<MULT9>

Desirability Function with Principal Component Analysis for Multi-response Optimization

Sri Winarni[a)], Budhi Handoko[b)], Yeny Krista Franty[c)]

Department of Statistics, Padjadjaran University

[a)]sri.winarni@unpad.ac.id
[b)]budhi.handoko@unpad.ac.id
[c)]yeny.krista@unpad.ac.id

Abstract. Multi-response optimization is the process of getting the combination treatment that produces optimum responses. In the industry manufacture it is needed to get the best quality product by considering some characteristics of products simultaneously. This research used optimization method desirability function combined with principal component analysis. This method is used when there is correlation among quality characteristics product. The approach used is to convert some of the responses into one objective variable is then used to determine the point of optimization. Case study in this research is the optimization of 12L14 free machining steel turning process. The result optimum point was found a cutting speed of 316 m/min, feed rate of 0.13 mm/rev and depth of cut of 0,5826 mm.

Keywords: multi-response optimization, desirability function, principal component analysis.

INTRODUCTION

In industrial environments, it is becoming more and more important that items can be produced to satisfy several requirements simultaneously, and many of them are related to its cost, quality and productivity. Thus, considering that the manufacturing processes must be configured to obtain the best results for a set of characteristics, the interest in employing multi objective optimization techniques has been increasing. the multiple objectives are optimized at the same time when the individual objective functions are combined in only one function, defined as the global optimization criterion of the process [1].

However, if the problem presents multiple correlated characteristics, the desirability function, does not consider the correlation structure between the responses. In attempt to offer a more adequate treatment to the optimization problems with multiple correlated responses, the Principal Component Analysis (PCA) has been shown as a good alternative [2].

In this study, the application of multi-response optimization with principal component analysis and desirability function will be performed in cases of free machining steel turning optimization [3]. The optimization process in the steel lathe is related to productivity and surface quality of steel produced. Experimental design used in this experiment is a central composite design. Factors to be tested consists of cutting speed, feed rate and depth of cut. This trial will optimize the surface quality characteristics of steel, which is the mean roughness (Ra) and total roughness (Rt). Namely productivity and optimize the characteristics of cutting time (Ct) and the material removal rate (MRR).

ISSN : 2579-4361

TABLE 1. Factors and Response of Experiment

Test	Factors			Response			
	V	F	D	Ra	Rt	Ct	MRR
1	220	0,08	0,7	1,36	9,49	2,11	12,32
2	340	0,08	0,7	1,65	10,7	1,36	19,04
3	220	0,12	0,7	1,78	10,08	1,4	18,48
4	340	0,12	0,7	1,84	10,41	0,91	28,56
5	220	0,08	1,2	2,22	14,71	2,11	21,12
6	340	0,08	1,2	2,2	13,47	1,36	32,64
7	220	0,12	1,2	1,82	11,13	1,4	31,68
8	340	0,12	1,2	2,24	13,2	0,91	48,96
9	180	0,1	0,95	1,9	12,51	2,06	17,1
10	380	0,1	0,95	2,08	12,49	0,98	36,1
11	280	0,07	0,95	1,85	10,73	1,89	18,62
12	280	0,13	0,95	1,85	10,78	1,02	34,58
13	280	0,1	0,53	1,68	8,89	1,32	14,84
14	280	0,1	1,37	2,3	13,37	1,32	38,36
15	280	0,1	0,95	2,32	12,57	1,32	26,6
16	280	0,1	0,95	2,23	12,84	1,32	26,6
17	280	0,1	0,95	2,26	12,92	1,32	26,6

Sources : Gomes at.al 2012

The fourth characteristic of the response will be optimized simultaneously. Limit the desired target specification of each response is given in Table 2.

TABLE 2. Limit the desired target specification of each response

Response	LSL	T	UCL
Ra	1.0	1.5	2.0
Rt	8.0	9.0	10.0
Ct	1.0	1.2	1.4
MRR	30	35	40

The purpose of optimization in this case is getting treatment composition which generates optimum response to the response characteristics of the fourth with a value of optimization approaches the desired target. The purpose of this study is to provide a study of the application of multi-response optimization using the principal component analysis and desirability function in the case of free machining steel turning optimization.

DESIRABILITY FUNCTION

Desirability function is a geometric transformation of the value of the response being worth 0 to 1 ($0 \leq d_i \leq 1$). This value indicates the level of the response to the proximity of the target. The responses are in the interval specified target value has a value desirability of zero to one ($0 < d_i < 1$). While the response is very close to the target value has a value desirability of one $(d_i = 1)$. In contrast to responses that are beyond the specified target interval then the desirability its value is zero $(d_i = 0)$ [4]. Function in an individual desirability function. This function will form a composite desirability function which is the geometric mean of the individual desirability function. Formula composite desirability function given in Equation 1:

$$D = (d_1 \times d_2 \times \ldots \times d_k)^{1/k} \quad \ldots \quad (1)$$

Where k is the number of responses were measured. Composite desirability function is the one that will be optimized.

Based on the objectives, desirability function can be categorized into three, namely: nominal-the-best (NB), larger-the-better (LB) and smaller-the-better (SB). If we let T is the desired target value, L is the lower limit of the target, and U is the upper limit of the target $(L \leq T \leq U)$, then form desirability function of each of these categories are as follows :

i. Larger-the-better (LB)

Used if the goal of optimization is to maximize the response, the shape of the individual functions of desirability as in Equation (2).

$$d_i = \begin{cases} 0 & ; \quad y_i < L \\ \left(\frac{y_i - L}{T - L}\right)^r & ; \quad L \leq y_i \leq T \\ 1 & ; \quad y_i > T \end{cases} \quad \ldots \quad (2)$$

r index on individual desirability function is weighted much emphasis nearby that showed a response to the target value. The value of $0 < r < 1$ indicates emphasis less on target. The larger the value of r, the further the response from the target value. The value r = 1 show the same interest to the target. At this value linear-shaped desirability function. The value $r > 1$ shows the emphasis is more on target. The ideal condition is a high desirability value indicates a value close to the target response.

ii. Smaller-the-better (SB)

Used if the purpose of optimization is to minimize the response form desirability function in this category is given in Equation (3).

$$d_i = \begin{cases} 1 & ; \quad y_i < T \\ \left(\frac{u - y_i}{U - T}\right)^r & ; \quad T \leq y_i \leq U \\ 0 & ; \quad y_i > U \end{cases} \quad \ldots \quad (3)$$

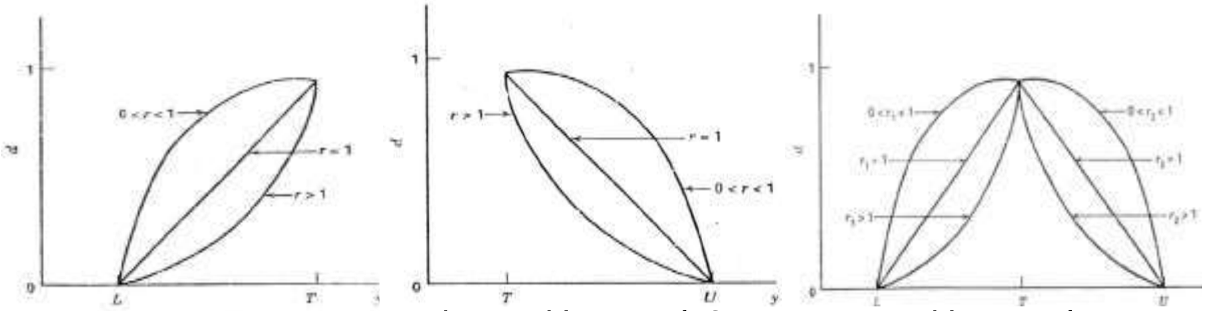
iii. Nominal-the-best (NB)

Used if the desired response is on certain target value. form desirability function as in equation (4).

$$d_i = \begin{cases} 0 & ; \quad y_i < L \\ \left(\frac{y_i - L}{T - L}\right)^r & ; \quad L \leq y_i \leq T \\ \left(\frac{u - y_i}{u - T}\right)^r & ; \quad T \leq y_i \leq U \\ 0 & ; \quad y_i > U \end{cases} \quad \ldots \quad (4)$$

Graph of desirability function for larger-the-better, smaller-the-better and nominal-the-best given in Figure 1.

ISSN : 2579-4361



a. Desirability Function for LB b. Desirability Function for SB c. Desirability Function for ND

FIGURE 1. Desirability Function for Larger-the-Better, Smaller-the-Better, Nominal-the-Best

In the individual desirability function equation, notation *y* is the response surface models for each response. In general if there are *k* independent variables the model of the first order can be written in Equation (5). [5]

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

General equation of the second order response surface models given in equation (6)

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j \epsilon \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

PRINCIPAL COMPONENT ANALYSIS

The PCA technique can account for most of the variation of the original *p* variables via *k* uncorrelated principal components, where $k \leq p$. Restated, let $x = x_1, x_2\ldots, x_p$ be a set of original variables with a variance-covariance matrix Σ. Through the PCA, a set of uncorrelated linear combinations can be obtained in the following matrix [1]:

$$Y = A^T x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Where $Y = (Y_1, Y_2\ldots, Y_p)^T$, Y_1 is called the first principal component, Y_2 is called the second principal component and so on; $A = (a_{ij})$ $p \times p$ and *A* is an orthogonal matrix with $A^T A = I$. Therefore, *X* can also be expressed as follows:

$$X = AY = \sum_{j=1}^{p} A_j Y_j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Where $A_j = [a_{1j}, a_{2j},\ldots, a_{pj}]$T is the *j*th eigenvector of Σ. Consequently, the secondary variables have following characteristics :

a. $\sum_{j=1}^{p} a_{ij}^2 = 1 \qquad \forall i = 1, \ldots, p$
b. $\sum_{k=1}^{p} a_{ik}\, a_{jk} = 0 \qquad \forall i, j | i \neq j$
c. Each secondary variable can be obtain from a linear combination of original variables.
d. The first secondary variable covers maximum deviation existing in original variables. $Var(Z_i = p_1' Y) = p_1' \sum p_i$ would be maximized subject to the constraint that $p_i' p_i = 1$. It was shown that the characteristic vector associated with the largest root of the following equation is the optimal solution for p_1 and the largest root λ_1 is the variance of Z_1. $|\Sigma - \lambda I| = 0$
e. the k^{th} secondary variable covers maximum deviation which is not covered by *k*-1*th* one. If the solution is expressed as $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$, the k^{th} component would be the characteristic vector associated to λk.
f. Secondary variables are independent.

ISSN : 2579-4361

PTIMIZATION BY DESIRABILITY FUNCTION BASE ON PRINCIPAL COMPONENT ANALYSIS

In multi response optimization process, principal component analysis is used to overcome the problem of multi response correlated. The following is the step analysis in principal component and desirability function [6].

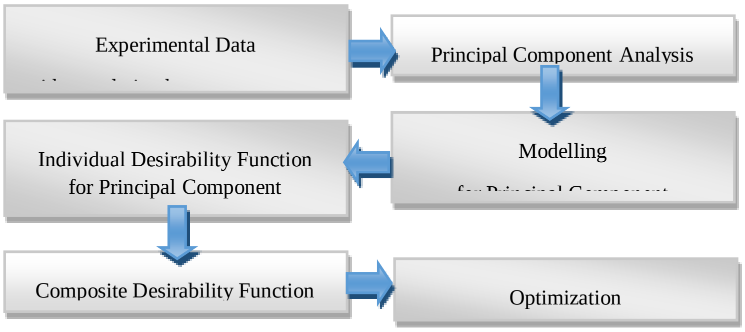


FIGURE 2. step analysis in principal component and desirability function

the analysis stage starts from the principal component analysis is used to solve the case of a correlation between the response. Principal component analysis is done by modeling the principal component that generates principal component coefficient [1]. The principal component coefficient used to construct individual desirability function. It will be convert to composite desirability function. Optimization point is found from composite desirability function.

RESULTS

Basically the analysis carried out in two stages, i.e. the formation of principal component analysis and desirability function. Analytical results obtained are as follows:

a. Analysis result of principal component analysis

By using equation (8) and (9) obtained principal component (PC) given on Table 3.

TABLE 3. The result of principal component analysis

Response	PC1	PC2	PC3	PC4
Ra	0.5706	-0.2868	-0.5478	0.5404
Rt	0.5004	-0.5298	0.1366	-0.6710
Ct	-0.3466	-0.7324	0.4229	0.4058
MRR	0.5513	0.3173	0.7088	0.3049

in the table 3 indicates that the PC is a linear combination of the four responses :

$$PC1 = 0{,}5706 \text{ Ra} + 0{,}5004 \text{ Rt} - 0{,}3466 \text{ Ct} + 0{,}5513 \text{ MRR}$$

$$PC2 = -0{,}2868 \text{ Ra} - 0{,}5298 \text{ Rt} - 0{,}7324 \text{ Ct} + 0{,}3173 \text{ MRR}$$

$$PC3 = -0{,}5478 \text{ Ra} + 0{,}1366 \text{ Rt} + 0{,}4229\text{Ct} + 0{,}7088 \text{ MRR}$$

$$PC4 = 0{,}5404 \text{ Ra} - 0{,}6710 \text{ Rt} + 0{,}4058 \text{ Ct} + 0{,}3049 \text{ MRR}$$

the proportions of variance explained by the four PC are given in Table 4.

TABLE 4. Proportion of variance from four PC

	PC1	PC2	PC3	PC4
Eigenvalue	2.5356	1.2145	0.2045	0.0454
Proportion	0.6340	0.3040	0.0510	0.0110
Cumulative	0.6340	0.9380	0.9890	1. 0000

From Table 4 its shows that PC1 explains largest proportion total of variance 63.4%, next 3.4% from PC2, and then 5.1% and 1.1% from PC3 and PC4 respectively. Cumulative proportion reaches 93.8% from PC1 and PC2, so it will use PC1 and PC2 for next analysis.

b. Analysis result of desirability function

From the linear combination obtained coefficient principal component is used to desirability function. desirability function is formed from the two largest PC. response surface models that form is as follows :

$$\widehat{PC}1 = 0.9710 + 1.3517V + 0.7662F + 2.124d - 1.4784F^2 - 0.9787d^2$$

$$\widehat{PC}2 = 1.05647V + 1.16987F - 0.75896d + 0.73392F^2 + 0.73489d^2 + 0.85563Fd$$

Both of model response surface to form a second order models, meaning that there is the influence of linear, quadratic and interaction between factors. The next is to determine the lower limit, the target and the upper limit for the PC1 and PC2. by normalizing the fourth target limits the response by subtracting the mean and dividing by the standard deviation are obtained result limit of the target for the fourth normalization response as showed in Table 5. By entries in PC equation than obtained PC target as follow:

TABLE 5. Normalization of target and PC target

Response	LSL	T	UCL
Ra	-3.4882	-1.6999	0.0884
Rt	-2.3307	-1.7144	-1.0981
Ct	-1.0550	-0.5505	-0.0460
MRR	0.3503	0.8654	1.3805
PC1	-2.5977	-1.1599	0.2780
PC2	1.0281	2.0736	3.1191

Use equation 4 obtained individual desirability function for PC1 and PC2 as follow:

$$d_{PC1} = \begin{cases} 0 & ;\ \widehat{PC}1_i < -2.5977 \\ \left(\frac{\widehat{PC1}_i-(-2.5977)}{(-1.1599)-(-2.5977)}\right)^r & ; -2.5977 \le \widehat{PC}1_i \le -1.1599 \\ \left(\frac{0.2780-\widehat{PC1}_i}{0.2780-(-1.1599)}\right)^r & ; -1.1599 \le \widehat{PC}1_i \le 0.2780 \\ 0 & ;\ \widehat{PC}1_i > 0.2780 \end{cases}$$

$$d_{PC2} = \begin{cases} 0 & ;\ \widehat{PC}2_i < 1.0281 \\ \left(\frac{\widehat{PC2}_i-1.0281}{2.0736-1.0281}\right)^r & ; 1.0281 \le \widehat{PC}2_i \le 2.0736 \\ \left(\frac{3.1191-\widehat{PC1}_i}{3.1191-2.0736}\right)^r & ; 2.0736 \le \widehat{PC}2_i \le 3.1191 \\ 0 & ;\ \widehat{PC}2_i > 3.1191 \end{cases}$$

And then both of individual desirability function composed in composite desirability function as follow.

$$D = (d_{PC1} \times d_{PC2})^{1/2}$$

Next the composite desirability function used to find optimization point.

c. Optimization result

The optimum point is obtained from maximize composite desirability function. with the help of software obtained optimum point as follows :

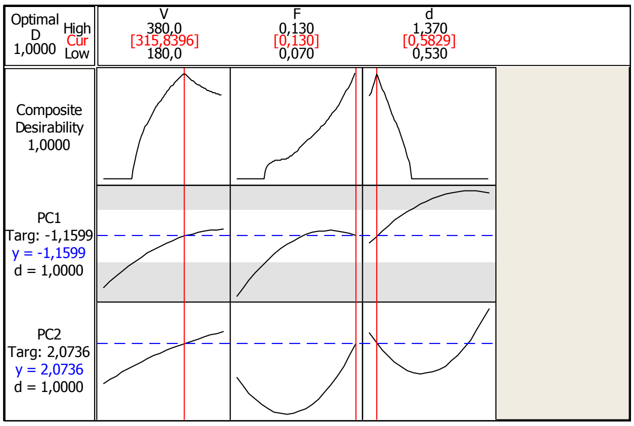


FIGURE 3. Optimization result

in Figure 3 shows that the optimum point obtained in factor V level 315.8396, factor F level 0.130 and factor d level 0.5829.

CONCLUSION

In cases of free machining steel turning optimization by principal component analysis and desirability function obtained optimum point is treatment combination at factor cutting speed level 315.8396, feed rate level 0.130 and depth of cut level 0.5829. This treatment will optimize the surface quality characteristics of steel, which is the mean roughness (Ra) and total roughness (Rt). Optimize the characteristics of cutting time (Ct) and the material removal rate (MRR).

REFERENCES

[1] T. H. Hejazi, A. Salmasnia, and M. Bastan, "Optimization of Correlated Multiple Response Surfaces with Stochastic Covariate," *Int. J. Comput. Theory Eng.*, vol. 5, no. 2, pp. 341–345, 2013.

[2] S. Datta, "Simultaneous Optimization of Correlated Multiple Surface Quality Characteristics of Mild Steel Turned Product," *Intell. Inf. Manag.*, vol. 2, no. 1, pp. 26–39, 2010.

[3] J. H. De Freitas Gomes, A. R. S. Júnior, A. P. De Paiva, J. R. Ferreira, S. C. Da Costa, and P. P. Balestrassi, "Global Criterion Method based on principal components to the optimization of manufacturing processes with multiple responses," *Stroj. Vestnik/Journal Mech. Eng.*, vol. 58, no. 5, pp. 345–353, 2012.

[4] D. C. Montgomery, *Design and Analysis of Experiments*, Internatio. John Wiley & Sons, 2013.

ISSN : 2579-4361

[5] S. Chatterjee, A. K. Mondal, and S. Mahapatra, "Combined Approach for Studying the Parametric Effects on Quality of Holes Using Rsm and Pca in Drilling of Aisi- 304 Stainless Steel," no. Aimtdr, pp. 5–10, 2014.

[6] A. Salmasnia, R. B. Kazemzadeh, and S. T. A. Niaki, "An approach to optimize correlated multiple responses using principal component analysis and desirability function," *Int. J. Adv. Manuf. Technol.*, vol. 62, no. 5–8, pp. 835–846, 2012.