Interference Cancellation in Coded OFDM/OQAM

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Abstract—This paper addresses the concatenation of a convolutional code and OFDM/OQAM modulation transmitted over a time-dispersive radio channel. The proposed receiver is composed of a suitable OFDM/OQAM filter-bank demodulator, followed by an intersymbol and inter-(subchannel) interferences canceller and a SISO (Soft Input – Soft Output) decoder put in Turbo-like mode. The iterative algorithms are based on message passing on bipartite graphs. As an example, performance comparison with the linear subchannel equalization is presented.

Keywords—OFDM/OQAM; Coded OFDM; Soft Successive Interference Cancellation; bipartite graphs, message passing

I. INTRODUCTION

Development of the Coded Orthogonal Frequency Division Multiplex (COFDM) QAM-based modulation for Digital Audio and Video Broadcasting [1] has paved the way for its adoption as physical layer accessing format into a number of WLAN, MAN and cellular systems, and has widely been adopted for the 4-th generation (4G) systems development and deployment in the form of WiMAX and LTE standards, as a “system of choice”. Although its advantages over the CDMA accessing format have been largely appreciated, it still may not represent an optimal system in terms of the overall system performance/complexity ratio, mainly due to an inherent spectral and power inefficiency caused by utilization of cyclic prefix (CP), inability to directly exploit the multipath diversity, and a considerable loss of performance due to various form of transmission and implementation imperfections, in particular in mobile environment and up-link direction of transmission. This paper deals with an alternative multi-carrier format based on orthogonal multiplexing of subchannels with the Staggered, or Offset QAM, here denoted COFDM/OQAM, and presents an iterative receiver for this type of signal, to be used in lieu of various forms of subchannels, or per-bin equalization developed and proposed thus far. Together with its rather straightforward extension toward modern MIMO configurations, it can serve as catalyst toward a continued interest in this form of signaling format, and its potentially ultimate adoption into newly developed standards, in particular for the Cognitive Radio applications, [2].

Since the introduction of orthogonally multiplexed subchannels with Nyquist-type spectral shaping [3], its Staggered- or O(-ffset)QAM form was proposed in [4], and the first digital implementation of modulation and demodulation by using (I)FFT and poly-phase filter banks was done in [5]. It was followed by the first sub-channel based linear equalization [6] to compensate for inter-symbol and inter-(sub)-channel interference arising from linear distortions introduced by a multi-path channel – the self-interference. Further development and extension of equalization concepts to MIMO systems have been undertaken recently, [7].

Application of traditional linear equalization, [6][8], or its non-linear DFE counterpart, [9], for combating transmission linear distortion impairments in the presence of appreciable (sub)channel frequency- and time-selectivity is tied with problems related to channel inversion and decision errors propagation, respectively. This suggests that the MLSE approach, which directly relies on the subchannel impulse response, could be much better, but its exponential increase in complexity with respect to the impulse response length presents a big disadvantage, so that it can be prohibitive for practical implementation. On the other hand, the invention of the Turbo codes by Claude Berrou at al., [10], and the iterative algorithms [11] started to be used to implement the Turbo equalization or Turbo interferences cancellation [12], with just linear complexity with respect to the channel impulse response lengths. Whosoever, the concatenation of the soft successive self-interference cancellation and channel decoding procedure in Turbo mode, [13], offers additional enhancement of the overall performance. It has been proposed and analyzed in [14] for an ISI/ICI canceller in COFDM/OQAM based on development of Turbo-codes and message passing decoding algorithms during the last two decades. In this paper we aim at a more detailed exposition of such a method, to bring it to attention of the wider scientific and engineering communities.

The transmission model1 for the OFDM/OQAM modulation in an interference cancellation framework is a multiple input and single output filter for each sub-channel. To enable modeling of this scheme by a hidden Markov chain with a reasonable number of states, to be used in the the Turbo-like self-interference cancellation, it is needed to re-formulate the OFDM/OQAM modulation so that the real-valued canceller impulse responses are used for even and odd sub-channels and corresponding cross-talk terms, as well as for all T/2 instants.

The demodulator is represented with a bipartite graph. control nodes are the ISI/ICI cancellers, in that they evaluate the extrinsic probability of the modulated symbols. The control nodes associated with the QAM modulation evaluate the extrinsic probability of the coded bits from the preceding

1 The system and its parts will be described in the conference presentation, and available on request sent to preferably 3rd author.
extrinsic probabilities of the modulated symbols. A SISO decoder evaluates the a posteriori probabilities of the information bits and returns the extrinsic probabilities of the coded bits, as for a more classical turbo-equalization algorithm [13]. Then the extrinsic probabilities of the modulated symbols are evaluated from the extrinsic probabilities of the coded bits, and used to determine soft decisions to be subsequently used for canceling the multiple-input single-output (MISO) interference, in combination with the estimated real-valued impulse response samples.

The paper is organized as follows. Section II formulates the modulator/demodulator part suitable for the interference cancellation method, with inclusion of channel linear distortions in Section III. After a brief outline of the channel impulse response estimation in Section IV, the estimation of the extrinsic probability is elaborated in Section V, followed by description of the iterative decoding procedure by message passing. Illustrative interference cancellation and equalization comparative results are provided in Section VI, and the paper finishes with some conclusions in Section VII.

II. COFDM/OQAM - COFDM WITHOUT CYCLIC PREFIX

The COFDM/OQAM modulation is a modulation with superior energy and spectral efficiency over the conventional COFDM/QAM system, as shown by an analysis and results in [15]. The Bit Interleaved Coded Modulation was introduced in [16]. The information bits \{a_k\} are encoded to produce coded bits \{a_n\}. The mappings of the 16QAM modulation dibits are defined for the real part and the imaginary part by the following mapping triplets: \{(0 0), -3\}; \{(0 1), -1\}; \{(1 1), +1\}; \{(1 0), +3\}, while mappings of a 4QAM and a 64QAM are defined in the similar manner.

In continuous time representation, the OFDM/OQAM modulated signal is defined by the relation

$$x(t) = \sum_{k=-k_2}^{k_2} \sum_{m=0}^{2M-1} j^{k+m} d_m^k \cdot g(t - mT/2) e^{j2\pi k(t-mT/2)/T}$$ (1)

where the symbols \(d_m^k\) are real valued (Real or Imaginary parts of M QAM symbols, belonging to a block of transmitted data). The index of the K subcarriers are denoted with numbers between \(k_1\) and \(k_2\). The function \(g(t)\) is a real, even, root Nyquist function [5] of norm 1, with the low-pass bandwidth \(B_g = (1 + \alpha)/T\), where \(\alpha\) is the roll off factor. It has finite time duration \([-L_gT, L_gT]\), with \(T\) denoting the QAM signaling interval. The bandwidth of a subchannel is equal to \(B_g\), and the frequency distance between two adjacent subchannels is \(1/T\). The total signal bandwidth amounts to \(B_s = (K + \alpha)/T\). The spectral efficiency of the OFDM/OQAM modulation is \(K/(K + \alpha) \approx 1\), and for even moderate number of subchannels it can be very close to one real valued symbol per signal dimension, or one QAM symbol per second per Hertz.

The transmit signal may be represented as a modulation, with real valued symbols, of an orthogonal basis \(\phi_m^k(t) = g(t - mT/2) e^{j2\pi k(t-mT/2)/T}\) and (1) becomes

$$x(t) = \sum_{k=-k_2}^{k_2} \sum_{m=0}^{2M-1} j^{k+m} d_m^k \phi_m^k(t).$$

After a transmission over an AWGN channel, the maximum likelihood, ML, receiver is obtained by the real part of the scalar product between the received signal \(r(t) = x(t) + b(t)\) and the basis functions \(\phi_m^k(t)\), with

$$r_m^k = \text{Re} \left\{ \int_{-L_gT}^{L_gT} r(t) e^{j2\pi k(t/mT/2)} dt \right\}$$ (2)

where the over-bar denotes complex conjugation. By using orthonormality of basis functions, the sampled version of (2) becomes \(r_m^k = d_m^k + b_m^k\). It should be clear from the context, that index \(m\) corresponds to \(T/2\) instants, as the index \(m\) in (1).

III. TRANSMISSION OVER A DISPERSIVE CHANNEL

The time-dispersive channel is defined by a finite impulse response \(h(t)\) for \(t \in [-L_kT, L_kT]\) with an AWGN noise at the FIR filter output. The received signal is represented by

$$r(t) = \int_{-L_kT}^{L_kT} x(t - \xi) h(\xi) d\xi + b(t),$$ (3)

or

$$r(t) = \sum_{k=-k_2}^{k_2} \sum_{m=0}^{2M-1} j^{k+m} d_m^k g(t - mT/2) e^{j2\pi k(t-mT/2)/T} + b(t)$$ (4)

Here

$$g_k(t) = \int_{-L_kT}^{L_kT} h(t - \xi) d\xi$$

is a function of time with duration over the interval \(t \in [-L_kT, L_kT]\), and has bandwidth \(B_g = (1 + \alpha)/T\). Projection of the received signal over the orthogonal basis \(\phi_m^k(t)\) is defined by (2).

With \(\phi_m^k(mT/2 + t) = j^{k+m} g(t) e^{j2\pi k(t/2)}\)

$$r_m^k = \text{Re} \left\{ \int_{-L_kT}^{L_kT} \sum_{k=-k_2}^{k_2} \sum_{m=0}^{2M-1} e^{j2\pi k(t-mT/2)} j^{k+m} g(t) e^{-j2\pi k(t/2)} dt \right\}$$ (5)

whereby \(b(t) = 0\), and by using \(k' = k + k''\) and \(n' = m - n''\)

$$r_m^k = \sum_{k=-k_2}^{k_2} \sum_{n=0}^{2M-1} d_m^{k''} \text{Re} \left\{ j^{k''} g_k(t) e^{j2\pi (k' + n')T/2} \right\}$$

Due to the finite length duration in time and frequency of
the impulse functions of the modulator and the channel, the received signal is the output of a linear matrix filter represented by real valued samples plus a white Gaussian noise, not accounted for after equation (4). The 3x1MISO model is described by the following relation \((m\) replaced by \(n\))

\[
    r_n^k = \sum_{k'=-1}^{2L} \sum_{n''=-2L_{n''}}^{2L_{n''}} d_{n-n''}^k h_{n''}^{k,k'} + b_n^k ,
\]

with reinserting (uncorrelated) noise samples, where \(h_{n}^{k,k'}\) are the three impulse responses of the global filters. \(h_{n}^{k,0}\) is the impulse response on the subchannel \(k\), \(h_{n}^{k,1}\) represents the interferences from the upper sub carrier of index \(k + 1\) and \(h_{n}^{k,-1}\) interferences from the lower subchannel of index \(k - 1\).

Note that the samples \(h_{n}^{k,k'}\) and the data \(d_{n}^k\) are all real-valued.

If the duration of the transmitter impulse were fixed to \(4T\) and the duration of the channel impulse response is less than \(T\), then the size of the matrix \(H\) is 3 times 10. Due to the very notation used in (9), same filters are used for even and odd \(T/2\) instants, since their definition does not depend on index \(n\).

IV. CHANNEL ESTIMATION

For the subchannels impulse response identification, the sum of squared noise terms from (9) is minimized over a training period of length \(N\), which by re-shuffling \(h_{n}^{k,k'}\) and \(d_{n-n''}^k\) becomes the one dimension vector form

\[
    \sum_{n=1}^{N} \left( r_n^k - \sum_{n''=n}^{n-1} d_{n-n''}^k h_{n''}^{k,k'} \right)^2 .
\]

Then by applying this method for each sub carrier of index \(k\), the \(H_k\) matrices of size 3x\((L + 1)\), \(L=2L_{n''}\), with impulse responses representing the inter-subchannel (1st and 3rd raw) and intersymbol interferences (2nd raw) can be estimated by product of the inverse data autocorrelation matrix and the signal-data cross-correlation vector, in line with conventional least-squares (LS) criterion for minimization of metric in (10).

V. ESTIMATION OF THE EXTRINSIC PROBABILITY

The main idea in interference cancellation is to suppress the interferences coming from the other symbols by using an estimation of their conditional expectation \(\hat{d}_n^k = E \left\{ d_n^k \mid r' \right\}\), where \(r'\) is the received signal after suppression of the interferences produced by all symbols except \(d_n^k\). Then, based on the transmission model defined by (9)

\[
    r_n^{k+k'} = r_{n-n''}^k - \sum_{k'=-1}^{L} \sum_{n''=-2L_{n''}}^{2L_{n''}} d_{n-n''}^k h_{n''}^{k+k',k'}
\]

for \(k+k' \neq 0\), \(n-n'' \neq 0\), \(k' = -1,0,1\) and \(n'' = -L, \ldots, 0, \ldots, L\).

For implementation it becomes much simpler to define \(r_n^k\) as the sum of residual interference and the AWGN term, so that

\[
    r_n^{k+k'} = d_n^k h_{n}^{k+k',k'} + r_{n}^{k+k'} .
\]

The interference cancellation algorithms are based on message passing over bipartite graphs. In the bipartite graph, the variable nodes are the observations \(r_n^k\), and the modulated symbols \(d_n^k\) nodes, while the control nodes are defined from the observation equation (12). A control node \(\eta_n^k\) is associated with each set of variable nodes \(\{r_n^k\}; d_n^k\). The joint probability of this set is defined by

\[
    P(r_n^k, d_n^k) = P(r_n^k \mid d_n^k) \cdot P(d_n^k)
\]

The extrinsic probability is determined by \(P(r_n^k \mid d_n^k)\) through this: the a posteriori probability of a random variable is the product of its a priori probability and its extrinsic probability. Thus the extrinsic probability represents the information provided by a control node on a random variable.

The a posteriori probability itself is proportional to the joint probability \(P(r_n^k, d_n^k) = P(d_n^k \mid r_n^k) \cdot P(r_n^k)\), and from (13) the extrinsic probability of the random variable \(d_n^k\) is proportional to the conditional probability or the likelihood \(P(r_n^k \mid d_n^k)\), defined by considering that the interference from the other symbols has been completely removed, and only the AWGN term remains. Then by using the observation equation

\[
    r_n^{k+k'} = d_n^k h_{n}^{k+k',k'} + h_{n}^{k+k',k'} ,
\]

where \(h_{n}^{k+k',k'}\) is a Gaussian random variable, the expression for the conditional probability is

\[
    P(r_n^k \mid d_n^k) = \exp \left\{ -\frac{\sum_{k'=-1}^{L} \sum_{n''=-2L_{n''}}^{2L_{n''}} (r_n^{k+k'} - d_n^k h_{n}^{k+k',k'})^2}{2\sigma_k^2} \right\},
\]

or

\[
    P(r_n^k \mid d_n^k) = \exp \left\{ -\frac{\sum_{k'=-1}^{L} \sum_{n''=-2L_{n''}}^{2L_{n''}} (d_n^k - h_{n}^{k+k',k'})^2 \cdot E_k / 2}{2\sigma_k^2} \right\} ,
\]

where \(f_n^k = \sum_{k'=-1}^{L} \sum_{n''=-2L_{n''}}^{2L_{n''}} (r_n^{k+k'} - d_n^k h_{n}^{k+k',k'})\) is the output of the matched filter, and \(E_k = \sum_{k'=-1}^{L} \sum_{n''=-2L_{n''}}^{2L_{n''}} (h_{n}^{k+k',k'})^2\) is the symbol energy on the sub carrier \(k\). The output of the matched filter may be evaluated with the signal \(r_n^k\) in (12) as

\[
    f_n^k = \sum_{k'=-1}^{L} \sum_{n''=-2L_{n''}}^{2L_{n''}} (r_n^{k+k'} - d_n^k h_{n}^{k+k',k'}) + d_n^k E_k ,
\]

where \(d_n^k\) is the previously estimated value of the conditional expectation of the symbol \(d_n^k\). The resulting extrinsic probability, produced by an additional normalization, will be denoted \(P_{\eta_n^k \rightarrow d_n^k}(d_n^k)\).

To compute the intermediary received signal \(r'\), the node \(\eta_n^k\)
uses the a priori probabilities \( P_{api}^{a_{n} \rightarrow n_{k}}(d_{n}^{k}) \) for \( n \neq n' \) and \( k \neq k' \) in computing conditional data symbols expectation by

\[
\hat{d}_{n}^{k} = E\{d_{n}^{k} \mid r\} = \sum_{\{d_{n}^{k}\}} d_{n}^{k} P_{api}^{a_{n} \rightarrow n_{k}}(d_{n}^{k}) . \tag{17}
\]

Note that a control node \( \eta_{n}^{k} \) does not use the a priori probability of the symbol \( d_{n}^{k} \) to evaluate its estimate in (17).

Since in the case without coding or with decoupled decoder iterations only one edge terminates at the variable node \( d_{n}^{k} \), the a priori probability \( P_{api}^{a_{n} \rightarrow n_{k}}(d_{n}^{k+1}) \) is equal to the extrinsic probability \( P_{ext}^{ext \rightarrow n_{k}}(d_{n}^{k+1}) \). The particularity of the related bipartite graph is that the control node \( \eta_{n}^{k} \) sends information to only one variable node \( d_{n}^{k} \) but it receives information from all connected variable nodes \( d_{n+k}^{k+k'} \) for \( k' = -1,0,1 \) and \( n' = -L,\ldots,0,\ldots,L \), except \( k' = n' = 0 \).

If an error correcting code is used, the variable node \( d_{n}^{k} \) is connected to the control node defined by the transformation of QAM symbols into bits, a deinterleaver, and a soft input soft output (SISO) decoder. The message passed from the SISO decoder to the variable node \( d_{n}^{k} \) is defined by \( P_{ext}^{ext \rightarrow n_{k}}(d_{n}^{k+1}) \), so that the a priori probability passed from the variable node \( d_{n+1}^{k} \) to the control node \( \eta_{n}^{k} \) is defined by

\[
P_{api}^{a_{n+1} \rightarrow n_{k}}(d_{n+1}^{k}) = P_{ext}^{ext \rightarrow n_{k}}(d_{n+1}^{k+1}) \cdot P_{ext}^{ext \rightarrow d_{n}^{k}}(d_{n}^{k+1}) . \tag{18}
\]

Thus the interference cancellation is obtained by using the information coming from the decoder and from the other control nodes \( \eta_{n'}^{k} \).

VI. DECODING BY MESSAGE PASSING

The convolutional code is decoded by using a forward-backward algorithm or any SISO algorithm. The inputs of the SISO algorithm are the a priori probabilities of the coded bits, \( P_{a_{n} \rightarrow code}(a_{n}^{k}) \), while the outputs of the SISO decoder are the a posteriori probabilities of the information bits for the sink \( P_{frmbit}^{app}(a_{n}^{k}) \) and the extrinsic probabilities of the coded bits \( P_{code \rightarrow a_{n}^{k}}(a_{n}^{k}) \). For the 16-QAM modulation, the two bits \( a_{n,1}^{k}, a_{n,2}^{k} \) are associated with the quaternary symbol \( d_{n}^{k} \) with a mapping indicated in section II.

A third group of control nodes represents the AM modulation, the real or imaginary component of a QAM modulation. In the case of a multidimensional QAM modulation, a 32 QAM, or an 8-dimensional coded modulations, these modulation nodes represent the coded modulation mapping. For a 16-QAM, i.e. a 4-AM modulation for each of the quadrature components, the control node AM is connected to the variable nodes \( d_{n}^{k}, a_{n,1}^{k} \) and \( a_{n,2}^{k} \). The corresponding a priori probabilities are defined by using the variable nodes input-output relations defined as follows:

\[
P_{api}^{a_{n} \rightarrow code}(a_{n}^{k}) = P_{code \rightarrow a_{n}^{k}}(a_{n}^{k}) . \tag{19}
\]

and

\[
P_{api}^{a_{n} \rightarrow 0}(d_{n}^{k}) = P_{ext}^{ext \rightarrow a_{n}^{k}}(d_{n}^{k}) . \tag{20}
\]

The message passing is divided in two directions. The forward direction from the ISI canceller to the decoder, and the backward direction from the decoder to the ISI canceller. The relations are

\[
P_{code \rightarrow a_{n}^{k}}(a_{n}^{k}) = \begin{cases} P_{code \rightarrow a_{n}^{k}}(a_{n}^{k} = 0) & P_{code \rightarrow a_{n}^{k}}(a_{n}^{k} = 3) + 1 \end{cases} \tag{21}
\]

and similar definitions apply for the second bit \( a_{n,2}^{k} \).

The backward relations are given by

\[
P_{code \rightarrow a_{n,1}^{k}}(a_{n,1}^{k} = -1) = P_{code \rightarrow a_{n,1}^{k}}(a_{n,1}^{k} = 0) P_{code \rightarrow a_{n,1}^{k}}(a_{n,1}^{k} = 1) \tag{23}
\]

etc., for dibits 11 and 10 with respective PAM levels 1 and 3.

VII. SIMULATION RESULTS

The performance of iterative interference cancellation and FEC decoding was comparatively evaluated with the per-bit linear equalization of [8], using the indoor channel models developed for development of the IEEE 802.11n WLAN standard [17]. The equalization is performed for each sub-channel as in regular QAM single-carrier, but by calculating only purely real and purely imaginary outputs at consecutive T/2 instants, and by adapting the complex coefficients at T/2 instants by using corresponding purely real and purely imaginary errors. Since the number of sub-channels can be varied flexibly, without significant constraints regarding spectral efficiency, the parameter \( N \) was set to 8, with seven active sub-channels, well fitting the IEEE 802.11 spectral mask in 20MHz channel bandwidth, achieving 7:8 instead of 6:8 basic spectral efficiency, [15]. The encoder is a \( R = 1/2 \) (133,171) convolutional code followed by an interleaver of length 56, and 16-QAM, with a Gray mapping. Soft metric in case of equalization was provided by the inverse of the MSEs after equalization. The simulated channel model was D, with a moderate impulse response spanning about two QAM symbol intervals. The channel parameters are estimated with a training period long enough (60 4-QAM symbols here) to ensure a reasonably well LS-type channel estimation for interference cancellation and RLS-type linear equalizer adaptation.
The length of equalizer and the length of the estimated channel impulse response are 16 and 15 (T/2 – spaced) taps respectively, with 6 iterations for the SIC case. To provide a reasonably good statistics, 300 independent fading instantiations were used. Signal power normalization was done on each instantiation. 16-QAM constellation was used. The soft interference cancellation was performed with five iterations. As a comparison, simulation results for the OFDM/QAM in configuration of the IEEE 802.11a standard are also provided. The coding rate of 3/4 was used, to have the same targeted data rate, as a compensation for the 25% CP and the 1/8 of the reduced spectral efficiency in the presence of nonlinear HPA. For each of the SNR values, the worst 5% of fading instantiations were excluded from the BER calculation.

From the results in Fig. 1 it can be seen that the soft interference cancellation method gives better performance for both un-coded (raw) and coded cases, in comparison with linear equalization. Note that there was no explicit attempt to provide symbol synchronization, and the symbol clock was fixed with three samples delay w.r.t. the ideal channel case.

![BER Comparison of OFDM/QAM with EQ and SIC (N=64), and OFDM/QAM (N=64)](image)

Fig. 1. BER performance comparison in case of noise-free adaptations.

VIII. CONCLUSIONS

After deriving the soft interference cancellation framework for the OFDM/QAM system, this paper shows the performance obtained by using turbo equalization with moderately long channel impulse responses in comparison with the performance obtained by application of per-bin linear equalization with subsequent, decoupled decoding. Although the comparative implementation complexity was not conducted, it is clear that for the same length of canceller/equalizer structures, the soft interference cancellation itself incurs much higher complexity. While the number of real multiplications per coefficient is four times smaller than in equalization case, its repetition and in particular the repetition of the FEC decoding stage contributes to the complexity increase. However, compared to conventional MLSE, the complexity of the presented cancellation algorithm does not increase with the size of the QAM modulation. A good side of the presented interference cancellation approach is a considerable simplification of the demodulator filter-bank, in that the calculation of only the information bearing quadrature components is needed.

By its inherent removal of the causality constraint, the interference cancellation may offer possibility for further performance enhancements based on noise prediction and cancellation. It would be also worth considering to separate the systematic (modulator/demodulator) part of subchannels impulse responses to possibly simplify the SIC procedure by benefiting from the channel impulse response sparsity.

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