International Journal of Systems Science

Optimal payment time with deteriorating items under inflation and permissible delay in payments
Chun-Tao Chang *, Shuo-Jye Wu *, Li-Ching Chen *
* Department of Statistics, Tamkang University, Taiwan 251, R.O.C.

Online Publication Date: 01 October 2009

To cite this Article Chang, Chun-Tao, Wu, Shuo-Jye and Chen, Li-Ching(2009)'Optimal payment time with deteriorating items under inflation and permissible delay in payments', International Journal of Systems Science, 40:10, 985 — 993

To link to this Article DOI: 10.1080/00207720902974561
URL: http://dx.doi.org/10.1080/00207720902974561

PLEASE SCROLL DOWN FOR ARTICLE
Optimal payment time with deteriorating items under inflation and permissible delay in payments

Chun-Tao Chang*, Shuo-Jye Wu and Li-Ching Chen

Department of Statistics, Tamkang University, Tamsui, Taipei, Taiwan 251, R.O.C.

(Received 9 November 2007; final version received 13 March 2009)

This article develops an inventory model under a situation in which the supplier provides the purchaser with a permissible delay of payments. Shortages are not allowed and the effect of the inflation rate, deterioration rate and delay in payment are discussed as well. As a result, in this article, we establish a mathematical model to determine the optimal payment period and replenishment cycle. Using Taylor series approximation, we characterise the optimal solution and provide an easy-to-use algorithm to find the optimal solution. Finally, the proposed models are illustrated through numerical examples and the sensitivity analysis is reported.

Keywords: inventory; finance; delay payments; payment period; deteriorating items

1. Introduction

In real life, deterioration of many items during storage period is a fact, such as chemicals, IC chip, volatile liquids, blood banks, medicines, electronic components and so forth. In general, deterioration is defined as the decay, damage, spoilage, evaporation and obsolescence of stored items and it results in decreasing usefulness. Therefore, the control and maintenance of inventories of deteriorating items becomes an important issue for decision makers in modern organisations. Several researchers have studied deteriorating inventory in the past. Ghare and Schrader (1963) were the first proponents to establish a model for an exponentially decaying inventory. Covert and Philip (1973) extended Ghare and Schrader’s constant deterioration rate to a two-parameter Weibull distribution. Goyal and Giri (2001) also provided a detailed review of deteriorating inventory literatures. Other interesting papers dealing with the same issue include Shah and Jaiswal (1977), Aggarwal (1978), Dave and Patil (1981), Sachan (1984), Hariga (1996), Shah and Shah (1998), Chang and Dye (2001), Skouri and Papachristos (2003), Chang (2004b), De and Goswami (2006), Shah (2006), Yang and Wee (2006) and Taso and Sheen (2007).

In the conventional economic order quantity (EOQ) model, it is implicit that the purchaser must pay for the items as soon as the items are received. However, in practice, the supplier may offer his/her customers a certain credit period, that is, during the permissible delay time period, the customer does not need to pay any interest. Thus, the delay payments to the supplier become a kind of price discount. Since paying later indirectly reduces the purchase cost, it can motivate customers to increase their order quantity. The related articles of permissible delay in payments have been considered by several researchers. Goyal (1985) derived an EOQ model under the conditions of permissible delay in payments. Aggarwal and Jaggi (1995) then extended Goyal’s model to allow deteriorating items. Jamal, Sarker and Wang (1997) further generalised the model to allow shortages. Teng (2002) amended Goyal’s model by considering the difference between unit price and unit cost, and believed that it makes economic sense for a well-established buyer to order less quantity but increase the times of placing order so as to take the benefits from the permissible delay. Chang, Ouyang and Teng (2003) then extended Teng’s model, and established an EOQ model for deteriorating items: in the model, the supplier provides a permissible delay for the buyers only when the order quantity is greater than or equal to a predetermined quantity. For other studies of trade credits please refer to Davis and Gaither (1985), Arcels and Srinivasan (1993), Shah (1993), Khouja and Mehrrez (1996), Hwang and Shinn (1997), Liao, Tsai and Su (2000), Sarker, Jamal and Wang (2000a), Chang and Dye (2001), Chang, Hung and Dye (2002) Chang and Wu (2003), Huang (2005), Ouyang, Chang and Teng (2005), Teng, Chang and Goyal (2005), Ouyang, Teng and Chen (2006), Shah (2006), Teng, Chang, Chern and Chan (2007) and Huang and Huang (2008).

*Corresponding author. Email: chuntao@stat.tku.edu.tw

ISSN 0020–7721 print/ISSN 1464–5319 online
© 2009 Taylor & Francis
DOI: 10.1080/00207720902974561
http://www.informaworld.com

Downloaded By: [Tamkang University] At: 02:32 23 October 2009
The effects of inflation are disregarded in some inventory models, because most decision makers think that inflation does not have significant influence on the inventory policy. However, the inventory systems always need to invest large capital to purchase inventories, and this purchasing is highly correlated to the return of investment. Thus, it is important to consider the effects of inflation on the inventory system. Buzacott (1975), Bierman and Thomas (1977) and Misra (1979) investigated the inventory decisions under an inflationary condition. Liao et al. (2000) presented a model with deteriorating items under inflation when a delay in payment is permissible. Chang (2004a) derived an EOQ model with deteriorating items under inflation when the supplier credits are linked to order quantity. There are some other papers related to this topic, such as Brahmbhatt (1982), Chandra and Bahner (1985), Datta and Pal (1991), Hariga (1995), Ray and Chaudhuri (1997), Chen (1998), Sarker, Jamal and Wang (2000b), Chang et al. (2002), De and Goswami (2006), Hou and Lin (2006) and Shah (2006).

Most previous studies (such as Goyal (1985), Teng (2002), Ouyang et al. (2006) and others) dealing with the inventory problems under permissible delay in payments discussed the case that the purchaser pays the supplier at the end of the credit period and the interest charges are always greater than the interest earned. However, in today’s business transaction, the purchaser may invest the money on the stock markets or to develop new products, and get a return from the investment which may be higher than interest charges. Thus, the purchaser may not pay the supplier at the end of the credit period; instead, he/she will invest the money until the interest payable to the supplier is larger than the interest earned. Consequently, it is more practical for the purchaser either to pay the supplier at the end of the credit period or to incur interest charges on the unpaid balance for the overdue period. However, the issue of how purchasers determine their payment time is disregarded in the aforementioned articles. As a result, we focus on this issue and incorporate all phenomena described above to develop an extended EOQ model. In the developed model, the effect of the inflation rate, deterioration rate and delay in payment are discussed simultaneously. The objective of the proposed model is to determine the optimal payment time and replenishment cycle for a purchaser to minimise the total cost. Therefore, our proposed research topic is an interesting and relevant issue in inventory management system. It is unprecedented as well.

The rest of this article is organised as follows. We describe the notation and assumptions used throughout this study in Section 2. The limitations of this model are implied in these assumptions. In Section 3, the mathematical models are derived under two different circumstances in order to minimise the total cost in the planning horizon. Furthermore, we study the necessary and sufficient conditions for finding the optimal solution to the problem. An algorithm is thus developed to find the optimal payment time and replenishment cycle. In Section 4, numerical examples are provided to demonstrate the applicability of the proposed model and the sensitivity analysis is conducted. In Section 5, we draw some conclusions.

2. Notation and assumptions

The following notation and assumptions are used throughout this article.

Notation:

- \( H \) = the length of planning horizon
- \( D \) = the demand per unit time
- \( h \) = the holding cost rate per unit time excluding interest charges
- \( r \) = constant rate of inflation per unit time, where \( 0 \leq r < 1 \)
- \( p(t) = pe^{rt} \) = the selling price per unit at time \( t \), where \( p \) is the unit selling price at time zero
- \( c(t) = ce^{rt} \) = the unit purchasing cost at time \( t \), where \( c \) is the unit purchasing price at time zero and \( c < p \)
- \( S(t) = Se^{rt} \) = the ordering cost per order at time \( t \), where \( S \) is the ordering cost at time zero
- \( I_d \) = the interest charged per $ in stocks per year by the supplier
- \( I_e \) = the interest earned per $ per year
- \( M \) = the permissible delay in settling account (i.e. the trade credit period)
- \( Q \) = the order quantity
- \( K \) = the period of payment (i.e. the payment time)
- \( \theta \) = the constant deterioration rate, where \( 0 \leq \theta < 1 \)
- \( I(t) \) = the level of inventory at time \( t, 0 \leq t \leq T \)
- \( T \) = the replenishment cycle

\( TC(T, K) \) = the total relevant cost over \( (0, H) \).

Note that the total relevant cost consists of (a) cost of placing orders, (b) cost of purchasing, (c) cost of carrying inventory excluding interest charges, (d) cost of interest charges for unsold items at the initial time or after the permissible delay \( M \),
and (e) interest earned from sales revenue during the permissible period.

**Assumptions:**

(1) The demand for the item is known and is a constant.
(2) The inflation rate \( r \) is a constant. In reality, the value of \( r \) is very small.
(3) Shortages are not allowed.
(4) Replenishment is instantaneous.
(5) A constant deterioration rate \( \theta \) is a fraction of the on-hand inventory, in reality, the value of \( \theta \) is very small. It deteriorates per unit time and the on-hand inventory, in reality, the value of \( \theta \) is very small, hence, the high inflation rate and the perishable items with fast deterioration rate are not considered in the article.

(6) During the trade credit period \( M \), the account is not settled, and generated sales revenue is deposited in an interest-bearing account. At the end of the permissible delay, purchasers have two choices when they pay the supplier: they can determine to pay off either at the end of the credit period \( M \) (i.e. \( K = M \)) or at any time between \( M \) and \( T \) (i.e. \( M < K < T \)). The purchaser pays off for all units ordered, and starts paying for the interest charges on the items in stocks when he/she pays the supplier at time \( M \). However, he/she must pay the supplier for the interest charges if he/she chooses the payment time between \( M \) and \( T \).

Note that, in reality, the value for the deterioration rate \( \theta \) or inflation rate \( r \) is usually very small, hence, the limitations are adopted in this article. Thus, the high inflation rate and the perishable items with fast deterioration rate are not considered in the article.

### 3. Model and analysis

#### 3.1. Mathematical model

We assume that the length of planning horizon \( H = nT \), where \( n \) is an integer for the number of replenishments to be made during period \( H \), and \( T \) is an interval of time between replenishments. The level of inventory \( I(t) \) gradually decreases mainly to meet demands and partly due to deterioration. Hence, the variation of inventory with respect to time can be described by the following differential equation:

\[
\frac{dI(t)}{dt} + \theta I(t) = -D, \quad 0 \leq t \leq T, \tag{1}
\]

with the boundary conditions: \( I(0) = Q \) and \( I(T) = 0 \). Consequently, the solution of (1) is given by

\[
I(t) = \frac{D}{\theta} \left[e^{\theta T} - 1\right], \quad 0 \leq t \leq T, \tag{2}
\]

and the order quantity is

\[
Q = I(0) = \frac{D}{\theta} \left(e^{\theta T} - 1\right). \tag{3}
\]

Since the lengths of time intervals are all the same, we have

\[
I(iT + t) = \frac{D}{\theta} \left[e^{\theta (T-t)} - 1\right], \quad 0 \leq i \leq n - 1, \quad 0 \leq t \leq T, \tag{4}
\]

by using (2) and (3).

The total relevant cost in \((0, H)\) consists of the following elements.

(i) Cost of placing orders

\[
S(0) + S(T) + S(2T) + \cdots + S((n-1)T) = S \left(\frac{e^H - 1}{e^T - 1}\right). \tag{5}
\]

(ii) Cost of purchasing

\[
Q[c(0) + c(T) + c(2T) + \cdots + c((n-1)T)] = \frac{cD}{\theta} \left(e^{\theta T} - 1\right) \left(e^{H} - 1\right). \tag{6}
\]

(iii) Cost of carrying inventory

\[
h \sum_{i=0}^{n-1} c(iT) \int_{0}^{T} I(iT + t) dt = \frac{hcD}{\theta^2} \left(e^{\theta T} - \theta T - 1\right) \left(e^{H} - 1\right). \tag{7}
\]

where \( I(iT + t) \) is defined in (4).

(iv) Regarding interests charged and earned (i.e. costs of (d) and (e)), there are two possible cases: (1) \( T < M \), and (2) \( T \geq M \). These two cases are depicted graphically in Figure 1.

**Case 1**: \( T < M \)

Since the replenishment cycle \( T \) is shorter than the permissible period \( M \), the purchaser will pay the supplier at the end of the trade credit period \( M \) (i.e. \( K = M \)). Hence, no interest charge is paid for the items. At the same time, the purchaser uses the sales revenue to earn interest at the rate of \( I_d \) during the period \([0, T]\). As a result, the interest earned in \((0, H)\) is

\[
I_d \sum_{i=0}^{n-1} p(iT) \left[\int_{0}^{T} Dt dt + DT(M - T)\right] = pI_d D \left(\frac{T^2}{2} + \frac{e^{H} - 1}{e^T - 1}\right). \tag{8}
\]
In addition, the interest earned during the positive inventory in \((0, H)\) is
\[
I_d \sum_{i=0}^{n-1} c(iT) \int_{iT}^{i+1} l(t) dt = \left[ \frac{c_l I_d}{\theta^2} \left( e^{\theta(T-K)} - e^{\theta(T-M)} \right) \right] \left( e^{\frac{H}{rT}} - 1 \right).
\]

Therefore, the total relevant cost in \((0, H)\) is
\[
TC_1(T) = \left[ S + \frac{cD}{\theta} (e^{\theta T} - 1) + \frac{hcD}{\theta^2} (e^{\theta T} - \theta T - 1) - \frac{c_l I_d}{\theta^2} D \left( TM - \frac{T^2}{2} \right) \right] \left( e^{\frac{H}{rT}} - 1 \right).
\]

**Case 2:** \(T \geq M\)

In this case, the replenishment cycle \(T\) is longer than or equal to \(M\). Because the purchaser pays the supplier at time \(K\), where \(M \leq K \leq T\), he/she must pay the supplier for the interest charges during \([M, K]\). Therefore, the interest payable for the inventory not being sold after the due date \(M\) in \((0, H)\) is
\[
I_d \sum_{i=0}^{n-1} c(iT) \int_{iT}^{i+1} l(t) dt \approx \left[ \frac{c_l I_d}{\theta} (e^{\theta(T-M)} - e^{\theta(T-K)}) \right] \left( e^{\frac{H}{rT}} - 1 \right).
\]

**Theoretical results and solution**

The problem is to find an optimal payment time and replenishment cycle, which minimise the total relevant cost. Although the function of total relevant cost is differentiable, the resulting equation is intractable. Therefore, it is difficult to find an optimal solution and it is impossible to obtain the optimal solution in explicit form. Note that, in reality, the value for the deterioration rate \(\theta\) or inflation rate \(r\) is usually very small and, hence \(\theta T, \theta M, \theta K\) or \(rT\) is very small. Hwang and Shinn (1997), Liao et al. (2000) and Teng et al. (2005) all used a truncated Taylor series expansion for the exponential term. Thus, we have
\[
e^{\theta T} \approx 1 + \theta T + (\theta T)^2/2,
\]
\[
e^{\theta(T-M)} \approx 1 + \theta(T - M) + \theta^2(T - M)^2/2,
\]
\[
e^{\theta(T-K)} \approx 1 + \theta(T - K) + \theta^2(T - K)^2/2,
\]
and
\[
e^{rT} \approx 1 + rT + (rT)^2/2.
\]

Using the above approximation, the total relevant cost for Case 1 \(TC_1(T)\) can be rewritten as
\[
TC_1(T) \approx \left[ S + D(c - p I_d M)T + D(c \theta + hc + p I_d) \frac{T^2}{2} \right] \times \left[ 2(e^{\frac{H}{rT}} - 1) \right].
\]
The first-order condition for $TC_1(T)$ in (17) is $dTC_1(T)/dT = 0$, which leads to
\[
T_1 = \frac{Sr + \sqrt{(Sr)^2 + 2SD\Delta_1}}{D\Delta_1},
\]
(18)
where $\Delta_1 = c(h + \theta - r) + pI_d(1 + Mr)$. The second-order condition is
\[
\frac{d^2TC_1(T)}{dT^2} \bigg|_{T=T_1} = \frac{2(\varepsilon^H - 1)}{r(2T_1 + rT_1^2)^2}[2D\Delta_1T_1 - 2Sr] > 0.
\]
(19)
Therefore, $T_1$ is the optimal value of $T$ for Case 1. Substituting (18) into (3), we get the optimal economic order quantity
\[
Q^*(T_1) = \frac{D}{\theta}(e^{\theta T_1} - 1).
\]
(20)
Substituting (18) into (9), the total relevant cost for Case 1 $TC_1(T_1)$ is obtained. To ensure that $T_1 < M$, we substitute (18) into inequality $T_1 < M$, and obtain that
\[
\text{if } 2S(1 + rM) < D\Delta_1M^2, \text{ then } T_1 < M.
\]
(21)
By using an analogous argument for Case 2, the total relevant cost for Case 2 $TC_2(K, T)$ can be rewritten as
\[
TC_2(K, T) \approx \left\{ \begin{array}{ll}
S + cDT + & \frac{cD(\theta + h)}{2}T^2 - \frac{cI_d D}{2}[|(T - K)^2}
\end{array} \right.
\]
\[
- (T - M)^2 + I_d \left[ (cDT - pDK)(T - K)
\right]
\]
\[
- \frac{pD}{2}T^2 \right) \left\{ \left( \frac{2(\varepsilon^H - 1)}{r(2T + rT^2)^2} \right) \right\},
\]
(22)
where $M \leq K \leq T$.

Now the problem is to determine the optimal values of $K$ (say $K^*$) and $T$ (say $T_2$) so that $TC_2(K, T)$ is minimum. The necessary conditions for $TC_2(K, T)$ to be minimum are
\[
\frac{\partial TC_2(K, T)}{\partial K} = 0 \quad \text{and} \quad \frac{\partial TC_2(K, T)}{\partial T} = 0,
\]
(23)
which in turn imply
\[
(2pI_d - cI_2)K - [(p + c)I_d - cI_2]T = 0,
\]
(24)
and
\[
\Delta_2 + \Delta_3\frac{2(1 + rT)}{(2T + rT^2)} = 0,
\]
(25)
where $\Delta_2 = D[c + (c(\theta + h) + (2c - p)I_d)T + (cI_c - (p + c)I_d)K - cI_2M]$, and
\[
\Delta_1 = \left\{ \begin{array}{ll}
S + cDT + & \frac{cD(\theta + h)}{2}T^2 - \frac{cI_d D}{2}[|(T - K)^2}
\end{array} \right.
\]
\[
- (T - M)^2 + I_d \left[ (cDT - pDK)(T - K)
\right]
\]
\[
- \frac{pD}{2}T^2 \right) \left\{ \left( \frac{2(\varepsilon^H - 1)}{r(2T + rT^2)^2} \right) \right\},
\]
\[
\frac{2I_d/I_c > c/p,}
\left\{ \begin{array}{ll}
[c(\theta + h) + (2c - p)D + \Delta_2/[(1 + rT)(2T + rT^2)] > 0,
\end{array} \right.
\]
\[
(2pI_d - cI_2)[(c(\theta + h) + (2c - p)D + \Delta_2 /
\right]
\end{array} \right.
\]
\[
/[(1 + rT)(2T + rT^2)] - [cI_c - (p + c)I_d]D > 0.
\]
(26)

Hence, if the solution $(K, T)$ satisfies Equation (26) and the constraint $M \leq K \leq T$, then the optimal solution $(K^*, T_2)$ is obtained. Since $TC_2(K, T)$ is a complicated function, it is not possible to show analytically the validity of the above sufficient conditions. Therefore, the sign of the quantities in Equation (26) are assessed numerically. Substituting $K^*$ and $T_2$ into Equations (3) and (12), the optimal economic order quantity is
\[
Q^*(T_2) = \frac{D}{\theta}(e^{\theta T_2} - 1),
\]
(27)
and the total relevant cost for Case 2 $TC_2(K^*, T_2)$ can be found.

3.3. An algorithm
Based on the discussion in Section 3.2, we have developed the following algorithm to determine the optimal payment time, replenishment cycle and order quantity.

**Step 1:** If $2S(1 + rM) < D\Delta_1M^2$, then the optimal replenishment cycle $T^* = T_1$ and the optimal payment time $K^* = M$. Hence, the optimal economic order quantity and the total relevant cost can be obtained by substituting $T_1$ and $M$ into (20) and (9), respectively. Otherwise, go to Step 2.

**Step 2:** Solving (24) and (25), we get the solution $(K, T)$.

**Step 3:** If the solution $(K, T)$ satisfies Equation (26) and the condition $M \leq K \leq T$, then the optimal replenishment cycle $T^* = T_2$ and the optimal payment time $K^* = K$. Hence, the optimal economic order quantity and the total relevant cost can be obtained.
Table 1. Sensitivity analysis on $S$.

<table>
<thead>
<tr>
<th>Ordering cost $S$</th>
<th>Payment time $K^*$</th>
<th>Replenishment cycle $T^*$</th>
<th>Economic order quantity $Q(T^*)$</th>
<th>Total relevant cost $TC(K^<em>, T^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.08192</td>
<td>$T_1 = 0.048805$</td>
<td>48.8407</td>
<td>979.5953</td>
</tr>
<tr>
<td>10</td>
<td>0.08192</td>
<td>$T_1 = 0.069042$</td>
<td>69.1131</td>
<td>1392.0624</td>
</tr>
<tr>
<td>15</td>
<td>0.092387</td>
<td>$T_2 = 0.138580$</td>
<td>138.8688</td>
<td>2799.1109</td>
</tr>
<tr>
<td>20</td>
<td>0.102785</td>
<td>$T_2 = 0.154178$</td>
<td>154.5347</td>
<td>3120.1583</td>
</tr>
<tr>
<td>25</td>
<td>0.112230</td>
<td>$T_2 = 0.168345$</td>
<td>168.7707</td>
<td>3412.8240</td>
</tr>
</tbody>
</table>

Table 2. Sensitivity analysis on $M$.

<table>
<thead>
<tr>
<th>Credit period $M$</th>
<th>Payment time $K^*$</th>
<th>Replenishment cycle $T^*$</th>
<th>Economic order quantity $Q(T^*)$</th>
<th>Total relevant cost $TC(K^<em>, T^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.045739</td>
<td>$T_2 = 0.068608$</td>
<td>68.6788</td>
<td>1382.2366</td>
</tr>
<tr>
<td>10</td>
<td>0.047922</td>
<td>$T_2 = 0.071884$</td>
<td>71.9611</td>
<td>1447.0425</td>
</tr>
<tr>
<td>15</td>
<td>0.051359</td>
<td>$T_2 = 0.077039$</td>
<td>77.1279</td>
<td>1549.9145</td>
</tr>
<tr>
<td>20</td>
<td>0.054795</td>
<td>$T_1 = 0.048814$</td>
<td>48.8493</td>
<td>982.1773</td>
</tr>
<tr>
<td>30</td>
<td>0.082192</td>
<td>$T_1 = 0.048805$</td>
<td>48.8407</td>
<td>979.5953</td>
</tr>
<tr>
<td>40</td>
<td>0.109589</td>
<td>$T_1 = 0.048796$</td>
<td>48.8321</td>
<td>977.0174</td>
</tr>
</tbody>
</table>

by substituting $T_2$ and $K^*$ into (27) and (12), respectively.

4. Numerical examples and sensitivity analysis

The proposed model can be applied to some examples of deteriorating items in real life. For example, in order to produce a 64-Mega DRAM, an IC packaging factory needs a moulding compound. The consumption rate of the moulding compound depends on the stock and the deterioration rate resulting from inappropriate storage, temperature or expiration date. Hence, the inventory of the moulding compound gradually decreases mainly to meet demand and partly due to deterioration. Now, let us consider the following numerical examples to illustrate the proposed model and study sensitivity of the optimal solution to change in the values of the different parameters.

**Example 1:** The total relevant cost is a function of ordering cost. Thus, it is affected by the ordering cost. In order to find the influence of different ordering costs on the optimal solution, we set ordering costs $5, 10, 15, 20$ or 25 per order. In addition, let $H = 1$ year, $D = 1000$ units/year, $h = 0.12$/year, $I_c = 0.09$/year, $I_d = 0.06$/year, $r = 0.03$ per unit, $c = $20 per unit, $p = $30 per unit, $\theta = 0.03$ and $S = $5 per order. When $M = 5, 10$ or 15 days (i.e. 0.013699, 0.027397 or 0.041096 years), it is easy to see that $2S(1 + rM) > \Delta_1 M^2$. Using the algorithm, we obtain the optimal replenishment cycle $T^* = T_2$ and the optimal payment time $K^*$ can be determined by Equations (24) and (25). When $M = 20, 30$ or 40 days (i.e. 0.054795, 0.082192 or 0.109589 years), we get that $2S(1 + rM) < \Delta_1 M^2$. Hence, the optimal replenishment cycle $T^* = T_1$ and the optimal payment time $K^* = M$. The computational results are shown in Table 2. The following phenomena are indicated in Table 2. (1) A higher value of credit period $M$ causes higher values of total relevant cost $TC(K^*, T^*)$, order quantity $Q(T^*)$, payment time $K^*$ and replenishment cycle $T^*$ when $M = 5, 10$ or 15 days. (2) A higher value of credit period $M$ implies lower values of total relevant cost $TC(K^*, T^*)$, order quantity $Q(T^*)$ and replenishment cycle $T^*$, but a higher value of payment time $K^*$ when $M = 20, 30$ or 40 days.

**Example 3:** In order to examine the inflationary effect on the optimal solution, we consider the inflation
rate $r = 0.01, 0.02, 0.03, 0.04$ or $0.05$. Given $H = 1$ year, $D = 1000$ units/year, $h = 0.12$/year, $I_c = 0.09$/S/year, $I_d = 0.06$/S/year, $c = 20$ per unit, $p = 30$ per unit, $\theta = 0.03$, $M = 30$ days $= 0.082192$ years and $S = 15$ per order. Using the proposed algorithm, the computational results are obtained and shown in Table 3. They indicate that a higher value of inflation rate $r$ causes higher values of order quantity $Q(T^*)$, payment time $K^*$, replenishment cycle $T^*$ and total relevant cost $TC(K^*, T^*)$.

### Table 3. Sensitivity analysis on $r$.

<table>
<thead>
<tr>
<th>Inflation rate $r$</th>
<th>Payment time $K^*$</th>
<th>Replenishment cycle $T^*$</th>
<th>Economic order quantity $Q(T^*)$</th>
<th>Total relevant cost $TC(K^<em>, T^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.082192</td>
<td>$T_1 = 0.080777$</td>
<td>80.8749</td>
<td>1634.2565</td>
</tr>
<tr>
<td>0.02</td>
<td>0.088422</td>
<td>$T_2 = 0.132633$</td>
<td>132.8973</td>
<td>2678.7836</td>
</tr>
<tr>
<td>0.03</td>
<td>0.092387</td>
<td>$T_3 = 0.138580$</td>
<td>138.8688</td>
<td>2799.1109</td>
</tr>
<tr>
<td>0.04</td>
<td>0.096931</td>
<td>$T_4 = 0.145396$</td>
<td>145.7135</td>
<td>2937.1320</td>
</tr>
<tr>
<td>0.05</td>
<td>0.102209</td>
<td>$T_5 = 0.153314$</td>
<td>153.6669</td>
<td>3097.6374</td>
</tr>
</tbody>
</table>

5. Conclusions

The proposed model incorporates some realistic features that are likely to be associated with some types of inventory. These features include deterioration, inflation and delay payment. First, deterioration of many items during storage period is a real fact. Next, from a financial point of view, inventory represents a capital investment and must compete with other assets because of a firm’s limited capital funds. Hence, the effects of inflation on the inventory system cannot be ignored. Finally, in real-life situations, the supplier frequently offers a permissible delay in payments to the customers especially when the economy turns sour. As a result, we develop an extended EOQ model under inflation for deteriorating items to determine the optimal inventory policy, when the supplier provides a permissible delay in payments. In addition, the inflation rate and deterioration rate are assumed to be very small. We then use Taylor series approximation to find the optimal solution. Moreover, an easy-to-use algorithm is developed to find the optimal payment time and optimal replenishment cycle. Some numerical examples are studied to illustrate the proposed model. The sensitivity of the solution to changes in the values of different parameters has also been discussed. It is seen that a higher value of ordering cost causes higher values of order quantity, replenishment cycle and total relevant cost. Moreover, a higher value of inflation rate implies higher values of order quantity, payment time, replenishment cycle and total relevant cost.

For further research, this article can be extended in several ways. For instance, we may extend the constant deterioration rate to a two-parameter Weibull distribution. Also, we could consider the demand as a function of quality as well as price. Finally we could generalise the model to allow for shortages, quantity discounts, and others.

### Acknowledgements

The authors are grateful to the Associate Editor and two anonymous referees for their encouragement and constructive comments. The work of the first author was partially supported by the National Science Council of ROC Grant NSC 97-2410-H-032-023.

### Notes on contributors

**Shuo-Jye Wu** is the Professor of Statistics at Tamkang University. He obtained his PhD in Statistics from the University of Wisconsin, Madison. His professional interests are in the development and application of statistical methodology for problems in reliability. He has published many papers in refereed journals including IEEE Transactions on Reliability, Reliability Engineering & System Safety, Statistics Sinica, International Journal of Advanced Manufacturing Technology and Production Planning & Control. He is a member of the American Statistical Association.


Shuo-Jye Wu is the Professor of Statistics at Tamkang University. He obtained his PhD in Statistics from the University of Wisconsin, Madison. His professional interests are in the development and application of statistical methodology for problems in reliability. He has published many papers in refereed journals including IEEE Transactions on Reliability, Reliability Engineering & System Safety, Statistics Sinica, International Journal of Advanced Manufacturing Technology and Production Planning & Control. He is a member of the American Statistical Association.
References


Li-Ching Chen is a Assistant Professor at Tamkang University. She obtained her PhD in Statistics from National Central University. Her research interests are in the statistical inference and application for biostatistics and genetic epidemiology. She has published some papers in Journal of Statistical Planning and Inference.

C.-T. Chang et al.
Appendix

To ensure that the objective function $TC_3(K, T)$ is convex, the solution $(K, T)$ from Equations (24) and (25) must satisfy the following sufficient condition:

$$
\begin{align*}
\frac{\partial^2 TC_3(K, T)}{\partial K^2}|_{(K, T)} & > 0, \\
\frac{\partial^2 TC_3(K, T)}{\partial T^2}|_{(K, T)} & > 0,
\end{align*}
$$

(A1)

From (22), we get

$$
\frac{\partial^2 TC_3(K, T)}{\partial K^2} = \frac{2(e^h - 1)}{r(2T + rT^2)} D(2pI_d - cI_f),
$$

(A2)

and

$$
\frac{\partial^2 TC_3(K, T)}{\partial T^2} = \left[ \frac{2(e^h - 1)}{r(2T + rT^2)} \right] D[cI_f - (p + c)I_d].
$$

(A3)

Substituting $(K, T)$ into (A1) and using (A2)-(A4), we know

$$
(2pI_d - cI_f) > 0 \Rightarrow 2I_d/I_f > c/p,
$$

(A5)

and

$$
(2pI_d - cI_f)[c(\theta + h) + (2 - p)I_d]D + \Delta_2/((1 + rT)(2T + rT^2)) > 0,
$$

(A6)

and

$$
(2pI_d - cI_f)[c(\theta + h) + (2 - p)D + \Delta_2/((1 + rT)(2T + rT^2))] - [cI_f - (p + c)I_d]^2 D > 0.
$$

(A7)

Hence, the sufficient condition is

$$
\begin{align*}
2I_d/I_f > c/p, \\
[c(\theta + h) + (2 - p)D + \Delta_2/((1 + rT)(2T + rT^2))] > 0, \\
(2pI_d - cI_f)[c(\theta + h) + (2 - p)D + \Delta_2/((1 + rT)(2T + rT^2))] - [cI_f - (p + c)I_d]^2 D > 0.
\end{align*}
$$


