Analysis of Two Different DOA Algorithms for the Estimation of Desired Signal Using Smart Antenna Technology

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Abstract - Employment of powerful Direction-of-Arrival (DOA) estimation and beam-forming algorithms with smart antenna systems enables continuous tracking of desired signal irrespective of the signal environment there by, enhancing the quality of wireless services. In this paper emphasis is laid mainly on the performance of the various powerful high-resolution subspace algorithms like MUSIC, root-MUSIC and another fast DOA estimation algorithm making use of pseudo-covariance matrix, taking into the consideration, the number of snap-shots required to build an array correlation matrix. The aim is to compare their performances by varying the number of input signal snap-shots and to find the best algorithm that gives optimum performance with reduced number of snap-shots from the results obtained. Their performance in cases of high signal correlation and rapidly changing signal DOAs is also discussed.


I. INTRODUCTION

In wireless communication system, the design, orientation and performance of the antenna decides the quality of the wireless services. And so, the performance of the antenna systems deployed for such purposes is of prime importance. Hence the Omni-directional[1] were replaced by the smart antenna array systems [2] to reduce multipath and co-channel interference and to offer preferential gain for the signals of served users by rejecting the signals that interfere with those of desired ones. This enables a higher capacity, in addition to spectral efficiency and frequency reuse [3]. This is achieved by focusing the radiation only in the desired direction and adjusting itself to changing traffic conditions or signal environments. Smart antennas employ a set of radiating elements arranged in the form of an array (ULA, in this case). The signals from these elements are combined to form a movable or switchable beam pattern that follows the desired user.

The very purpose of DOA estimation is to use the data received by the array to estimate the direction of arrival of the signal. The results of DOA estimation are then used by the array to design the adaptive beam-former [4], which is used to maximize the power radiated towards users, and to suppress interference. As a result, we can infer that a successful design of an adaptive array depends highly on the performance of the DOA algorithm. Direction of arrival algorithms are usually complex and their performance depends on many parameters such as number of mobile users and spatial distribution, the number of array elements and their spacing and the number of signals samples.

In this paper, the performance comparison of two DOA Estimation algorithms is done based on number of snap-shots required for each of them to give a good resolution. The number of snap-shots required determines the effectiveness of the system to rapidly update DOAs of the signals and enabling the continuous tracking of the targets. In the following sections, the algorithms considered for comparison are analyzed in detail and the results obtained are discussed.

II. MUSIC AND ROOT-MUSIC DOA ESTIMATION ALGORITHMS (SUB-SPACE BASED METHODS)

In Subspace based method [5], the observed covariance matrix is decomposed into two orthogonal spaces: signal subspace and noise subspace. The DOA estimation is calculated from any one of the subspaces. The subspace based DOA estimation algorithms MUSIC root-MUSIC and ESPRIT provide high resolution, more accuracy and are not limited to physical size of array aperture. The steering vectors corresponding to the directional sources are orthogonal to the noise subspace. As the noise subspace is orthogonal to the signal subspace, these steering vectors are contained in the signal subspace.

The noise subspace is spanned by the eigenvectors associated with the smaller eigen-values of the correlation matrix, and the signal subspace is spanned by the eigenvectors associated with its larger eigen-values.

A. MUSIC Algorithm

MUSIC stands for MUltiple SInal Classification, one of the high resolution subspace DOA algorithms, which gives an estimate of a number of arrived signals, hence their direction of arrival. Estimation of DOA is performed from either signal or noise subspace[6], assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal.

If the number of signals impinging on M element array is D, the number of signal eigenvalues and eigenvectors is D and number of noise eigenvalues and eigenvectors is M-D. The array correlation matrix with uncorrelated noise and equal variances is then given by

\[ \mathbf{R}_{ss} = \overline{\mathbf{A}} \mathbf{R}_{si} \overline{\mathbf{A}}^H + \sigma_n^2 \mathbf{T} \]

Where \( \mathbf{A} = [a(\theta_1), a(\theta_2), a(\theta_3), \ldots, a(\theta_D)] \)

is an M x D array steering matrix, and

\[ \mathbf{R}_{ss} = [s_1(k), s_2(k), s_3(k), \ldots, s_D(k)]^T \]
is $D \times D$ source correlation matrix.

$R_{\Sigma}$ has $D$ eigenvectors associated with signals and $M - D$ eigenvectors associated with the noise. We can then construct the $M \times (M-D)$ subspace spanned by the noise eigenvectors such that

$$E_{\Sigma} = [e_1, e_2, \ldots, e_{M-D}]$$

Noise eigen-vectors are orthogonal to steering vectors at the angles of arrival $\theta_1, \theta_2, \theta_3, \ldots, \theta_D$. Because of this orthogonality condition, the Euclidean distance $d^2 = a(0)^H E_{\Sigma} E_{\Sigma}^H a(0) = 0$ for each and every arrival angle. The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals $\theta_1, \theta_2, \theta_3, \ldots, \theta_D$ and placing this distance expression in the denominator gives sharp peaks at the angles of arrival. The MUSIC Pseudo-spectrum is given as:

$$P_{MU}(\theta) = \frac{1}{|a(0)^H E_{\Sigma} E_{\Sigma}^H a(0)|}$$

However, when the signal sources are coherent or as the noise variances vary, the resolution of MUSIC diminishes. Also when the DOAs of signals change rapidly, the performance of the MUSIC algorithm is severely degraded. To overcome this, many array input data snapshots are required to obtain a good performance.

B. Root-MUSIC Algorithm

The MUSIC algorithm in general can apply to any arbitrary array regardless of the position of the array elements. Root-MUSIC implies that the MUSIC algorithm is reduced to finding roots of a polynomial as opposed to merely plotting the pseudospectrum or searching for peaks in the pseudospectrum [7]. Barabell simplified the MUSIC algorithm for the case where the antenna is a ULA. Recalling that the MUSIC pseudospectrum is given by

$$P_{MU}(\theta) = \frac{1}{|a(0)^H E_{\Sigma} E_{\Sigma}^H a(0)|}$$

The denominator expression can be simplified by defining the matrix $\mathcal{C} = E_{\Sigma} E_{\Sigma}^H$ which is Hermitian. This leads to the root-MUSIC expression

$$P_{RMU} = \frac{1}{|a(0)^H \mathcal{C} a(0)|}$$

In case of a Uniform Linear Array (ULA), the $m^{th}$ element of the array steering vector is given by:

$$a_m(\theta) = e^{jkd \sin \theta}$$

where $m=1, 2, M$

The denominator argument of the root-MUSIC expression can be written as

$$a(0)^H \mathcal{C} a(0) = \sum_{m=1}^{M} \sum_{n=1}^{M} e^{jkd \sin \theta} C_{mn} e^{jkd \sin \theta}$$

$$= \sum_{m=-M+1}^{M-1} c_m e^{jkd \sin \theta}$$

where $C_j$ is the sum of the diagonal elements of $\mathcal{C}$ along the $j^{th}$ diagonal such that

$$C_j = \sum_{m=n-m}^{n} C_{mn}$$

The matrix $\mathcal{C}$ has off-diagonal sums such that $c_j = |c_j|$ for $j \neq 0$. Thus the sum of off-diagonal elements is always less than the sum of the main diagonal elements. In addition, $c_0 = c_1$.

For a $6 \times 6$ matrix we have 11 diagonals ranging from diagonal numbers 1 = -5, -4, 0, 4, 5. The lower left diagonal is represented by l = -5 where the upper right diagonal is represented by l=5. The $c_i$ coefficients are calculated by $c_0 = c_5$, $c_1 = c_5$, $c_2 = c_5 + c_6$, $c_3 = c_5 + c_6 + c_7$, $c_4 = c_5 + c_6 + c_7$, $c_5 = c_6$, $c_6 = c_6$, $c_7 = c_6$, $c_8 = c_7$, $c_9 = c_7$, $c_{10} = c_7$, $c_{11} = c_7$.

The above equation can be simplified to the form of a polynomial whose coefficients are $c_i$, thus

$$D(z) = \sum_{i=-M+1}^{M-1} c_i z^i$$

where $z = e^{-jkd \sin \theta}$

The roots of $D(z)$ that lie closest to the unit circle correspond to the poles of the MUSIC pseudospectrum. Thus, this technique is called root-MUSIC. The above polynomial is of order $2(M-1)$ and thus has roots of $z_1, z_2, \ldots, z_{2(M-1)}$. Each root can complex and using polar notation can be written as

$$z_i = |z_i| e^{-j \arg(z_i)}$$

where $i = 1, 2, 3, \ldots, 2(M-1)$ where $\arg(z_i)$ is the phase angle of $z_i$.

Exact zeros in $D(z)$ exist when the root magnitudes $|z_i| = 1$. We can calculate the Angle of Arrival (AOA) by comparing $e^{j \arg(z_i)}$ to $e^{j \arg(\theta)}$ to get

$$\theta_l = -\sin^{-1}(\frac{z_i}{|z_i|} \arg(z_i))$$

III. FAST DOA ESTIMATION ALGORITHM USING PSEUDO-COVARIANCE MATRIX

The MUSIC algorithm can also estimate the DOAs of signals with adjacent incidence angles. However, many array input data snapshots are required to obtain a good performance. In addition, the DOAs of the signals must not be changed while the input data is being received to acquire the covariance matrix, plus the incidence signals must be uncorrelated. Thus, when signals are correlated or the DOAs of signals change rapidly, the performance of the MUSIC algorithm is severely degraded. It is also impossible to estimate the DOAs of signals quickly, because the MUSIC algorithm cannot perform a DOA estimation until it has received all the snapshots of the array for the covariance matrix.

In contrast, a direct data domain adaptive (DDDA) beam-former uses a single snapshot to acquire a pseudo-covariance matrix [8], which performs similar function to a covariance matrix. A beam-former creates a beam pattern that allows the look direction signal to pass, while removing the rest of the signals. As such, for the rapid identification of the DOAs of incidence signals to array systems based on less sample data, even under coherent signal conditions, this paper presents a DOA estimation algorithm that uses a pseudo-covariance matrix based on a single snapshot. The bearing response and the directional spectrum are then obtained using the pseudo-covariance matrix.

The proposed DOA estimation algorithm operates based on the following two steps: First, rough incidence angle ranges for the signals are obtained by using the bearing response. Second, the exact incidence angles are obtained by combining the bearing response and directional spectrum. The proposed algorithm only uses a few
snapshots to estimate the DOAs of signals, whereas the MUSIC algorithm requires hundreds of snapshots, the proposed algorithm can estimate the DOAs of signals with rapidly changing incidence angles. Moreover, the DOAs from coherent signals can be discriminated from different DOAs by taking advantage of the pseudo-covariance matrix.

Direct Data Domain Adaptive (DDDA) Beam-forming Technique.

Consider a uniformly spaced linear array with \( N \) array elements. Assume that \( L \) narrow band signals impinge on array from \( L \) directions \( \theta_1, \theta_2, \ldots, \theta_L \). The angles \( \theta_i \), \( i = 1, 2, L \), are defined from the broadside direction of the array.

The signal vector \( X (k) \) impinging on the array at time \( k \) is defined as:

\[
x(k) = A(\Theta) s(k) + n(k) = [x_1(k), x_2(k), \ldots, x_N(k)]^T
\]

where superscript \( T \) denotes a transpose. \( A(\Theta) \) is a matrix consisting of steering vectors, \( s(K) \) is the signal vector consisting of \( L \) different incidence signals and \( n(K) \) is the white noise vector generated at each array element with a zero mean and variance of \( \sigma_n^2 \).

\[\begin{align*}
A(\Theta) s(K) & \quad \text{and} \quad n(K) \text{ can be expressed as:} \\
A(\Theta) & = [a(\theta_1), a(\theta_2), \ldots, a(\theta_L)] \\
s(K) & = [s_1(k), s_2(k), \ldots, s_N(k)]^T \\
n(K) & = [n_1(k), n_2(k), \ldots, n_N(k)]^T
\end{align*}\]

respectively, where \( a(\theta_i) = \) \( i \)th steering vector \( a(\theta) \) coming from \( \theta_i \). The eigenvalue equation given by

\[
a(\theta_i) = [1, e^{-\frac{2\pi d}{\lambda} \sin \theta_1}, \ldots, e^{-\frac{2\pi d}{\lambda} (N-1) \sin \theta_1}]^T
\]

The Fig.1 shows a DDDA beam-former using a pseudo-covariance matrix. The pseudo-covariance matrix generator builds \( X(k) \) as:

\[
X(k) = \begin{bmatrix}
x_1(k) & x_2(k) & x_3(k) & \cdots & x_M(k) \\
x_2(k) & x_3(k) & x_4(k) & \cdots & x_{M+1}(k) \\
x_3(k) & x_4(k) & x_5(k) & \cdots & x_{M+2}(k) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{M}(k) & x_{M+1}(k) & x_{M+2}(k) & \cdots & x_{N}(k)
\end{bmatrix}
\]

It performs a similar function to a covariance matrix, where \( M \) is equal to \( N+1/2 \). The degree of freedom (DOF) of the array is reduced to \( M \) because the size of a pseudo-covariance matrix is only \( M \times M \) in spite of an original DOF of \( N-1 \). When the look direction of the beam-former is \( \theta_S \), \( a(\theta_S) \) can be expressed as:

\[
a(\theta_S) = [a_1(\theta_S), a_2(\theta_S), \ldots, a_N(\theta_S)]
\]

\[
a(\theta_i) = [1, e^{\frac{2\pi d}{\lambda} \sin \theta_1}, \ldots, e^{\frac{2\pi d}{\lambda} (N-1) \sin \theta_1}]^T
\]

where \( \lambda \) and \( d \) are the wave-length of the carrier and inter element spacing, respectively.

The look constraint generator transposes \( a(\theta_S) \) into:

\[
S(\theta_S) = \begin{bmatrix}
a_1(\theta_S) & a_2(\theta_S) & \cdots & a_M(\theta_S) \\
a_2(\theta_S) & a_3(\theta_S) & \cdots & a_{M+1}(\theta_S) \\
\vdots & \vdots & \ddots & \vdots \\
a_M(\theta_S) & a_{M+1}(\theta_S) & \cdots & a_N(\theta_S)
\end{bmatrix}
\]

The pseudo-covariance matrix \( X(k) \) is the signal received at the antenna elements, and it is assumed that \( X(k) \) consists of thermal noise and interferences. The vector \( v \) for minimizing the noise power is defined to satisfy

\[
[X(k) - X(\theta_S)] v = 0
\]

The above equation can be expressed as a generalized eigen value equation given by

\[
X(k) v = \lambda X(\theta_S) v
\]

The eigenvalue-decomposition produces a diagonal matrix consisting of generalized eigenvalues and full matrix whose columns are the corresponding eigenvectors, where the smallest eigenvalue and \( v \) is an \( M \times 1 \) eigenvector corresponding to the eigenvalue defined as a noise subspace. So, the eigenvector \( v \) is orthogonal to other steering vectors except for the steering vector for the look direction \( \theta_S \).

The constraint condition is determined as

\[
w(\lambda_S) = 1
\]

Where \( w(\lambda_S) = [w_1(\Omega_S), w_2(\Omega_S), \ldots, w_M(\Omega_S)] \) and \( \lambda(\Omega_S) \) is the steering vector for the look direction \( \theta_S \) with \( M \times 1 \) dimension.

The weight vector can be obtained from the eigenvector \( v \) and the steering vector \( \lambda(\Omega_S) \) as:

\[
w(\lambda_S) = \frac{v}{v^T \lambda(\Omega_S)}
\]

When the array is steered to the look direction \( \theta_S \), the output of the array \( y(k) \) is defined as:

\[
y(k) = \sum_{m=1}^{M} w_m(\lambda_S) x_m(k)
\]

The output power can be minimized by the weight vector \( w(\theta_i) \), which pattern nulls are formed based on the directions of the signals except for the incidence signal from the look direction.

The array should satisfy \( L \leq M \) to discriminate all incidence signals. If the look direction \( \theta_S \) is equal to the incidence angle, the weight vector \( w(\theta_i) \) can be obtained from the eigenvalue-decomposition of the pseudo-covariance matrix. Since the weight vector \( w(\theta_i) \), which has units gain in the look direction, has pattern nulls at the incidence angles for the rest of the signals.

The array output \( y(k) \) is
y(k) = [1, 0, ..., 0][s_1(k), s'_1(k), ..., s_L(k)]^T + w(\theta_k)^T\hat{n}(k)

Where s_1(k) and s'_1(k) are correlated to each other.

In the proposed algorithm, there is no signal cancellation problem, even though the signals are correlated. The output power of array P_0(\theta_k) is

P_0(\theta_k) = \frac{1}{K} \sum_{i=1}^{K} y(k)^T y(k)^* \tag{10}

Where K is the number of snapshots required to observe the output power. If a signal S_i(k) impinges on the array within a small level of deviation from the look direction, the bearing response P_0(\theta_k) can be given by

P_0(\theta) \equiv E[s_i(k)s_i(k)^H] = \sigma_i^2 \tag{11}

The bearing response only has the power of S_i(k) added noise. If there is no signal impinging on the array within a small level of deviation from the look direction, the bearing response P_0(\theta_k) only includes the noise power. Therefore, the bearing response around an incidence signal has a much higher power than that from a noise direction. To locate the possible angles of incidence, the algorithm must search for peaks that are higher than a preset threshold. This threshold is chosen based on the signal and noise power levels.

To get more accurate DOAs of incidence signals, the directional spectrum can be used. First of all, the angle with the smallest bearing response \theta P_{min} should be searched from the bearing response as

\theta_{P_{min}} = \arg \min_{\theta_k} P_0(\theta_k) \tag{12}

The corresponding weight vector w (\theta_{P_{min}}) can be obtained by performing the DDDA beam forming. Next, the normalized directional spectrum can be taken as

P_0(\theta) = 10 \log_{10} \left( \frac{1}{\left| w(\theta_{P_{min}})^T\hat{a}(\theta) \right|^2} \right) \tag{13}

Because the weight vector w (\theta_{P_{min}}) is orthogonal to the steering vectors of the incidence signals, the directional spectrum has the peaks at the DOAs of the incident signals. So, accurate DOAs can be obtained by using both the bearing response and the directional spectrum. First, rough DOA ranges can be obtained from the bearing response. Second, peak DOAs can be considered as accurate DOAs if peaks in the directional spectrum are within rough DOA ranges acquired from the bearing response.

**IV. RESULTS AND DISCUSSIONS**

Here, the results are obtained for above algorithms taking six array elements. The y-axis and the x-axis represent the spectral power P(\theta) and the range of angles respectively.
If we observe the above results this is similar to music algorithm. We got sharp peaks in case of 100 snapshots. From this we can estimate the DOA clearly which is not possible in case of 5 snapshots. In root-MUSIC, we have an advantage of reduced complexity over MUSIC that, we can find the DOA of the signal simply by finding the locations of the roots of polynomial rather than plotting the spectrum searching for peaks in it. Fig. 4(a) and Fig.5(a) give the location of roots on z-plane and the Fig.4(b) and Fig 5(b) give the peaks at angles of arrival corresponding to the location of the roots of polynomial for 100 and 5 snap-shots each respectively.

Here, in Fig.6 and Fig.7 we observe the same resolution for 5 snap-shots and for 100 snap-shots. Hence, the proposed algorithm is superior to MUSIC, root-MUSIC algorithms in its performance and speed of DOA Estimation with reduction in the number of required signal snap-shots.

V. CONCLUSIONS

The above results clearly show that the proposed algorithm which makes use of pseudo-covariance matrix gives fine resolution even for 5 snap-shots which is not possible in case of MUSIC and root-MUSIC algorithms. The reduction in the number of snap-shots required reduces the delay in scanning of the input data and processing of it. The time required for acquiring say 5 snap-shots is very less thereby enabling the system to adapt to the rapidly changing or correlated interference environments. The time taken for the DOA estimation will be very less giving scope for the system for continuous updating of signal DOAs and signal tracking in rapidly changing signal environments.

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