V2V-based Synchronous Intersection Protocols for Mixed Traffic of Human-Driven and Self-Driving Vehicles

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***Abstract*—Self-driving vehicles are expected to be at the core of future transportation systems. Over time, these vehicles might enhance traffic efficiency and safety, especially at road intersections. However, there will likely be a long transition period before human-driven vehicles will be completely replaced by automated vehicles. Intersection safety and efficiency might be barely improved if automated vehicles just follow the traffic light signals. In this paper, we present a decentralized intersection protocol named the Distributed Synchronous Intersection Protocol (DSIP) that is for mixed traffic environments, where human-driven and self-driving vehicles cooperate with one another in order to avoid vehicle collisions and possible deadlocks while improving traffic efficiency. In DSIP, all automated vehicles use both Vehicle-to-Vehicle (V2V) communications and traffic lights to traverse the intersection safely. On the other hand, human-driven vehicles simply follow the traffic lights just like they do today. Under the protocol, the automated vehicles synchronize when there are no human-driven vehicles around the intersection. In addition, *Cooperative Perception* is used to detect the presence of human-driven vehicles, where all automated and connected vehicles sense and share the presence of the human-driven vehicles at the intersection. Our simulation results show that DSIP increases the traffic throughput of the intersections compared to common signalized intersections and other V2V-based intersection protocols.**

I. INTRODUCTION

Connected and Automated Vehicles (CAVs) are becoming more practical with advances in sensing, computing, actuating, and communication technologies. These CAVs are expected in time to enhance the efficiency, convenience, and safety of transportation systems. In fact, the National Highway Traffic Safety Administration (NHTSA) [1] points out that more than 35,000 people die in motor vehicle-related crashes every year in the US alone. Automation technologies have the potential to reduce that number because of one critical fact: more than 90% of serious crashes occur due to human error.

Driving can be made safer particularly at road intersections. More than 44% of all automotive crashes involve intersections, and they constitute the second largest category of accidents [2].

To enhance the safety at road intersections, a family of synchronous intersection protocols [3], [4] was recently presented for the traffic environment where all vehicles are fully automated and connected. In a synchronous intersection protocol, all vehicles cross the intersection without stopping by following a strict but well-defined spatio-temporal pattern tailored to each intersection. The distance between vehicles can also be configured for human comfort [3]. To join the synchronous intersection protocol, each vehicle must be equipped with an appropriate wireless interface, must be the same physical sizes, and has to control itself very accurately. In addition, a synchronous intersection protocol requires the infrastructure to manage approaching vehicles around the road intersections.

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While a synchronous intersection protocol automates intersection management, it cannot accommodate Human-driven Vehicles (HVs) due to its strict requirements and inherent centralized nature. Since there will be a long transition period before all human-operated vehicles can be replaced with automated vehicles [5], automated vehicles have to co-exist with human-driven vehicles and have to safely navigate themselves around each other.

In this paper, we present a Distributed Synchronous Intersection Protocol (DSIP) that can be used at mixed traffic environments, where HVs and CAVs cooperate with one another to avoid vehicle collisions and possible deadlocks at road intersections, while improving intersection throughput. In DSIP, all CAVs traverse the intersection without stopping when all the vehicles are automated. Also, CAVs use both vehicular communications and perception systems to safely traverse road intersections in a mixed environment with HVs and CAVs. Specifically, DSIP modifies their decision-making policy at intersections in the presence of HVs. When the CAVs confirm that there are no HVs around the intersection, the CAVs negotiate with each other by exchanging messages wirelessly, control themselves, and enter and cross the intersection efficiently and continuously, without coming to a stop before or inside the intersection. On the other hand, when the CAVs detect HVs around the intersection, the CAVs follow the traffic light at the intersection just like the HVs do in today's traffic systems. Based on this rule, DSIP accommodates different levels of automation on public roads, and will naturally adapt as the traffic changes from HVs to CAVs.

In this protocol, each CAV uses Vehicle-to-Vehicle (V2V) communications for inter-vehicle cooperation. Unlike the existing synchronous intersection protocols using Vehicle-to-Infrastructure (V2I) communications, DSIP is decentralized and no additional road infrastructure is required. Also, since the vehicles in DSIP are synchronized in a distributed manner, the protocol accommodates different sizes of vehicles.

To detect the HVs around the intersection, CAVs use *Cooperative Perception* where each CAV locally senses its neighboring HVs and shares the information by V2V communications, in order to enhance the reliability and coverage area. Since the network capacity is limited, the cooperative perception is used only to share the presence of HVs and it transmits the processed data.

The contributions of this paper are as follows.

1) We present *DSIP*, a Distributed Synchronous Intersection Protocol for intersection management when there are both human-driven vehicles and self-driving vehicles on public roads.
2) We introduce a decentralized mechanism to support synchronous intersection protocols.
3) We evaluate our protocol using a simulator-emulator, and demonstrate superior intersection throughput.

The remainder of this paper is organized as follows. Section II describes the system assumptions and requirements. Section III presents our DSIP and examples to illustrate its safety and usage. Section IV evaluates the protocol and compares its performance against intersections with existing traffic lights. Section V discusses previous work related to our research. Finally, Section VI presents our conclusions and future work.

II. OUR SYSTEM ARCHITECTURE AND ASSUMPTIONS

In this section, we present our system architecture and our assumptions. We also provide a review of the synchronous spatio-temporal intersection protocols [3], [4], [6].

A. *Vehicle Requirements*

DSIP assumes that CAVs coexist with HVs, which are controlled by human drivers and may not be equipped with any communication devices or on-board sensors. In this paper, we assume that all HVs continue to follow the existing traffic rules at road intersections. The human drivers must always follow the traffic light operations and speed limits as today.

We also assume that each CAV includes a map database, a navigation system, a perception system, an autonomous vehicle controller, a wireless communication interface, and a localization system, which are typical in a CAV system, such as CMU's self-driving Cadillac SRX [7]. The map database contains accessible lane information, the turn restrictions associated with each lane and the geographical layout of intersections. The perception system senses the surrounding environment by fusing data from different types of sensors, such as vision cameras, radar, and/or LIDAR. The vehicle controller takes as its input the route generated by the navigation system, actuates the vehicle to traverse the chosen path and also follows the intersection protocol through intersections. The wireless interface enables the system to transmit and receive information wirelessly using one or more mechanisms, such as 4G/5G cellular interfaces, satellite communications, or an embedded infrastructure. We assume that each CAV has an On-Board Unit (OBU) that supports Dedicated Short-Range Communications (DSRC), the Wireless Access in a Vehicular Environment (WAVE) protocol stack [8], [9] or C-V2X. This OBU is used to exchange data among multiple automated vehicles. Finally, each CAV is equipped with a high-accuracy Global Positioning System (GPS) receiver. This receiver provides the vehicle with both localization and time-synchronization services. Reasonable positioning accuracy is important for CAV operation with the required level of accuracy achieved by Differential GPS techniques, correction services such as Real-Time Kinematics (RTK) [10], short-term navigation systems such as Inertial Measurement Unit (IMU) and/or other localization techniques.

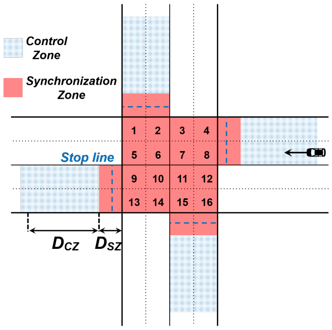


Figure 1: Illustration of Intersection Grid and Cells.

B. *Intersection Definition*

Road intersections have various shapes and geometries. Without loss of generality, we focus in this paper on the Four-way Perfect-Cross Intersection as shown in Figure 1. We represent an intersection as a large grid, which is divided into smaller cells [11]. Figure 1 shows an intersection grid of the multi-way intersection having two lanes along each direction. The intersection grid is divided into 16 small cells, and each cell has a unique identifier.

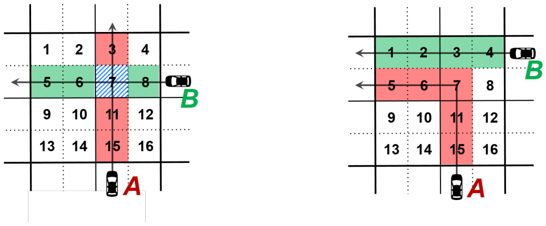
In addition, we define two dedicated zones, illustrated in Figure 1, as follows:

Control Zone: All approaching CAVs negotiate with each other within this area, and they control themselves to enter the intersection with a constant speed and with an assigned time.

Synchronization Zone: All CAVs run within this area with a constant speed and without stopping. Each CAV has the same speed from the entrance of the area to the exit of the road intersection.

To represent these two zones, we introduce two parameters: D_{SZ} and D_{CZ}. D_{SZ} is the distance between the intersection and the end of the Control Zone, and D_{CZ} is the length of the Control Zone. These two zones are defined in the map database. Further discussion is described in Section III-E.

DSIP uses the **Trajectory Cell List** (TCL) [12] to avoid vehicle collisions around the road intersection. The TCL includes the ordered cell numbers within the intersection that are traversed by a vehicle. Each CAV knows its TCL and keeps updating the TCL to share it with the neighboring CAVs. For example, in Figure 2 (a), the TCL of vehicle **A** includes cell numbers $\{15, 11, 7, 3\}$, and that of vehicle **B** includes cell numbers $\{8, 7, 6, 5\}$. Under this scenario, vehicles **A** and **B** have a potential collision at cell 7, and they negotiate to avoid a collision by using vehicular communications. Also, in the case shown in Figure 2 (b), vehicles **A** and **B** have no potential



(a) Path Conflict at Cell 7. (b) No Path Conflict.

Figure 2: Two Scenarios in the Cell-based Intersection.

collision points at the road intersection. Therefore, once these two vehicles confirm that they will not collide with each other, they terminate the negotiation and traverse the intersection.

C. Synchronous Intersection Protocols

DSIP is based on synchronous intersection protocols, in which all automated vehicles move in synchronized fashion at an intersection by using V2I communications [3], [6], [4]. In the synchronous intersection protocols, multiple vehicles from different directions enter and cross the intersection simultaneously without stopping before entering the intersection by following a spatio-temporal pattern[1]. The Federal Highway Administration (FHWA) analyzes traffic flow from a different perspective using quality and quantity measures. One of the main metrics for the quality of traffic flow is Intersection Stops [13]. Intersection Stops are defined as the number of stops made by vehicles approaching an intersection, and a smaller value is better in terms of the quality of traffic flow and fuel efficiency. In addition, fewer intersection stops are much more comfortable for passengers.

Under the synchronous intersection protocols, Ballroom Intersection Protocol (BRIP) [4] and Configurable Synchronous Intersection Protocol (CSIP) [3], [6], the Intersection Stops values become zero, and the throughputs of these protocols are significantly better than that of an intersection controlled by today's traffic lights. These synchronous intersection protocols, however, have two major drawbacks: (i) centralization and (ii) inability to accommodate human-operated vehicles. First, these protocols require the wireless communication device and controller at every road intersection in order to coordinate the approaching CAVs. The infrastructure might also become a single point of failure, and the CAVs cannot follow the protocols when the road infrastructure is down. Secondly, since the synchronous intersection protocols require strict and accurate adherence to the well-defined spatio-temporal arrival pattern, HVs are not accommodated.

On the other hand, in DSIP, the CAVs use Vehicle-to-Vehicle (V2V) communications to synchronize themselves at the intersection. These decentralized scheme improves the reliability and feasibility of the protocol. In addition, in DSIP, the CAVs follow the existing traffic rules when there is a need to cooperate with HVs at the intersections. We present our protocol in detail in the next section.

III. DSIP

In this section, we present the protocol for mixed traffic environments named *DSIP* for HVs and CAVs to co-exist with each other at road intersections. The protocol is designed to avoid vehicle collisions while keeping traffic throughput high.

First, we use *vehicle states* to guide the behaviors of each CAV. Each CAV is in a vehicle state Φ_{state} that dynamically transitions, and is used to command the vehicle to navigate the road intersection while accounting for the surrounding environment, the potential presence of HVs, and traffic congestion.

Secondly, to accommodate HVs while enhancing traffic throughput, CAVs modify their policy to either be in **Traffic-light mode** or **Synchronous mode**. When the CAVs are in the Traffic-light mode, they follow the existing traffic rules, and obey the operation of the traffic light. When the CAVs are in the Synchronous mode, the CAVs negotiate with each other and enter the intersection without stopping.

In the Synchronous mode, the CAVs control their speeds and arrival time within the Control Zone, entering and crossing the intersection efficiently and continuously, without coming to a stop before or inside the intersection. All CAVs around the intersection cooperate by using V2V communications and will be in the same mode.

In addition, all CAVs cooperatively use their perception systems and work to detect any surrounding HVs around the intersection. Once the CAVs detect any HVs around the intersection, the CAVs transition to **Traffic-light mode** and obey the dedicated traffic light to cooperate with the surrounding HVs at the intersection.

From a standards viewpoint, CAVs use Advanced Safety Messages for V2V communications [9], [14], and we use the second optional part of Basic Safety Messages (BSM) [9] for communicating protocol information. That is, Advanced Safety Messages are broadcast at 10 Hz, the same rate as the BSM.

A. Protocol Policy

In DSIP, CAVs use one of two modes as follows.

Traffic-light mode: CAVs in this mode follow the existing traffic lights where the green light allows traffic to proceed in the direction denoted and where the red light prohibits any traffic from proceeding.

Synchronous mode: CAVs in this mode enter and cross the intersection synchronously without coming to a stop before or inside the intersection. All approaching CAVs control their own speeds, accelerations, and time to enter the intersection within the Control Zone.

We now present the notation that we use in this paper.

- (r, r): Cell size. (lane width r)
- (l_α, w_α): Physical size of vehicle α (length l_α and width w_α).
- v_{sync}: Vehicle's velocity for the synchronization.
- v_{limit}: Speed limit.

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[1] Each intersection with its own geographical layout is assigned its own spatio-temporal pattern.

- ω: Safety gap.
- t_{α}: Original Arrival Time of vehicle α.
- $\hat{t_{\alpha}}$: Assigned Arrival Time of vehicle α.
- D_{SZ}: Distance for Synchronization Zone.
- D_{CZ}: Distance for Control Zone.
- D_{SL}: Distance from the intersection entrance to the Stop Line.

First, v_{sync} represents the constant speed of a CAV within the Synchronization Zone, when it is in the Synchronous mode. When the CAV is in the Synchronous mode and runs within the Control Zone, it controls its velocity to be close to v_{sync}. For safety purposes, v_{sync} has to be smaller value than the speed limit v_{limit}.

Also, DSIP provides a sufficient inter-vehicle gap ω to meet safety requirements to account for GPS inaccuracies and control system failure, and to keep human passengers comfortable. In particular, the protocol needs to maintain an appropriate longitudinal safety gap.

When each CAV enters the Control Zone, it calculates its Original Arrival Time t at the intersection and it exchanges the value of t with other CAVs to determine the timing and order to enter the intersection. Once the CAVs complete the negotiation, each gets its own Assigned Arrival Time $\hat{t}$, and controls itself to enter the intersection at time $\hat{t}$. The Assigned Arrival Time of each CAV can never be earlier than the Original Arrival Time (i.e. $t \leq \hat{t}$) because it may have no capability to safely accelerate and achieve the Assigned Arrival Time. We present the details of the negotiation in the next subsection.

B. Advanced Safety Messages

In our protocol, each CAV uses Advanced Safety Messages to interact with other CAVs within its communication range. The message contains 6 parameters as follows.

- Trajectory Cell List (TCL).
- Vehicle State Φ_{state}.
- Original Arrival Time t.
- Assigned Arrival Time $\hat{t}$.
- Visible HV Flag HV_{flag}.
- Timestamp of Last HV τ_{HV}

The first 4 parameters, TCL, Φ_{state}, t, and $\hat{t}$, are used for the negotiation among multiple CAVs. Each CAV holds its constant TCL and t, and it can change the other two parameters, Φ_{state} and $\hat{t}$. The HV_{flag} is the flag representing the presence of HV(s) visible to the transmitter CAV. τ_{HV} is the timestamp that is taken when a CAV detects any surrounding HVs. τ_{HV} will be updated whenever the neighboring CAVs detect HVs and the information is shared through V2V communications. These two values are used for cooperative perception to detect HVs around the intersection.

C. Vehicle State Transitions

The CAVs use the state transition diagram shown in Figure 3 where we have 7 vehicle states. All CAVs have to change their state Φ_{state} to either the *Traffic Light state* or the *Synchronized state* before entering the Synchronization Zone. We show the behaviors of the CAVs within each state next.

1) Not Around Intersection state: When the CAV is not near a road intersection, the vehicle is in the *Not Around Intersection state*. The vehicle state Φ_{state} transitions to the *Not Around Intersection* once the CAV exits the intersection.

2) Approaching state: When the CAV approaches a road intersection, the vehicle state Φ_{state} transitions to the *Approaching state* from the *Not Around Intersection state*. In this state, the CAV calculates its TCL and Original Arrival Time t.

3) Negotiating state: Once the CAV in the *Approaching state* receives the message via V2V communications, the vehicle state Φ_{state} transitions to the *Negotiating state*. A vehicle in the *Negotiating state* negotiates its priority in the intersection by comparing against its Original Arrival Time t.

4) Controlling state: When the CAV in the *Negotiating state* completes the negotiation and determines the Assigned Arrival Time $\hat{t}$, the vehicle state Φ_{state} transitions to the *Controlling state*. A vehicle in the *Controlling state* controls its speed and arrival time, in order to fit its speed into the velocity v_{sync} and to enter the intersection at time $\hat{t}$.

5) Synchronized state: When the CAV in the *Approaching state* confirms that there are no HVs around the intersection by using *cooperative perception* or when the vehicle in the *Controlling state* completes the required control for its speed and arrival time, the vehicle state Φ_{state} transitions to the *Synchronized state*. In this state, the CAV keeps a constant speed to cross the intersection without coming to a stop. A CAV in this state will change its speed and arrival time only for emergency situations.

6) Traffic Light state: When the CAV detects HVs around the intersection in the *Approaching state*, *Negotiating state*, *Controlling state*, and *Synchronized state*, the vehicle state Φ_{state} transitions to the *Traffic Light state*. Under this state, the CAV is in *Traffic-light mode* and obeys the traffic light.

7) Emergency state: When the CAV detects emergency situations, such as a traffic accident and/or a HV violating traffic rules at the intersection, the vehicle state Φ_{state} transitions to the *Emergency state*. Under this vehicle state, all of the CAVs slow down and/or stop to be safe.

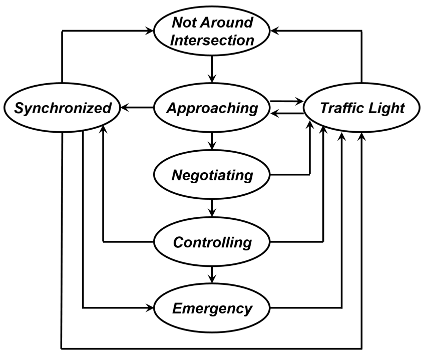


Figure 3: State Transition used by CAVs at Intersections.

The area between the intersection and the end of the Control Zone shown in Figure 1 is used for the CAVs transitioning from the *Synchronized state* to the *Traffic Light state*. When the CAV in the *Synchronized state* detects the surrounding HV(s) near the intersection and when the traffic light is red, the CAV has to decelerate and stop before the intersection. Overall, D_{SZ} represents the physical distance for the safety buffer to stop.

D. V2V-based Vehicle Synchronization

In DSIP, when there are no surrounding HVs at the intersection, all CAVs have the same constant speed and are not required to stop around and/or inside the intersection. The CAVs control their own speed and acceleration within the Control Zone and run at constant speed in the Synchronization Zone. We now present the DSIP priority assignment scheme and validate its freedom from deadlock.

1) Priority Assignment: The priority assignment is used to determine the Assigned Arrival Times, using the vehicle state Φ_{state} and the Original Arrival Time t. The priority is used to determine the order to calculate the Assigned Arrival Time for each CAV, and is not used to determine the order to enter the intersection.

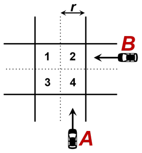
First, the CAVs in the *Controlling state* and in the *Synchronized state* have already had their own Assigned Arrival Times. Therefore, these CAVs will not change their Assigned Arrival Times and other surrounding CAVs have to select the appropriate Assigned Arrival Times for themselves. That is, when the CAV has at least one neighboring vehicle in *Controlling state* or in *Synchronized state* and they have a potential collision cell at the intersection, the second CAV has to control its speed and arrival time.

Next, the CAVs not in *Controlling state* and in *Synchronized state* negotiate with each other by using their Original Arrival Times to determine which CAV has the priority to calculate the Assigned Arrival Time. The CAV with the smallest Original Arrival Time gets precedence. Any tie of the Original Arrival Times is broken by using pre-appointed identifiers, such as a Vehicle ID and/or a Lane ID from the map database.

Once a CAV takes priority, it calculates its own Assigned Arrival Time so as to avoid the vehicle collisions with other neighbors in *Controlling state* and in *Synchronized state*. Knowing the Assigned Arrival Time of the neighboring vehicles with higher priority, the CAV calculates its Assigned Arrival Time at Cell β, $\hat{t}_{ego,\beta}$, as shown in Eq. (1). Here, $\hat{t}_{neigh,\beta}$ represents the Assigned Arrival Time at Cell β of the neighboring CAV. β is the potential collision cell between the ego vehicle and the neighboring vehicle.

$$\hat{t}_{ego,\beta} = max\{t_{ego,\beta}, \;\; \hat{t}_{neigh,\beta} + \frac{r}{v_{sync}} + \omega\} \tag{1}$$

The priority assignment scheme is presented in Algorithm 1. Here, $t'_{ego,\beta}$ represents the arrival time at Cell β of the ego vehicle, and keeps updating. Also, in Algorithm 1, when the *Priority* becomes 1, the ego vehicle takes the priority to determine the Assigned Arrival Time. Note that each CAV calculates its priority and determines its own control. No CAV commands other CAVs' behaviors.



(a) Two Vehicles Arrival.

Vehicle	Estimated Arrival Times at			
	cell1	cell2	cell3	cell4
A	✗	$t_A + \frac{r}{v_{sync}}$	✗	t_A
B	$t_B + \frac{r}{v_{sync}}$	t_B	✗	✗

(b) Estimated Arrival Times.

Figure 4: Scenario under *Synchronous mode*.

Consider the case when there are two vehicles, as shown in Figure 4 (a), vehicle **A** and vehicle **B**, approaching from different directions. Here, the Original Arrival Times of vehicle **A** and vehicle **B** are represented as t_A and t_B, respectively. Since vehicle **A** traverses cell 4 and cell 2, vehicle **A** transmits its TCL $\{4, 2\}$ and the Arrival Time t_A. Also, vehicle **B** transmits its TCL $\{2, 1\}$ and the arrival time t_B. The priority assignment follows Eq. (2). The vehicle with no priority has to control its arrival time to avoid collision.

$$t_A - t_B \begin{cases} \leq 0 & (\textbf{A} \text{ has priority}) \\ > 0 & (\textbf{B} \text{ has priority}) \end{cases} \tag{2}$$

When vehicle **A** has the priority (i.e. $t_A \leq t_B$), it does not need to change its arrival time ($t_A = \hat{t}_A$). On the other hand, vehicle **B** has to arrive at the intersection after $t_A + \frac{2r}{v_{sync}} + \omega$, as shown in Figure 5-(a). To satisfy this condition, vehicle **B** has to slow down within the Control Zone and delay its arrival time. When vehicle **B** has the priority (i.e. $t_A > t_B$), vehicle **A** has to arrive at the intersection after $\hat{t}_B + \omega$, as shown in Figure 5-(b). Since we give the Safety Gap ω between these

Algorithm 1: Priority Assignment for V2V-based Vehicle Synchronization.

```
foreach $k \in$ Neighboring CAVs do
    if $\Phi^k_{state}$ == Controlling or Synchronized then
        if potential collision then
            if $t'_{ego,\beta} < \hat{t}_{k,\beta} - \omega$ or $t'_{ego,\beta} > \hat{t}_{k,\beta} + \frac{r}{v_{sync}} + \omega$ then
                Priority = -1;
            else
                Priority = 0;
            end
        else
            Priority=-1;
        end
    else if $\Phi^k_{state}$ == Approaching or Negotiating then
        if $t_k > t_{ego}$ then
            if $Priority \neq 0$ then
                Priority =1;
            else
                Priority = 0;
            end
        end
    end
end
```

two vehicles, GPS inaccuracies and control system failure will not lead to vehicle accidents.

2) Freedom from Deadlocks: We now show that DSIP is a deadlock-free protocol. A deadlock represents a situation in which two or more CAVs are waiting for each other vehicle to take action, and nobody is able to progress.

As shown in Figure 6-(a), suppose vehicles $A_1, A_2, \cdots A_{n1}$, $B_1, B_2, \cdots B_{n2}$, $C_1, C_2, \cdots C_{n3}$, and $D_1, D_2, \cdots D_{n4}$ arrive from different directions. $(n1, n2, n3, n4 \in \mathbb{N}.)$ t_{A_1}, t_{B_1}, t_{C_1}, and t_{D_1} represent the Original Arrival Times of vehicles A_1, B_1, C_1, and D_1, respectively. In addition, as shown in Figure 6-(b) and -(c), there are two possible wait-for graphs representing the deadlock situations among vehicles A_{p1}, B_{p2}, C_{p3}, and D_{p4}. $(p1, p2, p3, p4 \in \mathbb{N}.)$

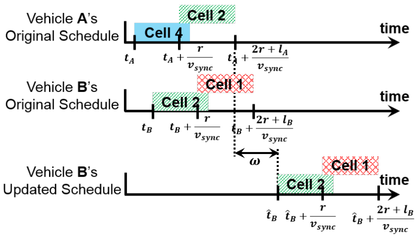
Theorem: DSIP is a deadlock-free protocol.

Proof: In DSIP, only the vehicles in *Approaching state* and *Negotiating state* negotiate with each other to determine which CAV has the priority to get the Assigned Arrival Time. Therefore, the vehicles in *Synchronized state* and *Controlling state* do not join the priority assignment procedure and do not lead the deadlock situation. Suppose $A_1, \cdots A_{k1}$, $B_1, \cdots B_{k2}$, $C_1, \cdots C_{k3}$, and $D_1, \cdots D_{k4}$ are in either *Synchronized state* or *Controlling state*, and we only need to consider the deadlock among $A_{k1+1} \cdots A_{n1}$, $B_{k2+1} \cdots B_{n2}$, $C_{k3+1} \cdots C_{n3}$ and $D_{k4+1} \cdots D_{n4}$. $(1 \leq k1 \leq n1$, $1 \leq k2 \leq n2$, $1 \leq k3 \leq n3$, $1 \leq k4 \leq n4.)$

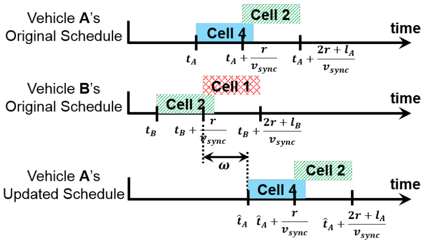
Next, suppose the leading vehicles of each road in the *Approaching state* or *Negotiating state*, A_{k1+1}, B_{k2+1}, C_{k3+1} and D_{k4+1}, are deadlocks like Case I in Figure 6-(b). The priority to determine the Assigned Arrival Times are calculated by the Original Arrival Time at the intersection. The conditions to cause the deadlock are captured in Eq. (3).

$$\begin{cases} t_{A_{k1+1}} < t_{D_{k4+1}} & \text{(Vehicle A has higher priority than D)} \\ t_{B_{k2+1}} < t_{A_{k1+1}} & \text{(Vehicle B has higher priority than A)} \\ t_{C_{k3+1}} < t_{B_{k2+1}} & \text{(Vehicle C has higher priority than B)} \\ t_{D_{k4+1}} < t_{C_{k3+1}} & \text{(Vehicle D has higher priority than C)} \end{cases} \tag{3}$$

Since these relationships are cyclic and no values of $t_{A_{k1+1}}$, $t_{B_{k2+1}}$, $t_{C_{k3+1}}$, and $t_{D_{k4+1}}$ can satisfy Eq. (3), we can conclude that there are no deadlock situations among these four vehicles like Case I in Figure 6-(b). Then, one of these vehicles takes the priority, determines the Assigned Arrival Time, and transitions to the *Controlling state*. Recursively, the negotiation between four vehicles happens and we can conclude that there are no deadlock situations like Case I in Figure 6-(b). We can similarly verify that there are no deadlock situations like Case II in Figure 6-(c). Similar arguments show that deadlocks with either two or three vehicles are not possible. □

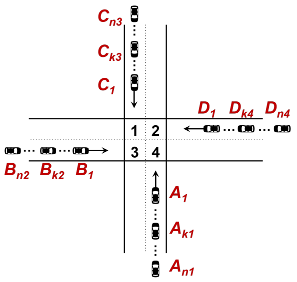


(a) Case I: Vehicle **A** has the priority.

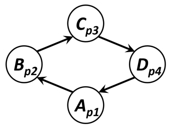


(b) Case II: Vehicle **B** has the priority.

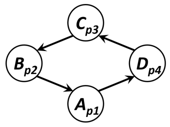
Figure 5: Timelines for Arrival Plans of Vehicles **A** and **B**.



(a) Consecutive Vehicle Arrivals.



(b) Deadlock Case I.



(c) Deadlock Case II.

Figure 6: Negotiation and Deadlocks.

3) Configurable Safety Gap: DSIP provides a configurable inter-vehicle gap ω to meet safety requirements to account for GPS inaccuracies and control system failure and to keep passengers comfortable. In this protocol, the longitudinal error is only considered because a CAV can calibrate the lateral error with the perception system by detecting lane markers [15]. Even though there are rarely markers within road intersections, short-term navigation systems by IMU might help to calibrate the lateral position.

We can give safer and more comfortable environments by increasing the value of ω. Conversely, the price to be paid is that, due to the larger gaps between vehicles, it offers lower intersection throughput. In DSIP, the gap ω between two vehicles can be customized for each intersection. The required value for ω can be determined by passenger comfort levels and localization accuracy.

A simulation video of the negotiation in DSIP can be seen at:

https://youtu.be/W0GSDhHHXi0

E. Map Configuration

The distances for Synchronization Zone D_{SZ} and for Control Zone D_{CZ} are explicitly defined in the map database. These distances are determined to decelerate to synchronize or to stop at the stop line safely. The required conditions are captured in Eq. (4).

$$\begin{cases} D_{SZ} & \geq & D_{SL} + \theta(v_{sync}, 0) \\ D_{CZ} & \geq & \theta(v_{limit}, v_{sync}) \end{cases} \quad (4)$$

, where $\theta(v_w, v_z)$ represents the deceleration distance from the initial velocity v_w to the destination velocity v_z. For human-driven vehicles, the deceleration distance is the sum of the reaction distance of human driver and the braking distance [16], [17]. Since automated vehicles have sufficiently short reaction time, this protocol only considers the braking distance. The deceleration distance $\theta(v_w, v_z)$ is calculated from the kinetic energy equation.

$$E = \frac{1}{2}mv_w^2 = \frac{1}{2}mv_z^2 + \mu mg \cdot \theta(v_w, v_z) \quad (5)$$

, where m is the vehicle's mass and μ is the coefficient of friction between the road surface and the tires. By using the Eqs. (4) and (5), the required distances for D_{SZ} and D_{CZ} are determined.

$$\theta(v_w, v_z) = \frac{v_w^2 - v_z^2}{2\mu g} \quad (6)$$

$$\begin{cases} D_{SZ} & \geq & D_{SL} + \frac{v_{sync}^2}{2\mu g} \\ D_{CZ} & \geq & \frac{v_{limit}^2 - v_{sync}^2}{2\mu g} \end{cases} \quad (7)$$

Note that the distance from the intersection entrance to the stop line D_{SL} is from the existing road designs [13]. Also, the value μ on public roads are widely studied and there are many discussions on the appropriate road designs [13], [17].

F. Cooperative Perception

Under DSIP, the CAVs determine and dynamically change their behaviors based on the presence of any surrounding HVs. To detect HVs at the intersection, *Cooperative Perception* is used and includes two different components: (i) Local Perception for the Surrounding HVs and (ii) Cooperative Sharing of Local Information. By using their perception systems cooperatively, the CAVs detect HVs that may be occluded by other objects around the intersection.

Due to the limitation of the network capacity, the protocol only shares HV_{flag} and τ_{HV}. The protocol never shares neighborhood tables and raw sensor data.

1) Local Perception for Surrounding HVs: First, each CAV around the intersection uses its local perception system to detect surrounding HVs in the visible area. Each CAV holds two neighborhood tables as follows:

- Ψ_{comm}: Neighborhood table built from vehicular communications. Each neighboring vehicle in this table has its location and operation (human-driven or CAV).
- $\Psi_{perception}$: Neighborhood table from the perception system. Each neighboring vehicle in this table has its approximate location.

Since Ψ_{comm} is available from the BSMs, the CAVs hold other CAVs' locations, speeds, and operation in Ψ_{comm}. The CAVs build $\Psi_{perception}$ from their local sensor systems, and $\Psi_{perception}$ contains only the locations of any detected vehicles.

Algorithm 2: Human-Driven Vehicle Detection with *Cooperative Perception.*

```
1  foreach $i \in \Psi_{perception}$ do
2      foreach $j \in \Psi_{comm}$ do
3          if $\Psi_{perception}$(i).location == $\Psi_{comm}$(j).location then
4              $\Psi_{CAV}$.add($\Psi_{perception}$(i));
5          end
6      end
7  end
8  if size($\Psi_{perception}$) $\neq$ size($\Psi_{CAV}$) then
9      $HV_{flag} = 1$;
10     $\tau_{HV}$ = Current Time;
11 end
12 if $HV_{flag} == 0$ then
13     if $HV_{flag,neigh} == 1$ then
14         $\tau_{HV} = \tau_{HV,neigh}$;
15     end
16 end
17 if $HV_{flag} == 1$ then
18     if $\tau_{HV}$ − Current Time $> \delta$ then
19         $HV_{flag} = 0$;
20     end
21 end
```

Each CAV compares the location of the vehicles in Ψ_{comm} and in $\Psi_{perception}$, and decides which vehicle is human-operated. The surrounding and visible CAVs might be stored both in Ψ_{comm} and in $\Psi_{perception}$, but the surrounding and visible HVs might be stored only in $\Psi_{perception}$.

We present the decision-making algorithm in the presence of HVs in lines 1 through 11 in Algorithm 2. DSIP also uses an additional table Ψ_{CAV} where all visible CAVs are stored. That is, by comparing the number of the visible vehicles and that of the visible CAVs, we can ensure whether there are HVs or not, by using the local perception system. In addition, DSIP updates a timestamp τ_{HV} when the CAV locally detects surrounding HVs, which is used in the cooperative sharing.

2) Cooperative Sharing of Local Information: In this step, the CAVs exchange the local information acquired in the previous step. To continuously maintain the information of the HVs (HV_{flag}), the CAVs keep exchanging and updating the timestamp τ_{HV}, which represents the last time when the CAVs detected the surrounding HVs at the intersection.

We use two symbols to represent the information obtained from the neighboring CAVs.

- $HV_{flag,neigh}$: HV_{flag} from the neighboring CAV.
- $\tau_{HV,neigh}$: τ_{HV} from the neighboring CAV.

In addition, DSIP uses a time threshold δ. When the CAV cannot detect any HVs for the period δ locally and cooperatively, the CAV considers there is no HV around the intersection and the flag HV_{flag} becomes 0. This logic is captured in Algorithm 2, from line 12 to line 21.

A simulation video of the cooperation between HVs and CAVs in DSIP can be seen at:

https://youtu.be/LtAGiDEfWlE

G. Component failures and Packet Loss

Our protocol assures safety by using V2V communications and the CAV perception systems cooperatively. It inherently accommodates the loss of transmitted packets, since safety messages are transmitted at 10 Hz and occasional packet losses are compensated by subsequent successful transmissions. Since the protocol data are transmitted with the BSM, any messages delayed over 0.1 sec are ignored [18].

In addition, cooperative perception helps to enhance safety when a CAV has a communication component failure. In DSIP, an automated vehicle without regular transmission of the safety beacon (BSM) will be regarded as HV due to cooperative perception. Therefore, even when a CAV having communication problems appears at the intersection, all other CAVs switch their modes from Synchronous mode to Traffic-light mode and follow the traffic light. The CAV having problems also changes its mode to Traffic-light mode, because it cannot receive the safety beacon from the neighboring vehicles.

IV. IMPLEMENTATION AND EVALUATION

In this section, we present the implementation and an evaluation of our intersection protocol for the mixed traffic environments DSIP in the *AutoSim*, a simulator-emulator [19]. We evaluate the traffic protocol in terms of traffic throughput, and compare against a common signalized intersection and the existing V2V-based intersection protocols [12].

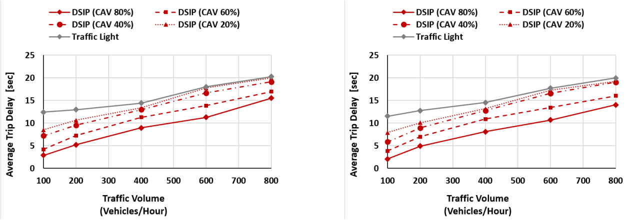
AutoSim [19] is an extended version of GrooveNet [20] with 3-D graphics and other capabilities. AutoSim consists of various core individual models, such as mobility, communication, control, localization, and pose estimation for each simulated vehicle. In the simulator-emulator, each simulated CAV can transmit and receive a Basic Safety Message (BSM). The message format used is defined by the SAE J2735 standard [18], DSRC Message Set Dictionary.

A. Performance Metric

We define the *Trip Time* as the time taken by a vehicle to go from a known start-point before the intersection to a known end-point after the intersection. We calculate the trip time for each simulated car and compare that against the trip time taken by the car assuming that it stays at a constant speed and does not stop at the intersection. The difference between these two trip times is considered to be the *Trip Delay* due to the intersection, and the *Average Trip Delay* across all CAVs in each simulation is compared in our evaluation. We also use Intersection Stops [13] as another metric, with smaller values representing better traffic quality.

Table I: Environmental Settings for the Experiments.

Communication range	400 (m)
Averaged vehicle length l	2.6 (m)
Averaged vehicle width w	1.6 (m)
Speed for Synchronization v_{sync}	25 (km/h)
Speed limit v_{limit}	40 (km/h)



(a) Exponential Distribution. (b) Log-normal Distribution.

Figure 7: Average Trip Delays for Heterogeneous Traffic.

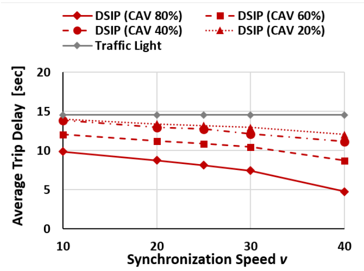


Figure 8: Average Trip Delays for Different Synchronization Speed v.

B. Scenarios

We present the evaluation in a four-way perfect-cross intersection, because we can get the similar results even when we change the forms of the intersection. The variables for the simulations are set by utilizing the DSRC standards [8], [9] and typical vehicle abilities, as shown in Table I. For example, the communication range and frequency for V2V communications are set as 400 m and 10 Hz, respectively. The speed for synchronized driving is set as 25 km/h [2]. Each simulation runs for 30 minutes, and we evaluate the time delay of the vehicles that are generated during the last 20 minutes to skip initial transients and measure steady-state behavior.

We evaluate DSIP in two traffic environments: *Heterogeneous Traffic Environment* and *Homogeneous Traffic Environment*. In heterogeneous traffic environment, HVs and CAVs co-exist. In homogeneous traffic contexts, all vehicles are fully automated and connected. We also use several probability distributions for the inter-vehicle distance: normal distribution, exponential distribution, and log-normal distribution. The inter-vehicle distance plays a significant role in traffic flow analysis and the models have been widely studied since the 1960s [21]. Our simulations evaluate DSIP by comparing against two baseline traffic protocols: *Traffic Light-based Protocol* and *Spatio-Temporal Intersection Protocol* (STIP) [12]. STIP is used only in the evaluation in homogeneous traffic contexts because it cannot accommodate HVs.

Traffic Light-based Protocol: Traffic signals control traffic movement at the intersection. In our baseline measurements,

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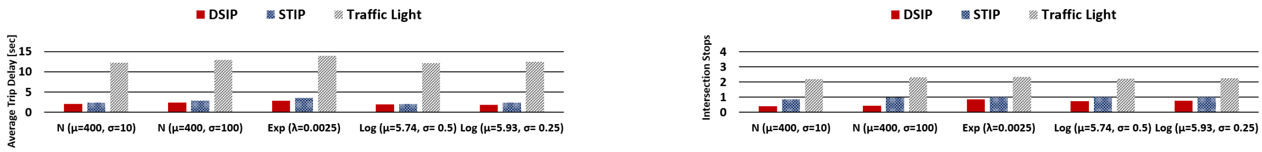
[2] Qualitatively similar results are obtained for other configurations and are not included here due to length considerations



(a) Evaluation for Average Trip Delay (100 (vehicles/hour)). (b) Evaluation for Intersection Stops (100 (vehicles/hour)).

(c) Evaluation for Average Trip Delay (400 (vehicles/hour)). (d) Evaluation for Intersection Stops (400 (vehicles/hour)).

Figure 9: Traffic Throughput with Various Inter-vehicle Distance Models.

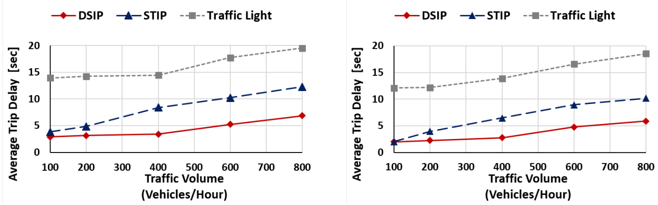
the time length for a green light is 15 seconds, and the time length for a yellow light is 3 seconds.

Spatio-Temporal Intersection Protocol (STIP): The vehicles exchange messages by V2V communications before approaching the intersection. The higher-priority vehicle does not necessarily stop before and/or within the intersection. The priority assignment is based on the arrival time at the intersection. This protocol works only for self-driving vehicles and do not assume the heterogeneous traffic contexts.

C. Evaluation for Heterogeneous Traffic

We first evaluate the Average Trip Delay in a heterogeneous traffic environment when we change the traffic volume from 100 to 800 vehicles per hour per lane. We range from very sparse traffic rural intersections to very congested urban intersections. Figure 7-(a) and (b) show the average trip delay in heterogeneous traffic contexts when the traffic arrival pattern follows the exponential distribution and the log-normal distribution ($\sigma = 0.5$), respectively. For both cases, we change the ratio of CAV penetration from 20 % to 80 %. Here, the parameters of the distributions are inherently determined by the traffic volume at the intersection. When the traffic volume is low, DSIP demonstrates the superior performance because many CAVs do not stop at the intersection. As the traffic volume is increased, the average trip delay becomes longer because more CAVs meet HVs at the intersection. When we increase the ratio of the CAVs, the average trip delay becomes shorter, because more CAVs can synchronize and do not need to stop to follow the traffic lights at the intersection. On the other hand, when we set the CAV penetration ratio as a low value, the trip delay as expected is closer to that in the Traffic Light-based Protocol, but it is definitely smaller.

Secondly, we show the relationships between the synchronization speed v and the speed limit in Figure 8. In this evaluation, we vary the synchronization speed v from 10 (km/h) to 40 (km/h). The traffic volume is 400 (vehicles/hour) and the inter-vehicle distance model follows the log-normal distribution ($\sigma = 0.5$). When we increase the value of v, the average trip delay becomes smaller, which means the traffic throughput is improved. When the CAV penetration ratio is higher, the average trip delay becomes much shorter because more CAVs can synchronize at the intersection. Conversely, when the CAV penetration ratio is low, the average trip delay is not dramatically changed because CAVs and HVs follow the traffic light for much longer times.



(a) Exponential Distribution. (b) Log-normal Distribution.

Figure 10: Average Trip Delays for Homogeneous Traffic.

D. Evaluation for Homogeneous Traffic

When all vehicles are fully automated, we evaluate the Average Trip Delay and Intersection Stops for different arrival patterns, as shown in Figure 9-(a), -(b), -(c), and -(d). We compare against the Traffic Light-based Protocol and the V2V-based intersection protocol, STIP. The traffic volume is set as 100 (vehicles/hour) in Figure 9-(a) and -(b), and set as 400 (vehicles/hour) in Figure 9-(c) and -(d). For the first case, in the rural intersection with 100(vehicles/hour), we use 5 different models for vehicle arrival patterns: (i) Normal distribution ($\mu = 400$, $\sigma = 10$), (ii) Normal distribution ($\mu = 400$, $\sigma = 100$), (iii) Exponential distribution ($\lambda = 0.0025$) (iv) Log-normal distribution ($\mu = 5.74$, $\sigma = 0.5$), and (vi) Log-normal distribution ($\mu = 5.93$, $\sigma = 0.25$). The mean of the inter-vehicle distance is 400 (m). For the second case with 400 (vehicles/hour), as shown in 9-(c) and -(d), we also use 5 different models: (i) Normal distribution ($\mu = 100$, $\sigma = 5$), (ii) Normal distribution ($\mu = 100$, $\sigma = 50$), (iii) Exponential distribution ($\lambda = 0.01$) (iv) Log-normal distribution ($\mu = 4.36$, $\sigma = 0.5$), and (vi) Log-normal distribution ($\mu = 4.54$, $\sigma = 0.25$). The mean of the physical distance between two consecutive vehicles is set as 100 (m). In all cases, DSIP has the shortest average trip delay and the smallest value for the intersection stops. The Traffic Light-based Protocol always has a long average delay because approximately half of the arriving vehicles have to stop and wait for the green period. STIP sometimes has the largest value for Intersection Stops, because up to 3 vehicles may need to stop before the intersection when 4 vehicles appear at almost the same time.

Secondly, we evaluate the Average Trip Delay when we change the traffic volume from 100 to 800 vehicles per hour per lane. Figure 10-(a) and -(b) present the average trip delay of all the CAVs when we use the exponential distribution and the log-normal distribution ($\sigma = 0.5$), respectively. We change the values λ and μ in order to change the traffic volume at the intersection. In all cases, DSIP has the shortest average trip delay even though the protocol is originally designed for the mixed traffic environments of HVs and CAVs. This is because, in DSIP, each CAV controls its speed and arrival time before approaching the intersection. On the other hand, in STIP, some vehicles have to completely stop at the intersection. Each stopped vehicle delays its travel time not only for slowing down and stopping but also for accelerating later.

From these evaluations, we show that DSIP is superior to the Traffic Light-based Protocol and STIP not only in the mixed traffic environment but also in the fully-automated traffic environment.

V. RELATED WORK

Cooperation among self-driving vehicles have been widely studied for intelligent intersection management, platooning, merging, and lane-changing maneuvers[22], [23], [24]. Most previous work on multi-vehicle coordination assume that all vehicles are fully autonomous, and few research studies have investigated the coordination between human-driven and self-driving vehicles.

Azimi et al. [11], [12] proposed several spatio-temporal intersection protocols. In this work, vehicles are assigned priorities based on their arrival times at the intersection, with vehicles reaching the intersection earlier assigned higher priorities than vehicles coming later. These priority-based intersection protocols were extended to the mixed traffic environment [11], where self-driving vehicles use their perception systems to predict other vehicles' future paths and behaviors. Although the protocol coordinates the vehicles when human drivers completely follow the traffic rules, vehicle collisions and deadlock might happen when human drivers select, for example, the wrong lane to enter the intersection. These priority-based intersection protocols were also extended to merge points, where two lanes with different priorities meet [25]. In this paper, self-driving vehicles use both vehicular communications and the local perception systems for cooperating with other CAVs and HVs to safely navigate merge points.

Second, some researchers focused on agent-based control protocols [26], [27], [28]. Dresner et al. [28], [29] proposed a multi-agent approach for intersection control. In this system, all vehicles call ahead to a reservation manager agent at the intersection to reserve conflict-free trajectories. The authors extended the system to the heterogeneous traffic environment [30] to enable efficient intersection management for early self-driving vehicles' penetration stages. Since the protocol heavily relies on V2I communications and on the manager agent, the infrastructure becomes a single point of failure. Also, the protocol requires new road infrastructure at each intersection and will be costly. In contrast, DSIP does not need additional infrastructure, and works in a decentralized manner.

Other previous studies focused on the maneuvers to avoid possible vehicle collisions on shared road segments [31], [32], [33]. The work in [31] defined and classified specific cases away from road intersection that can lead to possible vehicle collisions. It presented a cooperative dynamic intersection protocol where self-driving vehicles use both vehicular communications and perception systems. A longitudinal motion planning technique was also considered for lane-changing maneuvers in [33], where automated vehicles select an appropriate inter-vehicle traffic gap and time instants to perform the maneuver. This work assumed that all of the vehicles are connected and automated, and did not address the cooperation of HVs and CAVs.

A motion-planning approach has also been studied [34], [35], [36]. Kim et al. [35], [36] presented a motion-planning framework based on model-predictive control, in order to establish system-wide safety. Although these studies addressed the system-wide safety between HVs and CAVs, they are limited to the motion planning and platooning maneuvers, and did not study road intersection management. In contrast, DSIP specifically focuses on intersection management where multiple vehicles from different directions intersect.

Also, few works [37], [38], [39] studied cooperative perception where multiple vehicles share the sensor data for occluded objects. Kim et al. [38] presented a cooperative perception for platooning maneuvers, and Higuchi et al. [39] discussed data sharing protocols for the cooperative perception.

VI. CONCLUSION

We presented an intersection protocol named *DSIP* for the mixed traffic environments, in which Connected and Automated Vehicles (CAVs) and Human-driven Vehicles (HVs) cooperate with one another in order to improve the throughput and ensure safety at road intersections. In DSIP, CAVs change their decision-making policy at the intersection depending on the presence of the HVs. When there is no HV around the intersection, the CAVs negotiate with each other and enter the intersection efficiently and continuously, without coming to a stop before or inside the intersection. On the other hand, when there is at least one HV, the CAVs obey the dedicated traffic light for safety concerns.

We finally note several limitations of our work. First, we need to consider the effects between neighboring intersections in order to apply our protocol in the real world. Secondly, CAVs may transmit incorrect information with V2V communications. In future work, we will design more secured intersection protocols against false data provided by CAVs.

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