Resolution of Computational Aeroacoustics problems on unstructured grids with a higher-order finite volume scheme

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Abstract

CFD has become more and more used in the industry for the simulation of flows. Nevertheless, the complex configurations of real engineering problems make difficult the application of very accurate methods that only works on structured grids. From this point of view, the development of higher order methods for unstructured grids is desirable. The finite volume method can be used with unstructured grids, but unfortunately it is difficult to achieve an order of accuracy higher than two, and the common approach is a simple extension of the one-dimensional case. The increase of the order of accuracy in finite volume methods on general unstructured grids has been limited due to the difficulty in the evaluation of field derivatives. This problem is overcome with the application of the Moving Least Squares (MLS) technique on a finite volume framework. In this work we present the application of this method (FV-MLS) to the solution of aeroacoustic problems.

Key words: Computational aeroacoustics, Finite volume method, Moving Least Squares, Unstructured grids

1. Introduction

The simulation of sound propagation in the air is a very difficult numerical problem [1]. If we try to solve an acoustic problem with the same methods as developed for aerodynamics, a lot of numerical difficulties arise that are not present in the resolution of aerodynamic problems. The origin of such difficulties relies on the nature of the acoustic problem. The low magnitude of acoustic waves makes the use of low dissipation schemes mandatory, and it complicates even more the problem of the boundary conditions. Thus, the acceptable amplitude of reflections caused by waves leaving the domain is much smaller than in typical aerodynamic problems.

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Another feature of aeroacoustic problems is that the range of frequencies of interest is wider than in aerodynamics.

In computational aeroacoustics (CAA), the most successful numerical schemes have been spectral methods or high resolution finite differences [2, 3]. These methods work very well on structured grids, but unfortunately they present problems when applied to the resolution of problems with complex geometries. In this context, the development of methods that can solve CAA problems on unstructured grids is interesting. The finite volume method, widely and successfully used for the simulation of aerodynamics with unstructured grids, presents difficulties when it is applied to aeroacoustic problems in its most usual formulation (at most order two), due to the lack of resolution of the scheme. Even though rising the order is not the only (nor probably the best) way to improve the resolution of the schemes, it is the most usual approach on unstructured grids due to the difficulty in generalizing the methods developed for structured meshes [4]. But this approach is also not obvious, and the main problem is the evaluation of high order derivatives. The FV-MLS method [5, 6, 7] overcomes this difficulty by using the Moving Least Squares (MLS) technique [8] to compute the gradients and successive derivatives. Thus, it builds higher-order schemes in a finite volume framework without the introduction of new degrees of freedom.

The aim of this work is to extend the application of the FV-MLS method to the resolution of aeroacoustic problems, by focusing our attention on the resolution of the Linearized Euler Equations (LEE). Moreover, the multiresolution features of the MLS approach [9] allow the development of low-pass filters that could be used together with a grid-stretching technique to build an absorbing layer that avoid reflections at the boundaries, following the methodology exposed in [10].

2. The Linearized Euler Equations

Most aeroacoustic problems are linear, so it is possible to linearize Euler equations around a (mean) stationary solution \( U_0 = (\rho_0, u_0, v_0, p_0) \). Then, the 2D Linearized Euler Equations written in conservative form are the following:

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \mathbf{H} = \mathbf{S} \quad (1)
\]

where \( \mathbf{S} \) is a source term and

\[
\mathbf{U} = \begin{pmatrix} u' \\ v' \\ p' \end{pmatrix}, \quad \mathbf{E} = \begin{pmatrix} \rho' u_0 + \rho_0 u' \\ \rho_0 + u_0 u' \\ \rho_0 v' \\ u_0 p' + \gamma \rho_0 u' \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho_0 v_0 + \rho_0 v' \\ \rho_0 + v_0 v' \\ \rho_0 v_0 + \gamma p_0 v' \end{pmatrix}
\]

\[
\mathbf{H} = \begin{pmatrix} 0 & \frac{\rho_0 v' + v_0 p'}{\rho_0} & \frac{\rho_0 u' + \rho_0 u' + p'}{\rho_0} \\ \frac{\rho_0 u_0 + \rho_0 u_0'}{\rho_0} & 0 & \frac{\rho_0 v_0 + \rho_0 v_0'}{\rho_0} \\ \frac{(\gamma - 1) p'}{\rho_0} & \frac{(\gamma - 1) u'}{\rho_0} & 0 \end{pmatrix}
\]

where the velocity is \( \mathbf{v} = (u, v) \), \( \rho \) is the density, \( p \) the pressure and \( \gamma = 1.4 \). Subscript \( \bar{\cdot} \) is referring to mean values and \( ' \) indicates perturbation quantities around the mean. In case of an uniform mean flow, \( \mathbf{H} \) is null.
3. Numerical method

3.1. A MLS-based Finite Volume scheme

A method based on the application of Moving Least Squares to compute the derivatives in a finite volume framework (FV-MLS) [5, 6] has been used to discretize the Linearized Euler Equations (1). Fluxes are discretized with a flux vector splitting method. In order to increase the order achieved by the method, a Taylor expansion of the variable is performed at the interior of each cell. Next, the approximation of the higher order derivatives needed to compute the Taylor reconstruction is obtained by a Moving Least Squares approach. Thus, if we consider a function \( \Phi(x) \) defined in a domain \( \Omega \), the basic idea of the MLS approach is to approximate \( \Phi(x) \), at a given point \( x \), through a weighted least-squares fitting of \( \Phi(x) \) in a neighborhood of \( x \) as

\[
\Phi(x) \approx \hat{\Phi}(x) = \sum_{i=1}^{m} p_i(x) \alpha_i(z) \bigg|_{z=x} = p^T(x) \alpha(z) \bigg|_{z=x} \tag{4}
\]

\( p(x) \) is an \( m \)-dimensional polynomial basis and \( \alpha(z) \bigg|_{z=x} \) is a set of parameters to be determined, such that they minimize the following error functional:

\[
J(\alpha(z) \bigg|_{z=x}) = \int_{\Omega} W(z-x, h) \bigg[ \Phi(y) - p^T(y) \alpha(z) \bigg|_{z=x} \bigg]^2 d\Omega
\tag{5}
\]

being \( W(z-x, h) \bigg|_{z=x} \) a kernel with compact support (denoted by \( \Omega_x \)) centered at \( z = x \). The parameter \( h \) is the smoothing length, which is a measure of the size of the support \( \Omega_x \) [5]. In this work the following polynomial cubic basis is used:

\[
p(x) = \left( 1 \quad x \quad y \quad xy \quad x^2 \quad y^2 \quad x^2y \quad xy^2 \quad x^3 \quad y^3 \right)^T \tag{6}
\]

which provides cubic completeness. In the above expression, \( (x, y) \) denotes the cartesian coordinates of \( x \). In order to improve the conditioning, the polynomial basis is locally defined and scaled: if the shape functions are evaluated at \( x_i \), the polynomial basis is evaluated at \( (x - x_i)/h \). From a practical point of view, for each point \( I \) we need to define a set of neighbors inside the compact support \( \Omega_x \). Following [5], the interpolation structure can be identified as

\[
\hat{\Phi}(x) = p^T(x) \alpha(z) \bigg|_{z=x} = p^T(x) M^{-1}(x) P_{\Omega_x} W(x) \Phi_{\Omega_x} = N^T(x) \Phi_{\Omega_x} = \sum_{j=1}^{n_{x_i}} N_j(x) \Phi_j \tag{7}
\]

In the above, \( n_{x_i} \) is the number of neighbors of the cell \( I \). Moreover, \( M = P_{\Omega_x} W(x) P_{\Omega_x}^T \) is the moment matrix. We also define the matrices \( P_{\Omega_x} = \left( p(x_1) \cdots p(x_{n_{x_i}}) \right) \), \( \Phi_{\Omega_x} = \left( \Phi(x_1) \cdots \Phi(x_{n_{x_i}}) \right) \) and \( W(x) = \text{diag}(W_i(x)) \) with \( i = 1, \cdots, n_{x_i} \) (see [5]).

The approximation is written in terms of the MLS “shape functions” \( N^T(x) \). The derivatives of \( N^T(x) \) can be used to compute an approximation to the derivatives of the function. So, the gradient of \( \Phi(x) \) is evaluated as

\[
\nabla \hat{\Phi}(x) = \sum_{j=1}^{n_{x_i}} \Phi_j \nabla N_j(x) \tag{8}
\]
The equation to be solved results of the application of the finite volume discretization to equations (1):

$$A_I \frac{\partial \mathbf{U}_I}{\partial t} = \sum_{iedge=1}^{nedge_I} \sum_{i=1}^{nq_I} [-\mathbf{F} \cdot \mathbf{n}]_{iq} \mathbf{W}_{iq} + \mathbf{S}_I$$

(9)

where $\mathbf{F} = (E_x, F_y)$. $A_I$ is the area of cell $I$, $nedge_I$ the number of cell edges, $\mathbf{U}_I$ and $\mathbf{S}_I$ are the average values of $\mathbf{U}$ and $\mathbf{S}$ respectively, over the cell $I$ (associated to the cell centroid). $\mathbf{W}$ are the integration weights and $nq_I$ is the number of integration points. $\mathbf{F} \cdot \mathbf{n}$ is computed with a standard flux vector splitting technique.

We compute the first and second derivatives required for the Taylor reconstruction of the variables at quadrature points at the edges by using equation (8), in a context of generalized Godunov’s methods. In case of unsteady problems, this reconstruction needs to use correction terms in order to ensure that the average value of the reconstructed variables over a cell $I$ is the centroid value $\mathbf{U}_I$ [5, 6, 7]. The resulting scheme is a third order method.

The neighbors of each cell centroid $I$ of the grid are the centroids of the neighboring cells. For boundary cells, we add nodes (ghost nodes) placed in the middle of the edge defining the boundary. The definition of the stencil for each cell is done at the beginning of the calculations. An exponential kernel has been used, defined in 1D as:

$$W(x, x^*, \kappa_x) = \frac{e^{-s^2} - e^{-\left(\frac{d_{max}}{\kappa_x}\right)^2}}{1 - e^{-\left(\frac{d_{max}}{\kappa_x}\right)^2}}$$

(10)

with $s = |x - x'|$, $d_{max} = \max(|x_i - x'|)$, $i = 1, \cdots, n_{xx}$, and $x^*$ is the reference point (the point around which the stencil moves, in this case the centroid of each cell, $I$), $x$ is the position of every cell centroid of the stencil and $\kappa_x$ is a shape parameter. Moreover, we define $c = \frac{d_{max}}{2\kappa_x}$.

A 2D kernel is obtained by multiplying two 1D kernels:

$$W_j(x, y, \kappa_x, \kappa_y) = W_j(x, x^*, \kappa_x)W_j(y, y^*, \kappa_y)$$

(11)

In this work we have used the values of $\kappa_x = \kappa_y = 2.5$.

More details about the FV-MLS method can be found in [5, 6, 7].

### 3.2. Boundary Conditions

Absorbing boundary conditions have been implemented by using an absorbing layer based on grid stretching. Grid stretching transfers the energy of the wave into increasingly higher wavenumber modes and the numerical scheme removes this high-frequency content. With this process the energy of the wave is dissipated. This becomes clear by looking at figure 1, where the dispersion and dissipation properties of the FV-MLS scheme are shown. These properties are related to the real and complex part of the numerical wavenumber $\kappa'$. For a high wavenumber $\kappa$ the numerical method introduces more dissipation.

On the other hand, it is possible to increase the dissipation in the absorbing layer by using the MLS method as a filter. The filtering process is developed by the application of a MLS reconstruction of the variables, i.e:

$$\Phi(x) = \sum_{j=1}^{n_x} \Phi(x)N_j(x)$$

(12)
where $\Phi$ is a variable, $\bar{\Phi}$ is the filtered variable and $N$ is the MLS shape function. This reconstruction is performed by using a kernel with shape parameters with more dissipative behavior than the ones used to the approximation of the variables. The value of these parameters determines the range of frequencies to be filtered. In case of applying a MLS-based filter, as a general rule, it is suggested to filter progressively. The most aggressive filtering must be done near the outlet boundary at the end of the absorbing layer. Filtering may be progressively applied by modifying the shape parameters of the exponential kernel, in analogy to the method proposed in [11]. Dissipation properties of the FV-MLS method make the application of the explicit MLS-based filter not necessary. Thus, in this work the explicit filtering is not applied, but it may be useful for other methods working on unstructured grids with not enough implicit dissipation.

4. Numerical examples

In this section we present some aeroacoustic test problems solved on unstructured grids. Most of the examples in the literature are solved on cartesian grids by using spectral or high-resolution finite difference methods. These methods are the best in terms of spectral resolution, but present difficulties when applied to complex geometries. In this point the FV-MLS method becomes very interesting, because it allows the resolution of problems in unstructured grids with a higher order numerical scheme. An explicit fourth-order Runge-Kutta scheme was used for the computations.

4.1. Source Radiation in a Uniform Mean Flow

Here we reproduce the example of [12]. We compute the radiation of a periodic source in two cases: in a subsonic and in a supersonic mean flow. The source is located at $x_s = y_s = 0$, and it is defined as:

$$S_p = \frac{1}{2} \exp \left( -\ln(2) \frac{x^2 + y^2}{2} \right) \sin(\omega t) \times [1, 0, 0, 1]^T$$  \hspace{1cm} (13)
where the angular frequency is $\omega = 2\pi/30$ and $t$ is the time coordinate. The wave length is $\lambda = 30$ units, and the computational domain is the circle with radius $r = 100$ units. The source term is made dimensionless with $[\rho_\infty c_\infty/\Delta x, 0, 0, \rho_\infty c_\infty^3/\Delta x]^T$. In order to avoid spurious reflections at boundaries, an absorbing layer has been placed surrounding the computational domain. With the aim of testing the stability and the behavior of the proposed method for the boundary conditions, an unstructured absorbing layer has been constructed, although in general it is recommended to use a structured mesh. The absorbing layer is placed from the boundary of the computational domain to $x = \pm 300$ and $y = \pm 300$. Figure 2 shows the grid used for the resolution of this problem. To build this grid we have placed 632 equally spaced nodes at the circumference defining the computational domain, and 20 nodes on each edge at the outer boundary.

First, we analyze the subsonic case with Mach number $M_x = 0.5$. Two acoustic waves propagate upstream and downstream of the source. Due to the effect of the mean flow, the apparent wavelength is modified. Thus, it is different upstream ($\lambda_1 = (1 - M_x) \lambda$) and downstream ($\lambda_2 = (1 + M_x) \lambda$) of the source. In figure 3 pressure isocontours for different non-dimensional

![Figure 2: Periodic source in a subsonic ($M = 0.5$) uniform mean flow. Absorbing layer (grid for the computational domain is skipped for clarity) (left) and computational domain grid detail (right).](image)

![Figure 3: Periodic source in a subsonic ($M = 0.5$) uniform mean flow. From left to right, pressure contours at times $t = 90, t = 150, t = 210, t = 270$.](image)
times $t$ are shown. They are barely discernible of the results obtained in [12] on a cartesian grid with the Dispersion-Relation-Preserving (DRP) scheme [3]. The pressure profile along axis $y = 0$ at time $t = 270$ is reproduced in figure 6 (left). It is in good agreement with the analytical solution and it matches the results obtained in [12].

In order to check the stability of the boundary conditions, we let the computations to continue until $t = 5400$. This correspond to 180 periods of the source, a time long enough for the wave to travel until the boundary and to be reflected. Results are shown in figure 4 (left). Comparing the pressure field with the one corresponding to $t = 270$ (9 source periods), it is observed that there is no change in the solution. The good behavior of the absorbing layer is also shown in figure 4 (right) at time $t = 5400$. The acoustic wave is completely dissipated when it leaves the computational domain. We note the good results despite the use of a low-quality grid for the absorbing layer. We also note that we use a third order scheme in the absorbing zone. In this zone, it is possible to use a lower-order scheme, but this would generate oscillations. In order to avoid them, we have preferred to use the same scheme in all the domains.

![Figure 4: Periodic source in a subsonic ($M = 0.5$) uniform mean flow at time $t = 5400$. Pressure contours in the computational domain are shown in the left, and the behavior of the absorbing layer is shown on the right. The acoustic waves are dissipated when they left the computational domain indicated by the red square. The distorted waves that appear next to the front wave are located in the absorbing layer.](image)

On the other hand, it is interesting to check if the scheme can accurately reproduce the interactions between the traveling waves. To this end, we compute the same problem in a supersonic uniform flow. In this case, the radiated field is completely different. Now, the two pressure waves propagate downstream of the source with velocity $M \pm 1$, and interference phenomena take place. The location of the source given by equation (13) is $(x_s, y_s) = (-50, 0)$. We use the same grid than in the previous case. The results, (figures 5 and 6 on the right) agree quite well with the analytical solution and with those obtained in [12]. The disagreement between the analytical and the computed solution for the supersonic case in the neighborhood of the location of the source, has been reported by other authors (see [12]), and it could be related to the calculation of the convolution product of the analytical solution rather than the computed solution.

4.2. Propagation of a wall-bounded acoustic pulse

This example simulates the propagation of an acoustic pulse inside a duct, in an uniform mean flow $M = 0.5$. This problem is taken from [13], where it is solved by using a fourth order
seven-point DRP scheme on a cartesian grid, and with PML boundary conditions. This example is used to show the suitability of the method for turbomachinery applications and duct acoustics. The computational domain is defined by two solid walls placed at $y = -50$ and $y = 50$, with $x$ coordinate varying from $-100$ to $100$. An absorbing layer with length 50 units is placed at inflow and at the outflow. Figure 7 shows the grid used in this example. It has been built by placing a circle with radius 40 units at the centre of the duct, and meshing it with a paving scheme, with 252 points over the circumference. The total number of elements of the grid is 20330. Each absorbing layer has 1000 elements. Only implicit filtering has been considered.

The following initial pressure disturbance is placed at $(-50, 0)$:

$$p'(x, y) = \exp\left( -\ln(2) \frac{(x + 50)^2 + y^2}{36} \right)$$

(14)

The acoustic pulse is convected downstream, and it is reflected at the walls. In order to compare with the results presented in [10, 13], we show the pressure contours at non-dimensional times $t = 60, t = 110, t = 150$ and $t = 200$ in picture 8. The results are comparable to those of [10, 13], and no spurious reflections have been noticed.

Figure 5: Periodic source in a supersonic ($M = 1.5$) uniform mean flow. From left to right, pressure contours at times $t = 114, t = 190, t = 266, t = 304$.

Figure 6: Pressure profile along axis $y = 0$ for a periodic source in a uniform mean flow. Left: subsonic flow ($M = 0.5$) at $t = 270$. Right: supersonic flow ($M = 1.5$) at $t = 304$. The circles show the analytical solution given by Bailly and Juvé [12].
4.3. Vortex convection in a uniform mean flow

The Linearized Euler Equations support the propagation of entropy, vorticity and acoustic waves. Entropy waves consist of density fluctuations, whereas vorticity waves consist of velocity fluctuations. Acoustic waves involve fluctuations in all the physical variables. When the vorticity and entropy waves arrive at boundaries, they can generate strong spurious acoustic waves that propagate and spoil the solution. We are going to check the ability of the method to deal with these waves. The example taken from the Acoustic Database [14] is reproduced here: the convection of a vortex in a subsonic ($M_x = 0.5$) uniform mean flow. The vortex is defined as follows:

$$p_0(x) = \frac{1}{\gamma}, \quad u(x) = u_0 + u' = M_x + \epsilon y \exp\left(\frac{-\ln(2)x^2 + y^2}{b^2}\right)$$

$$\rho_0(x) = 1, \quad v(x) = v' = -\epsilon x \exp\left(\frac{-\ln(2)x^2 + y^2}{b^2}\right)$$

(15)

with $b = 5$ and $\epsilon = 0.03$.

An explicit Low Dispersion and Dissipation Runge-Kutta scheme (LDDRK) [2] with four steps has been used for the calculations. We solve this problem in both, a structured and an
unstructured grid, for comparing the effect of the second one on the generation of spurious waves. In order to evaluate the magnitude of the reflections generated by the vortex when it leaves the computational domain, it is useful to compute the time evolution of the residual fluctuating pressure, $L_p$, defined by:

$$L_p = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (p - p_0)^2}$$  \hspace{1cm} (16)$$

where $N$ is the number of cells in the interior of the computational domain.

4.3.1. Computations on a structured grid

For the cartesian grid, the computational domain is defined such as $-50 \leq x \leq 50$, $-50 \leq y \leq 50$, with $\Delta x = \Delta y = 1$. An absorbing layer is placed from $x = 50$ to $x = 200$.

The configuration parameters of the absorbing layer are critical for vorticity waves. Unfortunately, there is not an universal optimal configuration of the absorbing layer, since it is problem dependent. In figure 9 (right) the absorbing layer built with 30 cells gives better results than the one defined with 50 cells. In finite difference methods spurious oscillations will appear if the metrics are not smooth enough. With the FV-MLS approximation, metrics are not so critical, and the more dissipative effect of a bigger cell size predominates. However, although the metrics dependency is not so important, it is advisable to build the absorbing layer with a smooth transition from the end of the computational domain to avoid the generation of spurious acoustic waves. To this end, we place an absorbing zone divided in two parts. In the first part, a very smooth rate of growth has been applied to the 20 first cells. A more aggressive rate of growth has been applied to the last 10 cells.

The results are comparable to those obtained in [14] by using a DRP scheme. The bigger difference in the magnitude of $L_p$ occurs at the beginning of the calculations, and it could be related to the fact that in [14] damping terms have been used in the time integration scheme that have not been used here.
4.3.2. Computations on an unstructured grid

We solve the problem of the convection of a vortex on an unstructured grid. Due to the irregularity of the grid, the magnitude of the spurious oscillations produced by initial transitional acoustic waves is bigger than in the previous case. The unstructured grid used in this case is shown in figure 10. It has 11476 cells in the calculation domain and the same absorbing layer than for the structured case.

Figure 10: Unstructured grid used for the vortex convection in a uniform mean flow problem. The absorbing layer is not shown.

Results are shown in figure 9 (left). As expected, the magnitude of pressure fluctuations is bigger than those obtained on a structured grid, but the results are satisfactory. In figures 11 we show the dissipation of the vorticity inside the absorbing layer. In figure 12 we plot the spurious acoustic waves entering in the computational domain. The results may improve with the use of a smoother grid, made with triangular elements.

Figure 11: Vorticity evolution in the absorbing layer. We plot 8 vorticity contours, from $1 \cdot 10^{-4}$ to $36 \cdot 10^{-4}$ for different times. The line at $x = 50$ indicates the start of the absorbing layer.
Figure 12: Evolution of the spurious pressure waves in the computational domain of the unstructured grid. We plot 8 pressure contours, from \(-8 \cdot 10^{-6}\) to \(8 \cdot 10^{-6}\) for different times. In \(t = 100\) pressure waves related to numerical noise at the beginning of the computations can be still observed.

5. Conclusions

The application of the FV-MLS method to solve aeroacoustic problems on unstructured grids has been presented. Some numerical tests for the resolution of the linearized Euler equations with uniform mean flow have been performed. The results obtained for the propagation of acoustic waves are in good agreement with the results obtained with finite difference methods on Cartesian grids.

The dissipation of the highest frequencies performed by the FV-MLS method implicitly, allows the development of an absorbing layer technique based on grid stretching. This approach has proved to be very stable and robust, and good results are obtained for the dissipation of both acoustic and vorticity waves, that could be present in many aeroacoustic problems. For acoustic waves, the proposed method works even with an unstructured low-quality grid. For this kind of waves, the absorbing layer presents low grid metrics dependency. Vorticity waves are more sensitive to the variation of the cell size in the absorbing layer, and the use of structured sponge zones is advisable. On the other hand, by using the FV-MLS scheme there is no need of using explicit filtering. However, a MLS-based filter may be used with other less dissipative methods on unstructured grids.

It is worth to note that although all the examples in this paper solve the linearized Euler Equations, both the FV-MLS method and the MLS-based absorbing layer are able to work with non linear equations.

References


