Computer simulations of table tennis ball trajectories for studies of the influence of ball size and net height

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Abstract

One possible measure to increase the medial appeal of table tennis is to slow down the game by using bigger balls or higher nets. Usually, an empirical approach is followed to study the effect of such changes on the players and the game. In this work, a different approach is taken, namely solving numerically the equation of motion for table tennis balls for systematical, statistical studies of the impact of ball size and weight as well as of net height on the distribution functions of successful strokes.

The analysis confirms the empirical observation that the change of the ball in the year 2000 from a 38-mm to a 40-mm-ball can be compensated with other parameters such that their resulting trajectory distribution functions are nearly identical. This was also observed in reality, where adaptation of the player's technique compensated the larger ball size. A larger ball of 44 mm with small weight is one option for suppressing high velocities, coupled also to a reduction of the influence of spinning. As an alternative an increase of the net height is possible. A small increase of the net height could be one future option, where the basic character of the game is not strongly modified, but especially the influence of the service could be reduced.

KEYWORDS: SPORTS EQUIPMENT, PHYSICS COMPUTING, MONTE CARLO METHODS

Introduction

The medial appeal of table tennis seems to go down in terms of TV hours, at least outside Asia. One of the reasons is the fact that the speed of the game is nowadays so high that it is very hard for spectators to follow the balls (Nelson 1997, Djokic 2007). Possible counteractions to slow down the game are to use bigger balls or higher nets. Usually, empirical studies are done to study the effect of such changes on the players and the game. An alternative approach, followed in this work, is the use of computer simulations. The equation of motion for table tennis balls is solved numerically to allow systematical, statistical studies of the impact of ball size and weight as well as of net height on the distribution functions of successful strokes.

One key problem for the medial appeal of table tennis is that the spin of the ball, the rotation, is not visible for spectators, because they see only its effect. This makes it difficult to understand why a simple looking ball of the opponent leads to a mistake for the other player. Therefore, one intention of possible rule changes is to reduce the impact of spin on the game. Another goal is to reduce the speed of the balls to allow a better visual tracking during the rallies (Djokic 2007). Some rule changes, like a larger ball, different counting system, stricter limits for rubbers or new service rules, were already implemented and new modifications are under discussion (Djokic 2007). For the players all rule or technical changes have strong impacts on their techniques and strategies, requiring usually adaptations of their individual training programs. Therefore, players are rather hesitant to new rules.

The 40-mm-ball played today is 2 mm larger and 0.2 grams heavier than the 38-mm-ball used before. It has a larger air drag due to its larger cross sectional area reducing the maximum velocities (Bai 2005). The mass distribution of the larger ball is shifted further away from the center compared with the 38 mm ball. This creates a larger inertial moment and reduces the spin. The larger 40-mm-ball results in a velocity and spin reduction of about 5 to 10 percent (Li 2005, Iimoto 2002). However, the larger ball had practically no impact on the characteristics of table tennis, because larger exertions of forces by the players compensated the effects of the size increase (Liu 2005, Li 2005). As a consequence of the modified technique, the fitness of the individual player got more important. In modern table tennis the forces for a stroke are created not only by the arms but the whole body is used to support this. A stronger athletics allows more pronounced use of the legs producing larger forces on the ball, which are needed to compensate the size increase. In addition, the wrist has to be used more effective to produce spin. For the larger ball only the use of the forearm is no longer sufficient for spin, as it was the case for the 38-mm-ball. The needs for larger exertion of forces amplify possible technical mistakes, because the individual movement execution gets extended (Kondric 2007).

One obvious strategy to reduce the maximum velocity in table tennis rallies is to increase the net height. However, such a change will have a severe impact on the characteristics of table tennis, because this will limit very directly fast spins, shots and service. Therefore, up to now this change of rule was avoided and ball size was the preferred correction action. Nevertheless, a scientific data base is still missing for a decision.

In this work the impact of larger balls or higher nets on table tennis trajectories is studied using computer simulations. A data base is created to quantify the influence of such changes. Modifications in technique, tactics, strength and fitness are not considered in this analysis. For a huge number of initial conditions the effect on successful strokes is studied. This delivers the maximum amount of possible strokes for different conditions in terms of statistical distributions which can be compared and analyzed. This represents already the best possible adaptation to the changes, independent of what this would mean for the players in terms of changes in their training. In particular the impact of the changes on the ball velocity distributions will be discussed as motivated before.

After a short discussion of the effects of larger balls and higher nets as measures to slow down table tennis, the forces acting on a moving ball are introduced. The computer code solving the equation of motion is described and statistical analysis of trajectory distribution functions for different balls and net heights is done. Using for this a GPU (Graphics Processing Unit) by CUDA (Compute Unified Device Architecture, CUDA 2013) coding gives a very large speed-up compared to CPUs. Results for different cases are compared and analyzed. Finally, the results are summarized and discussed.

Methods

For a quantitative analysis of ball size and net height effects a computational approach is followed. The basic element of the simulation is the solution of the equation of motion for table tennis balls. The equation of motion needs a mathematical description of the acting forces. The flight trajectory of a table tennis ball is determined by the gravitational force of the earth and aero dynamical forces.

The gravitational force

$$\vec{F}_G = -m \cdot \vec{g}$$

alone results in a parabolic trajectory. This force acts towards the centre of the earth and depends on the mass m of the ball and the gravitational constant g (9.81 m/s²).

The aero dynamical forces modify the simple parabola by air drag and lift. Air drag acts as friction force against the direction of the movement of the ball. A simple example for this force is the back pushing of a hand held out of a driving car. A larger velocity gives stronger force acting against the direction of the car. This force also gets larger if one puts out not only a part of the hand, but the full hand. It scales with the cross sectional area. The mathematical expression is

$$\vec{F}_D = -\tfrac{1}{2} \cdot C_D \cdot \rho \cdot A \cdot v \cdot \vec{v},$$

with the density of air ρ, the cross sectional area A for a ball with radius r ($A = r^2 \cdot \pi$), the ball velocity v and an air drag coefficient C_D. This coefficient can be measured, e.g. in wind tunnel experiments.

The second important aero dynamic force is the air lift. The so-called "Magnus effect", named after his discoverer Heinrich Gustav Magnus (1802-1870), is the reason that a rotating ball experiences a deviation from its flight path. A famous example for this is a free kick goal from the Brazilian soccer player Roberto Carlos in a friendly game with France at the 3rd of June 1997. Carlos gave a lot of spin to the ball during the free kick hitting the ball right from the center of gravity with his left foot. The flight path of the ball got extreme passing around the defenders who formed a wall into the goal.

The Magnus effect is a surface effect, because around the spinning ball a co-rotating air layer is formed at the surface of the ball. The flying and spinning ball induces a pressure imbalance, because on one side the ball is rotating with the air flow created by the movement of the ball in the air, the other side opposite to it. On the side where counter-rotation exists, the total velocity of the air flow is reduced, because both velocities compensate partly. On the co-rotation side a larger flow velocity is created, because both velocities add up. Higher velocity in a flow means lower pressure and the pressure differences on the two sides lead to the deviating Magnus force, mathematically expressed with an air lift coefficient C_L as

$$\vec{F}_L = \tfrac{1}{2} \cdot C_L \cdot \rho \cdot A \cdot v \cdot \overrightarrow{e_\omega} \times \vec{v}$$

The air lift force acts perpendicular to the axis of rotation $\overrightarrow{e_\omega}$ and to the velocity $\vec{v}$.

Air drag and lift coefficients of a rotating ball (see Figure 1) as a function of the ratio of spinning velocity to translational velocity are implemented into the computer code as a fit of experimental data (Achenbach 1972, Bearman 1976, Davies 1949, Maccoll 1928, Mehta 1985) as a rational function y(x)

$$y(x) = \frac{a + b \cdot x + c \cdot x^2 + d \cdot x^3}{1 + e \cdot x + f \cdot x^2 + g \cdot x^3}$$

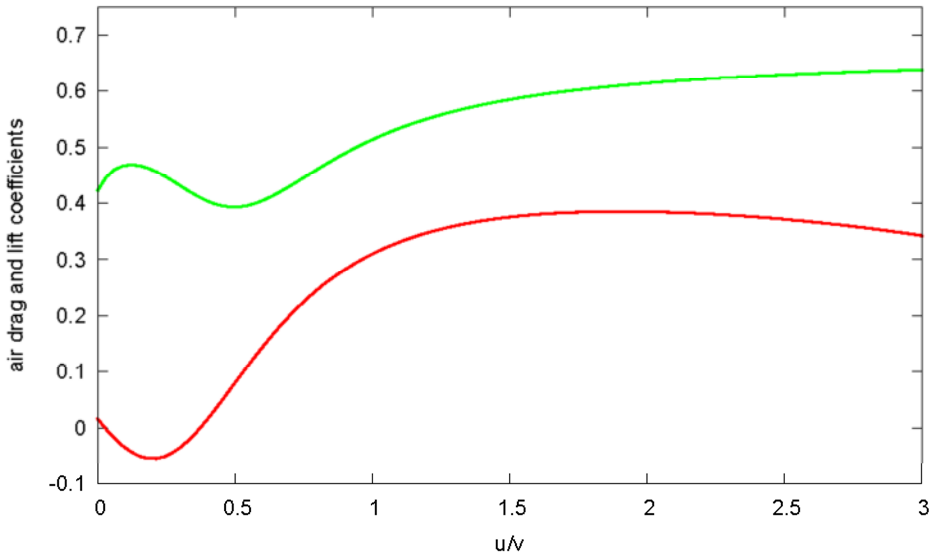


Figure 1: Air drag coefficient CD (upper green curve) and air lift coefficient CL (lower red curve) as a function of the ratio of spinning velocity u to the translational velocity v.

During a topspin shot with forward rotation the lift force acts downwards, during a backspin with backward rotation it acts upwards.

Swirling balls, often quoted in soccer and volleyball, can be created when the ball is hit with a critical velocity leading to the access of the inverse Magnus effect. It shows up in Figure 1 for low spinning velocities as a negative value of the air lift coefficient. This can lead also in table tennis to swirling balls, because during the flight path the regime of positive and negative air lift coefficients can change resulting in a swirling. However, for table tennis balls negative air lift coefficients exist only where the coefficient itself is already quite small. Therefore, the effect exists, but gives only deviations of some millimeters. The frequently quoted swirling balls with long pimples are therefore more a psychological effect than physics: the pre-programmed movement of the player anticipates a flight path of a strongly rotating ball from a normal rubber sponge. The balls from the long pimples with reduced rotation have a different flight path with less lift and fall down earlier such that the player is missing the ball and he complains, that the ball was swirling.

The computer code solves the equation of motion of table tennis balls for given initial positions, velocities and spins. An Euler solver was used, because its algorithmic simplicity allowed an easy transfer onto the GPU with CUDA. A commonly used Runge-Kutta algorithm was not chosen, because it has larger computational costs. A fourth order Runge Kutta approach needs to calculate four times the forces, which slows down the code performance in our case compared to the simple Euler method. This was not compensated by the larger time step possible with the Runge-Kutta method compared to the Euler method. The dependence of the aero dynamic forces on the velocity also does not allow the use of a Verlet algorithm. Therefore, we decided to stay with the Euler method.

One example of a table tennis ball trajectory is shown as a red line in Figure 2. The table tennis table region is marked in green, the net is blue. The orange sphere is the initial point of the trajectory, where the ball is hit. The spinning of the ball is taken constant during the flight. x and y are the spatial coordinates within the plane of the table tennis table. z is the height coordinate above the table. A time step of 0.0001 seconds was used.

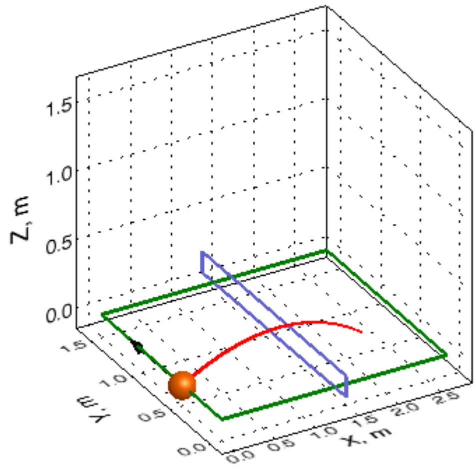


Figure 2: 3D trajectory of a table tennis ball

The ball in Figure 2 is hit at the baseline (x = 0 m) in the forehand part of the table (y = 0.6 m) on the height of the table (z = 0 m). The black arrow shows the rotation axis of the ball, which is here purely pointing into positive y-direction: the ball was a pure topspin without any sidespin.

Results

For a statistical analysis of the effects of ball sizes and net heights on trajectories of table tennis balls a Monte Carlo procedure was used. Many different initial conditions were solved: x was varied between 0.3 m to -3 m, representing hitting locations from 30 cm above the table to 3m behind the table. y was kept constant at 0.381 m, which is ¼ of the width of the table tennis table. This was chosen as a representative position, the exact location of the hitting point in y (forehand or backhand position) is not important for this numerical test. Initial height z was sampled from 0.4 m to -0.4 m. The direction of the initial velocity was determined in the following way: the horizontal angle was sampled between the limiting angles of the starting point to the net posts, the elevation angle was chosen randomly. The spin axis was also sampled randomly, that means topspin, backspin and sidespin were included.

The analysis was particularly aiming at fast shots. Therefore, only balls passing the net within 30 cm height distance were accepted. The absolute values of the translational velocities were limited from 20 to 200 km/h, the spinning velocities from 0 to 150 turns/s (which is equal to 9000 turns/min). These values were determined empirically before as limits for 38 mm balls (Wu 1993). These limits are probably different for other balls sizes and net heights, but in all case studies successful hits were not restricted by the accessible parameter space chosen here. A ball is counted as a successful ball if it passes the net within the height limit and hits the other side of the table tennis table.

Monte Carlo studies using random numbers were done for the 38-mm-ball with a weight of 2.5 g, used in tournaments until end of 2000, the actual 40-mm-ball with 2.7 g and a 44-mm-ball with a weight of 2.3 g, which was tested already in Japan. For the 40-mm-ball an increase of the net height for 1 and 3 cm was analyzed, too.

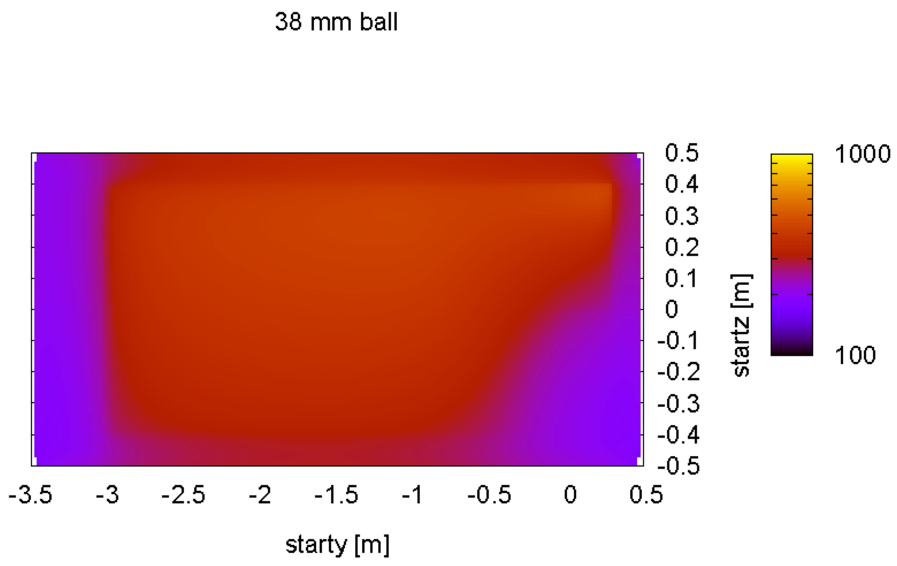
The sampling of such a large number of initial conditions guarantees to cover all possible combinations of initial parameters (positions, translational and spinning velocities) for the different cases creating a successful stroke. Clearly, for different balls and net heights the parameter space of initial conditions leading to successful strokes will be different. The database created in this study allows also an analysis of this effect.

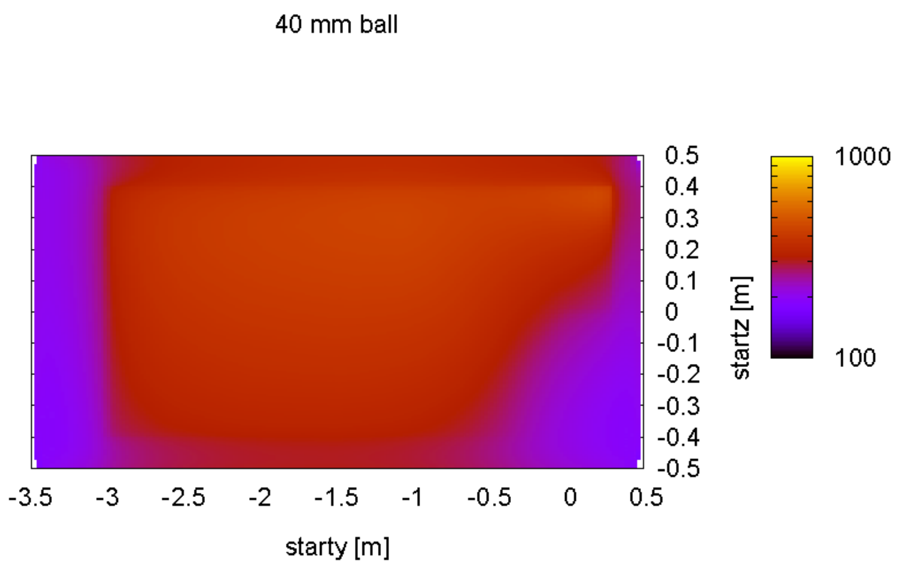
For each case $5*10^8$ initial conditions were sampled and trajectories calculated. Initially this was done on a Linux Cluster with 32 cores. The run-time for each core was 20 hours resulting in a total run time of 640 hours. Alternatively, GPU computing with CUDA was used on a Dell Precision T7500 Desktop with NVIDIA Quadro FX3800. Here, only 3 hours for the same calculation are needed. CUDA (CUDA 2013) is a programming interface to use the parallel architecture of NVIDIA GPUs for general purpose computing. CUDA library functions are provided as extensions of the C language, which allows for convenient and rather natural mapping of algorithms from C to CUDA. A compiler generates executable code for the CUDA device. The CPU identifies a CUDA device as a multi-core coprocessor. For the programmer, CUDA consists of a collection of threads running in parallel. A collection of threads, which is called a block, runs on a multiprocessor at a given time. The blocks form a so-called grid. They divide the common resources, like registers and shared memory, equally among them. All threads of the grid execute a single program called the kernel. All memory available on the device can be accessed using CUDA with no restrictions on its representation. However, the access times vary for different types of memory. Shared and register's memory are the fastest, as they locate on the multiprocessor (on chip).The shared memory has the lifetime of the block and it is accessible by any thread on the block from which it has been created. This enhancement in the memory model allows programmers to better exploit the parallel power of the GPU for general purpose computing. Additionally, the texture memory which is off-chip allows for faster reading compared to the global memory due to caching.

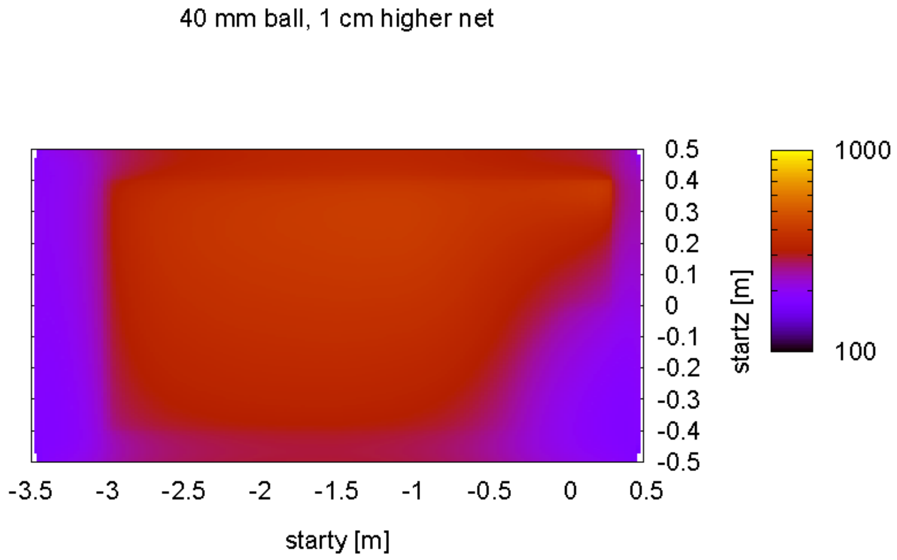
Our implementation consists of two main procedures. First, a predefined number of trajectories are initialized on the CPU side. Thereafter, the ball movements are implemented on the GPU. One step of the equation of motion for the ball's trajectory, which includes the speed and the position of the ball, is computed in a kernel. The input parameter of the kernel function is the previous trajectory point. The calculations run for a maximal number of iterations. In each iteration step, the updates of the ball's position and velocity are computed, if the trajectory has not stopped earlier, e. g., when the ball flew beyond the table.

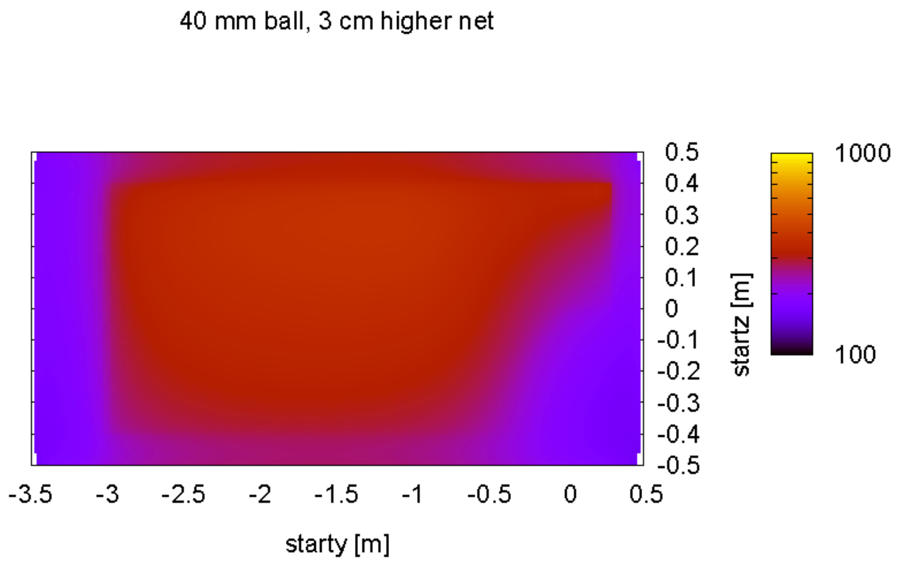
Figure 3 shows as a function of initial position in y and z the number of successful trajectories. The number of successful trajectories from half distance is nearly constant for all balls and net heights. Only for distances below one meter the number of successful strokes decreases continuously, because balls in this region have smaller probabilities hitting the table due to the smaller angle. Balls hit above the table can again reach easier the other side. There is practically no difference for the 38 and 40-mm-ball. Changes of the balls are compensated by other parameter changes. The 44-mm-ball allows more successful strokes even for negative height, because of its lighter weight and its higher air drag. A higher net affects strongly the balls hits above the table limiting there the number of successful trajectories.

In general, the differences between the different cases get more pronounced the higher the hitting point of the balls. A ball hit below the table must have a large spinning to reach the other table side within the height limit. Larger velocities are not possible, because then the balls are not able to reach the other table side and will pass beyond the baseline. Balls hit above the table, even above the height of the net, can be hit with much higher velocities for a successful strike.


38 mm ball
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
startz [m]
1000
100
-3.5 -3 -2.5 -2 -1.5 -1 -0.5 0 0.5
starty [m]


40 mm ball
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
startz [m]
1000
100
-3.5 -3 -2.5 -2 -1.5 -1 -0.5 0 0.5
starty [m]


40 mm ball, 1 cm higher net
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
startz [m]
1000
100
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
starty [m]


40 mm ball, 3 cm higher net
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
startz [m]
1000
100
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
starty [m]

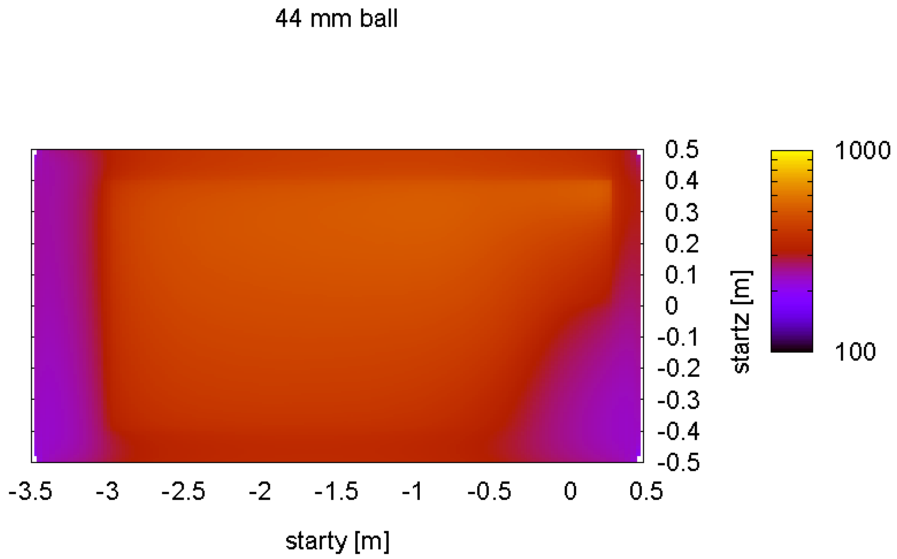


Figure 3: Number of successful trajectories as a function of initial y- and z-conditions

In Figures 4 and 5 the influence of the ball velocity on the distribution functions of the number of successful strokes is shown. Figure 4 shows the dependence on the initial velocity, Figure 5 the dependence on the final velocity. The velocity range used for sampling the initial velocity of 20-200 km/h is identical to 5.6-55.6 m/s.

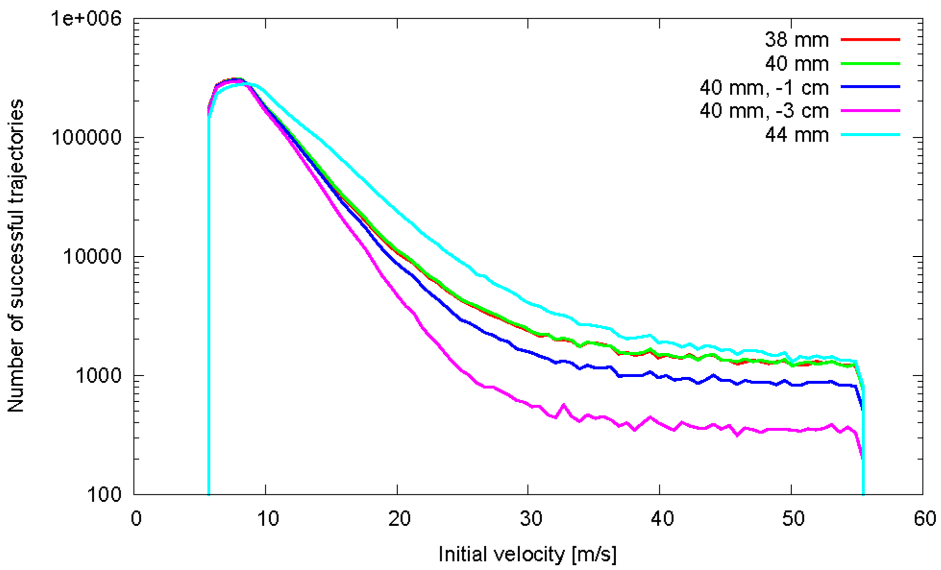


Figure 4: Number of successful trajectories as a function of initial velocity of the balls

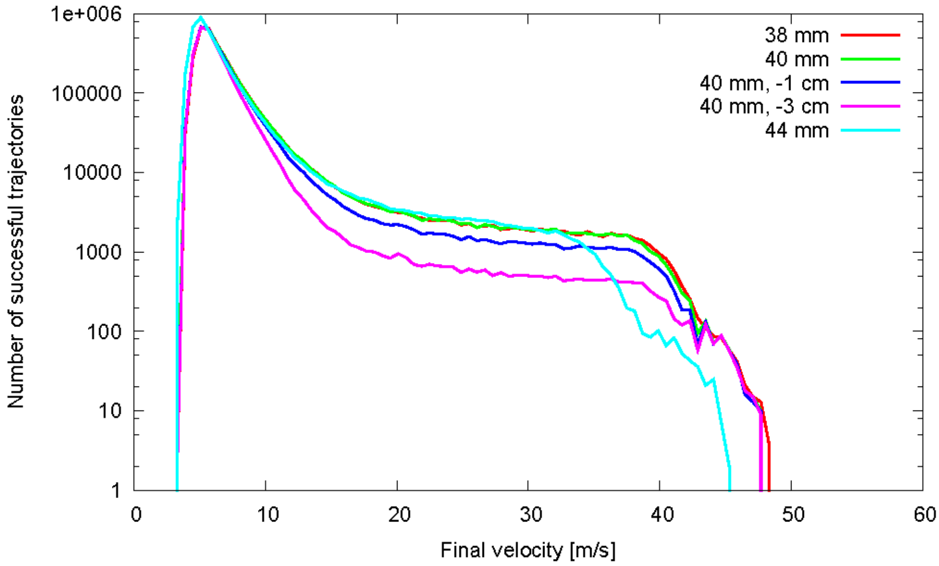


Figure 5: Number of successful trajectories as a function of final velocity of the balls

Again, the results for the 38 and 40 mm ball differ only marginally. For the 44-mm-ball one gets more successful trajectories compared to the 38 and 40-mm-ball for higher initial velocity, the distributions for the final velocities are nevertheless very close again. However, very high velocities above 35 m/s are suppressed earlier for the 44-mm-ball. A stronger influence is visible for the 40-mm-ball increasing the net height. Already for smaller initial and end velocities of about 10 m/s a reduction of successful trajectories shows up being equivalent to a slowing-down of the game. For very low velocities the impact of the air drag is not yet important resulting in larger number of successful trajectories.

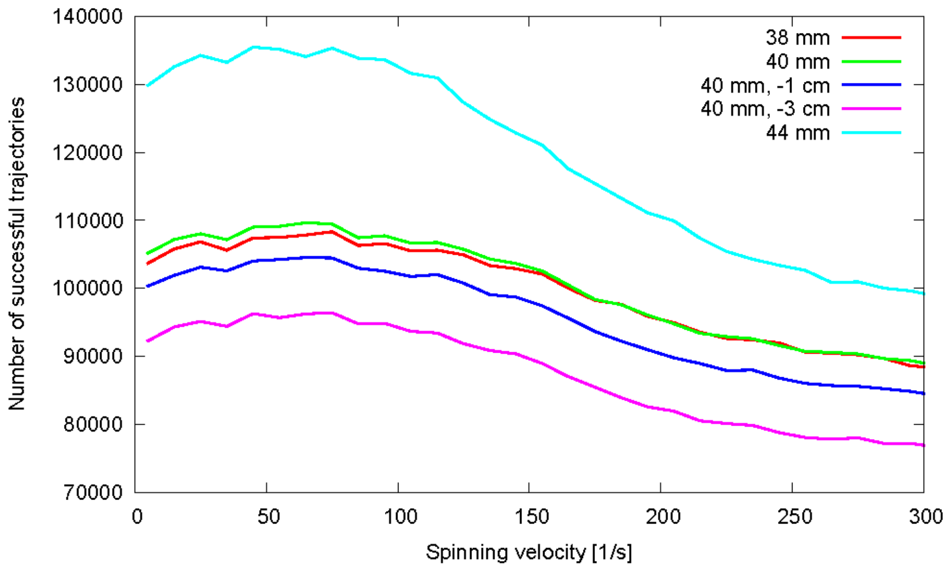


Figure 6: Number of successful trajectories as a function of spinning velocity

Figure 6 demonstrates that the influence of spin is rather weak, because all differences are within 20 percent. The number of successful trajectories is biggest for the 44-mm-ball, followed by nearly identical numbers for the 38 and 40 mm ball and the case with a 1 cm increase of the net height. As expected the highest net gives the smallest number of successful trajectories. The ratio of successful trajectories with strong spinning to those with little spinning is nearly the same in all cases with the exception of the 44-mm-ball. Here, the influence of spinning on the distribution is strongly reduced.

Conclusions

Statistical analysis of the influence of ball size and net height on the number of successful table tennis trajectories using computer modeling is used to quantify the effects on trajectory distribution functions. The analysis confirm the empirical observation that the change of the ball in the year 2000 from a 38-mm to a 40-mm-ball can be compensated such that their resulting trajectory distribution functions are nearly identical. This was achieved in reality by adaptations of the technique and the material. A larger ball of 44 mm with small weight is one option for suppressing high velocities, resulting also in a reduction of the influence of spinning. As an alternative option an increase of the net height is possible. For this, the character of the game will change more strongly, because the possibilities for successful trajectories are reduced limiting technical and tactical alternatives. A small increase of the net height could be one option, where the basic character of the game is not too strongly modified, but reducing especially the influence of the service.

Modifications of basic rules of table tennis like ball size and net height can reduce the maximum velocities, but such modifications will be linked with severe changes in the characteristics of table tennis: dynamics, technique and strategy will change strongly, too. The question is if a possible gain in attractivity of table tennis for TV by such changes is worth the loss of key elements of existing table tennis.

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