

Feasibility Study of Medium Span Spliced Spline Stressed Membranes

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ABSTRACT: This paper examines the feasibility of 3 medium span (16m – 32m) spliced spline stressed membranes. Medium span slender arch systems have been used for canopy structures of stadia cladding such as the Gottlieb-Daimler Stadium. Lateral bracing by the membrane means that the arch can be slender and flexible. Flexibility and lightness fit in well with the design of pre-stressed structures that are themselves flexible and adjust to applied loads. The presented membrane structures are designed bearing in mind their deployability (as necessary for temporary tents) and offer a good simple alternative to medium span high maintenance pneumatics covering squash courts or swimming pools.

Key Words: spline, splice, medium span.

1. INTRODUCTION

Arch structures are frequently used for the support of tensile membrane systems, and for small-scale structures they can be very slender because they are stabilized by the pre-stress in the membrane. Medium scale, slender arch systems have also been used for the canopy structures of stages or stadia cladding such as the arch ribs of the Don Valley Stadium and Gottlieb-Daimler Stadium [1]. For these medium spans, of say 16m, simple cable bracing in the arch plane might be used to prevent asymmetric distortion, but lateral bracing by the membrane means that the arch rib can be slender and flexible. The aspects of lightness and flexibility of supporting arches fits well with the concepts of pre-stressed surface structures, which are flexible and accommodate applied loads. Adriaenssens [2] showed that for a closed ring, arc or any spatial curve bent from an initially straight member with uniform second moment of area I about any axis perpendicular to its centroidal axis (such as a tube),

the effective torsional stiffness is due solely to the geometric stiffness, without any contribution from the classical 'torsion constant'. This of course assumes that no concentrated torsional moment is applied to the section, and with the detailing of most arch supported membranes this assumption will be correct in practice. A very important physical consequence of this is that the arch can be constructed as a series of straight tube segments which are simply slotted together with insert splices, and that this system would have identical stiffness to that of a continuous spline. The splices would normally be straight inserts, but it is equally possible to use branched splices to give much wider scope to the development of feasible structural systems as developed in this paper. The detailed erection procedure of these systems is beyond the scope of this study. Practically the initially straight spline could be inserted in a membrane pocket and bent into shape by applying the appropriate axial force to spline's extremities by a hydraulic jack. At

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the same time the structure could be guided by additional props until it reaches its final equilibrium position. The purpose of this paper however is to demonstrate that the approach of spliced splines can be used for medium span engineered structures provided that the correct materials are used. In this design study the form-finding and load analysis of a series of three medium span (16m – 32m) spline structures supporting a lightweight membrane are explored.

2. MATERIAL CONSIDERATIONS AND CONCEPTS

This section reviews and briefly assesses the materials used for the design study.

2.1. Medium span tubular splines

Glass Fibre Reinforced Plastics (GFRP) have been successfully used for small structures such as masts and electric power line poles but in the field of membrane structures their application has been limited despite their light weight and resistance to corrosion. The properties quoted in this section are taken from literature [3] and are representative of GFRP pultrusions manufactured by Tufnol. For this feasibility study the material is assumed to be linearly elastic. The material's low Young's modulus ($E_{GFRP} = 40 \text{ kN/mm}^2$) allows the spline to be bent into shape through the application of relatively small forces (i.e. smaller than 30kN). Once bent into shape, the GFRP spline can withstand higher tensile and bending stresses compared with an equivalent structural steel section ($\sigma_{allowable_tension} = \sigma_{allowable_bending} = 700 \text{ N/mm}^2$). However the compressive stresses associated with axial loading must be limited to $\sigma_{allowable_compression} = 100 \text{ N/mm}^2$. In order to examine the combined compressive and bending spline stresses present in the pre-stressed state and in the various loading conditions set out in a later section, the following formula is used:

$$c = \frac{\sigma_{axial}}{\sigma_{allowable_axial}} + \frac{\sigma_{bending}}{\sigma_{allowable_bending}} < 1 \quad (1)$$

For non-ductile fibrous systems this formula (1) represents a fairly common approach. The GFRP pultrusions are cost effective compared with metals and can be made to any desired length. Through selection of right type of resin, deterioration due to exposure to UV light can be prevented. In short, the mechanical properties of GFRP structural sections seem to lend themselves well to the design of medium span spline systems. The GFRP splines are

continuously restrained by the membrane and therefore stiffened through the membrane's pre-stress.

2.2. Membrane properties

The membrane properties chosen for the medium span systems are typical of a lightweight PVC coated polyester fabric such as Ferrari 502[4] which would be appropriate for temporary structures. The woven fabric material consists of two sets of crossing yarns with one set in the warp direction and the other set in the weft direction of the fabric material (direct warp stiffness $E_x = 0.8 \text{ MN/m}$, direct weft stiffness $E_y = 0.6 \text{ MN/m}$, and assumed cross stiffness $E_c = 0.3 \text{ MN/m}$). The flexible PVC coating provides low shearing stiffness with a shear modulus of 0.03 MN/m . These properties are on the conservative side in terms of providing the lowest likely stiffness to the spline arch. A pre-stress of 1 kN/m is applied in the warp and weft direction of the fabric material to ensure structural stability and to provide adequate stiffness against deflection (again this is a conservatively low value). A 502 Ferrari fabric with a factor of five on the virgin strength can be expected to just sustain a tension of 11.20 kN/m under the severe conditions that the membrane is slightly torn or vandalised and at the same time meets its full design loading. For the 32m span membrane supported with one arch and for the 32m span dome a heavier fabric such as Ferrari 702 [4] is chosen with the following properties; direct warp stiffness $E_x = 1.0 \text{ MN/m}$, direct weft stiffness $E_y = 1.0 \text{ MN/m}$, assumed cross stiffness $E_c = 0.5 \text{ MN/m}$ and shear modulus 0.05 MN/m . The fabric can be expected to sustain the same factor on virgin strength and under the same conditions at a value of 12.00 kN/m . For pre-stressed states and loading conditions of the various structures, the membrane stresses in the warp and weft direction are checked to ensure that they are within these limits. However, it is important to note that if membrane stresses did exceed these limits, they could be relieved with small design revisions as discussed in sections 5 to 7.

3. FORM-FINDING

The form-finding of the spline starts from an initial arbitrary shape which is then relaxed to a stable equilibrium position. In this study, the initial geometry of the various spline structures (for purposes of data preparation) is based on arcs of a circle. The initial radius of curvature is chosen on the basis of limiting the bending stresses in this hypothetical state to a maximum of 1/3 of the GFRP permissible bending stress.

4. LOADING CONDITIONS

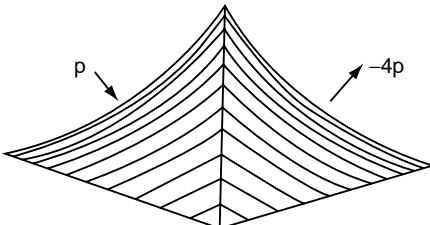
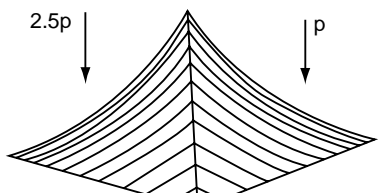
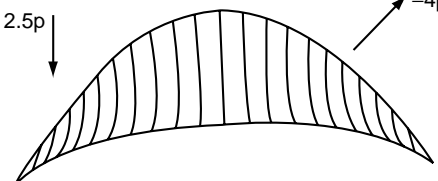
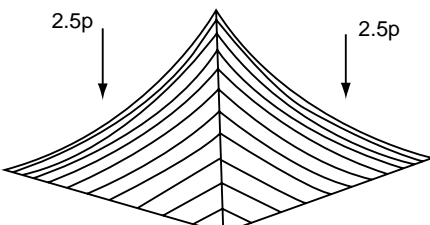
After the form of each stressed spline membrane system is found, it is subjected to a series of loading conditions, which are considered realistic for this study. These conditions are set out in table 1. for the medium span systems, and are expressed in terms of constant p , where $p = 0.20 \text{ kN/m}^2$. Four conditions are considered: the most severe condition in the context of a test for sway stability of the arch splines is condition C (asymmetric snow and wind loading and self weight), and is probably unrealistically severe. Conditions A, B, and D are however, realistic design states for permanent applications. For the spline dome (case study 2) the asymmetric wind loading varies from wind pressure to suction across the dome's

profile in the direction parallel to the wind. In the direction perpendicular to the wind, the wind loading is constant along planes perpendicular to the wind. Snow load does not vary in value and is applied vertically to half or the entire membrane.

5. CASE STUDY 1: MEMBRANE SUPPORTED WITH ONE SPLINE ARCH

A central spline supports a membrane of 16m span and 12m width (see Fig. 1). For purposes of defining an initial starting geometry for the form-finding process, the spline is relaxed from an initially specified 8.4m radius of curvature and a central height of 5m. The spline consists of straight GFRP segments slotted together with splices. The tubular segments have an

Table 1. Loading conditions expressed in terms of p

<p>condition A: asymmetric wind loading and self weight</p> <p>Wind pressure (p) is applied to half of the membrane parallel to the spline(s), while wind suction ($-4p$) is applied to the other half. Additionally self weight of the entire system is applied which takes into account the weight of the membrane, spline(s), plates or clamps.</p>	
<p>condition B: asymmetric snow loading and self weight</p> <p>Snow load ($2.5p$) is applied to half of the membrane parallel to the spline(s), while a different value of snow load (p) is applied to the other half. Self weight of the entire system is also applied.</p>	
<p>condition C: asymmetric snow and wind loading and self weight</p> <p>Snow load ($2.5p$) is applied to one side of the membrane. Wind suction ($-4p$) is applied to the other side of the membrane. The self weight of the entire system is also accounted for.</p>	
<p>condition D: uniform snow loading and self weight</p> <p>This condition applies a uniform downward snow load ($2.5p$) on the whole membrane region as well as the self weight of the entire system.</p>	

outside diameter of 100mm and wall thickness 5mm. This spline has a stiffness identical to a continuous spline with elastic stiffness $EA = 59.690 \text{ MN}$ and bending stiffness $EI = 0.067 \text{ MNm}^2$. The central arch is rather slender (slenderness ratio 599) but is continuously constrained by the pre-stressed membrane through direct contact/connection. The lightweight membrane stabilises the arch laterally and is bounded by two fixed arches. These arches are inclined 70° away from the central arch and have a central height of 2m. The design of the boundary

structure (in this case potentially a cable stiffened glass facade) is not considered within this study. The boundary arches distribute the membrane stresses more evenly than if the membrane was bounded by a few fixed points to the ground.

The form-finding process, the load and buckling analysis for all three case studies are carried out with a non-linear dynamic relaxation (DR) process with kinetic damping. A spline beam element was specifically developed to be incorporated in the DR method in order to study the favourable interaction between the spline and the stressed membrane ([2],[5],[6]).

The form-finding process for the membrane supported with one spline arch is straightforward, no nodes or structural elements besides the two fixed boundary arches need be fixed or restrained. The form of the membrane shapes the arch beneficially; at the spline's crown, where the radius of curvature needs to be reduced, the vertical component of warp membrane stress is larger than at the spline's end segments. It therefore tends to pull the spline down in this region. The system finds a stable equilibrium position which is symmetrical about the arch.

The load and buckling analysis are also carried out with this non-linear DR method. When this arch supported membrane is loaded with the various load

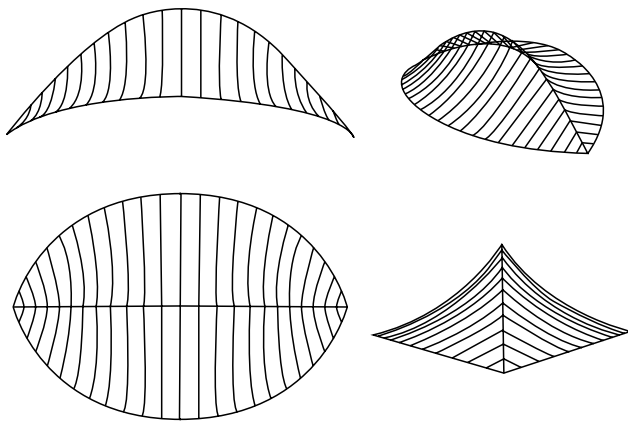


Figure 1. Front and side elevation, perspective view and plan of membrane supported with one arch.

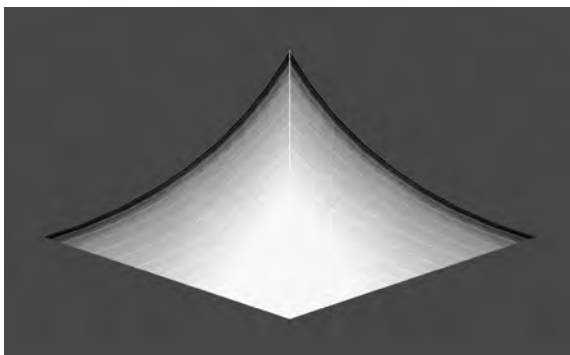


Figure 2. Membrane supported with one arch under self weight.

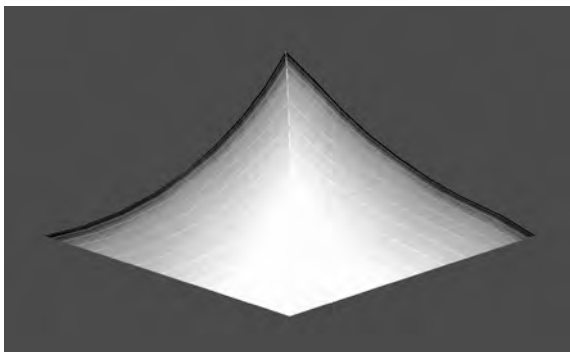


Figure 3. Membrane supported with one arch under loading condition A deflecting out of its initial plane.



Figure 4. Membrane supported with one arch.

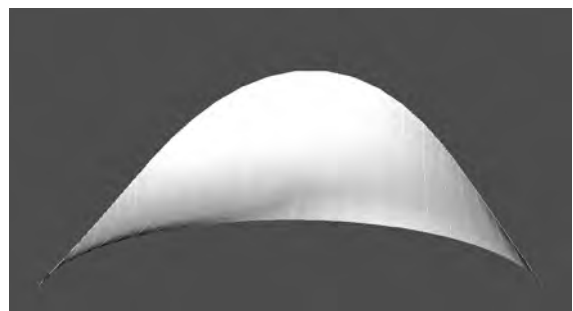


Figure 5. Membrane supported with one arch under loading condition C moving sideways.

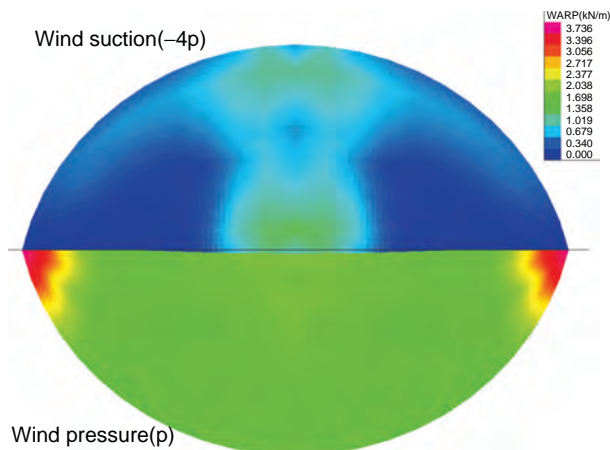


Figure 6. Membrane stress distribution plot under loading condition A. The warp stresses vary from 0.00 kN/m to 3.74 kN/m.

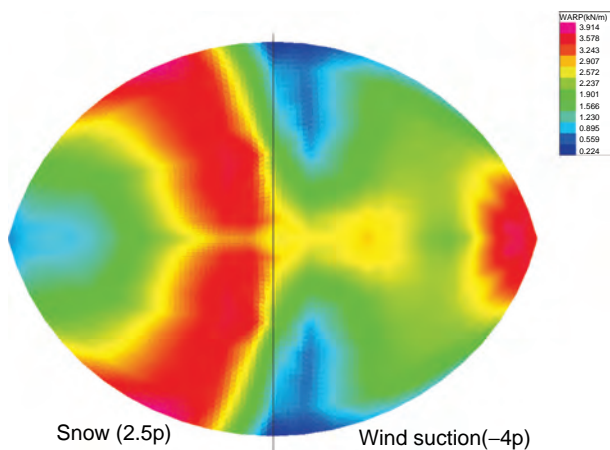


Figure 7. Membrane stress distribution under loading condition C. The warp stresses vary from 0.23 kN/m to 3.92 kN/m.

cases combined bending and axial stresses are found to lie within the allowable material stress limit according to formula (1). Under asymmetric wind loading and self weight (condition A) or asymmetric snow loading and self weight (condition B), the arch deflects laterally out of its initial plane as shown in Figs. 2 and 3. Figs. 4 and 5 show the whole structure moving sideways within the plane of the initial arch under condition C. With an increased value of p (in excess of 0.20 kN/m^2), a point of contraflexure develops and the arch sags. Any further increase in load causes the system to buckle through and adopt an inverted equilibrium position. Under condition D the spline experiences the highest compressive stress. The buckling load under condition C does not exceed twice the design load.

Scaling the structure up to 32m span and the unrelaxed central height to 10m, shows that when

using a central core of 200mm outside diameter and 10mm wall thickness, the system stands up to all loading conditions and the resulting combined axial compressive and bending stresses are within the material's stress limit.

The system's structural behaviour can be improved if the form found shape of the spline is more perfectly circular. This reduces the bending stresses in the pre-stressed state and under the various loading conditions, and can be achieved by varying the stress distribution in the membrane (so that the combination of membrane slope and tension at the spline arch pull it into a circular shape). However a much more beneficial stabilising effect can be achieved by varying the patterning of the membrane so that it acts radially/diagonally to restrict sway of the arch - the stiffer elastic warp stiffness of the membrane restrains the arch sway. When these two methods are combined for the same 32m span structure with the same properties and cross-sectional spline dimensions, it is found that in the pre-stressed state and under the various loading conditions, the spline experiences combined axial compressive and bending stresses well below the values found in the initial 32m span. Greater utility could be provided by extending the arched space with additional arches. For large scale structures (above 50m), a slender arch rib stabilised solely by a pre-stressed membrane is not feasible; the membrane fabric is too flexible to provide adequate bracing. Consequently the arch need to be stiff and can become quite massive. The membrane could be shaped by a spline, which is surrounded by a pre-stressed cable bracing system to provide additional bending stiffness [7].

6. CASE STUDY 2: MEMBRANE SUPPORTED BY SPLINES WITH BRANCHED SPLICES

The spline dome is made of straight splines but slotted together with branched splices onto which 4 or 6 tubular segments are fitted. Fig. 8 shows how this spline configuration triangulates the surface and increases the stiffness as it produces a form of grid shell dome with bending and in-plane stiffness. Two domes with different height/span ratios and member cross-sectional sizes are examined. The initial state geometry of the first dome has a plan diameter of 16m, an arc radius of 11.17m and a central unrelaxed height of 2.61m. Apart from the fixed horizontal boundary ring, no nodes or splines are restrained, fixed or released during form-finding or load analysis. The

maximum combined axial and bending stress occurring in any of the splines in the pre-stressed state is 232 N/mm^2 . The most deformed state which occurs under load condition B, is shown in Figs. 9 and 10. It is worth noting that in spite of the fact that the system is very shallow, under conditions A, B and C this particular dome of 16 m span stands up to over twice the design load before buckling through.

A larger version of the spline dome with a span of 32 m and a more useable unrelaxed central height of 7.85 m is likely to require a heavier membrane (Ferrari 702) with the following properties; direct warp stiffness $E_x = 1 \text{ MN/m}$, direct weft stiffness $E_y = 1 \text{ MN/m}$, assumed cross stiffness $E_c = 0.5 \text{ MN/m}$ and shear modulus 0.05 MN/m . The fabric is pre-stressed to 2 kN/m in the two major directions of the weave. The cross-sectional dimensions of the tubular segments are increased to 200 mm outside diameter and 10 mm wall thickness. The maximum combined spline stress occurring in the form-found shape is 264 N/mm^2 . The way in which the structure responds to the different loading conditions A–D is very similar to the smaller dome of 16 m span. Under conditions A, B and C the 32 m span dome stands up to more than twice the design load before buckling through.

When the radius of the spline dome is increased to 48 m and its unrelaxed central height to 11.25 m , the required GFRP sections sizes (outside diameter of 350 mm and wall thickness 20 mm) needed to carry the axial and shear forces are beyond manufacturing limits [3].

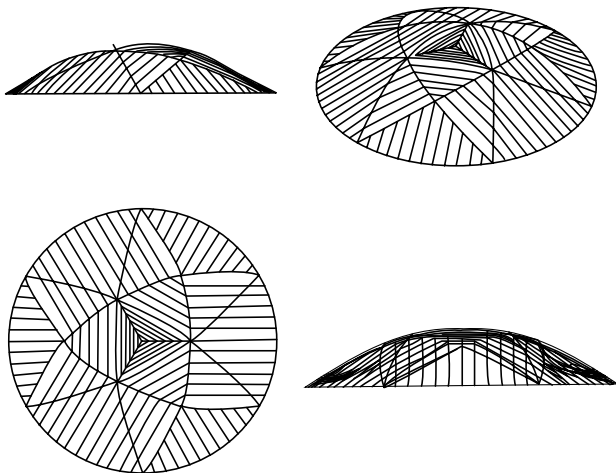


Figure 8. Front and side elevation, perspective view and plan of spline dome.

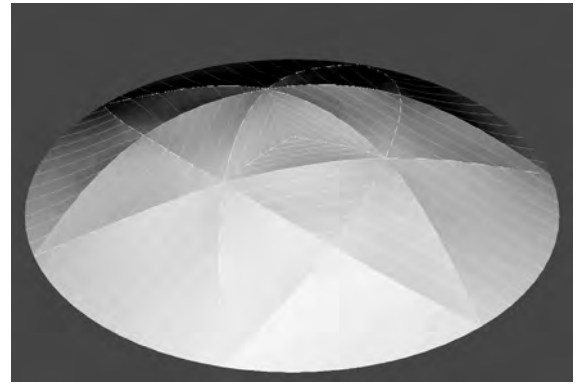


Figure 9. Perspective view of spline dome under self weight.

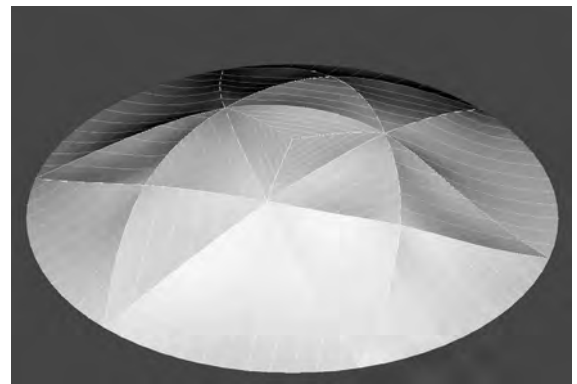


Figure 10. Perspective view of spline dome under loading condition B.

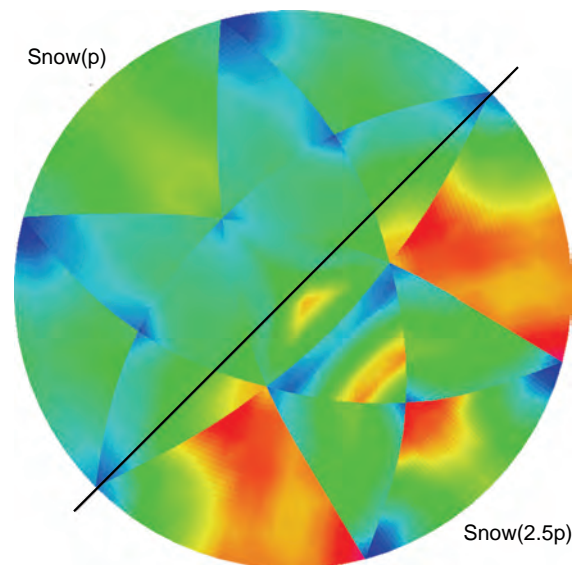


Figure 11. Stress distribution plot of spline dome under loading condition B. The warp stresses under condition B range from 0.00 kN/m to 7.48 kN/m .

7. CASE STUDY 3: CYLINDRICAL GRID SHELL

Attempting to create a cylindrical and useable volume, the splines can form a grid using branched splines. This spline configuration triangulates the lower area of the grid shell (as shown in Fig. 12) and in doing so increases its in-plane stiffness. The grid shell spans $16m$, has a central unrelaxed height of $5.06m$ and is unlimited in length provided it is stabilised by bracing vertical planes at regular intervals e.g. using a fan of cables similar to those developed by J. Schlaich for the Mineral Spa at Stuttgart-Bad Cannstatt [8] or by end diaphragm walls. Again the design of these structures is not considered in this study. A similar curved structural system was patented by Escrig [9]. However his foldable structure consists of pre-bent bars that are connected through scissors hinges. Makowski [10] states that a single barrel vault of this type seldom has a length to span ratio of greater than 2. In this design study two different bay lengths are examined; one of $14.57m$ length fixed between two vertical arches (length/span = 0.91) and one of $14.57m$ length bounded at one end by a fixed vertical arch and

restrained in only the x -direction at the other, which effectively represents a bay of $29.14m$ length between two vertical fixed arches (length/span = 1.82). To set up the initial state geometry, tubular segments of outside diameter $100mm$ and wall thickness $5mm$ are bent to a tight radius of curvature of $8.4m$. When the spline is inclined 60° from the horizontal plane, the maximum height of the grid shell is then $5.06m$ in its unrelaxed state.

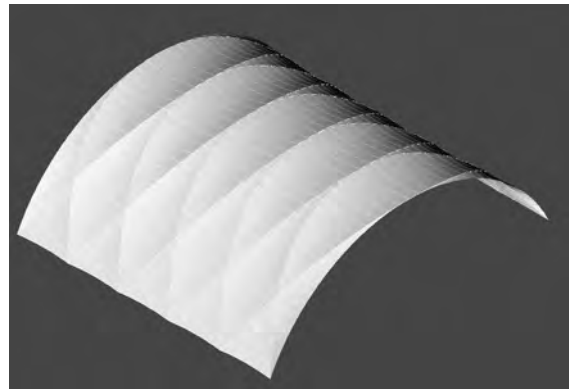


Figure 14. Perspective view of grid shell loading condition C.

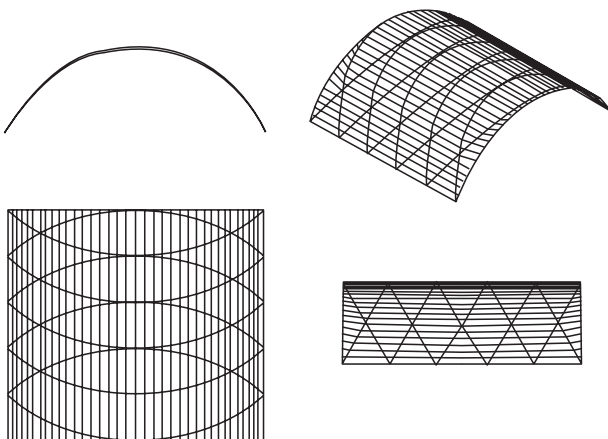


Figure 12. Front and side elevation, perspective view and plan of grid shell.

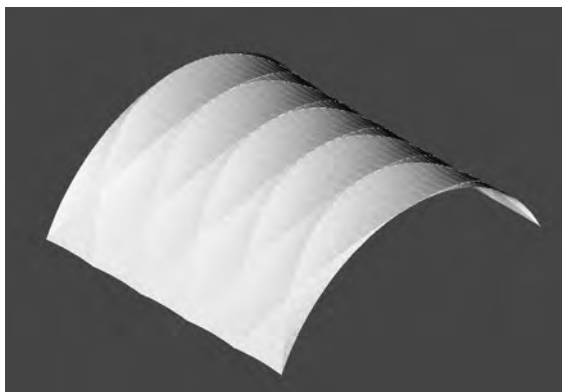


Figure 13. Perspective view of grid shell under self weight.

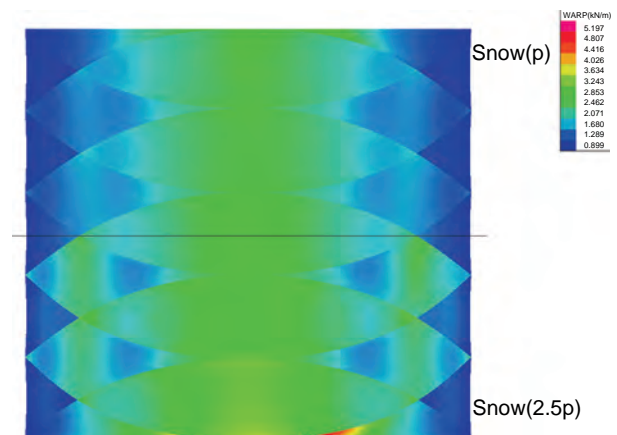


Figure 15. Stress distribution plot of grid shell under loading condition C. The warp stresses under condition B range from 0.699 to 5.197kN/m.



Figure 16. Front elevation of grid shell under self weight.



Figure 17. Front elevation of grid shell under loading condition C.

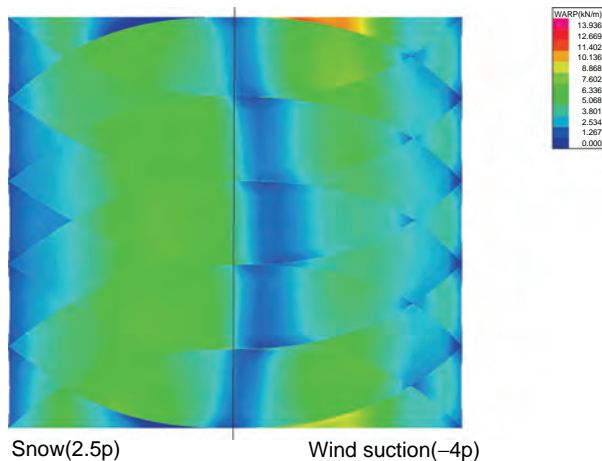


Figure 18. Stress distribution plot of grid shell under self weight and asymmetric snow and wind load (condition C). The warp stresses under condition C range from 0.00kN/m to 13.94kN/m.

As the grid shell does not have a funicular shape, the form-finding process must balance the axial forces and shear forces resulting from bending the splines into the equilibrium shape. The maximum combined spline stress that occurs in the equilibrium shape is 264 N/mm^2 for the 14.57m length grid shell and 239 N/mm^2 for the 29.14m length grid shell. The structural response to the various loading conditions A–D of the two different length grid shell bays is satisfactory. Under load condition B (that is self weight and asymmetric snow load), the splines directly subjected to downward snow load tend to flatten out at the apex of the grid shell and push the shell outwards at the bottom region as shown in Figs. 13–15 for the 14.57m length grid shell. Figs. 16–18 show the structural stability of this grid shell subjected to condition C (snow load on one side of the structure and wind suction on the other). The restrained lengths of the spline straighten out under downward load and deflect within their initial plane. Beyond a bay length of 29.14m , additional

structural elements would be necessary in order to increase the grid shell's bending stiffness and its resistance to load condition C. The layout of the splines in this cylindrical grid shell makes the introduction of diagonal cables within the shell plane difficult. In addition, the spline's bending EI and elastic stiffness EA cannot be much increased by using a larger cross-section for the tubular segments. This would increase the limiting radius of curvature and thus reduce the height of the grid shell. The structure could however be improved by moving the splines closer together so that there are more spline intersections and more triangulated areas. In this layout the splines can be more slender and the central unrelaxed height of the grid shell raised.

8. CONCLUSION

The introduction of GFRP as spline material opens a new realm of medium span (16m – 32m) spline supported membrane structures. The GFRP tubular splines can be rather slender (common slenderness ratio 600) as they are continuously restrained by the membrane through direct contact/connection. Due to GFRP's low Young's Modulus and high allowable bending stress, the spline configurations can be bent into the required shape without overstressing the splines. The three proposed systems exhibited good structural response under realistic loading conditions A–D. Structure 1 introduced the concept of splines fitted together with straight splices. It was found that small design changes to these systems could significantly improve their buckling behaviour especially under the severe sway condition C. The spline could be effectively stiffened by re-arranging the membrane patterning so that the warp direction of the fabric is orientated more radially/diagonally to the spline (rather than the admittedly more economic traverse patterning). Additionally the shape of the spline(s) could be structurally improved by varying the pre-stress throughout the membranes. Structure 2 improved the system's in-plane stiffness through triangulation using branched splices. The GFRP spline systems were successfully concluded with the design of a cylindrical space, as would be required for certain sport activities. The structural system of the spline grid shell consisted of closely spaced splines interlinked through branched splines. Through the various branched intersections, the unrestrained length over which the spline can buckle is greatly reduced. Thus the splines can be quite slender and therefore bent to a tighter radius of curvature. This design study of the three medium span spline systems presented elegant

shaping elements for membranes and demonstrated their feasibility.

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