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Ahargaṇa in *Makarandasāriṇī* and Other Indian Astronomical Texts

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Abstract

Ahargaṇa is a basic parameter used for calculating mean positions of planets and other elements. The number of civil days elapsed since a chosen fixed epoch is called '*ahargaṇa*', literally meaning 'heap of days'.The intercalary months (*adhikamāsa*) play an important role in calculating *ahargaṇa.* The present paper deals with different procedures for finding *ahargaṇas* according to different Indian astronomical texts in detail with concrete examples. It could be seen how easier it is to convert a given traditional lunar calendar date into Julian/Gregorian date by using the *vallī* components of *Makarandasāriṇī,* and also a given Julian or Gregorian date into *ahargaṇas* by using various tables.

Key words: *Ahargaṇa*, *Ahargaṇavallī*, *Ārdharātrika, Audāyika*, Bhāskara II, Ganeśa Daivajña, *Grahalāghava*, *Karaṇakutūhala,* Makaranda*, Makarandasāriṇī, Saurapakṣa*

1. Introduction

For the purpose of finding the mean positions of planets for any given day, first the total number of civil days elapsed since the beginning of a chosen epoch is calculated. Then it is multiplied by the mean daily motion of a planet which gives the mean angular distance covered by the planet during that period. From this motion, after removing the completed number of revolutions (multiples of 360°), the remainder is added to the mean position of the planet at the epoch to find the mean position of the specified day.

Literally the word '*ahargaṇa'* means 'heap of days'. According to *Siddhānta*s, it is the number of mean civil days elapsed at midnight or mean sunrise for the Ujjain meridian. This meridian passes through a point on the equator with the same longitude as Ujjain, called Laṇkā. The traditional Hindu calendar follows both Luni-solar and Solar systems. The former is pegged on to the later through intercalary months (*adhikamāsa*).

2. The General Procedure for Finding *Ahargaṇa*

The process of finding *ahargaṇa* essentially consists of the following steps:

i) Convert the solar year elapsed (since the epoch) into months by multiplying by 12.

ii) Add the number of *adhikamāsa*s during that period to give the actual number of lunar months that have elapsed up to the beginning of the current year.

iii) Add the number of lunar months in the given year.

iv) Convert these actually elapsed number of lunar months into *tithis* (by multiplying it by 30).

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v) Add the elapsed number of *tithis* in the current lunar month.

vi) Subtract the *kṣaya dinas* and finally convert the elapsed number of *tithis* into civil days.

Note: While finding *adhikamāsa*s, if an *adhikamāsa* is due after the lunar month of the current year, then 1 is to be subtracted from the calculated number of *adhikamāsa*s. This is because an *adhikamāsa* which is yet to come in the course of current year will have already been added.

3. *Audāyika* and *Ārdharātrika* Systems

In Indian astronomical texts, the *Kaliyuga* is said to have started either at the mean sunrise on February 18, 3102 BCE or at the midnight between 17th and 18th of February 3012 BCE. Accordingly the corresponding systems are called respectively *Audāyika* (sunrise system) and *Ārdharātrika* (midnight system).

Interestingly even the important astronomical parameters are somewhat different in the two systems. In fact the earliest available systematic text, *Āryabhaṭīya* of Āryabhaṭa I (b. 476 CE) belongs to the *Audāyika* system. It is believed that Āryabhaṭa wrote another text – popularly described as *Āryabhaṭa Siddhānta* which belongs to the *Ārdharātrika* system. The earliest text of *Ārdharātrika* system is *Khaṇḍakhādyaka* (*KK*) of Brahmagupta, which is available and very popular (b. 598 CE).

4. To find *Ahargaṇa* since the *Kali* Epoch

Before evolving a working procedure for finding the *Kali Ahargaṇa,* we shall list some useful data for the purpose according to *Sūryasiddhānta* (*SS*). In a *Mahāyuga* of 432×10^4 years, we have

i) Number of sidereal revolutions of the Moon: 577,53,336

ii) Number of revolutions of the Sun: 43,20,000

iii) Number of lunar months in a *Mahāyuga* of 432×10^4 years given by (i) - (ii) : 534,33,336

(iv) Number of days in a *Mahāyuga* : 1577917828

Number of *adhikamāsa*s in a *Mahāyuga*

= Number of lunar months – (12× Number of solar years)

= 534,33,336 - (12 × 43,20,000)

= 534,33,336 - 518,40,000

= 15,93,336.

Number of *kṣayatithis* in a *Mahāyuga*

= Number of *tithis* – Number of civil days

= 30×Number of lunar months – Number of civil days

= 30 × 53433336 – 1577917828

= 25082252.

Suppose we wish to find *ahargaṇa* for the day on which *x* luni-solar years, *y* lunar months and *z* lunar *tithis* have been elapsed. Then the number of *adhikamāsa*s in *x* completed solar years is given by $x_1 = \text{INT}\left[(x)\times\left(\frac{15,93,336}{43,20,000}\right)\right]$

where INT (i.e. integer value) means only the quotient of the expression in the square brackets is considered.

Now, since in the given luni-solar year, *y* lunar months and *z tithis* have elapsed, we have

Number of *tithis* elapsed since the epoch = $(12x+x_1+y)\times 30 + z$.

The number of *kṣayatithis* corresponding to this is found by the rule of three: If there are 25082252 *kṣayatithis* in a *Mahāyuga* corresponding to 534,33,336×30 *tithis*, then the number of *kṣayatithis* corresponding to $[(12x+x_1+y)\ 30 + z]$ *tithis* is given by

$$k = \text{Int}\left[\left(\frac{(12x+x_1+y)\times 30+z}{53433336\times 30}\right)\times 25082252\right]$$

Therefore the number of civil days (*ahargaṇa) N′* elapsed since epoch is given by

$$N' = (12x+x_1+y) \times 30 + z - k$$

$$= \mathrm{INT}\left[\left\{(12x+x_1+y) + \frac{z}{30}\right\}\times 30\left(1-\frac{2508252}{53433336\times 30}\right)\right]$$

$$= \mathrm{INT}\left[\left\{(12x+x_1+y) + \frac{z}{30}\right\}\times 29.530589\right]$$

Here, $30\left(1-\frac{2508252}{53433336\times 30}\right) = 29.530589$ is the average duration of a lunar month in days.

Since in our calculations we have considered only mean duration of a lunar month, the result may have a maximum error of 1 day. Therefore to get the actual *ahargaṇa N,* addition or subtraction of 1 from N' may be necessary.

This is decided by the verification of the weekday. The tentative *ahargaṇa* N' is divided by 7 and the remainder is expected to give the weekday counted from the weekday of the chosen epoch. For example, the epoch of *Kaliyuga* is known to have been a Friday. Therefore when N' is divided by 7, if the remainder is 0, then the weekday must be a Friday, if 1 then Saturday etc. However if the calculated weekday is a day earlier or later than the actual weekday, then 1 is either added to or subtracted from N' so as to get the calculated and actual weekday the same. Accordingly the actual *ahargaṇa* $N = N' \pm 1$.

It is important to note that the method described above is a simplified version of the actual procedure described variously by the *siddhāntic* texts.

Note: While finding the number of *adhikamāsas* x_1 in the above method if an *adhikamāsa* is due after the given lunar month in the given lunar year, then subtract 1 from x_1 to get the correct number of *adhikamāsa*s. Also, if the fractional part of $x\times\left(\frac{15,93,336}{43,20,000}\right)$ is close to 1, then we would have to add 1 to x, if an *adhikamāsa* has already occurred very close to the given date.

Example: Finding *Kali ahargaṇa* corresponding to *Caitra kṛṣṇa trayodaśī* of *śaka* year 1913 (elapsed) i.e., for 12 April 1991.

Number of *Kali* years = 3179 + 1913 = 5092, since the beginning of the *śaka* era i.e.,

78 CE, corresponds to 3179 years (elapsed) of *Kali yuga.*

∴ *Adhikamāsa*s in 5092 years
= (15,93,336/43,20,000) × 5092 = 1878.0710

Taking the integral part of the above value, x_1 = 1878.

Now, an *adhikamāsa* is due just after the *Caitra māsa* under consideration. Although the *adhikamāsa* is yet to occur, it has been included already in the above value of x_1. Therefore the corrected value of x_1 is 1878-1 = 1877.

Since the month under consideration is *Caitra,* the number of lunar months elapsed in the lunar year, $y = 0$. The current *tithi* is *trayodaśi* of *kṛṣṇa pakṣa* so that the elapsed number of *tithis* is 15 + 12 = 27 i.e., $z = 27$.

∴ Number of lunar months completed
= (5902×12) + 1877 +0 + 27/30 = 62,981.9

The number of civil days

$$N^1 = \mathrm{INT}[62{,}981.9\times 29.530589] = \mathrm{INT}[18{,}59{,}892.603] = 18{,}59{,}892.$$

Now, dividing N^1 by 7, the remainder is 6; counting 0 as Friday, 1 as Saturday etc., the remainder 6 corresponds to Thursday. But, from the calendar, 12 April 1991 was a Friday. Therefore, we have the actual *ahargaṇa* $N = N^1 + 1$ = 18,59,893 since the *Kali* epoch.

5. *Ahargaṇa* according to *Makarandasāriṇī*

Makarandasāriṇī (*MKS*) is one of the most popular texts among the Indian astronomical tables. These tables with explanatory *ślokas* are composed by Makaranda, son of Ānanda at Kāśī in 1478 CE. This *sāriṇī* belongs to *saurapakṣa.*

The author of *MKS* has incorporated many changes to yield better results during his time. He has given the *ahargaṇavallī* table for computing the *ahargaṇa* for the given day *in rāśī, amśa, kalā* and *vikalās* in which the *adhikamāsa* concept of a lunar calendar is incorporated, so that finding the *ahargaṇavallī* from *MKS* tables is easier when compared to the procedure for obtaining the *ahargaṇa* from other related astronomical texts belonging to *saura pakṣa.*

Ahargaṇa vallī expressed in *rāśī, amśa*, *kalā* and *vikalās* is equivalent to the *ahargaṇa* days expressed as a sum of power of 60. The *MKS Ahargaṇa* is counted from the beginning of the *Kaliyuga, Vaiśākha śuddha pratipath* Friday and is correct to the midnight of the central meridian.

Remark: At the beginning of the *Kaliyuga,* i.e., at the midnight between 17th and 18th February 3102 BCE, all the mean heavenly bodies were at 0^o (*Meṣa*). This means that was the instant of the mean *Meṣa sankrānti* and also the mean beginning of the lunar month.

Now, at that moment, *Mandakendra* of the Sun, MK = $78^o - 0^o = 78^o$

$\therefore$ *Mandaphala,* the equation of the centre
$= (14^o/2\pi) \sin 78^o = 2^o.17947836$

by taking the *manda* periphery = 14^o,

Converting this into days we get = $2^o.17947836/ 59'8'' = 2.21142111$ days.

Since the equation of the centre is positive, true *Meṣa sankrānti* occurs 2 days earlier. That is the beginning of *kaliyuga,* being the end of *amāvāsya* occurs 2 days after the true *Meṣa sankrānti*. This means that it is the beginning of *Vaiśākha* month. In other words, the beginning of *Caitra* will have occurred around January 19, 3102 BCE (30 days before).

Tables 5.1, 5.2, 5.3 give *ahargaṇa vallī* for a given date which is equivalent to the *ahargaṇa* days according to *Sūryasiddhānta* and other *saurapakṣa* texts.

In the table 5.1 *ahargaṇavallī* is given for the tabulated *śaka* years with an interval of 57 years, starting from *śaka* 1628 up to 2654 [i.e. 1706 CE to 2732 CE]. Incidentally while the text says that the commencing year is *śaka* 1400 (1478 CE) the table for *ahargaṇavallī* in the published version of *MKS* starts from 1628(1706 CE). This may be because the published work is based on *Viśvanātha*'s manuscript.

In the beginning, the first column gives *vallī* for 57 years (called *śeṣāñka kṣepaka*) in *rāśī, amśa, kalā* and *vikalās*. Also the last row gives *vāra* (weekday*)*. The table can be generated by adding *vallī* of *kṣepaka* year 57 i.e. 0|5|46|59 and *vāra* 1 to the previous entries correspondingly. This is shown in example 5.1 below.

Now, The length of a sidereal solar year is 365.2587565 days in *SS*. Hence, 57 sidereal solar years = 365. 2587565×57 = 20819.7491205 days. Actually a luni-solar system is used in *MKS* and

Table 5.1. *Ahargaṇa vallī* for a given date

śeṣāñka kṣepaka	57	1628	1685	1742	1799	1856	1913	1970	2027	2084	2141	2198	2255	2312	2369	2426	2483	2540	2597	2654
rāśī,	0	8	8	8	8	8	8	8	8	8	8	9	9	9	9	9	9	9	9	9
amśa,	5	7	13	19	25	30	36	42	48	53	59	5	11	17	22	28	34			
kalā	46	42	29	16	3	50	37	24	11	58	45	31	17	4	51	38	25			
vikalā	59	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35			
vāra	1	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	1	2	3	4

57 such years is very close to 20819 days. When multiples of 7 are removed, we obtain the *śeṣa vāra* 1 (remaining *vāra* after dividing 20819 by 7) including leap years.

Dividing 20819 by 60 we obtain Q_1=346 & R_1=59.

Now, dividing Q_1 by 60 we get Q_2=5 & R_2=46.

Dividing Q_2 by 60 we get Q_3=0 & R_3=5.

Dividing Q_3 by 60 we get Q_4=0 & R_4=0.

Thus *vallī* corresponding to 57 years (*Kṣepaka)* = $R_4|R_3|R_2\ |R_1$ = 0|5|46|59 and *vāra*=1

Example 5.1

Śaka year	*rāśī*	*amśa*	*kalā*	*vikalā*	*vāra*
1628	8	7	42	50	0
adding	0	5	46	59	1
1685	8	13	29	49	1
adding	0	5	46	59	1
1742	8	19	16	48	2

Table 5.2 gives *ahargaṇa vallī* for the balance years for 1 to 57 in *rāśī, amśa* , *kalā* and *vikalās* and also *vāra*.

Now, the number of days in a mean lunar month = 29. 53058795.

the number of days in a mean lunar year = 354.3670554

and the number of days in year having an *adhikamāsa* = 383.8976434 = 384 (approx.)

(since a lunar year having an *adhikamāsa* (intercalary months) will have 13 lunar months).

Number of *adhikamāsas* in 57 years $= 57 \times \left(\frac{15,93,336}{43,20,000}\right) = 21.0231$, rounded off to 21.

Then, in 57 years, there would be 36 luni-solar years with 12 lunar months and 21 luni-solar years with 13 lunar months. Hence duration of 57 years = 36×354.3670554 + 21×383.8976434 = 20819.06451, taken as 20819. However it is not suitable to have a fractional number of days in a year. This problem can be solved in the following manner: All the 21 years with 13 lunar months would have 384 days. Out of 36 years with 12 lunar months, 25 would have 354 days and 11 would have 355 days. Then the duration of a 57-year cycle is 25×354+11×355+21×384 = 20819 days.The *vallī* and *vāra* for an year with 354 days are 0|0|5|54 and 4. For an year with 355 days they are 0|0|5|55 and 5, and for an year with 384 days, they are *vallī* = 0|0|6|24 *vāra* = 6 respectively. We observe that these have been included in *ahargaṇa vallī* tables. In table 5.2 for the year 1, the number of days is taken as 384, since it had an *adhikamāsa* and the corresponding *vallī* components are given as 0|0|6|24 and *vāra,* 6. For the next year the number of accumulated days will be 384+354 = 738 and the *valli* components corresponding to 738 days is 0|0|12|18 and *vāra,* 4. Similarly for year 3 the number of accumulated days is taken as 384+354+355 = 1093. The *vallī* components are 0|0|18|3 and *vāra,* 1 and so on as shown in the example 5.2 below.

Example 5.2

Year	*rāśī*	*amśa*	*kalā*	*vikalā*	*vāra*
1	0	0	6	24	6
adding	0	0	5	54	4
2	0	0	12	18	3
adding	0	0	5	55	5
3	0	0	18	13	1
adding	0	0	6	24	6
4	0	0	24	37	0
adding	0	0	5	54	4
5	0	0	30	31	4
adding	0	0	6	24	6
6	0	0	36	55	3

Remark: In the formation of table 5.2, there appears to be a couple of incorrect values in the printed version.

Table 5.3 gives *pākṣikacālanam* (fortnightly values) of *ahargaṇa vallī* which is always additive.

Table 5.2

koṣṭaka (years)	*Ahargaṇa vallī* *rāśī*	*amśa*	*kalā*	*vikalās*	*vāra*
1	0	0	6	24	6
2	0	0	12	18	3
3	0	0	18	13	1
4	0	0	24	37	0
5	0	0	30	31	4
6	0	0	36	55	3
7	0	0	42	49	0
8	0	0	48	44	5
9	0	0	55	8	4
10	0	1	1	2	1
11	0	1	6	56	5
12	0	1	13	20	4
13	0	1	19	15	2
14	0	1	25	9	6
15	0	1	31	33	5
16	0	1	37	27	2
17	0	1	43	51	1
18	0	1	49	45	5
19	0	1	55	40	3
20	0	2	2	04	2
21	0	2	7	58	6
22	0	2	13	52	3
23	0	2	20	16	2
24	0	2	26	11	0
25	0	2	32	5	4
26	0	2	38	29	3
27	0	2	44	23	0
28	0	2	50	47	6
29	0	2	56	42	4
30	0	3	2	36	1
31	0	3	9	0	0
32	0	3	14	54	4
33	0	3	20	49	2
34	0	3	27	13	1
35	0	3	33	7	5
36	0	3	39	31	4
37	0	3	45	25	1
38	0	3	51	19	5
39	0	3	57	43	4
40	0	4	3	38	2
41	0	4	9	32	6
42	0	4	15	56	5
43	0	4	21	50	2
44	0	4	27	45	0
45	0	4	34	9	6
46	0	4	40	3	3
47	0	4	46	27	2
48	0	4	52	22	0
49	0	4	58	16	4
50	0	5	4	40	3
51	0	5	10	34	0
52	0	5	16	28	4
53	0	5	22	52	3
54	0	5	28	46	0
55	0	5	35	10	6
56	0	5	41	5	4
57	0	5	46	59	1

Table 5.3

Lunar Months	*Pakṣas*	*Ahargaṇa vallī* *rāśī*	*amśa*	*kalā*	*vikalās*	*vāra*
Caitra	*śukla*	0	0	0	15	1
	kṛṣṇa	0	0	0	30	2
Vaiśākha	*śukla*	0	0	0	44	2
	kṛṣṇa	0	0	0	59	3
Jyeṣṭha	*śukla*	0	0	1	14	4
	kṛṣṇa	0	0	1	29	5
Āṣāḍha	*śukla*	0	0	1	43	5
	kṛṣṇa	0	0	2	58	6
Śrāvaṇa	*śukla*	0	0	2	13	0
	kṛṣṇa	0	0	2	28	1
Bhādrapada	*śukla*	0	0	2	42	1
	kṛṣṇa	0	0	2	57	2
Āśvayuja	*śukla*	0	0	3	12	3
	kṛṣṇa	0	0	3	27	4
Kārtika	*śukla*	0	0	3	41	4
	kṛṣṇa	0	0	3	56	5
Mārgaśīrṣa	*śukla*	0	0	4	11	6
	kṛṣṇa	0	0	4	26	0
Puṣya	*śukla*	0	0	4	40	0
	kṛṣṇa	0	0	4	55	1
Māgha	*śukla*	0	0	5	10	2
	kṛṣṇa	0	0	5	25	3
Phālguṇa	*śukla*	0	0	5	40	4
	kṛṣṇa	0	0	5	54	4

In *pākṣikacālanam* of *ahargaṇa vallī* given in table 5.3, the last but one entry i.e., the fourth component of *pakṣa vallī* gives the number of civil days at the end of the *pakṣa* after removing the multiples of 60. Most of the *pakṣas* have 15 days, whereas some have 14. This would ensure that the average duration of a *tithi* is less than a civil day.

Example 5.3: At the end of 7 *pakṣa*s, the number

of civil days $= \frac{\textit{Duration of a Lunar month}}{2} \times 7$

$= \frac{29.53058795}{2} \times 7$

$= 103.3570578$

$\rightarrow 43.357057$

$\rightarrow 43$

(by removing multiples of 60 and taking the integer value)

Remark

(i) According to table 5.3, the duration of a normal luni-solar year is 0|0|5|54 = 354.

(ii) In table 5.2 an extra day is added to the luni - solar year to account for the accumulation of the fractional part of the days in a lunar year. In table 5.2 , while 354 days are considered for a normal lunar year additional 30 days are taken for lunar year with *adhikamāsa* once in 32 or 33 lunar months. This is not explicitly mentioned in the text.

6. Procedure for finding *Ahargaṇa Vallī* from *MKS* tables

The working procedure for finding *Ahargaṇa vallī* using *MKS* tables is as given bellow:

i) Subtract the nearest *śaka* year given in table 5.1 from *iṣa śaka* (given *śaka* year, for which *ahargaṇa vallī* is to be found) and obtain the difference called *śeṣa* (remainder).

ii) Find the *vallī* values corresponding to the nearest *śaka* year given in the table and also for the *śeṣa varṣa* (remainder) using tables 5.1 and 5.2 respectively. Also find the *vāra* corresponding to these given in the last columns of tables 5.1 and 5.2.

iii) Add the *vallī* and *vāra* for the *śaka* year and the remainder correspondingly. Remove the multiples of 7 from *vāra* (when it exceeds 7).

iv) The above sum gives *grahadina vallī* or *ahargaṇa vallī* for the *iṣṭa śaka* year (given *śaka* year).

v) Now, using table 5.3, obtain the *pakṣa vallī* and *vāra* for the given *pakṣa* of the running lunar month of the given *śaka* year.

vi) Add the *pakṣa vallī* and *vāra* obtained in the above step (*v*) to the *grahadina vallī* or *ahargaṇa vallī* of *iṣṭa śaka* year obtained in step (*iv*).

vii) Add the elapsed number of *tithis* of the running *pakṣa* of the lunar month to the sum obtained above in step (*vi*) in the fourth component of the *vallī*. This gives the *ahargaṇa vallī* or *ahargaṇa dina vallī* for the given day of the *śaka* year.

viii) Check for *vāra* and add or subtract 1.

Example 6.1: Given date: *Śā.Śaka* 1534 *Vaiśākha Śukla* 15 corresponding to 1612 May 15.

The nearest *śaka* year from table 5.1 is 1514.

Given *śaka* year - Nearest *śaka* year from table = 1534 – 1514 = 20 (*śeṣa)*

Now, *vallī* corresponding to 1514 is → 7|56|08|52 and *vāra* 5

vallī corresponding to *śeṣa varṣa*, 20 is → 0|02|02|04 and *vāra* 2

pakṣa vallī for *Vaiśākha Śukla* 15 is → 0|00|00|44 and *vāra* 2

Adding → 7|58|11|40 and *vāra* 2

Thus *ahargaṇa vallī* for the given day is 7|58|11|40 and *vāra* 2 (removing multiples of 7).

Note: (i) *vāra* is counted from Sunday as 0.

(ii) While adding the *vallī* components, multiples of 60 are removed.

Example 6.2*:* Given date: *Śā.Śaka* 1939 *Śrāvaṇa Kṛṣṇa Saptami* (7) corresponding to 2017 August 14, Monday

Nearest *śaka* year from table 5.1 is 1913

Given *śaka* year - Nearest *śaka* year from table = 1939– 1913 = 26 (*śeṣa).*

Now, *vallī* corresponding to 1913 is → 8|36|37|45 and *vāra* 5

vallī corresponding to *śeṣa varṣa*, 26 is → 0|02|38|29 and *vāra* 3

pakṣa vallī for *Śrāvaṇa Śukla* is → 0|00|02|13 and *vāra* 0

number of *tithis* in the given *pakṣa* is → 0|00|00|07 and *vāra* 0

Adding → 8|39|18|34 and *vāra* 1

Thus *ahargaṇa vallī* for the given day is 8|39|18|34 and *vāra* 1 (removing multiples of 7).

Remark: (i) If *Kali ahargaṇa* for a given date is known, then the *ahargaṇa vallī* can be obtained by taking the remainders after dividing *Kali ahargaṇa* days by 60 as shown in the following example.

Example 6.3: *Śaka* 1849*, Mārgaśira Śukla* 15 Thursday corresponding to 1927 December 8.

The *Kali ahargaṇa* for the given date, A = 1836758.

Now, dividing A by 60, integer part of the quotient Q_1= 30612 and Remainder R_1= 38

dividing Q_1 by 60, integer part of the quotient Q_2= 510 and Remainder R_2= 12

dividing Q_2 by 60, integer part of the quotient Q_3= 8 and Remainder R_3= 30

dividing Q_3 by 60, integer part of the quotient Q_4= 0 and Remainder R_4= 8.

The *ahargaṇa vallī* for the given date is $R_4|R_3|R_2|R_1$= 8|30|12|38.

(ii) The *ahargaṇa vallī* for the given date is of the form $V_1|V_2|V_3|V_4$. From these *vallī* components, the *Kali ahargaṇa* can be obtained by using the formula

$$A = (V_1\times60^3) + (V_2\times60^2) + (V_3\times60) + V_4$$

Example 6.4: Given date: *Śā.Śaka* 1939 *Vaiśākha Śukla* 15 corresponding to 2017 May 10, Wednesday.

Ahargaṇa vallī = 8|39|16|58 i.e. V_1=8, V_2=39, V_3=16, V_4=58

Kali ahargaṇa, $A = (V_1\times60^3) + (V_2\times60^2) + (V_3\times60)+V_4 = 8\times60^3+39\times60^2+16\times60+58 =$ **1869418.**

Using modern tables 9–9, *Ahargaṇa* for 2017 May 10 is 1**869418.**

7. *Ahargaṇa* according *Grahalāghava*

Grahalāghava (GL) of Gaṇeśa Daivajña is the most popular astronomical handbook (*karaṇa)* especially in Maharashtra, North Karnataka and major parts of North India. For the purpose of computations of planetary positions, eclipses, etc. Gaṇeśa Daivajña has adopted a contemporary date as the reference point (epoch) viz., the mean sunrise of Monday, *Caitra śukla pratipat, śaka* 1442 corresponding to March19, 1520. Ganeśa has simplified the method of computations of positions of planets which is otherwise laborious by the traditional methods followed by celebrated astronomers.

To avoid a large number for the *ahargaṇa*, he has adopted a cycle (*cakra*) of 4016 days, approximately 11 solar years. His modified *ahargaṇa* never exceeds 4016 days and hence it is very handy. Further, huge numbers for *ahargaṇa* by the conventional method, for multiplication etc., would result in numerical errors. This is avoided by Gaṇeśa's innovation.

The epoch chosen by Gaṇeśa Daivajña is *Śālivāhana śakavarṣa* (year) 1442 *Caitra śukla pratipat,* corresponding to March 19,1520 (Julian), Monday. The *ahargaṇa* according to *Grahalāghava* for a given lunar date is determined as follows:

i) Subtract 1442 from the *Śālivāhana śaka* year (elapsed) of the given date to get the years elapsed (*gatābdi*)

ii) Divide the remainder by 11. The quotient is called *cakra* (cycle) = C

iii) Multiply the remainder obtained in step (ii) by 12 and to the product add the number of lunar months elapsed, counting *Caitra* as 1 The sum thus obtained is called 'mean lunar months'(*madhyama māsa gaṇa*) denoted by M

iv) The number of *adhikamāsas* is given by the quotient of $(M + 2C + 10)/33$.

Remark: The number of civil days in 11 solar years is nearly 11×365.2586. As the average lunar month has 29.530589 days, the number of lunar months in 11 years is $\frac{11\times 365.2586}{29.530589} = 136.05704$. Now 136 = 11 × 12 + 4, which means that there are 4 *Adhikamāsas*. These 4 *adhikamāsas* are taken into account in $\frac{M}{33}$ as $\frac{11\times 12}{33} = 4$.

Consider the fractional part 0.05704. Now $\frac{2}{33} \approx 0.060 \approx 0.05704$. This accounts for the factor $\frac{2C}{33}$ (a factor $\frac{2}{33}$ for each cycle). Actually $\frac{2C}{35}$ would have been better. For the convenience of clubbing with M, $\frac{2C}{33}$ is used. The discrepancy would not matter, as 1 is added to or subtracted from the computed number of *adhikamāsas* depending on the actual occurance of an *adhikamāsa* close to the desired day.

v) True lunar months (*spaṣṭa māsa gaṇa*)

= Mean lunar months + *adhikamāsas*

= M + quotient of $(M + 2C + 10)/33$

= TM

vi) The mean *Ahargaṇa* (*madhyama ahargaṇa*)

$$MAH = (TM \times 30) + TI + \frac{C}{6}$$

where TI is the number of *tithis* elapsed in the given lunar month.

vii) *Kṣayadinas* = Quotient of $\left[\frac{1}{64} \times Madhyama\ ahargaṇa\right] \equiv KD$

viii) True *ahargaṇa* (*sāvana dinas*) i.e., the number of civil days,

TAH = Mean *Ahargaṇa* - *Kṣayadinas*

= $MAH - KD = MAH$ - Quotient of $\left[\frac{1}{64} \times Madhyama\ ahargaṇa\right]$

ix) However since the average values of parameters are considered in the above computations, 1 day may have to be either added to or subtracted from the results of (viii) to get the actual *ahargaṇa*

This is done by verifying the weekday as follows:

(a) Multiply *cakras, C* by 5 and add *ahargaṇa, TAH* to it i.e., find $(5C + TAH)$

(b) Divide result of (a) by 7 and find the remainder. Let R = remainder of $(5C + TAH)/7$.

If $R = 0$, then the weekday is Monday; R =1, then it is Tuesday and so on.

(c) If the calculated weekday is a day next to the actual weekday, then subtract 1 from *TAH* and if it is one day earlier than the actual weekday, then add 1 to *TAH*.

Note: 1. Some times when (*Śālivāhana śaka* year – 1442) is divided by 11, to get *cakras* the remainder could be 0. In that case even 2 may have to be added or subtracted from the obtained *sāvana dinas* to get the true *ahargaṇa* for the weekday.

2. Sometimes there could be an *adhikamāsa* in a particular given lunar year.

(i) If the given date is before the *adhikamāsa* of that lunar year, then subtract 1 from the number of *adhikamāsa* obtained in the calculations.

(ii) If the given date is after the *adhikamāsa* of that lunar year, then add 1 to the number of *adhikamāsas* obtained in the calculations if the fractional part of the computed *adhikamāsa* is close to 1 .

Remark: If the fractional part is close to 0, there is no need to add 1, as the computed number of *adhikamāsas* would have already included the occurred *adhikamāsa* of the desired lunar year.

These two cases are demonstrated in the examples.

Example 7.1: *Śālivāhana śaka* 1534, *Vaiśākha Pūrṇimā*, Monday corresponding to May 14, 1612.

i) Subtract 1442 from the *Śālivāhana śaka* year 1534: years elapsed (*gatābdi*) = 1534-1442 = 92

ii) Divide the remainder in (i) by 11.

The quotient *cakra* (cycle), $C = 8$ and remainder = 4

iii) Multiply the remainder obtained in step (ii) i.e. 4 by 12 and adding the number of lunar months elapsed in the given year, we get 'mean lunar months' (*madhyama māsa gaṇa*) denoted by M i.e. $M = (4\times12) + 1 = 49$

iv) The number of *adhikamāsas* is given by the quotient of $(M + 2C + 10)/33$

$= (49 + 2(8) + 10)/33 = 75/33$; quotient = 2

v) True lunar months (*spaṣṭa māsagaṇa*) = Mean lunar months + *adhikamāsas* = M+ quotient of $(M + 2C + 10)/33 = 49 + 2 = 51 = TM$

vi) The mean *ahargaṇa* (*madhyama ahargaṇa)*:

$MAH = (TM \times 30) + TI + \frac{C}{6} = (51\times30) + 14$ + INT (8/6) = 1545 = MAH

(Note: INT stands for the integer value)

vii) *Kṣayadinas*

$= \text{Quotient of } \left[\frac{1}{64} \times \textit{Madhyama ahargaṇa}\right]$

$= \text{INT}(1545/64) = 24 = KD$

viii) True *Ahargaṇa* (*sāvana dinas*) i.e., the number of civil days,

TAH = Mean *ahargaṇa* - *kṣayadinas*

$= MAH – KD = 1545 – 24 = 1521$

ix) Weekday verification:

$5C + TAH = 5(8) + 1521 = 1561$

R = Remainder of $(5C + TAH)/7$ = remainder of $1561/7 = 0$

That is weekday comes out as Monday.

Since the weekday obtained from calculation is the same as the actual weekday (known), nothing needs to be added to or subtracted from *TAH*.

Thus, True *ahargaṇa* = 1521 and number of *cakras* = 8.

Example 7.2: *Śālivāhana Śaka* 1530, *Kārtika Śukla pratipat,* Saturday corresponding to December 6, 1608. In this year *Bhādrapada* is the *adhikamāsa* which comes before the given date.

i) Years elapsed (*gatābdi*) = 1530- 1442 = 88

ii) Divide the remainder in (i) by 11.

The quotient *cakra* (cycle), C = 8 and remainder = 0

iii) Mean lunar months (*madhyama māsa gaṇa*), $M = (0\times12) + 7 = 7$

iv) The number of *adhikamāsas* is given by the quotient of $(M + 2C + 10)/33$

$= (7 + 16 + 10)/33 = 33/33$; Quotient = 1

Since the given date occurs after the *adhika Bhādrapada māsa,* add 1 to the number of *adhikamāsas* obtained above. Therefore, the number of *adhikamāsas* elapsed for the given year = 1+1 = 2

v) True lunar months (*spaṣṭa māsa gaṇa*) = Mean lunar months + *adhikamāsas* = M + quotient of $(M + 2C + 10)/33$ = 7+ 2 = 9= TM

vi) The mean *ahargaṇa* (*Madhyama ahargaṇa*)

$$MAH = (TM \times 30) + TI + \frac{C}{6}$$

= (9×30) + 0+ INT (8/6) = 271= MAH

(Note: INT stands for the integer value)

vii) *Kṣayadinas* = Quotient of

$$\left[\frac{1}{64}\times Madhyama\ ahargaṇa\right]$$

= INT(271/64) = 4 = KD

viii) True *ahargaṇa* (*sāvana dinas*) i.e., the number of civil days,

TAH = Mean *ahargaṇa* - *kṣayadinas*

$= MAH - KD = 271 - 4 = 267$

Weekday verification:

$5C + TAH = 5(8) + 267 = 307$

$\therefore R$ = Remainder of $(5C + TAH)/7$ = remainder of 307/7 = 6

This is Sunday. But the actual weekday is Saturday. Therefore subtract 1 from *TAH* to obtain *ahargaṇa* for the given day i.e *ahargaṇa* = TAH - 1 = 267- 1 = 266.

Thus, *ahargaṇa* = 266 and the number of *cakras* = 8.

8. Relation among *Ahargaṇas of Karaṇakutṃhala* (*KRK*), *Grahalāghava* (*GL*) and *Makarandasāriṇī* (*MKS*)

Karaṇakutūhala (*KRK*) of Bhāskara II (born 1114 CE) is a *karaṇa* text (handbook) in astronomy which consists of 139 *ślokas*. The epoch chosen is the mean sunrise (at Ujjaynī) on Thursday, February 24, 1183 CE (Julian). This tract is also well-known as *Grahāgama kutūhala.* Some almanac makers are using this text even now for computations. In fact the voluminous work called *Brahmatulya sāriṇī* consists of ready–to-use tables based on Bhāskara's tract.

Subtracting 1687851 days from *MKS ahargaṇa* days or subtracting *vallī* 7|48|50|51 from *MKS ahargaṇa vallī* we obtain *ahargaṇa* days or *ahargaṇa vallī* correspondingly according to *GL* for the given date.

Adding 123114 days to *GL ahargaṇa* (*ahargaṇa* according to *GL*) we obtain *ahargaṇa* according to *KRK*

i.e., for a given date,

(i) *Ahargaṇa* according to *GL* = *MKS ahargaṇa* days -1687851

(ii) *Ahargaṇa vallī* for *GL ahargaṇa* = *MKS ahargaṇa vallī* - 7|48|50|51

(iii) *Ahargaṇa* according to *KRK* = *GL Ahargaṇa* + 123114

Note: Here 1687851 days are the *Kali ahargaṇa* for *GL* epoch (1520 CE, March 19 Monday)

Kali ahargaṇa for KRK epoch (1183 CE, February 24 Thursday) is 1564737.

Kali ahargaṇa for GL epoch - *Kali ahargaṇa* for *KRK* epoch = 1687851- 1564737 = 123114

Example: *Śaka* 1849*, Mārgaśira Śukla,* 15 Thursday corresponding to 1927 December 8.

The *Kali ahargaṇa* for the given date, *A* = 1836758.

For the given date, *MKS ahargaṇa vallī* =8|30|12|38

MKS ahargaṇa days = 1836758

(i) *Ahargaṇa* days of *GL* = *MKS ahargaṇa* days - 1687851 = 1836758-1687851 = **148907**

Converting 148907 days into *cakra* (*C*) and *ahargaṇa* (*A*), we get ***C*** **= 37** and ***A*** **= 315**

(ii) *Ahargaṇa vallī* of *GL* = *MKS ahargaṇa vallī* - 7|48|50|51=8|30|12|38 - 7|48|50|51 = **0|41|21|47**

(iii) *Ahargaṇa* of *KRK* = *GL ahargaṇa* + 123114 = 148907 + 123114 = **272021.**

9. Finding the Christian date from the *Ahargaṇa* and vice versa

In the table 9.1, the Julian days and *ahargaṇas* for the epochs of *Kaliyuga* and the *Grahalāghava* are given for the beginning of the Christian (Common Era)centuries from -3200 (J) to 2200 (G). Here **J** and **G** in brackets represent respectively Julian and Gregorian.

Note: Tables 9.1 - 9.3 are reproduced from *Indian Astronomy – Concepts and Procedures* by S. Balachandra Rao.

In the table 9.2, the days as also *cakras* and *ahargaṇas* for the year beginnings according to the *Grahalāghavam* are given from 0 to 99 years.

Note: For the beginning of century years before Christ (BCE, Before Common Era) for example — 3100 refers to 3101 BC etc. However this convention is not applicable to the positive years. This is since 1 BCE is taken as the zero year. For Julian century years consistently 36525 days are added. But for the Gregorian calendar (after 1582 CE) for century years for the usual century years 36524 days are added, since the tropical solar year is considered. However for the century year which are leap years (divisible by 400) one more extra day is added. Thus for positive century years, the listed year refers to that year itself and not its next year. For example 1900 and 2000 do not refer to 1901 and 2001.

Table 9.1. *Ahargaṇa: Kali, Grahalāghava* and Julian Days

Chris (common)	Julian Days (JD)	*Kaliahargana*	*Grahalāghava*	
			Cakra	*Ahargaṇa*
-3200(J)	552258	-36208	-430	2822
-3000(J)	625308	36842	-412	3584
-2800(J)	698358	109892	-393	330
-2600(J)	771408	182942	-375	1092
-2400(J)	844458	255992	-357	1854
-2200(J)	917508	329042	-339	2616
-2000(J)	990558	402092	-321	3378
-1800(J)	1063608	475142	-302	124
-1600(J)	1136658	548192	-284	886
-1400(J)	1209708	621242	-266	1648
-1200(J)	1282758	694292	-248	2410
-1000(J)	1355808	767342	-230	3172
-800(J)	1428858	840392	-212	3934
-600(J)	1501908	913442	-193	680
-400(J)	1574958	986492	-175	1442
-200(J)	1648008	1059542	-157	2204
0(J)	1721058	1132592	-139	2966
200(J)	1794108	1205642	-121	3728
400(J)	1867158	1278692	-102	474
600(J)	1940208	1351742	-84	1236
800(J)	2013258	1424792	-66	1998
1000(J)	2086308	1497842	-48	2760
1200(J)	2159358	1570892	-30	3522
1400(J)	2232408	1643942	-11	268
1500(J)	2268933	1680467	-2	649
1500(G)	2268923	1680457	-2	639
1600(G)	2305448	1716982	7	1020
1800(G)	2378496	1790030	25	1780
2000(G)	2451545	1863079	43	2541
2200(G)	2524593	1936127	61	3301

Table 9.2. *Ahargaṇa* for Year Beginnings

Year	Days	*Grahalāghava*		Year	Days	*Grahalāghava*	
		Cakra	*Ahargaṇa*			*Cakra*	*Ahargaṇa*
0	0	0	0	51	18627	4	2563
1	365	0	365	52	18993	4	2929
2	730	0	730	53	19358	4	3294
3	1095	0	1095	54	19723	4	3659
4	1461	0	1461	55	20088	5	8
5	1826	0	1826	56	20454	5	374
6	2191	0	2191	57	20819	5	739
7	2556	0	2556	58	21184	5	1104
8	2922	0	2922	59	21549	5	1469
9	3287	0	3287	60	21915	5	1835
10	3652	0	3652	61	22280	5	2200
11	4017	1	1	62	22645	5	2565
12	4383	1	367	63	23010	5	2930
13	4748	1	732	64	23376	5	3296
14	5113	1	1097	65	23741	5	3661
15	5478	1	1462	66	24106	6	10
16	5844	1	1828	67	24471	6	375
17	6209	1	2193	68	24837	6	741
18	6574	1	2558	69	25202	6	1106
19	6939	1	2923	70	25567	6	1471
20	7305	1	3289	71	25932	6	1836
21	7670	1	3654	72	26298	6	2202
22	8035	2	3	73	26663	6	2567
23	8400	2	368	74	27028	6	2932
24	8766	2	734	75	27393	6	3297
25	9131	2	1099	76	27759	6	3663
26	9496	2	1464	77	28124	7	12
27	9861	2	1829	78	28489	7	377
28	10227	2	2195	79	28854	7	742
29	10592	2	2560	80	29220	7	1108
30	10957	2	2925	81	29585	7	1473
31	11322	2	3290	82	29950	7	1838
32	11688	3	3656	83	30315	7	2203
33	12053	3	5	84	30681	7	2569
34	12418	3	370	85	31046	7	2934
35	12783	3	735	86	31411	7	3299
37	13514	3	1466	88	32142	8	14
38	13879	3	1831	89	32507	8	379
39	14244	3	2196	86	31411	7	3299
40	14610	3	2562	87	31776	7	3664
41	14975	3	2927	88	32142	8	14
42	15340	3	3292	89	32507	8	379
43	15705	3	3657	90	32872	8	744

Year	Days	*Grahalāghava*		Year	Days	*Grahalāghava*	
		Cakra	*Ahargaṇa*			*Cakra*	*Ahargaṇa*
44	16071	4	7	91	33237	8	1109
45	16436	4	372	92	33603	8	1475
46	16801	4	737	93	33968	8	1840
47	17166	4	1102	94	34333	8	2205
48	17532	4	1468	95	34698	8	2570
49	17897	4	1833	96	35064	8	2936
50	18262	4	2198	97	35429	8	3301
38	13879	3	1831	98	35794	8	3666
39	14244	3	2196	99	36159	9	15
40	14610	3	2562	89	32507	8	379
41	14975	3	2927	90	32872	8	744
42	15340	3	3292	91	33237	8	1109
43	15705	3	3657	92	33603	8	1475
44	16071	4	7	93	33968	8	1840
45	16436	4	372	94	34333	8	2205
46	16801	4	737	95	34698	8	2570
47	17166	4	1102	96	35064	8	2936
48	17532	4	1468	97	35429	8	3301
49	17897	4	1833	98	35794	8	3666
50	18262	4	2198	99	36159	9	15

In table 9.3, the *Ahargaṇa* for days of a year is given. In this table, the first two columns are headed by C and B which stand respectively for a common (non-leap) year and bissextile (leap) year.

Note: 9.1: In table 9.1, the Julian days refer to the noon (GMT) and the *Kali* and *Grahalāghava ahargaṇa* refer to the Ujjain mean sunrise of the day preceding January 1 for the non-leap century years. In the case of leap century year (e.g. 1600 and 2000 Gregorian), the number refers to January 1 itself.

9.2: In table 9.1, the letters J and G in brackets represent respectively the Julian and the Gregorian calendars. The Gregorian calendar came into effect from October 15, 1582, Friday.

9.3: The reckoning of the Julian days starts from the mean noon (GMT) on January 1, 4713 BCE, Monday. On that day, at the mean noon(GMT), the Julian day number = 0.

9.4: For a given date in a leap year, only for January and February, the column headed by B must be used in table 9.3. For other months even in a leap year and for all months in a common year the first column under C must be used.

The procedure for finding the Christian date from the *cakras* and the *ahargaṇa* of *GL*

(i) Multiply the number of *cakras* C by 4016 (the number of days in a *cakra*) i.e., find $4016C$. To this $4016C$ add the *ahargaṇa* A i.e., find ($4016C+A$). The *Kali ahargaṇa* for the *GL* epoch is 16, 87,850). Add this constant to ($4016\ C + A$) i.e., find ($4016\ C+ A+$ 16,87,850). This gives the *Kali ahargaṇa* for the required date.

From Table 9.1 to 9.3, for the thus obtained *Kali ahargaṇa,* the corresponding Christian date can be obtained as shown in the following example.

Table 9.3. *Ahargaṇa* for days of a year

Dates C	Dates B	Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
0	1	0	31										
1	2	1	32	60	91	121	152	182	213	244	274	305	335
2	3	1	33	61	92	122	153	183	214	245	275	306	336
3	4	3	34	62	93	123	154	184	215	246	276	307	337
4	5	4	35	63	94	124	155	185	216	247	277	308	338
5	6	5	36	64	95	125	156	186	217	248	278	309	339
6	7	6	37	65	96	126	157	187	218	249	279	310	340
7	8	7	38	66	97	127	158	188	219	250	280	311	341
8	9	8	39	67	98	128	159	189	220	251	281	312	342
9	10	9	40	68	99	129	160	190	221	252	282	313	343
10	11	10	41	69	100	130	161	191	222	253	283	314	344
11	12	11	42	70	101	131	162	192	223	254	284	315	345
12	13	12	43	71	102	132	163	193	224	255	285	316	346
13	14	13	44	72	103	133	164	194	225	256	286	317	347
14	15	14	45	73	104	134	165	195	226	257	287	318	348
15	16	15	46	74	105	135	166	196	227	258	288	319	349
16	17	16	47	75	106	136	167	197	228	259	289	320	350
17	18	17	48	76	107	137	168	198	229	260	290	321	351
18	19	18	49	77	108	138	169	199	230	261	291	322	352
19	20	19	50	78	109	139	170	200	231	262	292	323	353
20	21	20	51	79	110	140	171	201	232	263	293	324	354
21	22	21	52	80	111	141	172	202	233	264	294	325	355
22	23	22	53	81	112	142	173	203	234	265	295	326	356
23	24	23	54	82	113	143	174	204	235	266	296	327	357
24	25	24	55	83	114	144	175	205	236	267	297	328	358
25	26	25	56	84	115	145	176	206	237	268	298	329	359
26	27	26	57	85	116	146	177	207	238	269	299	330	360
27	28	27	58	86	117	147	178	208	239	270	300	331	361
28	29	28	59	87	118	148	179	209	240	271	301	332	362
29	30	29		88	119	149	180	210	241	272	302	333	363
30	31	30		89	120	150	181	211	242	273	303	334	364
31		31		90		151		212	243		304		365

Example 9.1 Suppose for a date *cakra* = 12 and *ahargaṇa* = 30

Number of civil days since the epoch
$= 4016\,C + A = 4016 \times 12 + 30 = 48{,}222$

Kali ahargaṇa of *GL* epoch = 16, 87,850

$\therefore$ *Kali ahargaṇa* for the given day = 17, 36,072

From tables 9.1 to 9.3, for this *Kali ahargaṇa*, we have

1600 (G)	→	17,16,982
Year 52	→	18,993
April 7	→	97
Total	→	17,36,072

Therefore the given date corresponds to **April 7, 1652(G).**

Finding the weekday from the *ahargaṇa*

Let *C* and *A* respectively be the *cakras* and the *ahargaṇa* according to *GL*. Multiply *C* by 5 and to this product add *A* i.e., find (5*C*+*A*).

Dividing (5*C*+*A*) by 7, let the remainder be *R*. If *R*=0, then the given date falls on a Monday; if *R*=1, Tuesday etc.

Example 9.2*:* In the example considered above (example 7.1), *C*=8, and *A*=1521. Therefore, 5*C*+*A*=5(8) + 1521 = 1561. When 1561 is divided by 7, the remainder *R*=0. Therefore, the given date is a Monday.

Finding the *ahargaṇa* from the Christian date

In Table 9.1, for the beginning of the Christian century (column 1) — in which the given date lies – *Kali ahargaṇa* (column 2) and the *cakras* and the (balance) *ahargaṇa* according to *GL* are given in column 3 and 4. In Table 9.2 the days elapsed at the beginning of each year are given. Table 9.3 provides the cumulative days corresponding to each day of a year. For example, consider Friday, August 04, 2017 CE. For this year the century beginning year is 2000(G). In Table 9.1, against 2000(G), we have

Kali ahargaṇa		*GL ahargaṇa*		Julian Day
		cakra	*ahargaṇa*	
2000(G)	18, 63,079	43	2541	24,51,545

In table 9.2 the total elapsed days are given for the beginning of the years of a century. In the example against 17 years from table 9.2, we have

	Days	*GL ahargaṇa*		Julian Day
		cakra	*ahargaṇa*	
Year 17	6209	1	2193	6209

In table 9.3, the accumulated days for the dates of different months in a year are given. In the example for August 4, the number of days elapsed in the year is 216.

Now the total JD and the different *ahargaṇas* are obtained by adding the corresponding number of days as shown below:

Kali ahargaṇa		*GL ahargaṇa*		Julian Day
		cakra	*ahargaṇa*	
2000(G)	18, 63,079	43	2541	24,51,545
Year 17	6209	1	2193	6209
August 4	216	0	216	216
Total	18, 69,504	45	934	24,59,970

Note 9.5: While adding the *ahargaṇa* numbers for the epoch of *GL*, if the *ahargaṇa* is greater than or equal to 4016, then the number should be divided by 4016 and the quotient (an integer) must be added to the *cakras* and the remainder retained under *ahargaṇa.*

Weekday from *GL ahargaṇa*

Here *C* = 45 and A = 934.

$\therefore$ 5*C*+*A*=225+934=1159

Dividing (5*C* + *A*) by 7, remainder *R*=4, we get Friday.

Note 9.6: In Table 9.1, column 2 gives the Julian day number (JDN). To find the weekday from JD of the given date, divide JDN by 7 and let R be the remainder. If R=0, it is Monday if R=1, Tuesday etc.

10. Conclusion

In the present paper we have presented the procedures for determining *ahargaṇa* according to *Makarandasāriṇī* and *Grahalāghava,* since the *Kali* epoch with concrete examples. We have also provided tables for determining *ahargaṇa* from a Julian or Gregorian date. It is shown how easy it is to convert a given traditional lunar calendar date to *kali* days using *vallī* components of *Makarandasāriṇī* and this in turn can be used to obtaining the Julian or Gregorian date.

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