Abstract— In the sprouting paradigm of interoperable radio networks, wideband spectrum sensing is a challenging task for analog-to-digital converters (ADC) incorporated at the prevailing wireless radio systems because of the necessities of high sampling rates functioning at or above Nyquist frequencies. In order to cope with current ADCs, compressive sampling (CS), a promising scheme in signal processing arena, can be employed to search for the spectrum holes in the sparse wideband signals which are then opportunistically used by the cognitive radios (CR). In CS, transform coding as well as measurement matrix selection is an essential tool and it plays a vital role in the acquisition of wideband signals which come out with a few number of random measurements. In this paper, two types of transform coding (Discrete Cosine Transform, DCT and Discrete Walsh-Hadamard Transform, WHT) is analyzed in the context of sparse wideband estimation via a well-known CS approach e.g., $l_1$-norm optimization problem which could be used for spectrum sensing in wideband CR. Through the engagement of those measurement matrices, detection performance, execution time to sense PU bands and achievable capacity are investigated and analyzed at a single CR node. Finally, WHT coded CS scheme has been proposed for the wideband CR spectrum sensing as the simulation results arrange for the validation of our choice.

Keywords— compressive sampling; $l_1$-minimization; wideband spectrum sensing; analog-to-information converter; mean square error.

I. INTRODUCTION

Recent research shows that at any particular spatial region and time, spectrum is present which is not being occupied by the licensed or primary users (PUs); especially, in the VHF-UHF bands licensed to television broadcasting, spectrum is often poorly utilized [1-2]. CRs exploit the opportunistic spectrum access and can occupy the licensed bands while the PUs data transmissions are inactive, thus augmenting better radio resource utilization. The enhancement of the spectrum efficiency can feasibly be done opportunistically by means of automatic frequency switching [3]. There are couples of conventional ways to perform wideband spectrum sensing for CR network [4]. First, wideband spectrum sensing can be performed by using a bank of tunable narrowband band-pass filters at the radio frequency (RF) front-end to scan one narrowband frequency for a particular sensing period which involves a variable number of RF components and the tuning range of the RF filters are preselected. Alternatively, a wideband circuit utilizes a single RF chain followed by a high-speed digital signal processing unit to flexibly search over multiple frequency bands simultaneously. In the later scheme, extreme sampling rates would be required which operates at or above the Nyquist sampling rate. The advantage of wideband spectrum sensing is that there is greater probability of having opportunity to a CR node to utilize the licensed bands which exploits higher achievable throughput to a CR. As the Nyquist rate sampling appears to be the best conciliation for wideband sensing however, it requires high-rate ADCs and challenging to handle a large number of samples in the outputs resulting greater memory capacities, higher RF signal acquisition costs and prohibitive energy requirements. Hence, there is a need for wideband sensing algorithms with lower complexity and lower power consumption as well [5]. In practice, wideband radio signal is sparse in some basis functions and capitalizing on the sparsity, a promising tool termed as CS, has been exploited in signal acquisition systems. The sparse wideband spectrum is estimated by means of CS approach that enables sub-Nyquist data procurement via an analog-to-information converter (AIC) [6] thus, relaxing the requirement of the high speed ADCs. An AIC directly relates to the idea of sampling at the information rate of the signal.

The CS scheme can overcome those difficulties in an efficient way. CS is a method of acquisition of sparse signals at considerably lowering sampling rates than that of Nyquist; spectrum reconstruction is performed by $l_1$-norm regularization scheme [7-8]. This scheme provides an effective way to perfectly (or near perfectly) estimate the discrete-time sparse signals (sparsity in frequency domain) by considering only few random measurements. CS relies on the empirical observation and in practice; many types of signals or images can be well-approximated by means of a sparse spreading out in terms of a suitable basis, i.e., considering only a small number of non-zero coefficients. The non-stored coefficients are simply set to zero while estimating the wideband sparse signal. This is certainly a reasonable strategy when full information of the signal is available. To date, random matrices whose rows are taken from orthonormal matrices (e.g. DCT matrices, WHT matrices, etc.) behaves well as to form the measurement matrix. The WHT matrix is a square array of elements that comprises only $+1$ or $-1$ whose rows and columns are orthogonal to one another. It is computationally simpler substitute for other transform coding (Fourier, DCT), since it requires no multiplication/division operations [9], hence time and energy efficient and would require less computational burden for CR transceivers. In the study anticipated in CS, while estimating the signal via $l_1$-norm
regularization, a few types of transform coding (DCT, WHT) are used to customize the measurement matrix. Traditionally WHT coding is used for image compression techniques [9].

To the best of our knowledge, it is the first time we propose the WHT based CS scheme for wideband CR spectrum estimation and analysis. In the simulation results, it is revealed that WHT coding can likely be exploited for the sparse wideband radio signal compression. Therefore, the WHT coded measurement matrix in wideband radio signal is formed at the aim of CS recovery scheme followed by spectrum detection probability and later, it is compared the outcomes with its DCT coded counterparts. First, the performance of normalized mean square error (MSE) is computed by means of the wideband spectral estimation via WHT and DCT coded CS scheme. Second, we give emphasis to address the probability of detection, estimated by exploiting normalized MSE of change in power spectral density (PSD) of the original and recovered signal and average execution time to detect the presence of PUs signal by employing those two transform coding. It is shown that though PU detection performances [10-11] and normalized mean square error (NMSE) are comparable, however the spectrum sensing by WHT coded CS scheme proceeds less execution time to discover data transmission opportunity for a CR terminal. Eventually, less sensing time gives higher achievable rate and those aspects are discussed in the rest of the paper.

The remainder of the paper is structured as follows. In section II, CS basics are discussed then in section III, compressive spectrum sensing through AIC is pointed out. The performance analysis of the wideband spectrum sensing is presented by way of simulations in section IV and as a final point, some conclusions are drawn in section V.

II. COMPRESSIVE SAMPLING: PREFACE

We shall first follow [5], [7-8] to recall the basic CS principles. In CS framework a real-valued, finite-length, one-dimensional time varying signal \( x(t) \), \( 0 \leq t \leq T \), can be represented as a finite weighted sum of basis functions (e.g., DCT, Fourier, Walsh-Hadamard) as follows

\[
x(t) = \sum_{i=1}^{N} s_i \psi_i(t) \quad \text{or} \quad x = \psi s \quad (1)
\]

And also only a few basis coefficients \( s_i \) are much larger than zero due to the sparsity of \( x(t) \), where \( s \in \mathbb{R}^{N \times 1} \) is a column vector of weighting coefficients \( s_i = (x, \psi_i) = \psi_i^T x \) and \((·)^T\) denotes the (Hermitian) transpose operation. For simplicity, let us consider the basis function is orthonormal. Using the \( N \times N \) sparsity basis matrix \( \Psi = [\psi_1, \psi_2, \ldots, \psi_N] \) by stacking the vectors \( \{\psi_i\} \) as columns, any signal \( x \) can be expressed as \( x = \Psi s \).

Now we concentrate on the signals which have sparse representations, where \( x \) is a linear combination of just by \( K \)—sparse representations, such that \( K \ll N \) i.e. only \( K \) elements of the \( s_i \) in (1) are non-zero and the rest \( (N - K) \) of the values be equal to zero; this is the basis of the transform coding [8]. Sparsity is influenced by the fact that many natural and man-made signals are compressible in the sense that there exist a basis function, \( \Psi \) where the equation (1) has just a few large coefficients and many small coefficients [6-8].

It has been established that the original signal \( x(t) \) can be recovered using \( M = K O(\log N) \) non-adaptive linear projection measurements on to a basis matrix \( \Phi \) which is incoherent with \( \Psi \) [5], [8]. The formation of \( \Phi \) is given by choosing elements that are drawn independently from a random distribution (e.g. Bernoulli, Gaussian, etc.). Hence, the measuring expression, \( y \) (which is identical to \( x \)) can be written as

\[
y = \Phi x = \Phi \psi s = \Theta s \quad (2)
\]

where \( \Theta = \Phi \psi \) is a \( M \times N \) matrix. It is obvious that the dimension of \( y \) in (2) is much lower that of \( x \), so there are theoretically infinite solutions to the equation which means that the problems is uncertain and it is difficult to reconstruct the original signal. Nevertheless, satisfying the condition that the signal \( x \) of having adequate sparsity, \( K \) and setting proper condition of measurement matrix, \( \Phi \), it is possible to estimate the signal in both time and frequency domain from the measurement, \( y \) and this is achieved by solving the following minimum \( l_1 \)-norm optimization problem [5-7]

\[
\hat{s} = \arg \min \|s\|_1 \quad \text{such that} \quad \Theta s = y \quad (3)
\]

This is a convex optimization problem that conveniently reduces to a linear program known as iterative greedy algorithms [12], basis pursuit (BP) [13], etc. Customary linear programming techniques can be employed to solve the BP problem whose computational complexities are polynomial in \( N \) [14]. CS scheme [6] offers the guarantee of recovering the original signal (accurately or near accurately) with a much lower sampling ratio than Nyquist rate sampling which outfits with the ADC’s available in the wireless systems.

III. COMPRESSIVE SAMPLING VIA AIC

![Fig. 1. AIC for the CS acquisition scheme](image)

Fig. 1 illustrates the acquisition under the proposed method. The baseband (at CR receiver) signal \( x(t) \) is sampled using an AIC [6], [11]. An AIC is conceptually similar to an ADC operating at Nyquist rate subsequent to compressive sampling. Let the output of the ADC is the sampled signal of \( x(t) \), denoted by

\[
x_k = [x_{kn} \ x_{kn+1} \ x_{kn+2} \ldots \ x_{kn+N-1}]^T, \quad k = 0, 1, \ldots, K \quad (4)
\]

is a \( N \times 1 \) vectors and the size of the measurement matrix \( \Phi \) is \( M \times N \), such that

\[
y_k = \Phi x_k \quad (5)
\]

So the output of the AIC denoted by the size of \( M \times 1 \) vectors

\[
y_k = [y_{kn} \ y_{kn+1} \ y_{kn+2} \ldots \ y_{kn+N-1}]^T, \quad k = 0, 1, \ldots, K \quad (6)
\]

Recovery of the compressive sampling can be done by solving \( l_1 \)-norm optimization problem in (3).
IV. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

In this section, it is compared the detection performance of a single CR node based on the wideband spectral estimation by the use of WHT and DCT transform coded CS approach. We consider, at baseband, the wideband signal, \(x(t)\) falls in the range of \([1,64]\Delta\) Hz that can accommodate a maximum of 32 non-overlapping PU sub-bands whose bandwidths \(B\) is set to \(2\Delta\) Hz each and encoded as \(\{c_{\Delta n}\}_{n=1}^{32}\), where \(\Delta\) is the frequency resolution. Therefore, the bandwidth of the wideband signal is \(64\Delta\) Hz that may accommodate a maximum number of \(\frac{W}{B} = 32\) active PU radios with no sparsity. While observed the burst of transmissions in the network, there are a total of 03 non-overlapping PUs with different carrier frequencies \(\{f_n\}_{n=1}^{3}\) inside the wideband \(W\) such that the three PUs communicating with the center frequency of \([20.7, 45.3, 59.5]\Delta\) Hz while the individual PU bandwidth is \(2\Delta\) Hz each. Let, every active channel possibly be occupied with a PU terminal using digital modulation scheme either 16-PSK or 16-QAM. The number of Nyquist rate samples \(N\) are taken into account from the wideband signal \(x(t)\) for an observation time \(T\) (e.g., the chosen frequency resolution \(\Delta = 1\) MHZ and \(N=8192\) samples to satisfy \(T = 256\mu s\)).

We consider, the received RF signal at the secondary terminal is corrupted by the additive white Gaussian noise (AWGN) of unit variance. The received SNR of the active channels is considered 20dB. In order to make the problem simple, it is anticipated that the primary radios deploy uniform power transmission strategy and the upper layer (e.g., medium access control (MAC) layer) arrange for the guaranty that the CRs keep quiet during every detection period while the PUs are in data transmission state. The compression ratio, \(M/N\) is varying from 2.5\% to 60\% meant for the wideband spectral estimation. Based on the estimated spectrum we then determine the normalized MSE w.r.t. PSD, the detection performance and average execution time for the sensing job. The statistical average of NMSE and execution time have set after 1000 experimental realizations were taken into consideration.

a. NMSE performance: we compute the normalized MSE of the PSD obtained by Welch periodogram which is defined by:

\[
MSE = E \left( \frac{||S_x - \hat{S}_x||^2}{||S_x||^2} \right)
\]

where \(S_x\) denotes the average of the PSD estimates based on Welch power periodogram, where the original signal sampled at Nyquist rate and \(\hat{S}_x\) is the average PSD estimate from periodogram of same type of the reconstructed signal through compressed sampling. It is obvious from the Fig. 2 that the higher the compression ratio \(M/N\), the better the signal reconstruction quality.

b. Detection performance versus Compression ratio: We evaluate detection probability, \(P_d\) based on the averaged PSD estimate \(\hat{S}_x\) from the WHT coded CS approach. Here, detection performance (in Fig. 4) has been tested to a band of interest which is opportunistically accessible for a CR usage. Moreover, detection performance of the same RF band of interest is compared with spectral estimation of the wideband signal \(x(t)\) preceded over a single RF chain. In that case, number of samples \(N\) remain the same to fix the sampling time \(T = 256\mu s\). The decision of the presence of a licensed transmission signal in a certain channel is made by an energy detector used in paper [11].
where \( X_q(K) \) is the Fourier transform of the \( q \)-th block of the received time-domain signal \( x_q(n) \), \( n \) denoting the sampling index, each block contains 8 PSD samples and the total number of blocks, \( Q = 1028 \). The probability of detection, \( P_d \) is calculated as:

\[
P_d = \Pr(N > \gamma | \mathcal{H}_1)
\]

where \( \gamma \) is the decision threshold and is centralized Chi-square distributed function which is found by fixing the probability of false alarm, \( P_f = 0.05 \) and \( \mathcal{H}_1 \) represents the presence of PUs. Fig. 4 describes the \( P_d \) with different values of compression ratios, \( M/N \). From simulation results (Fig. 4), it is seen that the detection probability of two diverse types of transform coding is comparable with respect to the considered digital modulation schemes.

![Detection probability versus compression rate, M/N](image)

**c. Achievable throughput of a single CR node**

To compute the achievable throughput for CR network we consider a simple problem which is collision free (as PU is absent and so no false alarm is caused by the CR) achievable throughput for CR network. Let us consider, \( \tau \) is the time slot reserved for sensing operation and \( (T-\tau) \) is the data transmission slot duration as shown in Fig. 5 [15-16]. Let us denote \( C_0 \) as the achievable capacity of a CR network considering PU data transmission off and \( C_0 \) can be inscribed as \( C_0 = \log_2(1 + SNR_\alpha) \), where \( SNR_\alpha \) denote the signal-to-noise ratio of the CR link. Inside an interoperable network, we also consider PU data transmission, CR data transmission and reception are Gaussian, white in nature and independent to each other. For a particular band of interest, \( P(\mathcal{H}_0) \) signifies the probability for which the PU data transmission is absent. Therefore, we recall the optimal achievable rate \( \mathcal{R}(\tau) \) from [15-16] is

\[
\mathcal{R}(\tau) = C_0 P(\mathcal{H}_0) \left( 1 - \frac{2}{\alpha} \right) \left( 1 - Q(\alpha + \sqrt{N} \gamma) \right)
\]

where \( \alpha = \sqrt{2\gamma + 1} Q^{-1}(P_f) \). From equation (14), it has been noticed that the achievable rate of a CR node varies with the sensing slot duration as well as frame duration e.g., the throughput is greater for shorter sensing time period \( \tau \) with a fixed frame length, \( T \). Hence, we try to sort out a trade-off between the sensing length and frame length. As the miss detection probability, \( P_m \) can obligate with the possibility of data collision (a collapse of achievable throughput) with the PU transmission while the probability of false alarm, \( P_f \) recommends the CR to stop packet transmission during the frame interval though PU channel is idle at that instant which also decrease the throughput performance. We assume MAC layer of CR network guarantees that only one CR can have the accessibility of a PU sub-channel at a particular time to avoid the collisions among the CR nodes inside the network [16]. Therefore, collisions can only be possible between the CR and the PU.

Later, to testify the effectiveness of the proposed approach with a CR system which estimates the entire spectrum, the throughput performance is investigated. To make easily understandable, we choose low regime SNR value of the PU system, e.g., \( SNR = -10 dB \) and probability of detection \( P_d = 0.90 \) and the probability of PU transmission is absent, \( P(\mathcal{H}_0) = 0.90 \) when a CR node wants to transmit. Intuitively, the sensing time, \( \tau \) is engaged for the full spectrum estimation with a single RF chain followed by CS method which is considered during simulation environment. Meanwhile, this sensing time, \( \tau \) is applied in eq. (10) to find the optimum throughput for a fixed frame length of 50ms and different SNR values as illustrated in Fig. 6.

![Influence of the Throughput on \( \tau \): T=50 msec](image)

**V. CONCLUSIONS**

In this paper, two types of transform coding is employed to form measurement matrices in CS scheme are analyzed and discussed when a single CR radio terminal wants to discover the spectrum white spaces for opportunistic practice of a wideband radio signal. It is exposed in the simulation results that the CS scheme by means of WHT measurement matrix involves modest mathematical computation which provides faster sensing
decision as a consequence of enhanced achievable throughput while the detection performance is comparable to its DCT coded counterparts. Moreover, WHT coding in measurement matrix gives flexibility to the current wireless radio devices due to their limited ADC capabilities. In future endeavor, the cooperative CR nodes will be employed under the same analytical platform to obtain a reliable detection performance which will stunned with the issues related to the hidden terminal problems, multipath fading, etc.

REFERENCES


