drexl and Kimms (1997) summarize work in the field of simultaneous lot-sizing and planning until 1996. A comprehensive overview is provided in Sürie (2005). An extensive overview of planning and scheduling problems with a special focus on the process industry is given by Kallrath (2002b). Jans and Degraeve (2005) give an overview of recent developments in the field of modeling single-level dynamic lot sizing problems and various industrial extensions.

Most of the models operate in discrete time, either on a *large-bucket* time scale where multiple items can be produced in each period or on a *small-bucket* time scale where only one item is produced in each period. The *Capacitated Lot Sizing and Scheduling Problem* (CLSP) is the most basic large-bucket lot-sizing problem. Scheduling decisions, however, are not included into the CLSP so that the scheduling problem is usually solved separately from the the lot-sizing problem (Karimi et al., 2003). In order to integrate the lot-sizing and scheduling problem into large-bucket problems, Lasserre (1992) develops a two-level hierarchical approach. On an upper level (lot-sizing level) lot-sizes are optimized given a fixed production sequence, while a second-level (scheduling level) determines a new production sequence given the previously determined lot-sizes. Various works take similar approaches, e.g., Potts and Van Wassenhove (1992), Dauzere-Peres and Lasserre (1994) and Dauzere-Peres and Lasserre (1997).

The *Discrete Lot Sizing and Scheduling Problem* (DLSP) is the most fundamental small-bucket model. It provides the ability to completely integrate lot-sizing and scheduling decisions. The DLSP assumes that the quantity produced in each period is either zero or full capacity (*all-or-nothing production*) and was introduced by Lasdon and Terjung (1971). A lot of papers have examined theoretical and computational aspects of the DLSP in general and of the DLSP with sequence dependencies (DLSDSD), among others Salomon et al. (1991 and 1997), Fleischmann (1994), and Jordan and Drexl (1998). Haase (1996) formulate a *Capacitated Lot Sizing Problem With Sequence Dependent Setup Costs* (CLSD). The main difference to the DLSDSD is that continuous lot-sizes are allowed and the setup state can be preserved over idle time. The *Continuous Setup*

Lot-Sizing Problem (CSLP) is a small-bucket approach which is closely related to the DLSP. It varies from the DLSP in that it allows to produce any quantity which does not exceed the available production capacity and it relaxes the all-or-nothing production constraint (Salomon, 1991). CSLP-like models with additional constraints are investigated by Smith-Daniels and Smith-Daniels (1986) and Karmarkar et al. (1987). A third class of small-bucket models is the *Proportional Lot Sizing and Scheduling Problem* (PLSP). The basic idea behind the PLSP is that if capacity of a particular period is not completely depleted the remaining capacity can be used for scheduling of a second item (Drexl and Haase, 1995). J. (1999) introduces various extensions of the PLSP. For multi-site production networks, a PLSP is presented in Timpe and Kallrath (2000) and Kallrath (2002a).

The main advantage of using a discrete time scale is that it provides a reference grid for all operations sharing the same resource which achieves a relatively straightforward and simple formulation of various constraints of the planning problem (Floudas and Lin, 2004). However, the primary limitation of using discrete time scales is the unnecessary increase of the overall problem size due to the introduction of additional binary variables associated with each discrete time interval. This inherent limitation has attracted a significant amount of attention and the development of models based on a continuous time scale (*Continuous time models*).

The basic idea of continuous time models is that beginning and/or ending of a period, usually defined as events, are endogenous parameters with the consequence that the duration of periods, i.e., the time between two consecutive events, is not necessarily equal. Due to the variability of events, the scheduling process becomes challenging since the mathematical model is more complicated compared to a discrete-time model. However, continuous-time approaches require much less computational effort compared to discrete-time models (Floudas and Lin, 2004). Kallrath (2002b) characterizes special features of planning and scheduling problems in the process industry. Planning problems usually consider material flows and balance equations connecting sources and sinks of a production or supply chain network. Time-indexed models use a relative coarse discretization of time, e.g., a year, quarter, month, or week. The focus on time in scheduling problems, on the other hand, is more detailed and requires a finer or continuous-time formulation.

Hybrid models refer to a class of models which combine large-bucket and small-bucket time scales. Holding cost and demands are updated according to the large-bucket time scale while setup and production operations base on the small-bucket time scale. The *Capacitated Lot Sizing Problem with Sequence Dependent Setup Costs* (CLSD) is a hybrid model which extends the CLSP by considering sequence dependent setup costs (and times), e.g., Haase and Kimms (2000) and Timpe (2002). Marinelli et al. (2007) formulate a hybrid *Continuous Setup Capacitated Lot Sizing Problem* (CSLP-CLSP) and develop a two-stage heuristic based on a decomposition into a lot sizing and a scheduling problem.

The most general hybrid model is the *General Lot Sizing and Scheduling Problem* (GLSP) introduced by Fleischmann and Meyr (1997). The GLSP is essentially a large bucket model but also includes an internal variable time scale within each particular large bucket period that determines the size and position of production lots. Koçlar and Süral (2005) show that the GLSP of Fleischmann and Meyr (1997) is strongly limited. The production state between two consecutive periods is only conserved if the available capacity exceeds the minimum production quantity. They generalize the model by an additional constraint. More variants of the GLSP are provided by Meyr (2000 and 2002), Kimms and Motta Toledo (2003), and Fandel and Stammen-Hegene (2006). The most comprehensive study of the GLSP with sequence-dependent changeovers is provided by Koçlar (2005) who discusses the impact and validity of some commonly encountered assumptions. The author proposes an alternative formulation for the problem using an entended model with additional valid inequalities. Computational experiments involve testing the impact of problem size parameters and the level of minimum batch sizes. Westerlund et al. (2007) propose a hybrid scheduling model which combines discrete-time and continuous-time formulations to model a multi-product, multi-stage production process with intermediate storage and non-linear optimal storage profiles.

Many authors have investigated tighter formulations of capacitated lot-sizing and planning problems. "Tighter" in this context means that the respective LP relaxation is closer to the optimal objective value. MIP models, algorithms, and reformulations are presented by Pochet and Wolsey (2006). Models with variable redefinitions as a network representation are the Simple Plant Location Problem (SLP) introduced by Krarup and Bilde (1977) and the Shortest Path Problem (SP) introdued by Eppen and Martin (1987). Stadtler (1996) introduces a new formulation of the multi-item, multi-level dynamic lot-sizing problem which models the changes of end-of-period inventory levels and provides tighter bounds than the traditional formulation. Koçlar (2005) presents an equivalent alternative formulation of the GLSP based on the Transportation Problem (TP). In various numerical studies the superiority of the TP formulation compared to the traditional formulation including production and inventory variables is shown. Denizel and Süral (2006) consider several strong formulations for the CLSP with sequence independent setup times and no setup costs, including SLP, TP, and SP. Further references in the lot sizing literature that make use of SP and TP formulations are Karmarkar and Schrage (1985), Wolsey (1989), Wolsey (2002), and Stadtler (2003).

To the best of our knowledge Koçlar (2005) is the only study which analyzes reformulation of lot-sizing and planning problems in a GLSP context that basically uses test instances relying on random instance generators. Although there exists a wide range of papers studying lot-sizing and planning problems and efficient reformulations, the majority is tested on instances generated by random generators. Only few studies carry out computational experiments based on actual industry data.

3 Model

3.1 Model assumptions and notation

The problem is formulated as a mixed-binary model and includes several company-specific requirements. We assume that K products are processed on a single resource with limited production capacity. The production process splits these products into I articles where each article is assigned to exactly one product. Products and articles are indexed by $k \in \mathscr{K} := \{1, \ldots, K\}$ and $i \in \mathscr{I} := \{1, \ldots, I\}$, respectively with $K \leq I$. The product-article allocation is represented by an incidence matrix A_{ik} with $A_{ik} = 1$, if article i is an outcome of product k and $A_{ik} = 0$, otherwise.

The finite planning horizon is divided into a two-level time scale represented by macro and micro periods. Macro periods, representing months, are denoted by $t \in \mathscr{T} := \{1, \ldots, T\}$ and are assumed to be fixed and equidistant. In any period t multiple setup operations are allowed. D_{ti} denotes demand of article i, which has to be available at the end of macro period t. C_t denotes the production capacity of a macro period t.

In contrast to the macro-period time scale, the micro-period time scale, indexed by $s \in \mathscr{S} := \{1, \ldots, S\}$, is a continuous event-based representation of time and is characterized by variable and non-equidistant time periods. Any micro period s is terminated by the points in time τ_s and τ_{s-1}, the ending time of micro period s and τ_{s-1}, respectively. Hence, the production capacity of a micro period s is given by $C_t(\tau_s - \tau_{s-1})$. In any micro period s only a single setup operation is allowed and not more than a single product can be produced.

At the beginning of each micro period s the machine is set up on a particular product k. During each period s first the production process of product k is completed before the changeover operation, if any, is performed. The length of s depends on the production time of product k as well as the sequence dependent setup time if the machine is set up from product k to product l where P_k describes the capacity consumption to produce one unit of product k and Z_{kl} is the sequence dependent setup time.

The sum of production time and setup time must not exceed the production capacity of s. If a changeover operation to product k was performed in micro period $s - 1$, the minimum production requirement of product k in period s is Q_k. If no setup operation is performed, the machine setup is carried over into the next micro period. Also denote P_i as the capacity consumption to produce one unit of article i with $P_i = P_k$ for $A_{ik} = 1$.

As the changeover costs alone would not cover the full picture of reality related to changeovers (risk of startup, quality loss in part of the production, etc.), the number of changeover operations should be limited in each month. We assume that for any macro period t at most W setup operations are allowed. Accordingly, every period t contains exactly W micro periods and the total number of micro periods over the entire planning horizon is $S = WT$. The special case $W = K$ allows producing all products in one macro period, which is the most general formulation and provides the highest flexibility in terms of determining the optimal number of setups. Subset $\mathscr{S}_t \subset \mathscr{S}$ defines the set of all micro periods s that form macro period t with $|\mathscr{S}_t| = W$. Furthermore, subset $\bar{\mathscr{S}} \subset \mathscr{S}$ with $\bar{\mathscr{S}} = \{W, 2W, \ldots, TW\}$ contains all micro periods s which represent the last micro period of a period. To couple the exogenous macro period and the endogenous micro period time scale it is assumed that for all $s \in \bar{\mathscr{S}}$ the ending time τ_s is fixed and is equal to the ending time of its macro period. A graphical illustration is given in Figure 1.

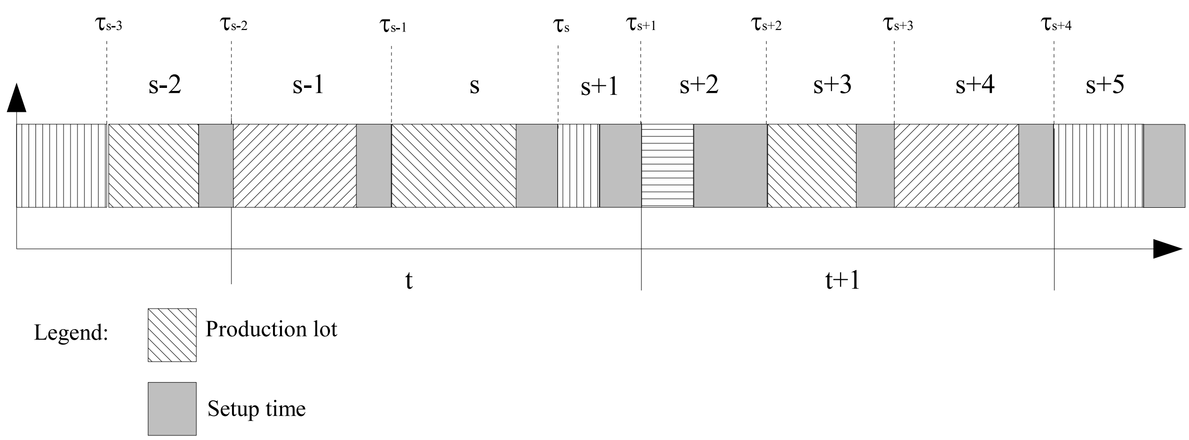

Figure 1: Illustrative example of macro-period and micro-period time scale

Several company-specific requirements have been included. According to the company's production philosophy it was required that the demand which has to be available at the end of a month is actually required to satisfy demand of the next month. The underlying strategy of this philosophy is to provide a higher flexibility and product availability. Due to this philosophy, the on-hand inventory at the beginning of the first production period still contains the demand of this period, which has been produced in the previous periods. To determine the initial inventory level actually being available to satisfy demand of the planning horizon, the demand of the first production period has to be derived from the initial on-hand inventory. Let $\bar{y}_{0i}$ define the initial inventory level of article i. A special company requirement was to allow that possibly unsatisfied demand of the first production period, which occurs if demand of the first production period exceeds the initial on-hand inventory, can be produced in the first production period regardless of the production philosophy. This allows $\bar{y}_{0i}$ to be negative and is reasonable because the model is used on a rolling horizon and deviations between a planned production schedule and finally produced quantities cannot be avoided.

Due to the fact that shortfalls cannot be entirely avoided, another company requirement was to determine additional costs associated with producing an unscheduled product (article). For that reason, penalty cost were introduced for each unit of unsatisfied demand. If an article could not be produced because of a capacity limitation, a minimum penalty cost was set to force a scheduling this article in the production plan and to determine additional cost. Additional cost in this context are lost contribution margins of other products as well as additional holding and setup cost.

The company further requested that the planning tool has to be able to fix production quantities in certain months. In these “fixed months” only a predetermined and exogenously fixed production plan can be produced. Macro periods with fixed production schedules are denoted by $\mathscr{T}_f := \{1, \ldots, T_f\} \subseteq \mathscr{T}$, the set of “fixed macro periods”. In any period $t_f \in \mathscr{T}_f$ production quantities for all products are exogenously given by $\bar{Q}_{t_f k} \geq 0$. Another requirement was to limit available capacity for certain products in order to be able to model exogenous constraints. For example, if the availability of a certain product is limited to 80%, then production capacity available to produce the associated articles can be reduced to 80% of maximum capacity. To model this aspect, let $G_{tk} \in [0, 1]$ be the fraction of capacity C_t which cannot be used for product k in macro period t. Then, $(1 - G_{tk})C_t$ is the maximum capacity in period t available to produce product k. If $G_{tk} = 0$, then the entire capacity is available for product k in period t, and if $G_{tk} = 1$ no capacity is available. We furthermore define $x(t)$ as the tth entry of set x, e.g., $\bar{\mathscr{S}}(t)$ is the tth entry of set $\bar{\mathscr{S}}$. Finally, we define the following revenue and cost parameters:

R_i : Revenue per unit of article i,

V_{kl} : Sequence-dependent setup cost, if a setup operation from product k to product l is performed,

H_i : Inventory holding cost for one unit of article i in a macro period t,

F_i : Penalty cost per unit of article i.

3.2 Model

We define the following decision variables:

q_{sk} : Production quantity of product k in micro period s,

q_{si} : Production quantity of article i in micro period s,

y_{ti} : Inventory level of article i at the end of macro period t,

x_{ti} : Sales of article i in macro period t which can be used to satisfy demand D_{ti},

γ_{sk} : Setup state variable; $\gamma_{sk} = 1$, if the system is set up for product k in micro period s; otherwise $\gamma_{sk} = 0$,

ξ_{skl} : Sequence dependent setup variable; $\xi_{skl} = 1$, if a setup operation from product k to product l is performed in micro period s; otherwise $\xi_{skl} = 0$,

τ_s : Ending time of micro period s,

f_{0i} : Unsatisfied demand out of negative initial inventories which could not be satisfied in the first production period.

The objective is to maximize the total profit Π over the entire planning horizon and is expressed by (1).

$$\Pi = \underbrace{\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} R_i x_{ti}}_{\text{Revenue}} - \underbrace{\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}}\sum_{l\in\mathscr{K}} V_{kl}\xi_{skl}}_{\text{Setup cost}} - \underbrace{\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} H_i(y_{ti}+x_{ti})}_{\text{Holding cost}} - \underbrace{\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} F_i(D_{ti}-x_{ti}) - \sum_{i\in\mathscr{I}} F_i f_{0i}}_{\text{Penalty cost}} \quad (1)$$

Π is total revenue minus total cost, i.e., inventory holding cost, setup cost, and penalty cost. Sequence dependent setup cost are charged in every micro period s where a product changeover occurs. Holding cost are computed at the end of a macro period t. Unlike classic lot-sizing problems where the demand of a period is not included in the final inventory of this period, in our problem the demand, which has to be available at the end of period t is actually needed to satisfy demand of period $t+1$. Accordingly, holding cost are also charged to sales of period t. There are two types of penalty cost. The first term represents penalty cost that occur if the inventory level at the end of a macro period t does not cover demand. The second term represents penalty cost for unsatisfied demand out of a negative initial inventory which could not be satisfied in the first production period. The idea behind penalty cost is to create additional cost associated with producing an article which was not scheduled without penalty cost, e.g. a product which production considers unprofitable but which is made for a key account. Another reason to introduce penalty cost on unavailable inventory is to guarantee feasibility of the mathematical program.

The following constraints have to be taken into account.

Inventory balance equation

$$\bar{y}_{0i} + \sum_{s \in \mathscr{S}_1} q_{si} - x_{1i} + f_{0i} = y_{1i} \qquad \forall i \in \mathscr{I} \tag{2}$$

$$\bar{y}_{0i} + \sum_{s \in \mathscr{S}_1} q_{si} + f_{0i} \geq 0 \qquad \forall i \in \mathscr{I} \tag{3}$$

$$y_{t-1,i} + \sum_{s \in \mathscr{S}_t} q_{si} - x_{ti} = y_{ti} \qquad \forall t \in \mathscr{T} \backslash \{1\} \text{ and } i \in \mathscr{I} \tag{4}$$

Constraints (2) and (4) are the standard inventory balance equations. Constraint (2) characterizes the inventory balance equation of macro period 1 for each article i. From constraint (3), non-negativity constraints (17), and the objective of minimizing penalty cost, it follows that if the initial inventory plus the production volume is negative, then $f_{0i} = -\left(\bar{y}_{0i} + \sum_{s \in \mathscr{S}_1} q_{si}\right) > 0$, and if $\left(\bar{y}_{0i} + \sum_{s \in \mathscr{S}_1} q_{si}\right) > 0$, then $f_{0i} = 0$. Constraint (4) represents the inventory balance equation for all following macro periods.

Sales restriction

$$x_{ti} \leq D_{ti} \qquad \forall t \in \mathscr{T} \text{ and } i \in \mathscr{I} \tag{5}$$

The sales constraint makes sure that sales of article i in macro period t cannot be larger than the demand of that particular period.

Capacity constraint of micro period s

$$\sum_{k \in \mathscr{K}} P_k q_{sk} + \sum_{k \in \mathscr{K}} \sum_{l \in \mathscr{K}} Z_{kl} \xi_{skl} \leq C_t(\tau_s - \tau_{s-1}) \quad \forall t \in \mathscr{T} \text{ and } s \in \mathscr{S}_t \tag{6}$$

Micro period s has a length of $(\tau_s - \tau_{s-1})$. The available production capacity of every s is given by $C_t(\tau_s - \tau_{s-1})$ which is the fraction of the available capacity in macro period t used in micro period s. Constraint (6) ensures that in every micro period s production time plus setup time never exceeds available capacity.

Coupling of endogenous micro-period and exogenous macro-period time scale

$$\tau_{\bar{\mathscr{S}}(t)} = t \qquad \forall t \in \mathscr{T} \tag{7}$$

To couple micro periods and macro periods the ending time of a micro period s which is also the last period of macro period t has to be equal to t where t is also the fixed ending time of macro period t.

Logic condition

$$q_{sk} \leq C_t \gamma_{sk} \qquad \forall t \in \mathscr{T},\, s \in \mathscr{S}_t, \text{ and } k \in \mathscr{K}, \tag{8}$$

This condition ensures that product k can only be produced in micro period s if the system is set up on product k at the beginning of this period.

Minimum production requirement

$$q_{sk} \geq Q_k(\gamma_{sk} - \gamma_{s-1k}) \quad \forall s \in \mathscr{S} \backslash \bar{\mathscr{S}} \text{ and } k \in \mathscr{K}, \tag{9}$$

$$q_{sk} + q_{s+1k} \geq Q_k(\gamma_{sk} - \gamma_{s-1k}) \quad \forall s \in \bar{\mathscr{S}} \text{ and } k \in \mathscr{K}, \tag{10}$$

Constraints (9) and (10) ensure that after a product changeover from any arbitrary product to product k a lot of at least Q_k units has to be produced. Constraint (10) ensures the production continuity across macro periods, i.e., if the time interval of the last micro period of a macro period is not sufficient to produce the minimum quantity, and production is continued in the next macro period, then the sum of q_{sk} and q_{s+1k} has to satisfy the minimum production quantity. These constraints are only necessary for all $s \in \bar{\mathscr{S}}$ since these periods have a fixed ending time.

Changeover logic

$$\gamma_{s-1k} + \gamma_{sl} \leq \xi_{s-1kl} + 1 \quad \forall s \in \mathscr{S} \text{ and } k, l \in \mathscr{K} \tag{11}$$

The left-hand side is equal to 2 if product k is produced in micro period $s-1$ and product l is produced in micro period s. The right-hand side is only equal to 2 if a product changeover from product k to product l is performed in period $s-1$. Hence, setup states γ_{s-1k} and γ_{sl} can only be equal to one at the same time if a product changeover from product k to product l is performed. If only γ_{s-1k} or γ_{sl} is equal to one, ξ_{s-1kl} has to be equal to zero.

Product-article balance

$$q_{sk} = \sum_{i \in I} A_{ik} q_{si} \quad \forall s \in \mathscr{S} \text{ and } k \in \mathscr{K} \tag{12}$$

Constraints (12) ensure that the production quantity of product k in micro period s is equal to the sum of all associated articles produced in s.

Setup existence

$$\sum_{k \in \mathscr{K}} \gamma_{sk} = 1 \quad \forall s \in \mathscr{S} \tag{13}$$

These constraints impose that a certain setup state must exist in every micro period s.

Limited production capacity available for each product

$$\sum_{s\in\mathscr{S}_t} P_k q_{sk} \leq (1-G_{tk})C_t \qquad \forall t\in\mathscr{T} \text{ and } k\in\mathscr{K} \tag{14}$$

These constraints exogenously limit the maximum available production capacity for each product in each period. If $G_{tk} = 0$, there is no additional capacity limitation and q_{sk} is bounded by (6) and (8).

Macro periods with fixed production quantities

$$\sum_{s\in\mathscr{S}_{t_f}} q_{sk} = \bar{Q}_{t_f k} \qquad \forall t_f\in\mathscr{T}_f \text{ and } k\in\mathscr{K} \tag{15}$$

Constraints (15) ensure that if a macro period t is in $\mathscr{T}_f$, for any product k only exogenously given production quantities can be produced.

Binary and non-negativity constraints

$$\gamma_{sk}\in\{0,1\} \qquad \forall s\in\mathscr{S} \text{ and } k\in\mathscr{K} \tag{16}$$

$$q_{sk},\, q_{si},\, x_{ti},\, y_{ti},\, \tau_s,\, \xi_{skl}\geq 0,\; f_{0i}\geq 0 \quad \forall t\in\mathscr{T},\, s\in\mathscr{S},\, k,l\in\mathscr{K},\, \text{and } i\in\mathscr{I} \tag{17}$$

The last two constraints, (16) and (17), define the domains of the binary and continuous variables, respectively. Although the variables ξ_{skl}'s have to be binary, it is sufficient to declare these variables as non-negative because constraints (11) ensure the binary property.

4 Reformulation

In this section we present two alternative reformulations of our model. Previous research has been shown that equivalent reformulations of lot-sizing and planning problems may provide stronger bounds, and hence, better solutions by LP relaxation. Classical lot-sizing and inventory models use production and inventory variables assigned to every period. The basic idea of the reformulations is to decompose production variables of each production period related to the demand period in which the produced quantity is required. Both reformulations are based on the *Transportation Problem* which has been applied to many lot-sizing problems and provide tighter bounds by linear programming relaxation. A *quantity-based* and a *proportional-based* reformulation are presented.

4.1 Quantity-based transportation problem (QTP)

The QTP reformulation disaggregates decision variables for every production quantity in micro period s related to the demand period at which the quantity is required. This formulation requires a transformation of production decisions on the article level:

q_{sti} : amount of the article-i demand in macro period t which is produced in micro period s, for $s = 1, \ldots, S$, $t = 1, \ldots, T$, and $s \leq tW$.

The constraint $s \leq tW$ is necessary because demand cannot be backordered but has to be produced before the quantity is required. Demand which is satisfied out of initial inventory level is denoted by the decision variable $y0_{ti}$ defining the demand of article i in period t, which is satisfied out of the initial inventory level. By this reformulation there is no longer the need to have additional inventory decision variables because the inventory at the end of a macro period t can be formulated by the initial inventory level and the quantity produced up to period t for a demand in period t or for a later period. The optimization problem is then formulated as follows:

$$
\begin{aligned}
\Pi_{QTP} \quad = \quad & \sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} R_i \left(y0_{ti} + \sum_{s=1}^{tW} q_{sti} \right) - \sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} H_i \left(\bar{y}_{0i} - \sum_{l=1}^{(t-1)W} y0_{li} + \sum_{s=1}^{tW}\sum_{l=t}^{T} q_{sli} \right) \\
& - \sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}}\sum_{l\in\mathscr{K}} V_{kl}\xi_{skl} - \sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} F_i \left(D_{ti} - \left(y0_{ti} + \sum_{s=1}^{tW} q_{sti} \right) \right) - \sum_{i} F_i f_{0i} && (18) \\
s.t. \qquad & y0_{1i} + \sum_{s\in\mathscr{S}_1} q_{s1i} + f_{0i} - D_{ti} = 0 \quad \forall i \in \mathscr{I} && (19) \\
& \bar{y}_{0i} + \sum_{s\in\mathscr{S}_1} q_{s1i} + f_{0i} \geq 0 \ \forall i \in \mathscr{I} && (20) \\
& \sum_{t\in\mathscr{T}} y0_{ti} \leq \bar{y}_{0i} \qquad \forall i \in \mathscr{I} && (21) \\
& y0_{ti} + \sum_{s=1}^{tW} q_{sti} \leq D_{ti} \qquad \forall t \in \mathscr{T} \text{ and } i \in \mathscr{I} && (22) \\
& \sum_{i\in\mathscr{I}}\sum_{l\in\mathscr{T}} P_i q_{sli} + \sum_{k\in\mathscr{K}}\sum_{l\in\mathscr{K}} Z_{kl}\xi_{skl} \leq C_t(\tau_s - \tau_{s-1}) \ \ \forall t \in \mathscr{T} \text{ and } \ s \in \mathscr{S}_t && (23) \\
& \tau_{\mathscr{S}(t)} = t \quad \forall t \in \mathscr{T} && (24)
\end{aligned}
$$

$$A_{ik}q_{sti} \leq \gamma_{sk}C_t \quad \forall s \in \mathscr{S},\, k \in \mathscr{K}, i \in \mathscr{I},\ \text{and}\ \ t \in \mathscr{T} \tag{25}$$

$$\sum_{i \in \mathscr{I}} \sum_{t \in \mathscr{T}} A_{ik}q_{sti} \geq Q_k(\gamma_{sk} - \gamma_{s-1k}) \quad \forall s \in \mathscr{S} \backslash \bar{\mathscr{S}} \text{ and } k \in \mathscr{K} \tag{26}$$

$$\sum_{i \in \mathscr{I}} \sum_{t \in \mathscr{T}} A_{ik}(q_{sti} + q_{s+1ti}) \geq Q_k(\gamma_{sk} - \gamma_{s-1k})\ \forall s \in \bar{\mathscr{S}} \text{ and } k \in \mathscr{K} \tag{27}$$

$$\gamma_{s-1k} + \gamma_{sl} \leq \xi_{s-1kl} + 1 \qquad \forall s \in \mathscr{S} \text{ and } k, l \in \mathscr{K} \tag{28}$$

$$\sum_{k \in \mathscr{K}} \gamma_{sk} = 1 \qquad \forall s \in \mathscr{S} \tag{29}$$

$$\sum_{s \in \mathscr{S}_t} P_i \sum_{i \in \mathscr{I}} \sum_{l \in \mathscr{T}} A_{ik}q_{sli} \leq (1 - G_{tk})C_t \qquad \forall t \in \mathscr{T} \text{ and } k \in \mathscr{K} \tag{30}$$

$$\sum_{s \in S_{t_f}} \sum_{i \in \mathscr{I}} \sum_{l \in \mathscr{T}} A_{ik}q_{sli} = \bar{Q}_{t_f k} \qquad \forall t_f \in \mathscr{T}_f \text{ and } k \in \mathscr{K} \tag{31}$$

$$\gamma_{sk} \in \{0, 1\} \qquad \forall s \in \mathscr{S} \text{ and } k \in \mathscr{K} \tag{32}$$

$$y0_{ti}, q_{sti},\, \tau_s,\, \xi_{skl},\, f_{0i} \geq 0 \quad \forall t \in \mathscr{T},\, s \in \mathscr{S},\, k, l \in \mathscr{K},\ \text{and}\ i \in \mathscr{I} \tag{33}$$

As in the previous section the total profit (18) is total revenues minus total costs which is the sum of inventory holding cost, setup cost and penalty cost. The revenue of a particular article i and a macro period t depends on the quantity sold from initial inventory and the quantity produced in previous periods to satisfy demand of this particular period t. Holding costs are charged for the final inventory whereas (due to production philosophy) the ending final is the initial inventory minus the initial inventory which is sold until period $t - 1$ plus the quantity produced until period t but not yet sold. Constraints (19) and (20) are related to (2) and (3) but have to be slightly reformulated. If the initial inventory sold in period 1 plus the production volume produced in period 1 to satisfy demand is strictly negative, then $f_{0i} > 0$, otherwise $f_{0i} = 0$. Inventory balance equations (4) are not needed anymore. (21) makes sure that initial inventory sold over the planning horizon does not exceed the actual inventory level. Constraints (22)-(28) are related to constraints (5)-(11), due to the disaggregation, there is no need for constraint (12), and (29)-(31) are related to (13)-(15).

Despite that the LP relaxation of this model formulation does provide tighter bounds, a drawback is that the number of production and setup variables as well as the number of constraints grows quadratically with the number of micro periods. In the previous formulation the increase was merely linear. Since the number of micro periods depends on the number of setups allowed per macro period, the model size quadratically increases with the number of allowable setups.

4.2 Proportional transportation problem (PTP)

The proportional transportation problem disaggregates production quantities related to the fractional ratio of demand of that period in which the quantity is required.

q'_{sti} : proportion of the demand of period t and product i produced in micro period s for $s = 1, \ldots, S$ and $t = 1, \ldots, T$.

Compared to the QTP where decision variables define quantities, decision variables of the PTP define fractional ratios which are real numbers in a range of [0,1]. We get the PTP by altering equations (18) - (33) of the QTP and substituting all decision variables q_{sti} by $q'_{sti}D_{ti}$ for all s, t, i. Although QTP and PTP are characterized by rather equivalent formulations, the PTP provides a tighter bound because all q'_{sti} are bounded between 0 and 1.

5 Computational illustrations and implementation

In this section we describe a computational experiment which compares the performance of the hybrid GLSP formulation of Section 3 with the QTP and PTP formulation presented in Section 4. The experiment is based on confidential industry data which will not be provided in detailed.

5.1 Numerical design

Information on demands, capacities, setup times, setup costs, profit margins, inventory holding costs, unit production requirements, initial inventories, and minimum batch sizes are used exactly as given by the industry application. A typical optimization run is done for a planning horizon of 13 months, 15 products, and a product-article allocation which results in 24 articles. Sequence dependent setup times of a single product changeover range between 0 and 5% of the monthly capacity. Production capacity which is required for producing the minimum batch size across all products is between 3% and 14% of monthly capacity. Typically, no more than 4 product setups are allowed per month. Setup and penalty cost are set equal to zero. Furthermore, capacity reductions as well as periods with fixed production quantities are not considered here. The computational complexity of lot-sizing and planning problems with sequence-dependent setup cost and setup times is strongly sensitive to various parameters and their dependencies, e.g., monthly capacity, length of the planning horizon, number of allowable product setups, product structure (number of products, articles, and their allocation), demand characteristics (sparse or dense demand matrix), as well as sequence dependent setup times and costs. For the numerical design we therefore identified eight main factors that influence the performance and selected

	Parameter	Low level	Medium level	High level
1.	Capacity level (in production days)	20	25	30
2.	Number of setups per macro period	4	6	8
3.	Minimum lot-size	base level-50%	base level	base level+50%
4.	Profit margin	base level-50%	base level	base level+50%
5.	Demand per macro period	base level-50%	base level	base level+50%
6.	Number of products and articles	(7,11)	(11,17)	(15,24)
7.	Setup time	base level-50%	base level	base level+50%
8.	Holding cost	base level-50%	base level	base level+50%

Table 1: Numerical design

three factor levels (low, medium, high) for each factor. Table 1 shows the selection of parameters derived from the base level instance.

Since a full factorial design would be too large (3^8=6,561 instances) we used a Latin Hypercube Design (LHD) to draw 21 factor combinations out of the 3^8 possible combinations. LHDs are space-filling designs where the minimum distance between all design points is maximized. Because the problem of finding an LHD is NP hard we resort to design tables published online by Van Dam et al. (2009). The resulting detailed experimental design of the 21 test instances is presented in Table 2.

We implemented the model in Xpress Mosel 2.4 and solved it with Xpress Optimizer 19.0 on a Intel Core2Duo with 2.33 GHz and 2 GB memory. Default options concerning presolving and branch and cut options were used as provided by the software and not altered. For a typical optimization run with 13 months, 15 products, 24 articles, and 4 allowed setups per month, the hybrid GLSP has 23'585 constraints and 15'162 variables, 780 of which are binary. To evaluate the reformulations we carried out computational experiments with different time limits of 600, 1800, and 5400 seconds after which the solution process of th MIP solver was terminated.

Run	Capacity level	Number setups	Minimum lot size	Profit margin	Demand	No. of products	Setup times	Holding cost
1	low	medium	high	medium	medium	medium	medium	high
2	low	high	low	medium	medium	high	high	high
3	low	low	medium	high	high	high	medium	low
4	low	high	medium	low	medium	medium	low	low
5	low	high	medium	medium	low	low	high	medium
6	low	low	low	medium	low	low	low	high
7	low	medium	medium	low	high	low	high	low
8	medium	low	high	low	low	medium	medium	low
9	medium	high	high	high	high	high	medium	medium
10	medium	high	medium	low	low	high	medium	medium
11	medium	low	medium	high	low	medium	high	high
12	medium	medium	high	high	medium	low	medium	low
13	medium	medium	low	low	high	high	low	high
14	medium	medium	low	high	low	medium	high	low
15	high	medium	medium	high	high	low	medium	high
16	high	low	high	medium	medium	high	low	medium
17	high	high	low	high	low	medium	low	medium
18	high	low	low	low	medium	medium	high	high
19	high	medium	high	medium	high	high	high	medium
20	high	high	high	low	medium	low	low	medium
21	high	low	low	medium	high	low	low	low

Table 2: LHD design of 21 test instances out of 3^8 factor combinations

Run	**600 sec.**						**1800 sec.**						**5400 sec**					
	Hybrid-GLSP		QTP		PTP		Hybrid-GLSP		QTP		PTP		Hybrid-GLSP		QTP		PTP	
	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.
1	8.92	600	4.38	600	3.74	600	8.00	1800	3.38	1800	3.40	1800	7.96	5400	3.20	5400	3.06	5400
2	3.84	600	3.10	600	0.36	86.81	3.73	1800	2.57	1800	0.36	86.53	3.73	5400	2.49	5400	0.36	87.08
3	8.04	600	4.38	600	3.87	600	7.99	1800	4.06	1800	3.34	1800	7.40	5400	4.00	5400	3.09	5400
4	3.21	600	2.54	600	1.85	600	3.02	1800	2.40	1800	1.74	1800	2.65	5400	2.23	5400	1.65	5400
5	14.71	600	8.95	600	8.77	600	13.55	1800	8.61	1800	7.78	1800	13.47	5400	8.15	5400	7.55	5400
6	2.40	600	0.43	6.86	0.16	15.19	2.22	1800	0.43	6.88	0.16	15.11	2.18	5400	0.43	7.14	0.16	15.03
7	1.00	600	0.83	600	0.55	600	0.97	1800	0.78	1800	0.50	981.03	0.89	5400	0.76	5400	0.49	979.87
8	14.21	600	3.62	600	4.56	600	13.90	1800	3.22	1800	3.96	1800	13.54	5400	2.91	5400	3.16	5400
9	11.99	600	10.49	600	68.10	600	9.45	1800	8.44	1800	10.90	1800	9.19	5400	7.67	5400	7.34	5400
10	28.83	600	19.48	600	17.68	600	24.29	1800	17.51	1800	17.09	1800	23.99	5400	16.22	5400	15.35	5400
11	7.57	600	1.49	600	1.36	600	7.35	1800	1.33	1800	1.04	1800	6.94	5400	1.26	5400	1.00	5400
12	2.45	600	1.27	600	1.37	600	2.26	1800	1.08	1800	1.18	1800	2.00	5400	1.01	5400	1.07	5400
13	5.77	600	4.39	600	0.36	12.86	5.72	1800	3.97	1800	0.36	13.02	4.88	5400	3.89	5400	0.36	13.03
14	0.82	600	0.50	55.81	0.37	15.19	0.76	1800	0.50	56.39	0.37	15.58	0.69	5400	0.50	56.75	0.37	15.03
15	1.01	600	0.94	600	0.61	600	0.96	1800	0.87	1800	0.54	1800	0.95	5400	0.85	5400	0.49	5249
16	11.38	600	0.85	600	0.80	600	10.73	1800	0.61	1800	0.69	1800	10.19	5400	0.52	5400	0.65	5400
17	2.01	600	1.54	600	1.16	600	1.85	1800	1.34	1800	1.01	1800	1.85	5400	1.27	5400	0.97	5400
18	4.94	600	0.43	5.47	0.00	2.09	4.52	1800	0.43	5.55	0.00	2.11	4.43	5400	0.43	5.47	0.00	2.05
19	13.17	600	12.48	600	8.61	600	11.35	1800	11.39	1800	8.31	1800	10.83	5400	10.32	5400	8.28	5400
20	21.00	600	15.14	600	12.31	600	20.49	1800	14.30	1800	11.90	1800	18.67	5400	13.38	5400	11.17	5400
21	0.64	600	0.24	4.94	0.31	3.36	0.61	1800	0.24	5.14	0.31	3.42	0.59	5400	0.24	5.08	0.31	3.31

Table 3: Numerical results

5.2 Numerical results

Table 3 shows the numerical results for the 21 test instances obtained with the hybrid-GLSP as well as the QTP and PTP formulation. As a further termination criterion, we stopped the model optimization as soon as the integrality gap dropped below 0.5% and returned the currently best solution with respect to CPU time and integrality gap.

The first observation is that the classical hybrid-GLSP is outperformed by both TP reformulation. Table 4 shows the results that compare the sample means of the gaps for all model formulations and cut-off times using paired two-tailed t-tests. We report in column 3 the mean gap for all model formulations and cut-off times. While in column 4 the mean difference in gaps and the 95% confidence interval between QTP (and PTP, respectively) formulation to the hybrid-GLSP is presented, column 5 illustrates the mean difference in gaps and the 95% confidence interval between PTP and QTP approach.

In general, both QTP and PTP perform better than the hybrid-GLSP formulation with a p-value less than 0.01. Note that the significance of a t-test is typically given by its *p-value*, i.e. the probability of rejecting the null hypothesis – of sample means being equal – although it is true. The difference in gaps between PTP and hybrid-GLSP as well as QTP and PTP are not significant for cut-off times of 600 seconds (column 4 and 5 are presented in brackets). For cut-off times of 1800 seconds the gap difference becomes significant with a p-value of less than 0.1 and for a cut-off time of 5400 seconds the p-value decreases to 0.01, i.e., the PTP approach performs significantly better than the QTP.

While the hybrid-GLSP could not find any solution with an integrality gap below 0.5%, the QTP and PTP approach found a solution with a lower gap for several test instances. The QTP approach was able to find solutions with a gap below 0.5% in 4 (instance 6, 14, 18, 21) out of the 21 test instances within a time limit of 60 seconds. Moreover, 3 of these 4 instances found the solution within 10 seconds. However, it could be observed that an increase of the time limit does not yield to a significant performance improvement. An increase of the time limit from 600 seconds to 1800 decreases the mean gap only by 0.48%. A further increase to 5400 seconds decreases the mean gap by 0.3% (see Table 4). A characteristic that all these 4 test instances have in common is that the number of setups and the number of products is low or medium. Both parameters directly influence the model size, i.e., the number of variables and constraints.

The average performance of the PTP approach across all test instances is better than the QTP approach. While QTP could solve only 4 test instances with a gap below 0.5%, the PTP approach solved 6 instances within 600 seconds, 7 instances within 1800 seconds, and 8 instances within 5400 seconds. Even when the average PTP performance is better than the QTP performance, there are 4 test instances (if CPU time is limited to 5400 seconds, there are only 3 test instances) where QTP outperforms PTP. A characteristic that the 4 test instances (8,9,12,16) have in common is that the minimum lot-size is high.

Time in sec.	Formulation (#)	Mean GAP μ in %	Mean Difference in %	
600	Hybrid-GLSP	8.00	$\mu_{\#} - \mu_{GLSP}$	$\mu_{\#} - \mu_{QTP}$
	QTP	4.64	-3.36 ± 1.73	–
	PTP	6.52	(-1.48 ± 6.21)	(1.88 ± 5.84)
1800	Hybrid-GLSP	7.32	$\mu_{\#} - \mu_{GLSP}$	$\mu_{\#} - \mu_{QTP}$
	QTP	4.16	-3.16 ± 1.50	–
	PTP	3.57	-3.75 ± 1.53	(-0.60 ± 0.60)
5400	Hybrid-GLSP	7.00	$\mu_{\#} - \mu_{GLSP}$	$\mu_{\#} - \mu_{QTP}$
	QTP	3.86	-3.23 ± 1.47	–
	PTP	3.18	-3.82 ± 1.46	-0.68 ± 0.45

Table 4: Mean GAP and 95% confidence intervals of the difference

5.3 Implementation

The model is used on a regular basis once a month to support the short to mid term rough cut planning. To integrate the model in the planning process, an interface to MS EXCEL has been programmed which automatically uses on data from an EXCEL spreadsheet and returns the results back into an EXCEL spreadsheet. The supply chain planner who is responsible for the particular production campaign extracts information from the ERP, executes an optimization run and returns the production plan into the ERP. The tool is used in a rolling horizon scheme and the output is used as partial input to the short-term and mid-term (detailed) scheduling problem described in Janak et al. (2006a,b).

To interface Xpress with Microsoft Excel and implement the process logic, we used Microsoft Visual Basic for Applications (VBA) in combination with the Xpress Mosel Compiler and Runtime (XPRM) libraries for Visual Basic. This allows us to implement and test the model using Xpress' integrated development environment, but also run the model from within Microsoft Excel. Visual Basic handles model compilation, execution and generation of the solution workbook. XPRM accesses the two workbooks via ODBC to read input and write output data using SQL. When the planner initializes a new optimization run, a VBA method is called that prepares the input and output workbooks to exchange data with the Xpress model. It automatically adjusts the size and attribute labels of the SQL tables inside the worksheets to the number of articles and macro periods defined by the planner. Afterwards, the XPRM library is called to compile and execute the model. In Xpress, a sequence of SQL commands converts the worksheet data into Mosels native data structure. After the optimization is terminated, objective value, integrality gap and decision variables, i.e. production quantities, are written to the output workbook. A final VBA method then opens the workbook and presents the solution to the planner.

6 Conclusion

We developed a tailored hybrid general lot-sizing and scheduling model for a complex practical problem from the process industry. The production process is characterized by a two-stage product structure as well as sequence-dependent setup cost and setup times. The model thereby incorporates several company-specific features. For practical deployment we created an interface between a spreadsheet tool connected to a standard mixed-integer programming solver. In order to enhance the computational performance we developed and tested two reformulations of the initial model based on the Transportation Problem. The concept of the reformulations is to decompose production variables related to the demand period at which the produced quantity is required.

The reported numerical study gives a flavor of the performance of the hybrid GLSP and the two reformulations with respect to computational time and integrality gap for real world lot-sizing and scheduling problems. Even if for short cut-off times the reformulations do not perform significantly better, when the reformulations are given more time, the performance improves significantly. Our numerical study indicates that the remaining integrality gap may still be substantial, even after 5400 seconds.

For future work, it would be interesting to investigate whether the inclusion of valid inequalities will help to further improve this gap. Moreover, the applicability of the GLSP framework together with an exact mixed integer programming solver is rather limited, especially if the number of products and changeovers and thus the number of micro periods increases. As a natural next step, decomposition approaches and valid inequalities should be developed to extend this approach to larger or more detailed problems.

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