Generalized Net Model of the Cognitive and Neural Algorithm for Adaptive Resonance Theory 1

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Abstract: The artificial neural networks are inspired by biological properties of human and animal brains. One of the neural networks type is called ART [4]. The abbreviation of ART stands for Adaptive Resonance Theory that has been invented by Stephen Grossberg in 1976 [5]. ART represents a family of Neural Networks. It is a cognitive and neural theory that describes how the brain autonomously learns to categorize, recognize and predict objects and events in the changing world. In this paper we introduce a GN model that represent ART1 Neural Network learning algorithm [1]. The purpose of this model is to explain when the input vector will be clustered or rejected among all nodes by the network. It can also be used for explanation and optimization of ART1 learning algorithm.

Keywords: Neural Networks, Generalized Nets, ART1.

Introduction
Every learning system must be able to adapt in changing environment (it must be plastic). Constant changes can lead to the instability of the system, due to the fact, that the system can obtain information by learning, only when the old information is forgotten [3, 4].

The type of neural network that solves this problem is called ART [5, 6]. The essence of ART's predictive power is its ability to autonomously carry out fast, incremental, unsupervised and supervised learning in response to the changing world, keeping previously learned memories at the same time [4]. More generally, we can quickly learn about new environment, even if it has not been told about the difference of the rules in each environment. In a remarkable degree, humans can rapidly learn new facts without being forced to just as rapidly forget what they already know. As a result, we can confidently go out into the world without fearing that, in learning to recognize a new friend's face, we will suddenly forget the faces of our family and friends [4]. The ART method consist two layers. The first one is known as Comparison layer and the second is called Recognition layer, fully connect with "bottom-up" and "top-down" weights and reset module which control the degree of similarity between both layers. There are two methods of learning – supervised and unsupervised.

Unsupervised learning method is used for neural networks ART1, ART2, ART3, … [7, 8]. Supervised method is known as ARTMAP [7, 8].

The ART1 learning algorithm is described below, it operates with binary input vectors [3]. According to [2, 9, 10] learning algorithm is:

1. Notations
The set of input vectors
\[ I(x) = \{x(1), x(2), \ldots, x(N)\}, \quad x(t) \in \mathbb{R}^n, \quad t = 1, 2, \ldots, \]

where \( t \) (time) is a natural number.

Basically ART1 consists of two layers:
- \( F_1 \) is the input (Comparison field) with \( n \)-neurons \( F_{11}, F_{12}, \ldots, F_{1n} \);
- \( F_2 \) is the output (Recognition field) with \( M \)-neurons \( F_{21}, F_{22}, \ldots, F_{2M} \).

Weights from neuron \( i \) of \( F_1 \) layer towards the neuron \( j \) of \( F_2 \) layer
\[ w_{ij}(t), \quad i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, M; \quad W(t) = (w_{ij}(t)) \]

Each column in matrix \( W(t) \) is column vector \( w_j(t) \in \mathbb{R}^n, j = 1, 2, \ldots, M \).

Weights from neuron \( j \) of \( F_2 \) towards the neuron \( i \) of \( F_1 \)
\[ v_{ji}(t), \quad j = 1, 2, \ldots, M; \quad i = 1, 2, \ldots, n; \quad V(t) = (v_{ji}(t)) \]

Each row in matrix \( V(t) \) is column vector "\( v_j(t) \in \mathbb{R}^n, j = 1, 2, \ldots, M \"

\[ v_j(t) = (v_{j1}(t), v_{j2}(t), \ldots, v_{jn}(t))^T \]

\( o(t) \in \mathbb{R}^M \) – firing vector that consist one component for each neuron in \( F_{2j} \).

Vector \( \theta \in \mathbb{R}^M \) is null vector of the space.

2. Initialization of parameters
- \( n \) – number of neurons in \( F_1 \) layer;
- \( M \) – number of neurons in \( F_2 \) layer;
- \( N \) – number of input vectors;
- \( \rho \) – vigilance threshold.

\[ w_{ij}^0 = w_{ij}(1) = \frac{1}{1 + N}, \quad v_{ji}^0 = v_{ji}(1) = 1 \quad \text{for} \quad i = 1, 2, \ldots, n; j = 1, 2, \ldots, M \]

where \( w_{ij}^0, v_{ji}^0 \) are initialized weights at \( t = 0 \).

2.1. At the beginning of the work, the fire vector \( o(1) \) has all of its components equal to 1, i.e.,

\[ o_j = 1 \quad \text{in} \quad F_2. \]
2.2. Read an input vector $x(t)$.

2.3. Compute the activation values for all the enabled neurons $j$ of $F2$:

$\text{"net}(j) = \langle w_j(t), x(t) \rangle$"  \hspace{1cm} (3)

Select the winner neuron that has

$\text{"k, } 1 \leq k \leq M \text{ max } \{ \text{net}_j(t) | o_j = 1 \} = \text{net}_k(t)$".  \hspace{1cm} (4)

If the maximum is not unique and there is a tie for the winning neuron $k$ in $F2$ layer, then choose a special rule to break the tie.

2.4. Process the vigilance test for $F2_k$ output neuron:

$\text{"r} = \frac{\langle v_k(t), x(t) \rangle}{\|x(t)\|}$  \hspace{1cm} (5)

If $r \geq \rho$ there is match go to step 2.5 otherwise go to step 2.6.

2.5. Update the vectors $v_k(t)$ and $w_k(t)$ as follows:

$\text{"v}_k(t+1) = v_k(t) * x(t)$"  \hspace{1cm} (6)

$\text{"w}_k(t+1) = \frac{v_k(t+1)}{0.5 + \|v_k(t+1)\|}$  \hspace{1cm} (7)

$\text{"v}_j(t+1) = v_j(t), w_j(t+1) = w_j(t)$ for $j = 1, 2, \ldots, M, j \neq k$"  \hspace{1cm} (8)

Put output $o_j(t+1) = 1, j = 1, M$ and store input pattern, go to step 2.7.

2.6. Disable the output neuron $F2_k$ i.e. "$o_k(t) = 0$" and go to 2.3. When $o(t) = \theta$ the network rejects the input vector $x(t)$ and it is stored in reject set $RI(x)$ go to step 2.7.

2.7. Put the old weights on the positions of new weights and enable all the output neurons, namely

$\text{"v}_j(t+1) = v_j(t), w_j(t+1) = w_j(t), o_j(t+1) = 1$ for $j = 1, 2, \ldots, M$"  \hspace{1cm} (9)

2.8. Step 2.2 has been repeated again with the input set $I(x)$.

2.9. Print the content of classes and the matrixes $W$ and $V$.

We introduced a static structure of ART1 Neural Network here is described ART1 learning algorithm.
GN-model
The GN-model is presented in Fig. 1. Initially the following tokens enter the generalized net:
- in place $L_1$ – a token with an initial characteristic "Input vectors" – $x_0^\alpha = "I(x)"$;
- in place $L_6$ – a token with an initial characteristic "Number of neurons in F1" – $x_0^\beta = n$;
- in place $L_{17}$ – a token with an initial characteristic "Vigilance threshold" – $x_0^\gamma = \rho$;
- in place $L_{22}$ – a token with an initial characteristic "Initialized weights $v_{f,i}^0$ in F2, Number of neurons in F2, Special rule (S)" – $x_0^\delta = "v_{f,i}^0, M, S"$.

The GN is introduced by the set of transitions:

$$A = \{Z_1, Z_2, Z_3, Z_4, Z_5\}$$

where the transitions describe the following processes:
- $Z_1 =$ "Distribution of the input vectors";
- $Z_2 =$ "Initializing of weights $w_{i,j}^0$ in F1";
- $Z_3 =$ "Determination of weights $w_{i,j}$ in F1";
- $Z_4 =$ "Determination of resonance state";
- $Z_5 =$ "Determination of weights $v_{f,i}$ in F2".

GN-model consists of five transitions with the following description:

$$Z_1 = \langle \{L_1, L_5, L_9, L_{25}\}, \{L_2, L_3, L_4, L_5\}, R_1, \lor(\land(L_5, L_9), \land(L_5, L_{25}), L_1, L_5) \rangle$$

$$R_1 = \begin{array}{cccc}
L_2 & L_3 & L_4 & L_5 \\
L_1 & False & False & False & True \\
L_5 & W_{5,2} & W_{5,3} & W_{5,4} & True, \\
L_9 & False & False & False & True \\
L_{25} & False & False & False & True \\
\end{array}$$

where:

$W_{5,2} = W_{5,3} = W_{5,4} =$ "There are input vectors".

The token $\alpha$ from place $L_1$ that enters in place $L_5$ does not change its characteristic. The token $\alpha'$ from place $L_5$ splits in to three identical tokens entering in places $L_2, L_3, L_4$. The tokens $\zeta, \delta'$ from places $L_9$ and $L_{25}$ enter in place $L_5$ obtaining a characteristic:

$$x_{\zeta, \delta'}^\alpha = "\alpha' + 1".$$
where:

\[ W_{8,7} = \text{"The weights are initialized"}. \]

The token \( \beta \) from place \( L_6 \) enters in place \( L_8 \) obtaining a characteristic:

\[ x_{cw}^\beta = \frac{1}{1+n}. \]

The token \( \beta' \) in place \( L_8 \) enters in place \( L_7 \) keeping its characteristic.

\[ Z_5 = \langle \{L_3, L_7, L_{14}, L_{15}, L_{16}, L_{23}, L_{24}\}, \{L_9, L_{10}, L_{11}, L_{12}, L_{13}, L_{14}, L_{15}, L_{16}\}, R_3, \forall(L_3(\forall(L_7, L_{23})), L_{14}, L_{15}, L_{16}, L_{24}) \rangle \]
where:
\[ W_{14,16} = "Weights W_k(t+1) are updated" ; \]
\[ W_{15,10} = W_{16,9} = W_{16,10} = W_{24,15} = "o(k) = \theta" ; \]
\[ W_{15,12} = W_{15,14} = "Weights V_k(t+1) are updated" ; \]
\[ W_{15,13} = "The value of winning neuron is defined" ; \]
\[ W_{16,11} = "The values to the neurons are defined" . \]

The token \( \alpha' \) from place \( L_3 \) splits in two identical tokens that enter in places \( L_{15} \) and \( L_{16} \). The tokens \( \alpha' \), \( \beta' \) from places \( L_3 \) and \( L_7 \) enter in place \( L_{16} \) obtaining a characteristic:

\[ x^\zeta_{cu} = "W_j * x(t)" . \]

The token \( \zeta \) splits in two identical tokens one of which enters in place \( L_{11} \) and in place \( L_{16} \) stays "(\( \alpha', \beta', \zeta \)". The tokens \( \delta'' \) and \( \delta'' \) from place \( L_{23} \) enter in place \( L_{15} \) and unite with token \( \alpha' \) from place \( L_3 \) obtaining a characteristic:

\[ x^\eta_{cu} = "V_k(t) * x(t)" . \]

In place \( L_{15} \) stays "(\( \alpha', \eta \)". The token \( \mu' \) from place \( L_{23} \) unites with token \( \eta \) from place \( L_{15} \) and obtains a characteristic:

\[ x^\nu_{cu} = "update(V_k(t) * x(t))" . \]

The token \( \nu \) from place \( L_{14} \) unites with token \( \zeta \) from place \( L_{16} \) and obtains a characteristic:

\[ x^\zeta_{cu} = "W_j(t)" . \]

The token \( \alpha' \) from places \( L_{15} \) and \( L_{16} \) enters in place \( L_{10} \) keeping its characteristic. The token \( \eta' \) from places \( L_{15} \) splits in two identical tokens that enter in places \( L_{12} \) and \( L_{14} \), keeping its characteristic in place \( L_{12} \). The token \( \eta' \) from place \( L_{15} \) enters in place \( L_{14} \) obtaining a characteristic:

\[ x^\nu_{cu} = \frac{v_k(t+1)}{0.5 + \|v_k(t+1)\|} . \]
The token $\eta$ from place $L_{15}$ enters in place $L_{13}$ keeping its characteristic. The token $\zeta$ from place $L_{16}$ enters in place $L_9$ also keeping its characteristic. The token $\sigma'$ from $L_{24}$ enters in place $L_{16}$, obtaining a characteristic $x^\zeta_{cu}$. The token $\sigma''$ from $L_{24}$ splits in two identical tokens that enter in places $L_{15}$ and $L_{16}$, obtaining characteristics:

$$x^\sigma_{cu} = RI(x).$$

$$Z_4 = \langle \{L_4, L_{13}, L_{17}, L_{20}, L_{21}\}, \{L_{18}, L_{19}, L_{20}, L_{21}\}, R_4, \lor(\land(L_4, L_{13}), \land(L_{17}, L_{21}), L_{20})\rangle$$

$$R_4 = \begin{array}{cccc}
L_{18} & L_{19} & L_{20} & L_{21} \\
L_4 & False & False & False & True \\
L_{13} & False & False & False & True \\
L_{17} & False & False & True & False' \\
L_{20} & W_{20,18} & W_{20,19} & True & False \\
L_{21} & False & False & W_{21,20} & True \\
\end{array}$$

where:

$W_{20,18} = "\rho \leq r";$

$W_{20,19} = \neg W_{20,18};$

$W_{21,20} = "\text{Resonant state is calculated}".$

The tokens $\alpha'$ and $\eta$ from places $L_4$ and $L_{13}$ unite in place $L_{21}$, obtaining a characteristic:

$$x^\lambda_{cu} = \frac{V_k}{\|x(t)\|^n}.$$

The tokens $\lambda$ and $\gamma$ from places $L_{17}$ and $L_{21}$ unites in place $L_{20}$, obtaining a characteristic:

$$x^\mu_{cu} = "(\rho = r)".$$

In place $L_{21}$ stays $\lambda$, in all time of life of GN. The token $\mu$ from place $L_{20}$ splits in two tokens $\mu'$, $\mu''$, that enter in places $L_{18}$ and $L_{19}$. The token $\mu'$ that enters in place $L_{18}$ obtains a characteristic:

$$x^{\mu'}_{cu} = "\rho \leq r".$$

The token $\mu''$ that enters in place $L_{19}$ obtains a characteristic:

$$x^{\mu''}_{cu} = "\rho > r".$$

$$Z_5 = \langle \{L_2, L_{11}, L_{12}, L_{18}, L_{22}, L_{26}, L_{27}\}, \{L_{23}, L_{24}, L_{25}, L_{26}, L_{27}\}, R_5, \\
\lor(\land(L_2, L_{11}, L_{22}), \land(L_{22}, L_{27}), \land(L_{12}, L_{26}), \land(L_{19}, L_{27})), L_{18}\rangle$$

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where:

\( W_{26,23} = \text{"Weights and winning neuron are defined"}; \)
\( W_{26,25} = \text{"Weight matrices are updated"}; \)
\( W_{27,24} = \text{"Winning neuron is disabled"}; \)
\( W_{27,26} = \text{"Winning neuron is defined"}. \)

The token \( \delta \) from place \( L_{22} \) splits in two tokens: \( \delta' \) enters in place \( L_{27} \) and \( \delta'' \) enters in place \( L_{26} \) obtaining characteristics:

\[ \delta' = M, S; \quad \delta'' = v_{j,i}^0. \]

In place \( L_{27} \) stays \( \delta' \), in all time of life of GN. The token \( \alpha' \) from place \( L_2 \) enters in place \( L_{27} \), obtaining a characteristic:

\[ x_{cu}^{\alpha'} = o(t) = 1. \]

The token \( \zeta' \) from place \( L_{11} \) unites with token \( \delta' \) in place \( L_{27} \) obtaining a characteristic:

\[ \left\{ \begin{array}{ll}
  x_{cu}^{\zeta'} &= \text{"max}(k), o(k) = 1" \\
  x_{cu}^{\alpha'} &= \text{"max}(k), o(k) = 1 \text{ if } S^n\end{array} \right\}. \]

The tokens \( \varepsilon \) and \( \omega \) from place \( L_{27} \) enter in place \( L_{26} \) obtaining a characteristic:

\[ x_{cu}^{\delta''} = \text{"max}(k), V(k)". \]

The tokens \( \delta'' \) and \( \delta''' \) from place \( L_{26} \) enter in place \( L_{23} \) keeping their characteristics. The token \( \eta' \) from place \( L_{12} \) unites with token \( \delta'' \) in place \( L_{26} \) obtaining a characteristic:

\[ x_{cu}^{\eta'} = v_j(t). \]

The token \( \mu' \) from place \( L_{18} \) splits in two identical tokens that enter in places \( L_{23} \) and \( L_{27} \), in place \( L_{23} \) obtaining a characteristic:

\[ x_{cu}^{\mu'} = \text{"o}_k(t) = \text{update"}. \]

The token \( \mu' \) from place \( L_{18} \) enters in place \( L_{27} \) obtaining a characteristic:
\[
\begin{align*}
\{ x_{cu}^\varepsilon = & \max(\varepsilon(k), o(k) = 0) \\
\{ x_{cu}^{\varepsilon'} = & \max(\varepsilon(k), o(k) = 0 \text{ if } S^\varepsilon = 0) \}
\end{align*}
\]

The token \( \mu'' \) in place \( L_{19} \) unites with token \( \varepsilon \) in place \( L_{27} \) obtaining a characteristic:

\[ x_{cu}^\varepsilon = o_k(t) = 0, o_M(t) = 1. \]

The token \( \mu'' \) in place \( L_{19} \) enters in place \( L_{27} \) obtaining a characteristic:

\[ x_{cu}^{\varepsilon'} = o_k(t) = 0, o_M(t) = 1 \text{ if } S^{\varepsilon'} = 0. \]

The token \( \psi \) from place \( L_{26} \) enters in place \( L_{25} \) keeping its characteristic. The token \( \sigma \) in place \( L_{27} \) enters in place \( L_{24} \) obtaining a characteristic:

\[
\begin{align*}
\{ x_{cu}^{\sigma} = & \text{ net}(\varepsilon) \text{ if } o_k(t) = 0 \\
\{ x_{cu}^{\sigma'} = & RI(x) \text{ if } o_M(t) = 0 \}
\end{align*}
\]

The token \( \omega' \) in place \( L_{27} \) enters in place \( L_{26} \) obtaining a characteristic:

\[ x_{cu}^{\sigma'} = \max(\varepsilon(k), V(k) \text{ if } S^{\sigma'} = 0. \]

**Conclusion**

In the present work a GN-model to describe the learning algorithm of ART1 Neural Network is used. It has been shown, how the input vector is recognized and the process of acceptance or rejection (if there are not uncommitted neurons) from neural network. In that matter, the method can be used to solve the “stability-plasticity dilemma”.

This paper is in the second series of papers devoted on the ART1 neural network.

**References**


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