Comparison of Friction Angle Measured in Square and Circular Direct Shearbox Tests

Ir. Dr Chan Swee Huat

Wong Soon Yee

Dr Wong Kim Yuen

This article deals with drained direct shearbox tests on sand. The objective is to investigate whether the shape and size of the shearbox affect the determined friction angle (Ø).

In a direct shearbox test, soil is laterally restrained (upper box) and sheared (lower box) along a mechanically induced horizontal plane, while subjected to a pressure applied normal to that plane, as shown in Figure 1. The shearing resistance offered by the soil as one portion is made to slide on the other is measured. Failure occurs when the shearing resistance reaches the maximum value which the soil can sustain. By carrying out tests on a set (usually three) of similar specimens of the same soil under different normal pressures, the relationship between measured shear stress at failure and normal applied stress is obtained. If it can be assumed that the relationship is linear, the slope of the line and its intercept with the shear strength axis can be derived from the line of best fit through the plotted points. The slope gives the friction angle, Ø (in degrees), and the intercept gives the apparent cohesion, c (in kPa) (BS 1377-7:1990+A1).

According to the Mohr-Coulomb failure criterion, the strength or shear resistance (τ) of soil is given by:

$$\tau = \sigma \tan(\phi) + c \qquad (1)$$

For a non-cohesive soil, e.g. sand, *c* can be ignored.

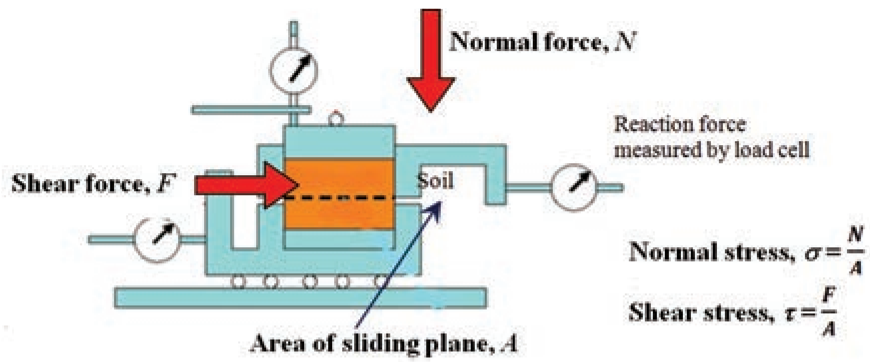

Figure 1: Principle of direct shearbox test

RELATIVE DENSITY OF SAND

One of the important variables that would affect the results of a direct shearbox test on sand is in the state of sand packing, which can be expressed in terms of relative density, void ratio, porosity or dry density. The term relative density, which is also known as density index, I_D, is used to describe the denseness or looseness of soil. The index relates dry density (or void ratio) of a soil sample or of an in-situ soil, to the limiting dry densities (or limiting voids ratios). The relationship can be defined in terms of maximum and minimum dry densities as follows:

$$I_D = \frac{\rho_d - \rho_{dmin}}{\rho_{dmax} - \rho_{dmin}} \times \frac{\rho_{dmax}}{\rho_d} \times 100\% \qquad (2)$$

where ρ_d denotes the dry density of the soil in question, ρ_d min the dry density at the least dense state, and $\rho_{d\ max}$ the dry density at the densest state.

The British Standard, BS 1377-4:1990+A2 contains the methods of test for determination of maximum and minimum dry densities of sand. Subclause 4.2 of the related standard is used for the determination of maximum density and Subclause 4.4 for minimum density. Table 1 shows the commonly used qualitative descriptions of granular soil deposits.

TEST STANDARDS FOR DIRECT SHEAR

1. BRITISH STANDARD

The British Standard (BS) describing

Table 1: Qualitative descriptions of granular soil deposits

DESCRIPTION	RELATIVE DENSITY, ACCORDING TO LAMBE AND WHITMAN (1969) (%)	RELATIVE DENSITY, ACCORDING TO DAS AND SOBHAN (2014) (%)
Very loose	0 - 15	0 - 15
Loose	15 - 35	15 - 50
Medium dense	35 - 65	50 - 70
Dense	65 - 85	70 - 85
Very dense	85 - 100	85 - 100

methods of test for soils for civil engineering purposes is BS 1377. This series contains nine parts, in which Part 7 contains the methods of test for direct shear. Clause 4 – Determination of shear strength by direct shear (small shearbox apparatus) in BS 1377-7:1990+A1 is applicable in this study.

2. ASTM STANDARD

The American Society for Testing and Materials (ASTM), since 1898, is one of the largest voluntary standards developing organisations in the world. The direct shearbox test reference is ASTM D3080/D3080M: 2011 Standard test method for direct shear test of soils under consolidated drained conditions.

3. CHOICE OF TEST STANDARD

In this study, the British Standard is used for shearbox of square shape and the ASTM Standard for the circular shape. It is noted that the BS covers only square shaped shearboxes while the ASTM covers both the square and circular shapes.

4. RESEARCH FRAMEWORK

The project problem is defined by deductive reasoning on how the box shape and size will influence the friction angle determined by a direct shearbox test. In a narrow sense, two types of sand with different grading are considered in order to effectively manage the project deliverables. The three relevant aspects (variables) of interest are: (a) box shape and size, (b) relative density, and (c) friction angle. The concepts relating to the variables can then be developed into a theoretical framework.

As a technical condition or pre-requisite, "relative density" can be a moderating variable that is related to the test method which delivers the result. The prevalent theory is that the "friction angle" is influenced by the "Test Method". Only the relative density seems to contribute to friction angle. In the preceding situation, friction angle is the dependent variable that will be positively influenced by the box shape and size (independent variable). Thus, the conceptual relationship between Test Method and Result has now become contingent on the Procedure (categorised as either loose or dense) in acting as a catalyst. This Procedure becomes the moderating variable. The concepts relating to variables and the theoretical framework are presented in Figure 1.

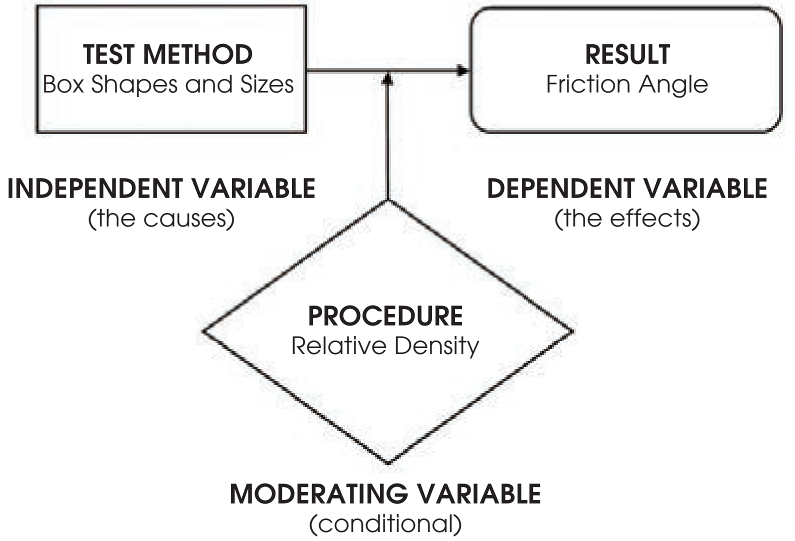

Figure 2: Concepts relating to variables and theoretical framework

While the study of friction angle (dependent) can influence box shape and size (independent) in a certain way, the possibility of the relative density (moderating) variable in modifying the independent and dependent relationship can also present a contingent effect. In other words, box shape and size can be selected independently with the expected or perceived friction angle and the relative density effects. However, to apply the potential benefits of box shapes and sizes, the friction angle will have to be analysed and verified by conducting laboratory tests at pre-determined or controlled cause-effect relationships.

RELATED LABORATORY TESTS

Other than the direct shearbox test, the study also involves other related laboratory tests to characterise the sand sample used. The five related laboratory tests are bulk density, particle density (or specific gravity), maximum dry density, minimum dry density and particle size distribution tests.

1. **Bulk Density Test:** This is performed according to BS 1377-2:1990+A1, Subclause 7.2 – Linear measurement method. This procedure applies to soils that can be formed into a regular geometric shape, the volume of which could be calculated from linear measurements of known area of the box and the height of the test specimen. The test specimen density, ρ (in Mg/m^3), is calculated from the following equation.

$$\rho = \frac{1000m}{AH} \quad (3)$$

where m denotes the specimen mass (in g), A the specimen area (in mm^2), and H the specimen height (in mm).

2. **Particle Density Test:** This is performed according to BS 1377-2:1990+A1, Subclause 8.3 – Small pyknometer method. This test is suitable for soils consisting of particles finer than 2 mm. The particle density of the soil, ρ_s (in Mg/m^3), is calculated from the following equation.

$$\rho_s = \frac{m_2 - m_1}{(m_4 - m_1) - (m_3 - m_2)} \quad (4)$$

where m_1 denotes the mass of density bottle; m_2 the mass of bottle and dry soil; m_3 the mass of bottle, soil and water; and m_4 the mass of bottle when full water only.

3. **Maximum and Minimum Dry Density Tests:** These are performed according to BS 1377-4:1990+A2,

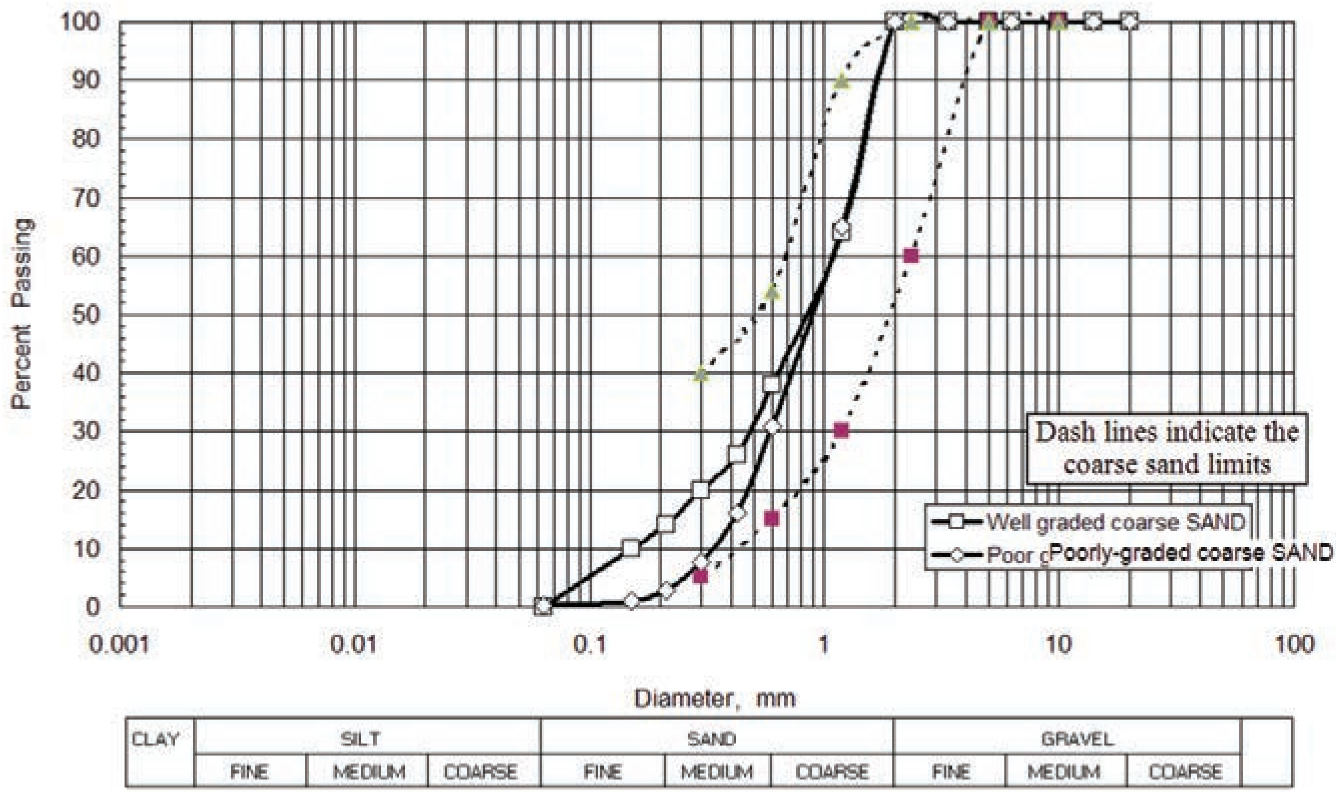

Figure 3: Particle size distribution of sand sample used

Table 2: Sand characteristics

Soil Type	ρ_s (Mg/m³)	$\rho_{d\,min}$ (Mg/m³)	$\rho_{d\,max}$ (Mg/m³)	d_{10} (mm)	d_{30} (mm)	d_{60} (mm)	$C_c = \frac{d_{30}^2}{d_{60} \times d_{10}}$	$C_u = \frac{d_{60}}{d_{10}}$	Remark
Poorly-graded sand	2.65	1.418	1.765	0.35	0.6	1.1	0.9	3.1	$C_c < 1$ and $C_u < 6$
Well-graded sand	2.65	1.460	1.822	0.15	0.5	1.1	1.5	7.3	$1 < C_c < 3$ and $C_u \geq 6$

Notes:

1. *d_{10}, d_{30} and d_{60} denote the diameters corresponding to 10%, 30% and 60% finer, respectively, in Figure 3.*
2. *C_c denotes the coefficient of gradation or coefficient of curvature, and C_u the uniformity coefficient.*

Table 3: Test specimen density computation

Poorly-graded	Box size (mm)	60 x 60			100 x 100			60 dia.			100 dia.		
	Specimen state	L	MD	D	L	MD	D	L	MD	D	L	MD	D
	Relative density (%)	22	52	72	22	52	72	22	52	72	22	52	72
	Specified dry density (Mg/m³)	1.482	1.580	1.652	1.482	1.580	1.652	1.482	1.580	1.652	1.482	1.580	1.652
	Soil weight (g)	112	119	125	311	332	349	88	94	98	244	261	273
Well-graded	Box size (mm)	60 x 60			100 x 100			60 dia.			100 dia.		
	Specimen state	L	MD	D	L	MD	D	L	MD	D	L	MD	D
	Relative density (%)	22	52	72	22	52	72	22	52	72	22	52	72
	Specified dry density (Mg/m)	1.527	1.628	1.704	1.527	1.628	1.704	1.527	1.628	1.704	1.527	1.628	1.704
	Soil weight (g)	115	123	129	321	342	358	91	97	101	252	269	281

Subclause 4.2 – Determination of maximum density of sands and Subclause 4.4 – Minimum density of sands, respectively. The density index, I_D (%) can be calculated from Eq. (2).

4. **Particle Size Distribution Test:** This test is performed according to BS 1377-2:1990+A1, Clause 9 – Determination of particle size distribution. The test specimen is verified to meet the requirements specified in BS882:1992, under the coarse sand limits (dash lines in Figure 3).

SAMPLE TYPE AND TEST SPECIMEN CRITERIA

1. **Sample Type:** The sand used in this project is characterised using the five related laboratory tests described in Section 4. The results obtained are summarised in Table 2 and Figure 3.

   The grain size of the coarse sand is verified to be suitable and meets the requirements of the small shearbox apparatus, as follows:

   - The size of the largest particle (2-2.5mm) shall not exceed one-tenth of the height of the specimen of 20 to 25mm box height.
   - The minimum specimen diameter for circular specimens, or width for square specimens, shall be 50mm or not less than 10 times the maximum particle size diameter, whichever is larger, and shall conform to the width to thickness ratio of 2.
   - The minimum initial specimen thickness shall be 0.5 in. (12 mm), but not less than six times the maximum particle diameter.

2. **Shearbox Test Specimen:** It is necessary to have a Procedure to replicate sand preparation to the specified dry density so that results of the friction angle in the Test Method can be compared and validated. A trial and error procedure is used in a test specimen preparation. The proportion of the sample to be well-graded in test specimen is prepared. The specified dry densities for loose (L), medium dense (MD) and dense (D) are then based on determined index of relative density. The computed density values are derived from relative density and are summarised in Table 3.

3. **Method of Sand Deposition:** A loose test specimen is prepared according to BS 1377-7:1990+A1, Subclause 4.4.4.2 – Dry Sand: Loose. The loose density state is achieved by rapid pouring of a test specimen into the shear box from a small height (25mm). The surface of the specimen is levelled using a suitable tool (extrusion dolly) to achieve the required thickness. The required soil weight for specified loose density is presented in Table 3. A medium dense or dense test specimen is prepared according to BS 1377-1:1990+A1, Subclause 7.7.4.2.2 – Compaction to specified density. The specimen is compacted in the shearbox in three different layers by light tamping for 25 times per layer. The required soil weight for specified dense density is presented in Table 3. The sample designation for each test specimen is presented in Table 4.

Table 4: Shearbox test specimen designation

Grading type	PG = Poorly-graded		WG = Well-graded
Prefix	S = Square shape		C = Circular shape
Size	60 = 60 mm		100 = 100 mm
Relative density	22% (L = Loose)	52% (MD = Medium dense)	72% (D = Dense)

RESULTS AND ANALYSIS

The friction angles obtained from all the shearbox tests performed are presented in Table 5. A total of 24 cases are studied, i.e. 2 (grading: poorly-graded & well-graded) × 2 (shape: square and circular) × 2 (size: 60 mm and 100 mm) × 3 (relative density: loose, medium dense and dense) = 24 cases. The friction angles are projected by best line fit, on the assumption that there is no cohesion for remoulded coarse sand. The ultimate shear strengths are taken at the end

Table 5: Direct shearbox test results

1	2	3	4	5
Test Specimen	γ_d (Mg/m^3)	I_D (%)	φ (°)	Range of Error ± (°)
PGS60	1.482	22 (L)	39 (39)	0.2 (0.2)
	1.580	52 (MD)	39 (45)	0.2 (0.1)
	1.652	72 (D)	39 (47)	1.3 (1.1)
PGS100	1.482	22 (L)	39 (39)	0.2 (0.2)
	1.580	52 (MD)	38 (44)	1.4 (1.1)
	1.652	72 (D)	37 (47)	0.1 (1.1)
PGC60	1.482	22 (L)	43 (43)	0.1 (0.1)
	1.580	52 (MD)	43 (49)	0.1 (0.1)
	1.652	72 (D)	43 (53)	0.3 (0.1)
PGC100	1.482	22 (L)	39 (39)	0.3 (0.3)
	1.580	52 (MD)	39 (46)	0.9 (1.3)
	1.652	72 (D)	39 (48)	1.1 (1.0)
WGS60	1.527	22 (L)	40 (40)	0.3 (0.3)
	1.628	52 (MD)	41 (50)	0.4 (1.3)
	1.704	72 (D)	41 (54)	0.1 (1.1)
WGS100	1.527	22 (L)	40 (39)	3.0 (3.0)
	1.628	52 (MD)	39 (44)	3.1 (1.0)
	1.704	72 (D)	36 (48)	0.6 (0.9)
WGC60	1.527	22 (L)	40 (40)	0.5 (0.5)
	1.628	52 (MD)	41 (47)	1.1 (0.4)
	1.704	72 (D)	40 (51)	4.5 (2.6)
WGC100	1.527	22 (L)	40 (40)	0.1 (0.1)
	1.628	52 (MD)	38 (44)	0.1 (0.7)
	1.704	72 (D)	38 (48)	1.2 (0.1)

Note: Values in parentheses in Columns 4 and 5 indicate peak friction angles

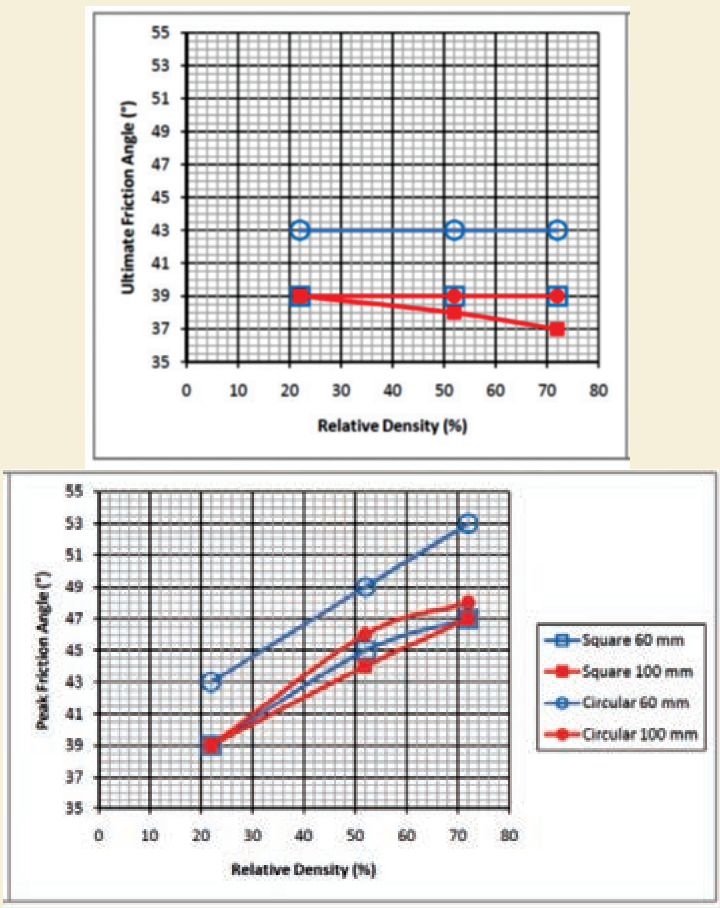

Figure 4: Ultimate and peak friction angles obtained in poorly-graded sand

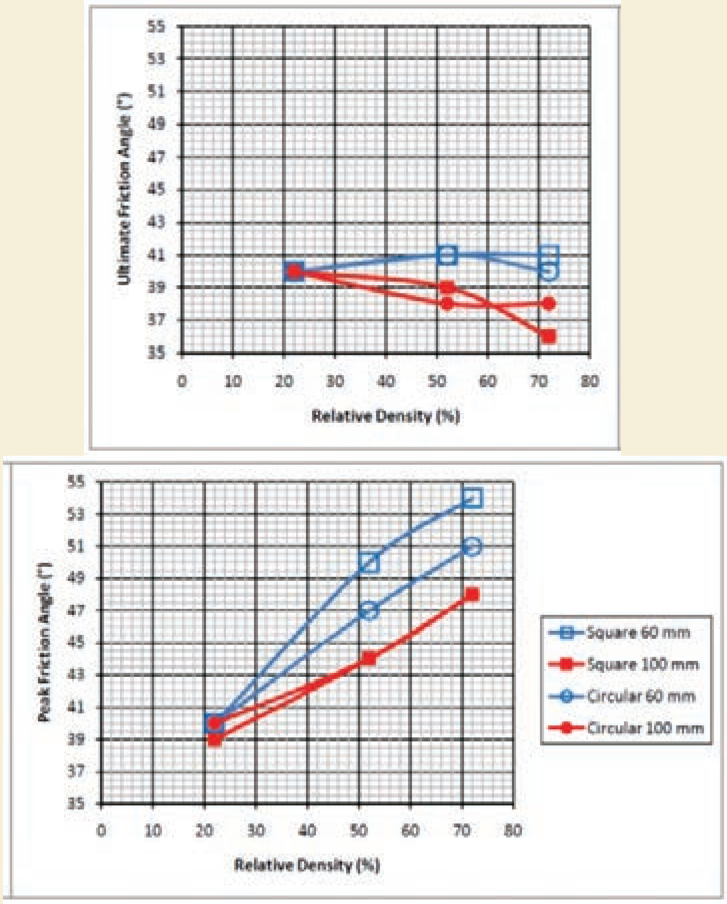

Figure 5: Ultimate and peak friction angles obtained in well-graded sand

of 10 mm travel (displacement) of the box capacity. From the stress paths analysis, noticeable peak shear strengths are observed in the medium dense and dense states.

The ultimate and peak friction angles of sand as a function of relative density are presented in Figure 4 for the poorly-graded sand, and Figure 5 for the well-graded sand. The following observations can be made from Figures 4 and 5.

a) In all cases, the peak friction angle increases with relative density.
b) The ultimate friction angle is not or slightly affected by relative density.
c) In most cases, the ultimate and peak friction angles decrease with shearbox size. Hence, the use of shearbox size of 60 mm in determining friction angle may be risky.
d) In general, for the shearbox size of 100 mm, square and circular shearboxes give approximately the same friction angles with an occasional maximum variation of about 2º.
e) However, for shearbox size of 60 mm, the square and circular shearboxes give different friction angles in most cases with a general variation range of about 3º to 6º.

CONCLUSION

The following conclusions can be drawn from this study:

1. The ultimate and peak friction angles decrease with shearbox size. The use of shearbox size of 60 mm in determining friction angles may be risky.
2. Compared to the shearbox size of 60 mm, the square and circular shearboxes of size of 100 mm give more similar friction angle values.
3. Shearbox size of 100 mm, either square or circular, is preferable to the size of 60 mm in determining ultimate and peak friction angles. ■

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Authors' Biodata

Ir. Dr Chan Swee Huat, *obtained his PhD degree from the National University of Singapore in 2003. He is a Committee Member of IEM GETD and the Hon. Treasurer of Malaysian Geotechnical Society. He is a Director of Geo-Excel Consultants Sdn. Bhd. Since graduation from Universiti Kebangsaan Malaysia in 1997, he has been involved in the design and construction of various geotechnical engineering works.*

Wong Soon Yee, *graduated in Master of Engineering with Honours in Civil Engineering from University of Nottingham Malaysia Campus. He then pursued a Doctor of Philosophy degree in Civil Engineering at the same University since September 2014. He has part-time industry experience at Soilpro Technical Services Sdn. Bhd.*

Dr Wong Kim Yuen, *graduated in Doctor of Business Administration from the University of South Australia. He is the Principal Partner and Managing Director of Soilpro Technical Services Sdn. Bhd., the Vice President (2016/18) of The Malaysian Site Investigators Association, MSIA and a member of Technical Committee on Geotechnical Works (TC/D/17) for ISC/D managed by SIRIM Berhad.*