Advances in the modelling and control of series hybrid electric vehicles

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Abstract—Using a first-principles approach, the constituent components of a series hybrid electric car are modelled and integrated to form an overall coupled dynamic model. Controllers for the individual components are constructed, and are combined with a load follower supervisory controller and driver model to enable simulation of the vehicle under general operating conditions. The powertrain includes AC/DC, DC/AC and DC/DC converters which are described by standard average models with realistic power efficiencies. The powertrain also includes permanent magnet synchronous machines that are modelled using conventional d-q frame equations. There is novelty in the use of a general purpose friction moment, acting on each of the electrical machines, that is included to capture the equivalent energy loss due to friction, eddy currents and hysteresis, thus providing a good match between the predicted steady-state behaviour of the machines and experimental data. A longitudinal vehicle dynamics model with realistic descriptions for the tyres, aerodynamic resistance, suspension and continuously variable transmission (CVT) is also included. A scheme for improving the motor efficiency under general operating conditions, by controlling the CVT, is devised. The overall vehicle model is used to track the New European Drive Cycle (NEDC), for two design scenarios, one with fixed and one with variable final drive ratio. The simulation results demonstrate the applicability of the model for control system design, realistic prediction of vehicle behaviour and energy losses in the various components, and design optimisation.

I. INTRODUCTION

Transportation alone is responsible for a substantial proportion of global energy related CO\textsubscript{2} emissions (approximately 23\% in 2007), and therefore electrical machine based vehicles are of prime importance in order to tackle environmental pollution and limited resources of fossil fuels [1]. Much work, especially by industry, has already gone into the design and development of hybrid electric vehicles (HEVs). Modelling and simulation are essential tools in the cost effective and rapid development of optimised or new HEV prototype designs. Studies on the subject of modelling and simulation of hybrid electric vehicles are too numerous to refer to individually but a survey of such work can be found in [2]. Overall, many modelling tools exist, such as ADVISOR, PSAT, PSIM and so on, and can be categorised as steady-state, quasi-steady and dynamic, or depending on the direction of calculation, as forward-looking and backward facing [2].

In any case, it is believed that existing platforms do not yet include sufficiently detailed component descriptions to allow HEV models to be used for the purposes for which they are built. These include to help identify which aspects of the design need to be modified for optimal overall performance, for example in respect of fuel economy, emissions, acceleration and so on, and also to improve performance by designing good control schemes. The present work is part of a research programme to develop high-fidelity hybrid electric vehicle models that are capable of both fast computation and accurate dynamic simulation. The task involves the modelling of the individual subsystems of the vehicle and its powertrain. The aim is to develop flexible subsystem models in the form of equations derived from first principles, rather than models that rely on static look-up tables and performance maps, or simple power-request dynamics that do not allow for the accurate scaling or for the adjustment of critical design parameters. A key issue is the accurate description of the interdependencies between the subsystems to enable integration to the full model.

The paper is organised as follows. In Section II the component models of the hybrid electric vehicle are described. Key characteristics of the models are descriptions for the hybrid-electric drive powertrain, regenerative braking, vehicle dynamics, aerodynamics, and component-level controls. In Section III the integration of the components models to an overall model is explained, while in Section IV the implementation of a CVT and its control is presented. Section V describes the supervisory controller used to regulate the energy flow in the various components. Simulation of the NEDC drive cycle is conducted and the results are presented in Section VI. Finally, conclusions are drawn in Section VII.

II. MATHEMATICAL MODELLING OF THE HEV COMPONENTS

The hybrid vehicle considered in this work utilises a series powertrain topology and represents a class of passenger vehicles that are appropriate for urban and extra-urban transportation. Its general structure, including the powertrain, is shown in Figure 1.

The powertrain comprises a diesel engine and a common DC-link of 700 V, on which there are connected a Permanent Magnet Synchronous Generator (PMSG) with the associated AC/DC converter, a Permanent Magnet Synchronous Motor (PMSM) with the associated DC/AC converter and a battery with the associated DC/DC converter. The car includes the body masses, the transmission, suspension system, tyres and aerodynamics. The supervisory controller decides the power flow between the various powertrain components, and acts in response to the commands of a driver model that is used to enable the following of speed demands corresponding to specific drive cycles. Simulink [3] is the main platform used...
to develop the overall model. The component models are first constructed in causal form, with appropriate choice of inputs and outputs, and are subsequently interconnected via their inputs and outputs, and control feedback loops, to obtain the overall model. The various component models are described in more detail next.

A. Engine

The engine dynamics are represented by a modelling approach that utilises emptying and filling models for the inlet and outlet manifolds, mean-value torque maps for the engine cylinders, the turbocharger turbine and turbocharger compressor, engine and turbocharger inertias, and fuel-injection valve dynamics. The engine parameter set is based on a turbocharged Puma 2.0L diesel engine. The fuel mass flow rate serves as an input and the engine torque, acting on the common engine-generator shaft (see Section II-D), is designated as an output. This model has been developed and validated recently at Imperial College [4].

B. Car

Accurate car response requires accurate representation of the longitudinal car behaviour. The model employed describes the longitudinal car dynamics and it is based on the multibody model presented in [5], capable of general motions. The constituent masses are introduced in a tree structure with the freedoms and forces between them specified. Thus the main body of the vehicle is allowed forward and vertical translation, and pitch rotation. The front and rear hub carriers are attached to it with vertical translation freedoms, with their motion restricted by spring and damper suspension forces. The model also has spinning wheels attached to the hub carriers. The rear wheel is connected via a crown wheel and pinion, and a CVT to the motor shaft. The model employed here includes also aerodynamic lift and drag forces which are proportional to the square of the speed. The tyres are treated as vertically compliant, with associated spring and damper forces, and the tyre longitudinal force is generated from normal load and longitudinal slip using standard ‘magic formulae’ [6]. The parameter values used in the model are representative of a contemporary European family saloon and are taken from [5]; the total mass is 1355.6 Kg, the pitch inertia is 2152.1 Kgm², and the drag coefficient is 0.35.

The car model is written in LISP and makes use of the multibody modelling code VehicleSim®, formerly called AutoSim [7]. The VehicleSim model can be configured to generate C/C++ MEX code that numerically integrates the nonlinear equations of motion. This code can be compiled into an S-Function for integration into the Simulink environment.

C. Permanent Magnet Synchronous Motor (PMSM)

In the present work a surface mounted PMSM motor has been used. This type of motor combines a number of attractive features when used in hybrid vehicle applications, as compared to induction or wound-rotor synchronous machines, such as higher torque-to-inertia ratio and power density.

The dynamic behaviour of the 3-phase star-connected PMSM is described by the standard nonlinear coupled differential equations in the rotor d-q reference frame [8], as follows:

\[
\frac{di_d}{dt} = \left( v_d - Ri_d + \omega_s L_q i_q \right) / L_d,
\]

\[
\frac{di_q}{dt} = \left( v_q - Ri_q - \omega_s (L_di_d + \lambda_f) \right) / L_q,
\]

where \(i_d\) and \(i_q\) are the d- (direct) and q- (quadrature) axis components of stator current, \(v_d\) and \(v_q\) are the d- and q-axis components of stator voltage, \(R\) is the stator resistance, \(L_d\) and \(L_q\) are the d- and q-axis stator inductances, \(\lambda_f\) is the flux linkage due to the permanent magnets, and \(\omega_s\) is the inverter frequency. The electromagnetic torque produced by the motor is given by [8]

\[
T_e = \frac{3}{2} p (\lambda_f i_q + (L_d - L_q) i_d i_q),
\]

where \(p\) is the number of pole pairs per phase in the stator. This torque is applied on the rotor shaft that is connected to the car transmission, thereby driving the car forward. The rotor dynamics are thus included inside the car model described in Section II-B, with \(T_e\) provided as an input to the car model and the rotor speed extracted from the car model. The rotor speed, \(\omega_r\), and the inverter frequency are related according to \(\omega_s = p\omega_r\).

1) Three-phase DC to AC converter (Inverter): A bidirectional pulse width modulated (PWM) inverter [9] is considered. In general, the electronic converter models that we use are average models and can be taken to be representative of a broader range of converter topologies. The overall idea is that electronic switches are assumed to respond to commands instantaneously, while their switching is occurring at a frequency much higher than the dynamics of the car-powertrain system, such that the voltage and current waveforms produced are averaged by the low pass filtering action that is naturally present in the system.

It can be shown that the average model of the PWM inverter in the d-q frame is given by the following equations
\[\begin{align*}
    i_{dc} &= \frac{1}{\eta_{inv}} \left( \frac{3}{2} (d_q i_q + d_i i_d) \right), \\
v_d &= d_q \nu_{dc}, \\
v_q &= d_q \nu_{dc},
\end{align*}\]
in which \(i_{dc}\) is the DC current drawn by the inverter from the DC-link, and \(d_d, d_q\) are continuous duty cycle functions in the \(d\) and \(q\) axis respectively, resulting by time averaging the discrete switching functions for each switching period of the inverter. Since the inverter model is assumed to apply smooth voltages to the motor windings, the instantaneous values of \(d_d\) and \(d_q\) can be obtained from Equations (5) and (6) and the actual values of \(\nu_{dc}, \nu_d\) and \(\nu_q\). Hence the inverter model requires inputs from \(\nu_d, \nu_q, \nu_{dc}, i_d\) and \(i_q\) and calculates \(i_{dc}\) according to Equation (4). It is also necessary to restrict the maximum amplitude of the phase voltages since the inverter cannot produce at any time an output voltage which is higher than its DC input voltage. In our model we use saturation functions that constrain \(\nu_d\) and \(\nu_q\) to achieve this physical limitation. The efficiency factor \(\eta_{inv} = 94.61\%\) \([11]\) is used to scale the DC current in Equation (4) to account for the power losses in the converter.

2) Control of the PMSM: In the present application it is desirable to control the forward speed of the car and consequently the speed of the motor which is driving the car. In general motor operation, proportional-integral (PI) loops are used to control independently three interactive variables of the motor, the rotor speed, the rotor flux and the rotor torque. Typically, this is achieved by utilising the vector control approach which is analogous to the control process of a separately excited DC motor \([8]\). In a DC motor the torque producing current generates armature flux which is perpendicular to the rotor flux produced by the field current. Vector control recreates these orthogonal components in an AC motor by controlling separately the quadrature current, \(i_q\), that is producing the torque, and the direct current, \(i_d\), that is producing the flux which is parallel to the rotor flux; \(i_d\) is typically used to weaken the airgap flux \([8]\). If \(i_d\) is forced to zero, the electromagnetic torque in Equation (3) becomes

\[ T_e = \frac{3}{2} p \lambda_f i_q, \]

which is directly proportional to the \(q\)-axis current. Under these circumstances optimal operation at unity power factor is achieved. In the present application, two inner current PI control loops are used to set the \(\nu_d\) and \(\nu_q\) values so that \(i_d\) becomes zero \((i_{d,ref} = 0)\) and \(i_q\) provides the required torque. The \(i_q\) reference is obtained from an outer PI loop that operates on the error between the speed reference signal of a specific drive cycle and the actual speed of the car, calculated in the car dynamics model described in Section II-B. The block diagram of the overall motor control scheme can be seen on the right part of Figure 2.

3) Power loss model of the PMSM: The PMSM parameter values used correspond to the AFM-140 axial flux PMSM developed and manufactured by EVO Electric \([12]\) and are shown in Table I. The dynamic model of the PMSM given by Equations (1)-(3) involves only losses due to the resistance of the copper windings. However, there are several other mechanisms of energy dissipation in the PMSM such as eddy current losses \((\propto \omega^2)\), hysteresis losses \((\propto \omega^2)\), and mechanical losses comprising bearing losses \((\propto \omega)\) and windage losses \((\propto \omega^3)\) \([13]\). In order to obtain a more accurate prediction of the motor efficiency in comparison to the steady-state experimental measurements provided by the manufacturer \([12]\), a rotor-speed-dependent friction torque is included in the model. This friction moment is acting on the rotor and under equilibrium conditions it is related to \(T_e\) and the load torque, \(T_l\), according to

\[ T_e = T_l + T_f. \]

In order to quantify \(T_f\), expressions for \(P_{in}\), the total power in (electrical) and \(P_{out}\), the total power out (mechanical), of the PMSM in steady state, are derived. Hence,

\[ P_{in} = \frac{3}{2} (v_d i_d + v_{dc} i_d) = \frac{3}{2} i_d^2 R + T_e \omega_r \]

\[ = \frac{3}{2} \left( \frac{T_e}{\frac{3}{2} \lambda_f p} \right)^2 R + T_e \omega_r, \]

in which it was assumed that \(i_d = 0\) and Equation (7) was used to substitute \(i_q\). Similarly,

\[ P_{out} = T_l \omega_r. \]

Power efficiency is defined as

\[ \eta = \frac{P_{out}}{P_{in}}. \]

By substituting Equations (8), (9) and (10) in (11) and rearranging, yields

\[ \frac{2R}{3 \lambda_f^2 p^2} T_l^2 + \left( \frac{4RT_l}{3 \lambda_f^2 p^2} + \omega_r - \frac{\omega_r}{\eta} \right) T_l + \frac{2RT_l^2}{3 \lambda_f^2 p^2} + \omega_r T_f = 0. \]

This equation can be written as a simple quadratic equation in \(T_l\) as follows:

\[ aT_l^2 + bT_l + c = 0, \]

where \(a = \frac{2R}{3 \lambda_f^2 p^2}, b = \frac{4RT_l}{3 \lambda_f^2 p^2} + \omega_r - \frac{\omega_r}{\eta},\) and \(c = \frac{2RT_l^2}{3 \lambda_f^2 p^2} + \omega_r T_f.\) \(T_l\) is therefore given by

\[ T_l(\eta, \omega_r) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \]
which is a function of efficiency and rotor speed. The friction torque is chosen so that constant efficiency contours on $T_f - \omega_r$ axes, at different efficiencies, fit the experimental results in [12], as shown in Figure 3.

The proposed friction torque is given in Equation (15) and its parameter values are found by trial and error to be $a_1 = 10$, $a_2 = 6.0$, $a_3 = 0.6$, $a_4 = 0.03$, $a_5 = 250$, $a_6 = -1.0 \times 10^{-5}$, $a_7 = 0.008$, $a_8 = 0.75$ and $a_9 = 2.1$. The variation of $T_f$ with positive rotor speed is illustrated in Figure 4 and as it can be seen, at low speeds it involves some familiar characteristics of mechanical friction, such as static and coulomb friction type behaviours. The first arctan term in Equation (15) is used to provide correct representation of the torque at negative speeds and also to smooth the discontinuity of the torque at zero speed, in order to prevent numerical instability in the simulation.

**D. Permanent Magnet Synchronous Generator (PMSG)**

The electrical and mechanical dynamic equations of the PMSG are the same as for the PMSM, the only difference being the reversal of the flow of current and torque. This reversal can be implemented either by changing the relevant signs in the equations or by using the PMSG equations and simply allowing the corresponding variables to change sign. We make use of the sign-reversed equations in the $d$-$q$ frame as follows [14, 15]:

$$\frac{di_d}{dt} = (-Ri_d - \omega_e L_d i_d + e_d - v_d) / L_d,$$

$$\frac{di_q}{dt} = (-Ri_q + \omega_e L_q i_q + e_d - v_d) / L_d,$$

in which $e_d (= 0)$, $e_q (= \omega_e \lambda_0)$ are the $d$ and $q$ components of the induced emf (source voltage) and $\omega_e$ is the angular frequency of the emf, while $\lambda_0$ is the rotor permanent magnet flux. The other symbols have the same meaning as in Equations (1)-(2). The electromagnetic torque, $T_e$, is given by Equation (3), with $\lambda_f$ substituted by $\lambda_0$.

The generator rotor is mechanically coupled to the engine inertia via a fixed gear ratio $G$. Therefore the mechanical dynamics of the generator are governed by the following equation:

$$T_{eng} - G(T_e + T_f) = (J_{eng} + G^2 J_f) \frac{d\omega_r}{dt},$$

with $T_{eng}$ the mechanical torque applied by the IC engine, $T_f$ the friction torque in the generator, $\omega_r$ the generator rotor speed, and $J_{eng}$ and $J_f$ the moments of inertia of the engine and generator respectively. The rotor speed and the source voltage frequency are related as:

$$\omega_e = p_g \omega_r,$$

with $p_g$ the generator number of pole pairs per phase.

1) **Three-phase AC to DC converter (Rectifier):** The generator is interfaced to the DC-link via a PWM rectifier [9]. Average modelling of the switching dynamics is again considered, with the average model of a rectifier being the same as the one for the inverter in Section II-C.1. Equations (4)-(6) are used to determine the DC output current of the rectifier, $i_{dc}$, that is provided to the DC-link. The maximum voltages that can be formed at the generator terminals by the PWM rectifier are physically limited by the actual value of the DC-link voltage. Therefore $v_{dc}$ and $i_{dc}$ are restricted with appropriate saturation functions to values less than $v_{dc}$.
The average efficiency of the rectifier is also assumed to be 94.61\% and it is built in the model in the same way as for the inverter, with 1/\( \eta_{\text{inv}} \) substituted by \( \eta_{\text{rec}} \) in Equation (4).

2) Control of the PMSG: Due to the series topology of the powertrain, the engine is only used to drive the generator and therefore these two components can be operated at a speed that is independent of the speeds of other rotating components. It is beneficial to choose respective speeds at which both the engine and the PMSG operate most efficiently, for the range of expected load torques. Gearing between engine and generator shafts can be employed, if these speeds are not the same; \( G \) was chosen to be 2 in this work. The angular velocity of the PMSG rotor is controlled via a PI feedback loop that sets the fuel mass rate input to the engine to produce the required torque on the common engine-generator inertia, to overcome the opposing electromagnetic and friction torques; see Equation (18). While the PMSG is driven at a constant angular speed, the DC-link voltage regulation can be achieved by the AC to DC converter. The DC-link voltage control scheme involves a fast inner current control loop and a slower outer voltage control loop, similarly to the vector control approach of the PMSM in Section II-C.2. The error between the actual and the desired value of the DC-link voltage in the outer loop is used to set the reference values for the \( d \)- and \( q \)-axis components of the PMSG current. The reference currents in the \( d \) and \( q \) axis will determine respectively the reactive and real power supplied by the generator. The inner current control loops act to establish \( i_d \) and \( i_q \) so that the current references are tracked, as in the case of the PMSM. Optimal performance of the converter in terms of power can be achieved when it draws from the PMSG sinusoidal current at unity power factor. This corresponds to setting \( i_{d,\text{ref}} = 0 \) and requesting all the current to come from the \( q \)-axis. The left part of Figure 2 illustrates the control strategy in diagrammatic form.

3) Power loss model of the PMSG: The PMSG parameter values used in our model are for the AFG-140 axial flux PMSG, also developed and manufactured by EVO Electric [16], and are shown in Table I. Under steady-state conditions, \( T_r \), the driving torque \( T_i \), and \( T_f \) are related as follows:

\[
T_c = T_i - T_f. \tag{20}
\]

Using a similar analysis to the one conducted for the PMSM in Section II-C.3, it can be shown that in the case of the PMSG the following relationship holds under steady-state conditions:

\[
\frac{2R}{3\lambda_0^2 p^2} T_f^2 + \left( \frac{4RT_f}{3\lambda_0^2 p^2} - \omega_r + \eta \omega_r \right) T_i + \frac{2RT_f^2}{3\lambda_0^2 p^2} + \omega_r T_f = 0. \tag{21}
\]

The efficiency is given by \( \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{\eta \omega_r} \), and by rearranging we obtain

\[
T_i = \frac{P_{\text{out}}}{\eta \omega_r}. \tag{22}
\]

Upon substitution of Equation (22) in Equation (21) we get a quadratic equation in \( P_{\text{out}} \) which can be solved to provide \( P_{\text{out}}(\eta, \omega_r) \). The friction torque is chosen so that the predicted \( P_{\text{out}} \) fits well the experimental data provided by the manufacturer [16] as \( \eta \) and \( \omega_r \) vary. The friction expression in Equation (15) is used here also and the optimal parameter values for the PMSG are found to be \( a_1 = 10, a_2 = 1, a_3 = 0.1, a_4 = -0.0019, a_5 = 0, a_6 = 1.38 \times 10^{-5}, a_7 = 0.02, a_8 = 1 \) and \( a_9 = 1.2 \). The variation of friction torque with rotational speed is shown in Figure 5 and the accuracy of the power loss model is demonstrated in Figure 6.

\[\text{Fig. 5. Variation of the PMSG frictional torque, } T_f, \text{ with rotor speed, } \omega_r.\]

\[\text{Fig. 6. PMSG steady-state power efficiency map for variations in output power, } P_{\text{out}}, \text{ and rotor speed, } \omega_r, \text{ for the experimental results in [16] (red dashed line) and for the PMSG model in Equations (21) and (22) (blue solid line). The contours correspond to constant efficiencies in the range 85%-96\%.}\]

E. Battery, DC to DC converter

A Li-ion battery model based on the work presented in [17, 18] is used. This model expresses the electrochemical parameters of the battery directly in terms of parameters of an equivalent electrical circuit. It is able to capture the generic dynamic response of a Li-ion battery, and it can establish a relationship among its state of charge (SOC), internal resistance, open circuit voltage and current demand. The model parameters are chosen to represent a stack of Li-ion cells arranged to provide a maximum rated capacity of 20
Ah and maximum rated voltage of 250 V. The battery is connected to the DC-link via a bidirectional DC/DC converter. We employ a converter that utilises a zero-voltage-switching (ZVS) bidirectional dual half-bridge topology. A switching-frequency-dependent average model for this converter has been derived in [19] to predict its dynamic characteristics in both power flow directions. The average efficiency of the converter is assumed to be 96%. The phase shift $\phi$ between the switching waveforms of the low and high voltage sides is an input variable and it is used to control the DC-link voltage according to the scheme shown in Figure 7.

**III. INTEGRATION OF COMPONENT MODELS**

In the present work, three paths of energy flow are involved. In the first case the engine drives the generator to produce electricity that powers the motor so that it drives the car. The integration of the various components in this case is shown in Figure 2. The DC-link between the AC/DC and DC/AC converters is a capacitor of value $C_0 = 1 \text{ mF}$ that receives DC current from the rectifier, provides DC current to the inverter and maintains a DC voltage, according to the standard capacitor differential equation. In the second energy flow path the battery is used as the energy source to drive the motor and consequently the car. In this case energy can flow in the reverse direction when the car is braking and energy is regenerated by the motor. The integration of the components for this arrangement is shown in Figure 7. In the third case the engine is charging the battery. Hybrid modes of operation are also possible in which both the engine and the battery are driving the motor, or the engine is charging the battery at the same time as driving the motor. These modes are enabled by enforcing a desired split of the current at the DC-link. In any case the directions of the currents involved are consistent with the direction of power flow.

**IV. CVT CONTROL**

The advantage of the stepless gear ratios offered by the CVT is that the PMSM can operate at its optimum speed while driving the wheels of the car at any speed. This is achieved by controlling the final drive ratio, $f_{d\text{rat}}$, to achieve operation of the motor in its most efficient operating region. The power loss model of Section II-C.3 allows us to define a relationship between load torque and motor speed for which the PMSM efficiency is high. We choose the following simple relationship that is possible to implement practically:

$$T_l = 0.47725 \omega_r.$$  

This will be recognised as a straight line on the load-torque vs PMSM-speed map in Figure 3, that passes through the middle of the contour of maximum achievable efficiency of 96%. Since $\omega_r = f_{d\text{rat}} \omega_{\text{wheel}}$ the effective final gear reduction ratio will be:

$$f_{d\text{rat}} = \frac{T_l}{0.47725 \omega_{\text{wheel}}}.$$  \hspace{1cm} (23)

Here $f_{d\text{rat}}$ is the total gear reduction from the motor shaft to the rear wheel of the car. This includes two stages of reduction, one stage from the CVT, typically in the range 3.117 : 1 – 4.27 : 1 [20], and one stage from the differential gear with a typical value of 3.42. Consequently we allow $f_{d\text{rat}}$ to take values anywhere in the range 10.66 – 1.42. The dynamic response of the CVT is assumed to be characterised by a first order lag with a corresponding time constant, $\tau$, of 200 ms [21]. The overall average power efficiency of the CVT typically ranges from 89% to 94% [22]. In our work the average efficiency of the toroidal CVT, 93%, is used by scaling the load torque on the transmission shaft appropriately. In the case of fixed gear ratio transmission we have assumed an efficiency of 96% [23]. The implementation of the CVT in the overall model is shown in Figure 8.

**V. SUPERVISORY CONTROL**

The main duty of the supervisory controller is to facilitate optimal utilisation of the available resources in the powertrain, thereby minimising the fuel consumption over a given driving schedule [24]. In the present work a ‘load follower’ control strategy is proposed. This strategy follows a similar approach to the power follower method [25–28], with the load current and the SOC being the main decision variables. There are four main and one transitory operating modes in the control scheme, which are designed...
according to the expected behaviour and off-line analysis of the efficiency of the powertrain components. The main modes, with self-explanatory names, are Battery Alone, Engine Alone, EngChargeBat and EngBat. The expected optimum performance of the battery occurs at 60%<SOC<65% and its behaviour deteriorates for load currents greater than 30 A. The utilisation of the engine at two different speeds, 800 and 2000 rpm, is related to the minimisation of the friction losses of the engine and generator when these are not being used. The engine alone cannot provide more than 48 A of load current. The state-flow chart in Figure 9 is used to generate the supervisory control signals for the different operating modes and it is self explanatory.

\[\text{Battery Alone during } I_{\text{gen}} = 0; \text{ during } I_{\text{bat}} = I_{\text{load}}; \text{ during } W_{\text{eng ref}} = 800; \]
\[\text{Start during } I_{\text{gen}} = 0; \text{ during } I_{\text{bat}} = 0; \text{ during } W_{\text{eng ref}} = \text{800}; \]
\[\text{after (1 sec); during } I_{\text{load}} = 30 \text{ & SOC<60} \]

**VI. Simulation Results**

Results for vehicle speed-profile tracking simulations are now presented. The tracking of prescribed driving patterns is enabled by the \( u_{\text{car}} \) PI controller, shown in Figures 2 and 7, which acts as the driver model. The standard New European Driving Cycle (NEDC) that consists of four repeated urban (ECE-15) and one extra-urban (EUDC) driving cycles, is followed; see Figure 10. This drive cycle corresponds to the typical usage of a car or any light duty vehicle in Europe, and it is also used for car engine emission level rating.

\[\text{Battery Alone during } I_{\text{gen}} = 0; \text{ during } I_{\text{bat}} = I_{\text{load}}; \text{ during } W_{\text{eng ref}} = 800; \]
\[\text{after (1 sec)}; \text{ during } I_{\text{load}} = 30 \text{ &SOC<60} \]

Fig. 10. NEDC speed profile.

Six different simulations are performed, in five of which the car has fixed final drive ratios of 2, 3, 4, 5, and 6, and in one it has variable transmission ratio provided by the CVT. The speed tracking in all cases is accurate. The energy provided by the sources and the energy dissipated in the various components, in each simulation, are shown in Figures 11 and 12. Aerodynamic drag produces the same amount of loss \((1.709 \times 10^6)\) J in all cases, due to the similar speed tracking quality, and it is not shown.

**Fig. 11.** Energy provided by the engine and battery, battery recharging energy (mostly from regeneration), and SOC equivalent energy while tracking the NEDC drive cycle.

**Fig. 12.** Energy dissipated by the engine, battery, DC/DC converter, PMSG and PMSM friction, rectifier, inverter, transmission, and tyres, while tracking the NEDC drive cycle.

The results in Figure 11 show that most of the energy is provided by the engine, while large amounts of energy can be regenerated, since mechanical brakes are not used. At the end of the driving cycle any deviation for the SOC of the battery from the initial value of 65% is taken into account by considering the equivalent amount of engine energy required to bring the SOC up to 65% (column labelled ‘SOC Equiv’). Figure 12 demonstrates that there is substantial energy loss due to friction both in the engine and in the generator; this loss is alleviated by operating the engine at two different speeds as described in Section V. The relative magnitude of other types of losses, such as converter, transmission and tyre slip losses, can also be seen in Figure 12. The benefits of using the CVT are demonstrated by the reduction in energy loss in the PMSM, caused both by friction and resistance of the windings, as compared with the fixed final drive ratio cases. The total energy used to complete the drive cycle in each case is shown in Figure 11. It is clear that the car with the CVT installed outperforms all the other cases, giving a 3.52% reduction in the total energy and 1.72% reduction in fuel consumption, as compared to the best case of fixed final drive ratio, \( f_{\text{drat}} = 3 \).

Figures 13 and 14 show the time histories of the DC-link current for \( f_{\text{drat}} \) of 2 and 6, and the case when the CVT is installed, for two segments of the NEDC cycle; one of the ECE-15 cycles and the EUDC cycle respectively. It is evident that when the car accelerates the current increases positively, and conversely while the car is decelerating regeneration takes place and the current becomes negative. It
can also been that the car with CVT requires less driving current and produces more regenerative current, as compared to the constant final drive ratio cases, which is consistent with the energy loss reduction brought about by the CVT.

VII. CONCLUSIONS

Mathematical models that are believed to provide accurate prediction of hybrid electric vehicle dynamic response are presented in this paper. A series powertrain topology is considered by integrating the relevant component models into an overall dynamic model that includes control systems for the individual components, a supervisory controller for the management of the power flow, and a driver model for the operation of drive cycle tracking simulations. Regenerative braking is also accommodated by utilising bidirectional electronic converters with reverse operation of the permanent magnet synchronous motor that normally drives the car.

Simulation results for different transmission configurations, in which the car follows standard test drive cycles, are presented. The power losses in the various components are quantified and their relative magnitude and importance is understood. The model is qualified for overall balance of the energy entering and leaving the HEV. It is demonstrated that the use of a continuously variable transmission helps to reduce the overall energy required to complete the full drive cycle, thus leading to fuel savings. The CVT control scheme is informed by a novel power loss model of the PMSM in which a speed-dependent frictional torque is included to inform a novel power loss model of the PMSM.

Overall, the modelling work has the potential to act as a platform for the development of sophisticated supervisory controllers that bring further fuel savings, and to enable overall design optimisation of the car and its powertrain. These are subjects of ongoing investigation.

REFERENCES