Decision Aiding

Violations of conjoint independence in binary choices: The equate-to-differentiate interpretation

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Abstract

The studies described in the present paper extend the author's earlier work on testing the “independence condition” for decisions under risk to the “conjoint independence” for multi-attribute decisions. The “equate-to-differentiate” approach, which is proposed as a means by which weak dominance can be made applicable in more general cases in binary choice, is presented to account for the observed violations of independence. [Journal of Business 59 (1986) S251] transparency hypothesis concerning violations of the independence axiom is compared with the equate-to-differentiate hypothesis. The findings for various decision tasks favor the equate-to-differentiate explanation. The results support the claim that the violation of independence should be observed when the equate-to-differentiate strategy is caused to be changed by the experimental conditions applied, but not otherwise.

Keywords: Decision analysis; Conjoint independence; Binary choice

The independence condition in decisions under risk, the sure-thing principle in decisions under uncertainty, and conjoint independence for multi-attribute decisions are three central ideas in choice theory. According to Fishburn and Wakker’s (1995) review, independence and the sure-thing principle are equivalent for decisions under risk, but in a less elementary way than has sometimes been thought. The sure-thing principle for decision under uncertainty and conjoint independence are identical in a mathematical sense.

While the independence condition has been considered as the most central idea in risk theory and while the expected utility (EU) theory of von Neumann and Morgenstern (1947) and the subjective expected utility (SEU) theory of Savage (1954) have been considered as the leading theories of choices in economics and psychology, the opponents, however, often direct their criticism toward independence and base alternative models on weakening the independence condition. Economists and psychologists have responded to this challenge by developing more general non-expected utility theories that are still utility theories. But as Friedman (1989) pointed out, the strategy of refining preference structures is a retrograde one because the
predictive power of the original theory is being weakened rather than strengthened. The challenge of the sure-thing principle can be described in a simple way by considering an example constructed by Allais (1953).

Formally, the sure-thing principle says that a preference between two gambles is independent of the states in which the two yield identical consequences. Therefore it requires \( f \succeq g \iff f' \succeq g' \) whenever \( S \) can be partitioned into two parts—\( I \) (the “irrelevant” event) and \( R \) (the “relevant” event)—such that, on \( I, f = g \) and \( f' = g' \), and on \( R, f = f' \) and \( g = g' \), where \( \succeq \) denotes preference-or-indifference. In the context of Allais’ (1953) choice problems (see Fig. 1 which is arranged by Fishburn and Wakker (1995) to associate states with probabilities in a particular alignment), the sure-thing principle asserts that an individual’s preference for \( f \) or \( g \) should not be influenced by the value of the common outcome produced by the probability of 0.89.

However, when being faced with the choices, many people prefer \( f \) to \( g \), but when the common consequence under \( s_1 \) is changed from 0 to 1M (1 million dollars), most of them prefer \( g' \) to \( f' \). Since the observed preference order changes depending on the value of the common outcome, which in this case is associated with the probability 0.89, the sure-thing principle is violated. The preference pattern of choosing \( f \) over \( g \) and \( g' \) over \( f' \) is referred to as the Allais paradox, which is then considered as a lever that moves EU.

Psychological models of risky decision making have struggled to explain the observed behaviour by proposing that rational decision makers operate through some form of expectation maximising process. This is the case from the EU theory in its traditional or Bernoullian form, to the von Neumann and Morgenstern EU theory (von Neumann and Morgenstern, 1947), to Savage’s subjective EU theory (Savage, 1954), to the weighted utility model (Edwards, 1962), to the rank-dependent utility model (Quiggin, 1982), to the sign-dependent utility model (Einhorn and Hogarth, 1986), to the rank- and sign-dependent utility model (Luce and Fishburn, 1991) and so on. All these models have one feature in common. That is, the final decision is simply a matter of comparing the totalled products of weights and values for each alternative while the computational overall values are cumulated in an interdimensionwise (holistic) way. As such, the independence is intuitively appealing and considered necessary for normative theories to hold, assuming that the addition of a common outcome will not change the overall maximisation for each option. This is analogously seen as using an equal-arm balance, where the bar holding the two pans will remain balanced if what is added to both pans is of the same weight.

As an alternative approach to human decision making, the equate-to-differentiate model (Li, 1994a, 1998a) is proposed as a means by which the dominance rule can be made applicable in more general cases. Weak dominance states that if alternative \( A \) is at least as good as alternative \( B \) on all attributes, and alternative \( A \) is definitely better than alternative \( B \) on at least one attribute, then alternative \( A \) dominates alternative \( B \) (cf. Lee, 1971; von Winterfeldt and Edwards, 1986). The model postulates that, in order to utilize the very intuitive or compelling rule of weak dominance to reach a binary choice between \( A \) and \( B \) in more general cases, the final decision is based on detecting \( A \) dominating \( B \) if there exists at least one \( j \) such that \( U_{Aj}(x_j) - U_{Bj}(x_j) > 0 \) having subjectively treated all \( U_{Aj}(x_j) - U_{Bj}(x_j) < 0 \) as \( U_{Aj}(x_j) - U_{Bj}(x_j) = 0 \), or, detecting \( B \) dominating \( A \) if there exists at least one \( j \) such that \( U_{Bj}(x_j) - U_{Aj}(x_j) > 0 \) having subjectively treated all \( U_{Bj}(x_j) - U_{Aj}(x_j) < 0 \) as \( U_{Bj}(x_j) - U_{Aj}(x_j) = 0 \), where \( x_j \) (\( j = 1, \ldots, M \)) is the objective value of each alternative on dimension \( j \) (for an axiomatic analysis, see Li, 2001).

The application of the equate-to-differentiate principle is straightforward, allowing the Allais

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<td>( g )</td>
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<td>( f' )</td>
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<td>5 M</td>
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<td>( g' )</td>
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Fig. 1. Allais Paradox, a counterexample to the sure-thing principle: If \( f \succeq g \) then \( f' \succeq g' \).
paradox to be accounted for and new phenomena to be predicted. In the light of a representative system (with best possible and worst possible outcome dimensions) to describe prospects involved in Allais’ choice problems (the description of Allais’ problem in its original verbal format is similar to the one illustrated in the succeeding paragraphs), Prospect A \(^1\) (C) is seen as better than Prospect B (D) on the worst possible outcome dimension, while Prospect B (D) is seen as better than Prospect A (C) on the best possible outcome dimension. In order to utilize weak dominance to reach a decision, people have to first try hard to “equate” less significant difference between prospects on either the best possible or the worst possible outcome dimension, thus leaving the greater one-dimensional difference to be differentiated as the determinant of the final choice. It is simply noted that the construction of the gambles will render the equating of difference on the “best possible outcome” dimension easier than that on the “worst possible outcome” dimension for the first pair of choices, but vice versa for the second, assuming a negatively accelerated (concave) utility function. This is most easily seen by way of graphical illustrations shown in Fig. 2. In other words, regardless of the fact that: the gambles in the second pair are the same as the gambles in the first pair; each minus a constant which is common to the gambles being considered, the gamble parameters are designed to encourage subjects to differentiate the subjectively greater difference (the difference between the bad-outcome ($0) of Prospect B and the certain outcome ($1 m) of Prospect A) on the worst possible outcome dimension in the first pair of choices, but to differentiate the subjectively greater difference (the difference between the good-outcome ($5 m) of Prospect D and the good-outcome ($1 m) of Prospect C) on the best possible outcome dimension in the second pair. The paradox arises because the final choice is not consistently based on a single fixed dimension in each pair of choices.

In searching for evidence of whether we are indeed guided by this simple rule in making our choices when faced with the Allais’ problems, a so-called “matching” task was designed (Li, 1994a) to help test the equate-to-differentiate account in further detail. Operationally, the outcomes of prospects on both the best and the worst possible outcome dimensions are paired in the task.

\(^1\) Prospect A, the sure thing option, itself can be seen as either the best possible outcome (when compared with the best possible outcome of the risky option) or the worst possible outcome (when compared with the worst possible outcome of the risky option).
Australian and Chinese students were asked to indicate their preferred choice in Allais’s two pairs of choice problems and then to choose the pair with outcomes which are, for them, the most different. If the equate-to-differentiate’s one-dimensional difference account is correct, then the knowledge of the chosen pair will permit explanation or prediction of option preference. That is, if Prospect A (C) is chosen, the subject should choose the pair of two “worst possible outcomes” as most different, thus leading to the minimization of the worst possible outcomes. On the other hand, if Prospect B (D) is chosen then the subject should choose the pair of two “best possible outcomes” as most different, thus leading to the maximization of the best possible outcomes. The choice and matching tasks reported in Li (1994a) are shown below:

First pair:

Choice
Prospect A: complete certainty of a good outcome, $1 m,
Prospect B: 0.10 probability of a very good outcome, $5 m; 0.89 probability of a good outcome, $1 m; 0.01 probability of a bad outcome, $0.

Matching (circle the one whose alternatives are most different)
F: “Win $1,000,000 with certainty” vs “0.10 probability of winning $5,000,000”,
G: “Win $1,000,000 with certainty” vs “0.01 probability of winning nothing”.

Second Pair:

Choice
Prospect C: 0.11 probability of a good outcome, $1m; 0.89 probability of a bad outcome, $0;
Prospect D: 0.10 probability of a very good outcome, $5m; 0.90 probability of a bad outcome, $0.

Matching (circle the one whose alternatives are most different)
I: “0.11 probability of winning $1,000,000” vs “0.10 probability of winning $5,000,000”,
J: “0.89 probability of winning nothing” vs “0.90 probability of winning nothing”.

The Allais-style preferences would suggest that there is a relationship between preference ranking and common outcome. That is, the majority of subjects choose the sure gain when the value of the common outcome is $1 m while the majority of subjects switch their choice when the value of the common outcome is reduced to $0. If the “third variable” (matching style) is taken into account, then more information is obtained and the data as seen in Table 1 is generated.

Based on the subjects’ responses, a total of 66 responses (60%) conformed to Allais’ position. A test reveals that there is a significant relationship between preference ranking and common outcome ($\chi^2(1) = 7.12, p < 0.01$). On the other hand, a total of 75 responses (68%) are consistent with the equate-to-differentiate theory and there is a significant relationship between common outcome, matching style and choice ($\chi^2(3) = 15.48, p < 0.01$).

An inference from these results is that if the matching strategy can be changed in each pair of choices, another way around the paradox for the EU theory could be generated. That is, perhaps it is the case that the sure gain option is not chosen in the first pair of choices and then the choice is reversed in the second pair of choices (where the second pair is obtained from the first by eliminating a fixed chance of winning a specific outcome from both prospects under consideration). For example, consider the following mountain-

<table>
<thead>
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<th>Value of common outcome</th>
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<tr>
<td>$1m (N = 55)$</td>
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<tr>
<td>Matching type</td>
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<tr>
<td>F</td>
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<td>C</td>
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<td>B</td>
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Table 1
Summary of making choice and matching decisions for Allais’ two pairs of choices

Note. The data underlined are those conforming to Allais’ position. The data in brackets are those predicted by the equate-to-differentiate model.
eering expedition problem \(^2\) \((N = 259)\), the associated percentages of subjects choosing each option are listed on the right, with two asterisks denoting significance at the 0.01 level:

Imagine that a mountaineering expedition of four members is caught in an avalanche. The base camp of the mountaineering expedition is faced with two alternative rescue programs. One is to send the wounded by helicopter to the provincial capital hospital for emergency treatment. The other is to send the wounded by vehicle to the local hospital for emergency treatment. Two programs (A and B) are under consideration using various combinations of these treatment alternatives. According to the exact estimates of experts, the possible outcomes of the two programs are as follows:

If Program A is adopted, one person will be saved with certainty \([35\%]^{**}\).
If Program B is adopted, there is an 11\% chance that three people will be saved, a 67\% chance that one person will be saved, but also a 22\% chance that no people will be saved \([65\%]\).

Imagine you are the base camp commander faced with deciding between programs A and B. Consider each carefully and circle your choice:

\[
\begin{array}{c}
A \\
B
\end{array}
\]

Now that you have made your decision as commander, conditions change. One change is that it is reported that the weather has worsened. Another is that there is a landslide on the road to the site of the accident. According to new estimates by the experts, the possible outcomes of the programs are now as follows:

If Program A’ is adopted, there is now a 33\% chance that one person will be saved, but a 67\% chance that no one will be saved \([63\%]^{**}\).
If Program B’ is adopted, there is now an 11\% chance that three people will be saved, but an 89\% chance that no one will be saved \([37\%]\).

Please reconsider each new program and circle your choice:

\[
\begin{array}{c}
A' \\
B'
\end{array}
\]

The inference was confirmed. When an effort is made to reduce the difference between the null-outcome and the sure gain in the first pair of choices, the preferred choice turns out to be risk seeking: “the preference for security in the neighbourhood of certainty” (Allais, 1991, p. 10) disappears. On the other hand, when an effort is made to increase the difference between the null-outcomes offered by the second pair of choices, the preferred choices turn out to be the opposite to the Allais pattern response.

This is the way the equate-to-differentiate model explains the violation of independence condition in both Allais’ risky problems dealing with monetary outcomes and Li’s risky problems dealing with human lives. Taken together, the violations alluded to are both interpreted by the equate-to-differentiate model as an indication that the equate-to-differentiate strategy (deciding which dimensional difference is to be equated and which is to be differentiated) is caused to change by the experimental conditions applied. The present model is developed on the grounds that the individual does not possess a stable ordinal utility \(v_i = V(u_{i1}, u_{i2}, \ldots, u_{ij}, \ldots, u_{iM})\) that allows trade-offs among individual utilities \(u_{ij}\) and is unable to perform a utility-integration calculation. The present model’s noncompensatory property will not allow deficiencies on one-dimension to be compensated for by high values on another in a choice process.

The present research was intended to further enhance our understanding of how the equate-to-differentiate strategy functions by extending the research from testing the “independence condition” for decision under risk to testing the “conjoint independence” for multiattribute decisions. Fishburn and Wakker (1995) define conjoint independence for preferences over \(n\)-tuples \((x_1, \ldots, x_n)\) in the product set \(X^n = \prod_{j=1}^n X_j\). Conjoint independence requires that, for all \(x, y \in X^n\), \(x \succ y\) is independent of common coordinate values of \(x\) and \(y\). In other words, it requires that \(x \succ y \iff x' \succ y'\) whenever \(\{1, \ldots, n\}\) can be partitioned into two

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\(^2\) Data was reported in Li (1993, Exhibit 3), but with Australian and Chinese data given separately.
parts $I$ and $R$ such that, on $I$, $x = y$ and $x' = y'$, and on $R$, $x = x'$ and $y = y'$. An example Fishburn and Wakker (1995) provide for $\succ$ on $\mathbb{R}^4$ is given in Fig. 3. Here $I = \{2, 4\}$ and $R = \{1, 3\}$.

The following experiments involving binary choice were designed to investigate the equate-to-differentiate explanation and prediction regarding the performance of subjects in multiattribute decisions. The riskless, or certain outcomes, rather than risk outcomes were employed in the present study for two main reasons.

One is that the common risk outcome, such as the one in Allais paradox, has been seen as not able to be cancelled because the outcomes were framed so as to simplify the presentation of the option but mask the presence of the common possible outcome under the (irrelevant) state. This line of thinking (e.g., Tversky and Kahneman, 1986; Shafir and Tversky, 1992) suggests that the violation of sure-thing principle be seen as a result of its application not being transparent. Tversky and Kahneman (1986) concluded that, like other normative principles of decision making, the sure-thing principle is generally satisfied when its application is transparent, but is sometime violated when it is not.

The second reason is that, even if the common risk outcome is transparently presented, such as the common outcome–probability pairs in Li’s (1994b) choice problems, the cancellation still could not be optimal because the common outcomes should be weighted differently with reference to their different rank order. Initiated by Quiggin (1982), theories in which the weights of stimulus components depend on their ranks have become more popular in recent years to explain violations of independence (Birnbaum and McIntosh, 1996). This family of rank-dependence theories (e.g., Allais, 1986; Lopes, 1984, 1987; Quiggin, 1982; Yaari, 1987) would see the inconsistent choices as a consequence of the fact that the decision weight for each outcome depends on the rank of the outcome with respect to the other outcomes in the lottery and the value of the outcome.

It is therefore expected that the conjoint independence can be tested by skirting round the ‘rank-dependent’ effect and by making the ‘transparent’ effect applicable in riskless choice situations, where the decision maker knows in advance which state of nature will occur. If the transparency account is right, or, as Birnbaum and McIntosh (1996) suggest that subjects can use cancellation if the experimental design promotes such a strategy, then the independence should be held when the irrelevant coordinate was transparently presented. In addition, in this case, the rank-dependent account will be mute on the question of whether the cancellation might be possible or not, when all outcomes are changed from risk ones to riskless ones, especially when the outcomes are changed from higher metric level (e.g., ordinal, interval or ratio measurement) to the lowest, not rankable, metric level (i.e., nominal measurement).

In what follows, just as it is believed that the presence of the sure-thing principle depends on whether the change of irrelevant event values will cause the final choice to be consistently based on a single fixed dimension in each pair of choices, similar consideration would suggest that giving rise to violation of the conjoint independence will also depend on whether the change of irrelevant coordinate values will cause the final choice to be consistently based on a single fixed dimension in each pair of choices. Applying the additive form of conjoint measurement (e.g., Luce and Tukey, 1964) to the simplest multiattribute decision where only two attributes are to be traded off (e.g.,

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<td>$y'$</td>
<td>4</td>
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Fig. 3. Conjoint independence: If $x \succ y$ then $x' \succ y'$.

3 The two pairs of choice problems are $(5, 0.998; 3000, 0.002)$ vs $(5, 0.998; 6000, 0.001)$ and $(5000, 0.998; 3000, 0.002)$ vs $(5000, 0.998; 6000, 0.001)$, where the common outcome $x$ is associated with a 99.8% probability. Of the subjects, 70% chose $(5, 0.998; 6000, 0.001)$ when $x = 5$, while 70% chose $(5000, 0.998; 3000, 0.002)$ when $x = 5000$. 

Alternative A is better than Alternative B on attribute x while Alternative B is better than Alternative A on attribute y, the overall utility \( U(x, y) \), from which a preference can presumably be derived, is the sum of two marginal utilities \( U_1(x) \) and \( U_2(y) \):

\[
U(x, y) = U_1(x) + U_2(y).
\]

Let \( \succ \) be a binary preference relation (is preferred to) on Alternatives A and B. In a normative decision model, \( A \succ B \) is said to hold if and only if \( U_A(x, y) > U_B(x, y) \). In the present model, however, Alternative A weakly dominating Alternative B is said to be detected if \( U_A(x) - U_B(x) > 0 \) is differentiated having treated \( U_A(y) - U_B(y) < 0 \) as \( U_A(y) - U_B(y) = 0 \), or, Alternative B weakly dominating Alternative A is said to be detected if \( U_B(y) - U_A(y) > 0 \) is differentiated having treated \( U_B(x) - U_A(x) < 0 \) as \( U_B(x) - U_A(x) = 0 \). In a contest between the value of \( |U_A(x) - U_B(x)| \) and the value of \( |U_A(y) - U_B(y)| \), it is quite possible that a common \( \Delta x \) shared by both A and B, a change of irrelevant coordinate value, could change the evaluation of \( U_A(x) - U_B(x) \) but not the evaluation of \( U_B(y) - U_A(y) \). It can easily be seen that, in an extreme case, \( U_A(x + \Delta x) - U_B(x + \Delta x) \) would turn out to be zero assuming a negatively accelerated (concave) utility function:

\[
\lim_{{\Delta x \to \infty}} U_A(x + \Delta x) - U_B(x + \Delta x) = 0.
\]

If this is the case, a decision of \( A \succ B \), which has presumably been reached by equating the difference of \( |U_A(y) - U_B(y)| \), would not be influenced by a common \( \Delta y \) but would be influenced by a common \( \Delta x \). This is because a common \( \Delta x \) could enhance the process of equating the difference of \( |U_A(x) - U_B(x)| \) rather than that of \( |U_A(y) - U_B(y)| \). In contrast, a decision of \( B \succ A \), which has presumably been reached by equating the difference of \( |U_A(x) - U_B(x)| \) would not be affected by a common \( \Delta x \) but would be affected by a common \( \Delta y \). Accordingly, in the equate-to-differentiate view, the violation of independence principle of normative decision theory could be produced, even though the irrelevant coordinate was transparently presented.

The aim of this research was to see whether the equate-to-differentiate approach could provide a possible explanation and prediction for the observed violations of the independence. The following choices dealing with common components shared by the two offered alternatives represent an attempt to carry out such a test.

1. Three decision problems and their results

1.1. Problem 1

1.1.1. Method

1.1.1.1. Subjects. 55 managerial students from the Northwest China Institute of Telecommunications Engineering, 60 managerial students from Min Jiang University, 60 engineering students from the Fujian Building Materials Engineering Institute participated, all as volunteers.

1.1.1.2. Stimuli and design. A hypothetical decision problem was prepared. It concerned a mission to purchase two saleable products from a factory. The number of respondents and the associated percentages of subjects choosing each option are listed on the right, with one asterisk denoting significance at the 0.05 level and two at the 0.01 level. The Purchasing Problem reads as follows:

Imagine you are a purchasing agent and your mission is to buy Product A and Product B produced in a factory. The product salesman in the factory tells you that both A and B are goods in great demand. If you want to buy either of them you must buy extra unsaleable goods for ¥300. In order to buy A and B, you agree to buy the extra unsaleable goods which you would otherwise be unwilling to buy. Furthermore, the salesman insists that both goods are indeed in such short supply that you can only buy one of them:

Which product are you going to buy? Please circle your choice.

Product A: cost ¥2500 18%**  
Product B: cost ¥7400 82%.  

N = 175  

1.1.2. Results and discussion

In the context of the present problem, Products A and B can be partitioned into two parts
and \( R \) where on \( I \), Product \( A = Product B \) as the amount of additional payment \( ¥300 \) is the same, and on \( R \), Product \( A \neq Product B \). The conjoint independence does not predict which product should be chosen, but it does predict that an individual's preference for Product \( A \) or Product \( B \) should be independent of the value of the additional payment (the common coordinate value of \( ¥300 \)). Like the conjoint independence's hypothesis, the equate-to-differentiate's hypothesis, in this case, does not predict which product should be chosen either. However, unlike the conjoint independence's hypothesis, the present model's dimensional and noncompensatory property regarding overall judgment predicts that an individual's preference for Product \( A \) or Product \( B \) should not be independent of the value of additional payment.

It is both reasonable and plausible to expect that, if decisions were based on the mission requirement or the degree of short supply, the decision makers would choose with equal frequency the two alternatives offered. Then, according to conjoint independence, such an even choice will remain unchanged even if the additional payment of \( ¥300 \) is added. The equate-to-differentiate's hypothesis, however, does not assert such an even choice. This is because, in spite of the fact that the price difference between \( A \) and \( B \) itself should not be the reason for a preferred choice considering the mission requirement and the degree of short supply, the price difference might be the cause which is most likely to be responsible for a resulting change in intra-dimensional evaluation (i.e., \( | U_A(x) - U_B(x) | \)) when the additional payment was added to the monetary dimensions on which \( ¥2500 \) of Product \( A \) and \( ¥7400 \) of Product \( B \) were represented. Such a change in intra-dimensional evaluation in terms of paid price, in the equate-to-differentiate view, could lead to an uneven choice if it was great enough.

It can be seen that, of all the subjects who answered the problem, we ended up with a total of 143 out of 175 subjects (82%) who were going to buy Product \( B \), the product of higher cost. The overwhelming majority choice suggests that the respondents tended to evaluate the additional \( ¥300 \) in relation to the price of the products \( A \) and \( B \), despite the fact that the additional \( ¥300 \) is presented transparently and could presumably be cancelled. In relative terms \( ¥300 \) added to \( ¥2500 \) is apparently seen as more impressive than \( ¥300 \) added to \( ¥7400 \), in spite of the fact that the two additional payments are exactly the same in terms of their physical amount (300) as well as their physical scaling unit (¥). This result is in full harmony with the psychophysical law that psychological response is a nonlinear function of the magnitude of physical change. Along the corresponding concave utility function of monetary value, the payment difference on the upper side is likely to be judged to be less than that on the lower side.

1.2. Problem 2

Problem 1 has suggested that the common coordinate values could cause a change in intra-dimensional difference thus leading to the failure of the cancellation operation. However, it can be argued that it is inclusive because it lacks the necessary control. In order to place observed choice behaviors in a controlled condition where the value of irrelevant coordinate is manipulated, the present problem was designed to make decision makers choose twice.

1.2.1. Method

1.2.1.1. Subjects. The subjects in this experiment were 50 undergraduate students from various disciplines at the University of New South Wales. They participated in order to fulfil course requirements.

1.2.1.2. Stimuli and design. A hypothetical decision problem was prepared. It concerned a choice between two candidates who were applying for a teaching post. The selecting problem reads as follows:

Imagine that you serve on the Search Committee of your Department and that there are two recent Ph.D. graduates applying for a teaching post in the Department. The choice between the two candidates is difficult because of ambiguous considerations regarding teaching experience, research ability, academic achievement, reasons
for applying and expectations about holding a job in the Department. You decide to base your decision entirely on the following information. To which candidate would you award the post? Tick the candidate (A or B) of your choice and indicate your confidence about your decision:

**Candidate A:** has published one paper in a “first rate” international academic journal,

**Candidate B:** has published three papers in a national academic journal,

**Confidence rating:** 1 2 3 4 5 6 7

Now that you have made your choice, additional information becomes available. That is, both A and B have one additional paper accepted by a “first rate” international academic journal for publication. Please reconsider each candidate's application and tick the candidate (A or B) of your choice and indicate your confidence about your decision:

Candidate A  Candidate B/Confidence Rating:  
1 2 3 4 5 6 7

### 1.2.2. Results and discussion

In the context of the present problem, the conjoint independence asserts that, if Candidate A (or B) is preferred to Candidate B (or A) without an additional paper accepted, then Candidate A (or B) should also be preferred to Candidate B (or A) with an additional paper accepted. Adding the same additional paper is not expected to reverse the original decision.

According to equate-to-differentiate view, however, a binary choice is seen as a process where people equate offered differences between options on one-dimension but differentiate another one-dimensional difference as the final determinant. The present problem is constructed so that A’s publication is better in quality while B’s publication is better in quantity. It is therefore reasoned that subjects who selected Candidate A (or B) first, tend to base their final choice on only the “quality” (or “quantity”) dimension, having treated the values on the “quantity” (or “quality”) dimension as if they were equal. 4 If the equate-to-differentiate hypothesis is correct, adding an additional “quality” paper to both candidates will render the equating of differences between the two candidates easier on the “quality” dimension (i.e., the common ability to publish in a first rate journal will lessen the “quality” difference between the two candidates). This leads to a particular kind of interaction: those who selected Candidate B (of better quantity) first, might obey the independence condition because each choice would be consistently based on the “quantity” dimension, while those who selected Candidate A (of better quality) first might violate the independence because the first choice was based on the “quality” dimension but the second might be based on the “quantity” dimension. On the other hand, if the transparency hypothesis is correct, the method of adding an additional paper to both candidates, which makes the common outcome clearly stand out or transparent by using wording which stresses the commonality, would encourage subjects to use cancellation to discard the common components, thus satisfying the independence.

For the purposes of data analysis, a variable called strength of preference, which was the product of the variables choice (+1 for selecting Candidate B and −1 for selecting Candidate A) and confidence of choice (ranging from 1–7), was constructed. The resulting variable ranges from −7 to +7. A positive score on the new variable indicates that Candidate B is more likely to be selected and a negative score indicates that Candidate A is more likely to be selected. Fig. 4 shows the mean strength of preference by publication evidence (with common publication vs without common publication) and subject group (subjects whose first choice was Candidate A vs subjects whose first choice was Candidate B). Note that there was indeed a significant interaction, $F(1,48) = 15.698$, $p < 0.001$. The character of the interaction was consistent

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4 It is quite possible that, as an anonymous referee argued, subjects' dimensionality was not quality vs quantity but energy, or intelligence, or prestige, or mobility. But however it was perceived, the added common outcome would be represented on that dimension which differs from the other.
with that predicted by the equate-to-differentiate hypothesis. That is, it was the choices made by subjects whose first choice was Candidate B that were unaffected by whether the common publication was absence or presence, \( t(27) = 0.563, \) ns, while the choices made by subjects whose first choice was Candidate A changed markedly when one common publication was added, \( t(21) = 4.908, p < 0.001. \) Thus, the hypothesis that shift in preferences can be attributed to the change of equate-to-differentiate strategy is supported.

The interesting interaction can also be seen from Table 2 that, among those who chose Candidate A first, it is apparent that 9 out of 22 shifted their choices when the common publication was added. On the contrary, among those who chose Candidate B first, only 1 out of 28 subjects shifted his choice when the common publication was added. A \( 2 \times 2 \) test revealed a significant relationship between common publication and choice for Candidate A first chosen subjects \( (\chi^2(1) = 11.31, p < 0.00) \) but not for Candidate B first chosen subjects \( (\chi^2(1) = 1.02, p = 0.313). \) A very similar result, which was obtained by using 40 Singaporean subjects but lacking an attempt to measure subjects’ confidence of choice, is also presented in Table 2.

The results confirmed the previously described expectations—the addition of an additional quality publication appears to be irrelevant to B-first-chosen subjects (whose qualitative difference is presumably equated) but relevant to A-first-chosen subjects (whose quantitative difference is presumably equated). The fact that the independence is likely to have been obeyed by those who selected B first but violated by those who selected A first corroborates the equate-to-differentiate line of reasoning.

### Table 2

<table>
<thead>
<tr>
<th>Change of common publication</th>
<th>Subject group</th>
<th>Candidate A</th>
<th>Candidate B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Candidate A</td>
<td>Candidate B</td>
<td></td>
</tr>
<tr>
<td>No common publication</td>
<td>22 [24]</td>
<td>0 [0]</td>
<td>0 [0]</td>
</tr>
<tr>
<td>( \chi^2(1) = 11.31 )</td>
<td>( \chi^2(1) = 1.02 )</td>
<td>( p &lt; 0.00 )</td>
<td>( p = 0.313 )</td>
</tr>
<tr>
<td>( p &lt; 0.00 )</td>
<td>( [\chi^2(1) = 2.13 )</td>
<td>( p = 0.144 )</td>
<td></td>
</tr>
</tbody>
</table>

Note. The data in brackets are those obtained by testing 40 undergraduate students from Nanyang Business School, Nanyang Technological University.

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1.3. Problem 3

1.3.1. Method

1.3.1.1. Subjects. Subjects were 96 undergraduate students enrolled in Introductory Psychology courses at the University of New South Wales, who participated for course credit.

1.3.1.2. Stimuli and design. The judging problem used involved a jury judgment. The custody scenario, with some changes in content, is borrowed from Shafir (1993). It reads as follows:

Imagine that you serve on the jury of an only-child sole-custody case following a relatively messy divorce. The facts of the case are complicated by ambiguous economic, social, and emotional considerations, and you decide to base your decision entirely on the following few observations. To which parent would you award sole custody of the child? Tick the parent (A or B) of your choice and indicate your confidence about your decision:

**Parent A:**
- stable income
- very good housing
minor education problems
extremely active social life

Parent B:
well educated and cultivated
reasonable social life
relatively unstable income
average housing

Confidence rating: 1 2 3 4 5 6 7

Now that you have made your judgment, additional evidence is provided. That is, it is reported that if the sole custody is awarded (each of the parents will receive extra financial support from the grandparents) each of the parents will reduce work-related travel to the limit so as to change the present situation of extensive absences due to travel. Please reconsider each parent’s features and tick your choice as to which parent (A or B) you would award sole custody of the child, and indicate your confidence about your decision:

Judgment: Parent A Parent B
Confidence rating: 1 2 3 4 5 6 7

The problems were presented in two different versions. Each of approximately half the subjects was randomly assigned to respond to either of the two versions. The two versions differ only in the bracketed statements.

1.3.2. Results and discussion

In the present problem, there are four statements describing each option (either Parent A or B). Two of them refer to evidence of material welfare while another two to evidence of psychosocial properties. The problem was designed so that Parent A is better than Parent B on material welfare but Parent B is better than Parent A on psychosocial properties. Now, if the equate-to-differentiate hypothesis as described previously is true, then a particular kind of interaction should have been observed. Specifically, the addition of financial support (which lessens the income difference) will only influence one group of respondents, while the addition of social support (which lessens the psychosocial difference) will influence another.

A variable called strength of judgment, which was the product of the variables judgment (+1 for selecting Parent B and −1 for selecting Parent A) and confidence of judgment (ranging from 1–7), was also constructed. Fig. 5 shows the mean strength of judgment by common evidence (with common evidence vs without common evidence) and subject group (subjects whose first choice was Parent A vs subjects whose first choice was Parent B). The analysis of “reduced travel” version (N = 51) yielded a significant interaction, $F(1, 49) = 5.536, p < 0.03$. As expected, subjects who selected Parent A first did not change their decisions whether the work-related travel was reduced or not, $t(12) = 1.028$ ns, while subjects who selected Parent B first changed their decisions when the “reduced travel” was present, $t(37) = 3.069, p < 0.01$.

On the other hand, the analysis of the “increased income” version (N = 45) also yielded a significant interaction, $F(1, 43) = 31.253, p < 0.001$. In this case, subjects who selected Parent A first, changed their decision when the extra income was added, $t(14) = 5.234, p < 0.001$, while subjects who selected Parent B first did not change their decisions whether the extra income was absent or present, $t(29) = 1.582$, ns.

It can also be seen from Table 3 that, among those who chose Parent A first, only 1 out of 13 shifted his choice when the common evidence of reduced travel was added but 10 out of 15 shifted their choices when the common evidence
of increased income was added. On the contrary, among those who chose Parent B first, we see that only 2 out of 30 shifted their choice when the common evidence of increased income was added but 9 out of 28 shifted their choices when the common evidence of reduced travel was added. In both cases, a $\chi^2$ test revealed a significant relationship between common evidence and preferential choice (for Candidate A first chosen subjects $\chi^2(1) = 10.15$, $p < 0.01$; for Candidate B first chosen subjects $\chi^2(1) = 3.58$, $p = 0.05$). Thus, the hypothesis that shift in choices can be attributed to the change of equate-to-differentiate strategy is supported.

2. General discussion

How people ought to use cancellation so as to obey independence has been a hotly disputed subject of study for a long time. Despite a great deal of research ostensibly proving that the cancellation is systematically violated, supportive evidence demonstrating that the cancellation has not been disproved by the experiment has been emerging constantly since Tversky and Kahneman's (1986) transparency account was proposed. Corresponding decision models, such as the cancellation-and-focus model (Houston and Roskos-Ewoldsen, 1998; Houston and Sherman, 1995; Houston et al., 1991), have also been developed. The cancellation-and-focus model is based on the differential treatment of shared and unique features when faced with a choice dilemma. This model describes that shared features will be canceled and will play little or no role in the choice process. Only the unique features of the candidate alternatives will be given consideration when preference judgments are made on the basis of a feature matching process. Although it is unlikely to detect cancellation in judgment, Houston and Sherman (1995) demonstrate that, under transparent conditions, their model's cancellation component is so robust that once the shared features are canceled for the purposes of choosing an alternative, these shared features seem to remain canceled when one subsequently judges the value of the alternatives.

In the equate-to-differentiate view, however, both Tversky et al.'s and Houston et al.'s experimental designs concerning transparency and cancellation-and-focus do not distinguish between conditions in which common outcomes (shared features) will or will not change the subjective intra-dimensional evaluation of the difference between unique outcomes (features). It can be seen that, in Tversky et al.'s experiments where independence is violated, the presence or absence of common outcomes actually does change the intra-dimensional evaluation of the difference between the best or the worst possible outcomes (e.g., Allais's type of choice). It is true that such a change in intra-dimensional evaluation is more likely to be enhanced when the possible outcomes were not transparently presented, but, without such a change in intra-dimensional evaluation, the transparency manipulation is not able to lead to independence (e.g., choices in Li (1994b) shown in Footnote 3). On the other hand, in Houston et al.'s experiments where independence is obeyed, it can be seen that the shared features actually do not change the intra-dimensional evaluation of the difference between unique good features or the difference between unique bad features (e.g., the automobile selecting problem in Houston and Sherman, 1995). That is, what Houston and Sherman manipulate is that, when a choice is a contest between unique good (bad) features of the alternatives, all the shared features are limited to bad (good) ones, which makes shared features completely irrelevant to the unique features. An inference from the previous analysis of Allais choice problems and Li's choice problems is that, if shared-good features (which lessen the "good"
differences) were added to the unique-good pair of alternatives, or if shared-bad features (which lessen the “bad” difference) were added to the unique-bad pair of alternatives, hence changing the intra-dimensional evaluation of the difference between the unique good (bad) features, another way of making a choice could be generated. It is therefore reasoned that, without taking the change in intra-dimensional evaluation into account, previous demonstrations by both Tversky et al. and Houston et al. might not be conclusive, hence there is little point in attempting an investigation of their explanations. The variation in selection of common outcomes (features) will leave their test result an open one.

The goal of the present research was to test the validity of the ‘equate-to-differentiate’ model further by examining its implications against those of other normative theories that are being proposed to explain and predict the “conjoint independence” for multi-attribute decisions. The main point of the model in the case of pairwise choice, where each alternative is generally better than the other on a single dimension, is that individuals seek to equate offered differences between alternatives on one-dimension, so as to differentiate the unequated one-dimensional difference as the determinant of the preferred alternative. That is, the offered alternatives are assumed to be equivalent on one-dimension, and the alternative with the greater utility on the other dimension will then be chosen. Dominance detecting would then be a special case of the equate-to-differentiate rule.

Guided by the equate-to-differentiate thinking, it has been assumed that all risk attitudes (e.g., either risk-seeking or risk-averse) should be observed when people's equate-to-differentiate strategy is caused to be changed by the experimental conditions applied. Such a manipulation of risky preference is empirically proposed and tested over a variety of decision axioms in decisions under risk, i.e., independence (Li, 1993, 1994b, 1995), invariance (Li and Adams, 1995; Li, 1998a; Li et al., 2000), and transitivity (Li, 1996). In the same vein, the dominance axiom is also tested in decision under certainty (Li, 1998b). Experimental evidence obtained is generally consistent with the predictions of equate-to-differentiate approach but is at variance with these normative axioms.

The three experiments reported in this paper, departing from the main trend, provide another avenue to investigate the conditions under which the axiom of independence is violated or not in binary choices. The data gathered in the present study shows that the common outcomes (shared features) selected in more transparent conditions to test independence does not lessen the violation of this basic rule of normative decision making. Such a finding, together with those obtained in the domain of risky choice, adds to the wealth of evidence pointing to fundamental limitations in people's capacity to process information. It is the present contention that the inconsistency between the decisions shown by human subjects and the ordering implied by any kind of utility-integration calculations is inevitable and predictable because the equate-to-differentiate rule best reflects actual human practice.

2.1. Marketing implications

Being a consumer in the Information Age is both exhilarating and mind-boggling. There are vast amounts of goods and services available to us, and the range is increasing day by day. We are faced with decisions every minute: To buy or not to buy? Which features to purchase? Which brand to use? Moreover, consumers are often asked to make difficult value tradeoffs, such as price versus safety in purchasing an automobile or environmental protection versus convenience in a variety of goods.

The issues raised in this paper may be seen to bear practical relevance for managers who seek to communicate value in their deals to prospective buyers. The present findings suggest that marketers can affect consumer preferences by manipulating the choice set of competing alternatives. Because few products achieve complete dominance in the marketplace, the choice of product comparisons also implicitly determines the nature of the trade-offs that consumers are asked to make in order to choose. In particular, when no dominant alternative exists, it is not imperative for managers to change all the features offered so as to make the
dominance rule applicable. The effect of shared features explored in this paper suggests that, for the consumer to choose from nondominated alternatives, marketers should consider not only the unique features of their brands but also the features common to all available alternatives. It would NOT be natural for ordinary people to reason that shared features provide no diagnostic information for preference judgments and are irrelevant for preferences. The shared features do matter.

With this in mind, it is quite possible for a marketer to alter the consumers’ decision even if he or she is about to add or subtract exactly the same feature that his or her competitors have already added or subtracted.

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