An Automated Acquisition Setup for the Evaluation of Intermittency Statistics

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Abstract - The design and realization of an automated acquisition setup, dedicated to experimental evaluation of intermittent chaotic phenomena and the related statistics, is presented. The setup was implemented in National Instrument’s LabView environment and it was structured in such a way that it is not dependent of the signal-registering devices used. The circuit evaluation is achieved by registering only one signal. An experimental intermittency example confirms the system’s effectiveness.

Keywords – Chaotic circuits, intermittency, on-off intermittency, in-out intermittency, automated measurement system.

I. INTRODUCTION

During the last twenty years, the problem of identifying and controlling system chaotic behavior has become the main issue in all applied science fields [1-4]. Any strongly nonlinear circuit or dynamical system tends to exhibit not only periodic behavior, but also more complicated behaviors, ranging from quasi-periodicity to chaotic behavior [5].

Potential applications of nonlinear chaotic circuits for secure or ultra-wideband data transmission, have raised the engineering society’s interest. It includes studying and investigating circuit chaotic behavior in all aspects, ranging from simple characterization to thorough investigation of their dynamics [6-12].

There are two main directions in investigating chaotic dynamics; the first one is computer simulation (mainly by numerically solving a circuit’s differential equations) and the second is conducting experimental measurements in real chaotic circuits. The first method is easy and of low-cost, but experimental verification is always essential.

Although each nonlinear circuit is a distinct case-study, requiring its own investigation, one can say that characterization of chaotic behavior is usually accomplished through certain steps. Phase portraits, power spectrum, bifurcation diagrams and Poincare sections, suggest very good criteria for deciding chaotic or periodic behavior. On the other hand, Lyapunov exponents, correlation and information dimension, as well as Kolmogorov-Sinai entropy are quantitative measures of chaotic behavior and they are very useful in defining nonlinear circuit features [13, 14].

A basic issue in the study of nonlinear circuit dynamics is the way that their behavior is evolving, according to their parameter changing. The development of qualitative and quantitative measures useful in the study of a circuit’s structural stability remains a crucial topic. Until today, there is no general framework determining the evolution of the behavior of nonlinear circuits or systems and many times, prediction of the mode of operation (periodic or chaotic) is not possible. The observed routes to chaos of nonlinear circuits (and dynamical systems) are the following [13]:

• The period doubling route to chaos
• Quasi-periodicity
• The intermittency route to chaos
• Crisis

All four scenarios demonstrate the feature of universality, which means that there are the same basic properties in each route to chaotic mode of operation, which are independent of the nonlinear circuit or dynamical system [5, 13-15].

Experimental study of nonlinear circuits is very important either in terms of circuit-model verification or independent experimental characterization of electronic circuit chaotic behavior. For this reason a number of automated setups for the experimental evaluation and analysis of chaotic systems have been recently presented [16, 17], having different capabilities, though.

Intermittency statistics have been theoretically studied and described and are of great importance in characterizing a nonlinear circuit. Although this kind of evaluation is not such a complicated procedure when the nonlinear circuit is studied by computer simulation, this is not so when the circuit or the system is experimentally studied.

In this paper, an automated acquisition setup for the experimental evaluation of intermittency statistics is presented, supplied by two application examples in the case of crisis induced intermittency and on-off intermittency.

II. INTERMITTENCY

Intermittency is an apparently random alternation of a signal, between a quiescent (or laminar) state and a bursting state. Intermittent behavior is one of the known routes from periodic to chaotic mode of operation [13-15]. This route to chaos was first studied by Pomeau and Manneville in 1979 [18, 19] and was experimentally
verified in the Bergé experimental study of the Reyleigh-Benard convection [20].

Models of intermittency are well known in nonlinear dynamics and include the three types introduced by Pomeau and Manneville [19], as well as the crisis induced intermittency [21, 22]. There have been also reports of a different type of intermittent behavior known as on-off intermittency [23] appearing in chaotic synchronization between nonlinear circuits (or systems) and its generalization, known as in-out intermittency [24, 25]. The first four intermittency phenomena regard transition from periodic to chaotic mode of operation, while the last two refer mostly in synchronization between coupled chaotic operating circuits.

Although, in the first three cases of intermittency, the main component is the transition from periodic to chaotic mode of operation, the kind of bifurcation, that the circuit’s dynamical system undergoes, serves as a criterion for their classification:

Type I intermittency:
It appears due to a saddle-node bifurcation.

Type II intermittency:
It appears due to a Hopf bifurcation.

Type III intermittency:
It appears due to inverse period doubling.

In all three types, the circuit initially operates in periodic mode; a state that corresponds to stable closed trajectories in the phase space. This periodic mode of operation is replaced by alternating transitions between chaos and periodicity, as a system parameter \( r \) is changing (bifurcation parameter). This results to replacement of regular periodic oscillations by irregular chaotic bursts. The parameter value for which intermittency begins is called critical parameter value \( r_{crit} \). During the chaotic bursts the stable periodic trajectories are becoming unstable and finally destroyed (for increasing the bifurcation parameter). As a result, the system’s trajectory orbits far away from the vicinity of the original stable periodic one. But it seems that the system retains a kind of memory, because when the orbit gets back at the neighborhood of the periodic trajectory it enclaves itself in it [5, 13-15].

At the onset of intermittent transition to chaos and for bifurcation parameter values \( r < r_{crit} \), the system spends more time in regular periodic mode of operation interrupted rarely by short chaotic bursts. The time spaces of periodic behavior are called laminar lengths and although the chaotic bursts seem to happen randomly, one can define a mean laminar length \(<\ell>\). Further increase of the bifurcation parameter leads to more frequent appearance of these chaotic bursts, which now go on for longer time spaces. Consequently, mean laminar length tends to zero \(<\ell>\rightarrow 0\), resulting to a fully chaotic behavior [18].

Crisis induced intermittency is an apparently sudden qualitative structure change between a nonlinear circuit’s chaotic attractors, due to a circuit parameter variation. Transitions between the different structures of the chaotic attractor lead to an intermittent behavior. There are three types of crisis induced intermittency:

**Boundary crisis:**
It is related to the sudden destruction of a chaotic attractor.

**Interior crisis:**
It is related to the attractor’s increase of volume, in the system phase space.

**Attractor merging crisis:**
It comes up when at least two different chaotic attractors collide (in order to form one attractor).

Resting in only one of the attractors defines laminar lengths, while transition to the other attractor defines irregular bursts. It should be noted that crisis induced intermittency is a phenomenon that can happen in both directions [5, 14].

Finally, there exists another kind of intermittency, which is mainly observed in the case of synchronization between coupled chaotic-operating circuits. On-off intermittency is related to the transverse instability of (chaotic) attractors confined to a lower dimensional manifold [26, 27]. Because of this low dimension of the manifold, every intermittency exhibits clear and distinct regular (laminar) and irregular (burst) stages.

A generalized form of the on–off intermittency is the so-called in–out intermittency [24, 25, 28]. It is also associated with some invariant sets in a lower dimensional manifold, but these invariant sets don’t need to be (minimal) chaotic attractors as in the on–off intermittency case. In in-out intermittency, trajectories escape occasionally from the relatively steady laminar state to irregular “burst” states and then quickly return back to the laminar state. As a result, the system attractor blows out from a lower dimensional subspace in a random-like fashion, due to transverse instability [24].

### III. INTERMITTENCY STATISTICS

Despite the “irregularity” demonstrated by intermittently operating circuits and generally speaking systems, statistics that are related to all kinds of intermittency obey to certain power laws, described later on in this section.

A general law holds for all intermittent phenomena either regarding routes to a circuit’s chaotic mode of operation or synchronization properties of coupled circuits. This general law describes the way that the mean laminar length \(<\ell>\) scales, accordingly to the deviation of the circuit’s bifurcation parameter \( r \) from its critical value \( r_{crit} \):

\[
\langle \tau \rangle \propto (r - r_{crit})^{-\gamma}
\]

where, \( \gamma \) is the critical exponent that characterizes the type of intermittency described.

Since each type of intermittency demonstrates different
transition behavior near the critical bifurcation parameter, Pomeau and Manneville have defined the value of the critical exponent $\gamma$ for the three types of intermittency as follows [14]:

**Type I intermittency:**

$$\langle \tau \rangle \propto (r - r_{\text{cr}})^{-3/2}$$  \hspace{1cm} (2)

**Type II & III intermittency:**

$$\langle \tau \rangle \propto (r - r_{\text{cr}})^{-1}$$  \hspace{1cm} (3)

It should be noted that although intermittencies of type II and III obey to the same power law for the mean laminar length, they do not share the same distribution of the probability of laminar length. As a result the following distributions hold for each type of intermittency.

**Type I intermittency:**

$$P(\tau) \propto \tau^{-3/2}$$  \hspace{1cm} (4)

**Type II intermittency:**

$$P(\tau) \propto \tau^{-2}$$  \hspace{1cm} (5)

**Type III intermittency:**

$$P(\tau) \propto \tau^{-3/2}$$  \hspace{1cm} (6)

In the case of crisis induced intermittency, the general power law of (1) still holds but the critical exponent $\gamma$ does not have any specific value. On the contrary, it can have any value in the range of [14]:

$$\frac{1}{2} \leq \gamma \leq \frac{3}{2}$$  \hspace{1cm} (7)

The distribution of the probability of transition duration $\tau$, from one sub attractor to the other, does not follow any of the equations (4)–(6), but obeys to the following power law:

$$P(\tau) \propto \tau^{-3/2} e^{-\tau/\langle \tau \rangle}$$  \hspace{1cm} (8)

where $\langle \tau \rangle$ is the mean transition time duration, between the two sub attractors for a certain value of the bifurcation parameter [29, 14].

Finally, in the case of on-off and in-out intermittency, the related statistics can be defined as those of the Pomeau-Manneville type III intermittency described by (3) and (6). It should be mentioned that, in the case of on-off intermittency, the laminar length distribution deviates from the power law described by (6), for very small and very large values of $\langle \tau \rangle$ [24].

### IV. THE PROPOSED SETUP

The previous section demonstrates the fact that any experimental evaluation of intermittent phenomena of all kinds, should be conducted on time domain. Since the nature of the necessary calculations is statistical, the central problem is the very large number of transitions for each bifurcation parameter that should be acquired, in order to ensure a safe evaluation.

Practically, in the case of intermittent behavior of any circuit, the main objective is to obtain the laminar length distribution $\tau$ vs $P(\tau)$ for each value of the circuit bifurcation parameter ($r$). These distributions lead to the calculation of the mean laminar length $\langle \tau \rangle$ for each value of the bifurcation parameter ($r$). According to (1) a proper fitting could calculate the critical exponent $\gamma$ and lead to conclusions about the kind or the type of intermittency. On the other hand, proper fittings of the laminar length distributions themselves [according to (4)–(6) and (8)], could give additional information, which combined with the critical exponent calculation could lead to safer conclusions about the type of intermittency.

Since, a very large number of transitions is demanded (usually more than $10^6$), in order to reach safe conclusions from the resulting fittings, a dedicated, automated acquisition setup accompanied by a complimentary evaluation software of the experimental data was created and it is briefly outlined in the rest of this section.

**The Automated Setup hardware**

The hardware of the proposed, in this paper, automated setup for the evaluation of intermittency statistics is presented in Fig. 1. It consists of a digital storage oscilloscope, connected to a PC or laptop, through GPIB communication port or just a USB or RS-232 connection, and a PC or laptop.

The oscilloscope used in the experimental example presented in this paper, was an 8-bit resolution Tektronix 1002B (100MHz/1Gs) with a USB connection capability. Every signal snapshot is quantized in 256 voltage levels with a time step of 2500 points (for all channels). As a result both resolution and maximum sampling frequency are not standard but they depend on the (voltage and time) scaling decided by the oscilloscope user. Since in this case only one signal is needed for the evaluation, all points are

![Fig. 1. Block diagram of the proposed setup.](image-url)
disposed (2500) in quantizing one channel signal. Thus, real sampling frequency is equal to the specified maximum sampling frequency.

Usually, no signal scaling is needed. In the case of large voltage signals or signals from physical dynamical systems (other than circuits), a signal conditioning circuit may intercede between the system under evaluation and the oscilloscope either to properly scale the signal or to convert it to a voltage signal proper to be captured.

This setup has been structured in such a way that any digital storage oscilloscope could be used with mere software modifications. Alternatively, instead of using a digital storage oscilloscope, one can use an A/D converter, such as NI’s PCI-6024E, that is much cheaper (but worse in maximum sampling frequency).

**The Automated Setup software**

The dedicated software was built in NI’s LabView 8.0 environment, which served to control the digital oscilloscope, as well as, to execute all necessary calculations, in order to create the statistical distributions considered necessary for the intermittency evaluation.

The user interface that was realized is presented in Fig. 2. It consists of two parts. The first part, on the left, is the program’s control panel, while the second part, on the right, contains the registered signal and the related distributions.

More specifically, the control panel is constituted of two folders:

In Fig 3(a) the “debugging” folder is shown. This folder is used to choose the instrument that is going to be used by the program (in the case of more than one registering devices connected to the PC). Moreover, one can send directly commands to the registering instrument without exiting the program, while raw data returned from it can be monitored.

In Fig 3(b) the “measurements” folder is shown, respectively. This folder contains all the necessary control buttons, indicators and inputs that are needed in order to operate the program.

The graphs contained in the second part of the user interface (Fig. 2) are:

- The captured time series \(x(t)\),
- the created [from \(x(t)\)] laminar lengths and the burst durations \(z(N)\), where \(N\) is the number of samples used by the oscilloscope to quantize the registered signal,
- both the laminar length and the burst duration and the mean values of laminar and burst durations, as depicted for each value of the bifurcation parameter and
- the mean values of laminar and burst durations, as depicted for each value of the bifurcation parameter.

There are three main steps in the procedure of evaluating the intermittent behavior of a circuit or system:

In the first step the user determines the comparison levels (up limit, down limit) that could lead to a proper discrimination between periodic laminar state and chaotic bursts. More specifically, these two levels identify the signal’s chaotic bursts, which exceed the levels in either direction.

The proper time delay (duration) between comparisons, should be entered to the program, so that the duration of the same chaotic burst wouldn’t be cut to false smaller ones. It should be noted that time delay is expressed in number of points.

Since a large number of measurements is needed, due to the statistical nature of intermittency, the user has to decide and enter the number of consecutive signal registrations (repetitions) needed. This discontinuous signal registration ensures a smooth distribution of initial conditions that is necessary for the proper evaluation of intermittency statistics and the avoidance of possible artifacts [5].

The second step is activated by the “GRAB” button command. In this step the oscilloscope captures the proper
signal (it should be intermittent) quantizes it and sends it to the connected PC together with all scaling information that is necessary to enable the software carry out the calculations needed, so that the signal values are the same with the real ones on the circuit. The captured signal is presented in the first graph (Fig. 2).

Every time, the program decides the duration (in points) of laminar and burst states, according to the voltage levels named by the user, in the control panel. These durations \( \xi(N) \) are delineated in the graph underneath the one of the captured signal. Zeros represent the laminar states and ones the bursts.

The durations (in points), of both the laminar and the burst parts, of the registered signal are gathered in two graphs, depicting the distribution of laminar lengths \( r \) vs. \( P(r) \) (on the left), as well as, the distribution of chaotic bursts (on the right). At the same time the mean laminar length \( \langle r \rangle \) and the mean burst duration are statistically calculated and their values appear in the control panel in the indicators labeled “Average UP” and “Average DOWN”.

The procedure described in the second step is repeated as many times as the user has predefined (repetitions indicator) and the calculated results from each new repetition are added to the previous ones.

At the end of the programmed repetitions, the final value of the mean laminar length \( \langle r \rangle \) and the mean burst duration, that correspond to a certain value of the circuit’s bifurcation parameter, is placed at the two last graphs. The program then saves both the mean durations and both the distributions in a tab delimited txt format file.

The whole procedure of the second step can be repeated as many times as the user wants, for progressively different values of the circuit’s bifurcation parameter. It should be noted that before activating the second step anew, the laminar length and burst duration distributions should be cleared by the “Clear Distribution” command.

Finally, the program enters the third stage when no more laminar length distribution plots for other bifurcation parameters, are asked by the user.

The value of the critical bifurcation parameter (where intermittency begins) is registered and the mean laminar length \( \langle r \rangle \) versus the deviation of bifurcation parameter from its critical value \( (r-r_{\text{crit}}) \) is presented in a double logarithmic plot, providing a sense of the value of the critical exponent \( \gamma \) and consequently the type of intermittency.

Both the graphs of mean laminar length and the mean burst duration can be saved in a tab delimited txt format file, by activating the button “SAVE averages”.

An example

The presented automated setup for the evaluation of intermittency statistics has been already used in a number of published studies [30-32].

In order to demonstrate its effectiveness, a typical example of an intermediate measurement of on-off intermittency statistics is quoted in Fig. 4 [33]. This kind of intermittent synchronization between two coupled nonlinear circuits.
of intermittent phenomenon appeared in the synchronization between two bidirectionally-coupled, identical, double-scroll nonlinear circuits. The registered timeseries is the difference of the signals from the same nodes, of the two circuits. Laminar lengths depict synchronization (difference signal is almost 0), while bursts depict desynchronization.

The laminar length distribution has the exponential form described by (6), but the mean laminar length vs. the deviation of the bifurcation parameter from its critical value, although in double logarithmic plot, is not a straight line yet, because the illustrated measurement is an intermediate one.

V. CONCLUSIONS

The measurement system presented in this paper, intends to automate a very elaborate experimental work and the related calculations, regarding intermittently operating chaotic electronic circuits and systems.

One decides the type of intermittency and obtains details regarding the related statistics, by capturing only one signal produced by the circuit or system that operates intermittently. This setup is useful not only for model verification but also for the pure experimental study of nonlinear circuits either they demonstrate intermittency or intermittent synchronization. All measured data and the related calculation results, are saved in txt files for further processing.

Finally, the whole measurement system is easily adapted to different signal-capturing devices, whether they are digital storage oscilloscopes or A/D converters.

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