ORTHOSYNTHESIS: ON THE BEAMFORMING FROM A SET OF UNIFORM LINEAR ARRAYS BY THE ORTHOGONAL METHOD

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ABSTRACT

A method of beamforming of uniform linear arrays by using the orthogonalization technique is presented. A set of composing functions similar to that used by Woodward – Lawson is overlapped. In our study, instead of sampling the desired pattern, the orthogonal method is applied. Depending on the form of the pattern, the number of composing functions may be the same or different than the elements of the array. Also the progressive phase of each function can be derived in several ways. A set of numerical examples for different array patterns will be presented and will show the usefulness of the method.

INTRODUCTION

Synthesis methods for linear antenna arrays have been studied by many researchers in the last decades; the existence of a long series of papers on this subject is enough to emphasize the importance of the area. Most of the procedures allow the synthesis of narrow-beam or low sidelobe patterns or the maximization of an index (gain, SNR) subject to one or several constraints. Excellent textbooks in the international literature, [1-3], present several treatments on the synthesis problem. Among the above treatments we notice the Fourier transform method [2], the Schelkunoff procedure [4], the Dolph-Chebyshev and the Riblet synthesis [5, 6], the Woodward - Lawson method [7-8] and the Orchard et al synthesis [9]. The orthogonal method was introduced by Unz [10] and has been extensively used by Sahalos [11] in many antenna synthesis problems. Our effort in this paper is the application of the above method for multiple beam beam-forming.

The procedure starts from a set of uniform, non-orthogonal progressively phased composing functions [8], which are orthonormalized and overlapped. The orthonormalized functions are weighted to form the desired pattern. Our method is general and able to interpolate between beams and to combine them in order to approximate the desired one in the least mean square error (MSE) sense. The method has no restrictions on the element numbers and the pattern symmetry. Also the degrees of freedom, in contrast to Woodward-Lawson are the maximum possible.

FORMULATION

Let a linear array (Fig. 1) be composed of N equidistant point sources. The array factor is given by:

\[ F_j(\theta) = \sum_{k=1}^{N} A_k e^{-j(N-k-1)\frac{\beta d \sin \theta}{2}} \]

where \( A_k \) is the excitation coefficient of the \( k^{th} \) element, \( \beta = \frac{2\pi}{\lambda} \) and \( d \) is the equidistance of the array elements.

We suppose that the array factor comes from a set of \( M \) different patterns of uniformly illuminated \( N \) element arrays. In this case \( F_j(\theta) \) can be expressed in the following form:

\[ F_j(\theta) = \sum_{m=1}^{M} I_m \Phi_m(\theta) \]

where
The functional base of (2) \( \Phi_m(\theta) \) is not orthogonal and, the inner product \( \langle \Phi_i, \Phi_j \rangle \) is given by:

\[
\langle \Phi_i, \Phi_j \rangle = \frac{1}{N^2} \sum_{n=1}^{N} \sum_{l=1}^{N} S_{ln} e^{j(N-(2j-1))\alpha_l - j(N-(2n-1))\alpha_n} \neq 0
\]

(3a)

where

\[
S_{ln} = 2\pi \sin(l-n)\beta d \quad \frac{l-n}{\beta d}
\]

(3b)

With the aid of (3), it is possible to construct \( M \) new functions, related to each other by a relation analogous to that of orthogonality, following the Gram-Schmidt procedure [11].

We thus get:

\[
Y_k = \Phi_k = \sum_{j=1}^{k-1} \langle Y_j, \Phi_k \rangle Y_j
\]

(4)

Equation (4) is divided by \( \|Y_k, Y_k\|^2 \) so that the resulting function will be orthonormalized. It is:

\[
\Psi_k = \frac{Y_k}{\|Y_k, Y_k\|}\]

(5)

\( F_r(\theta) \) can now be expressed as a function of \( \Psi_i(\theta) \) as follows:

\[
F_r(\theta) = \sum_{i=1}^{M} B_i \Psi_i(\theta)
\]

(6)

If we suppose that a pattern \( F_d(\theta) \) is desired and \( F_r(\theta) \) can approximate it, \( B_i \) will be given by:

\[
B_i = \langle F_d(\theta), \Psi_i(\theta) \rangle
\]

(7)

The quantities \( I_i \) in view of (6) are:

\[
I_i = \sum_{j=1}^{M} B_i C_i^{(j)}
\]

(8)

From (3) – (5) one can get the coefficients \( C_i^{(j)} \) in the following form:

\[
C_i^{(n)} = \frac{1}{\|Y_n, Y_n\|^2}, \quad C_k^{(n)} = -C_i^{(n)} \sum_{j=k}^{n} C_k^{(j)} \sum_{i=1}^{j} \langle \Phi_i, \Phi_j \rangle
\]

(9)

Taking into account expressions (1)-(2) the array element excitation are derived as:

\[
A_k = \sum_{m=1}^{N} \lambda_m I_m, \quad \lambda_m = e^{j(N-(2k-1))\alpha_m}
\]

(10)

\( a_m \) of (3) and (11) could be defined in several ways. A suitable form could be:

\[
a_m = \delta \left[ M - (2m-1) \right] \frac{\pi}{2N}
\]

(12)

For \( \delta = 1 \) the phase of the \( m^{th} \) composing function \( \Phi_m(\theta) \) makes the main lobe maximum coincident with the innermost null of the \( (m-1)^{th} \) corresponding one. In this case, where \( M=N \), the composing functions are the same with those used in the Woodward – Lawson method. It must be pointed out that the above method is based on the sampling concept of
the desired pattern and its defect is the lack of control over the sidelobe level in the unshaped region of the pattern. Our method makes use of composing functions of more general type and, instead of sampling, it applies the orthonormalization procedure. Taking different values of $\delta$ and $M$ in (12) we receive different solutions of the synthesis problem. It is also possible to specify certain values of $\alpha_m$ which could improve the solution. In the examples section several cases with different choices of $M$ and $N$ are presented.

**EXAMPLES**

As a first example we will apply the method to the design of Chebyshev patterns. The design cases correspond to the ones presented in [12]. It is supposed that an endfire $T_5(x)$ pattern with SLL=-20dB is desired. For the 1st case, where the progressive phase shift is $-\beta d$, using $M=N=11$ the excitation coefficients obtained by our method are exactly the same with those obtained by the Riblet analytical method for an array with equidistance $d=0.25\lambda$. Decreasing $M$ and keeping $N=11$ it is found that for up to $M=8$ different solutions give similar results. It must be pointed out that for $M=N=8$ the desired pattern can be produced. Fig. 2 shows the corresponding patterns. It is noticed that instead of 11 the problem can be solved with as few as 8 elements. Of course the MSE in the case of 11 elements is 100 times less than the corresponding one for $N=8$.

A design with a cosecant-squared pattern with constraints on the sidelobe envelope is presented. This case has to do with the control of the level of the close sidelobes as in the case of the Elliot’s method. Sixteen elements with half-wavelength distance are used. The close sidelobes are controlled from the value of the HPBW of the pattern. The beam is between $-60^\circ$ and $-12^\circ$. Also the SLL envelope from $35^\circ$ to $90^\circ$ is $\leq -30$dB. Up to $35^\circ$ there are sidelobes with level depended on the HPBW. A HPBW of 12.5$^\circ$ shows the close sidelobe level $-55$dB (see Fig. 3). Obviously the example gives equivalently acceptable results with these given by Elliot et al [13].

Comparing the result with the one obtained by
the Woodward-Lawson method one can see that our technique offers almost 3 times more accurate results in terms of the mean square error.

A 32 element linear array with required maximum at $\theta_o = -5^\circ$ and different levels of the sidelobes in the left ($\leq -40\,\text{dB}$) and right ($\leq -60\,\text{dB}$ and $\leq -45\,\text{dB}$) regions of the mainlobe has been designed. Fig. 4 shows the pattern of the array with $d = 0.45\lambda$. It is noticed that this case can be compared to that given by Elliot et al [3], where the sidelobe level of the array is controlled.

CONCLUSION

The method of orthosynthesis has been presented in this paper. It was supposed that the desired pattern comes from a set of $N$ element, uniformly illuminated arrays. The factor of each uniform array is the basis function of a non orthogonal $M$-dimensional vector space. The basis is orthonormalized by the Gram – Schmidt procedure and the current of the array is found by the well-known orthogonal method. Several examples for different arrays and different patterns have shown the applicability of the method.

REFERENCES