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TRENDS/CASES

"Doing fractions" and "understanding fairness": Examining the cultural scripts of a mathematics lesson through the eyes of Japanese and Malaysian educators

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Abstract This article analyzes Malaysian and Japanese educators' reactions to a mathematics lesson observed by a subset of researchers. The main aims are to reveal the cultural scripts about mathematical learning that are held by, but often invisible to, members of a culture; and to address educators' conceptions of teaching mathematics across cultures. This study's emphasis is placed largely on understanding the logic behind the teaching scripts and the underlying epistemology of teacher pedagogical decisions. A qualitative

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analytical approach identifies the idea that focus on a single problem during a whole lesson enables opportunities for deeper learning (Japanese participants) or may be boring for students (Malaysian educators), and explores the value of, or problems associated with, extrinsic rewards. This research stands to make an important contribution, both through its novel method, and in its specific ideas about desirable qualities of mathematics lessons and how these might differ from the viewpoint of individual cultures.

Keywords Cross-cultural analysis · Mathematics lesson · Fraction · Fairness · Elementary school · Teacher pedagogical decisions · Malaysia · Japan

As a large body of cross-cultural studies shows, teachers and researchers may often not be aware of the logic of teaching in detail (Dreher et al., 2021; Kaur et al., 2013; Mesiti et al., 2021). This means a culture of teaching and educators' conceptions of teaching cannot be extracted only from practitioner reflection or research analysis in the same cultural context and social background, even with after-lesson discussions, video studies, and lesson microscopic observations alone (Baldry et al., 2023; Jacobs & Morita, 2002). Over several decades, schools and the cultural script of teaching in classrooms have faced "a lot of change without improvement" (Cuban, 2013; Fey, 1979; Ravitch, 2020). Hiebert and Stigler's review shows that "[t]he core of teaching in the United States has not changed much, at least in the past 50 years" (2017, p. 169), and Tyack and Cuban (1997) assert that "[t]o bring about improvement at the heart of education-classroom instruction…has proven to be the most difficult kind of reform" (p. 134). Such findings indicate that empirical research to unearth measures of pedagogical understanding may be necessary.

This study examines a Malaysian elementary school mathematics lesson on "doing fractions" and "understanding fairness", through the eyes of Japan-based educators. The main aim is to understand in depth the logic behind the teaching and realize the underlying epistemology in teacher pedagogical decisions. We place emphasis here on a cross-cultural analysis to obtain an in-depth understanding of the cultural script of teaching in Malaysia. Furthermore, through obtaining data in the form of video and audio transcripts of lesson observations, we aim to interpret how mathematical pedagogy can be illustrated and improved.

In this cross-cultural study, "doing" and "understanding" mathematics refers to the two broad categories of knowledge-procedural knowledge and conceptual knowledge (Chapin et al., 2003; Hiebert & Lefevre, 1986). According to Bruner (2016) and Ryle (2000), procedural knowledge is knowledge of procedures, "know-how", or "knowing how", focusing on information required to understand problems. Conceptual knowledge is propositional knowledge or showing an understanding of the principles and relationships that underlie a domain (Hiebert & Lefevre, 1986). We argue that conceptual knowledge is the concept of "how to know", or "knowing that", which involves understanding the conceptual basis for solving mathematical problems (Putnam et al., 1990). Therefore, mathematical pedagogy is directly related to the promotion of student understanding of mathematical concepts and the development of their thinking regarding these concepts.

Research question

This study's main research question is how a cross-cultural analysis of a mathematics lesson in Malaysia can examine mathematical pedagogy and realize the underlying epistemology in teacher pedagogical decisions.

Theoretical context

The resistance phenomenon in classroom practices indicates that "we are dealing not just with psychological and pedagogical issues. We are dealing with cultural matters" (Gallimore, 1996, p. 230). Fullan (2015), in discussing the dimensions of educational change, also stressed the relevance of articulating beliefs and values of a particular community to achieve a "real change". What constitutes the beliefs and values that guide classroom activities are often unseen and unnoticed, perhaps because they are too familiar and widespread (Gallimore, 1996). Hence, the consequence to educational researchers is the difficulty of describing the unseen. This, however, should not discourage the effort to unveil the "unseen" elements that could be the key to the black box of educational practice. Stigler and Hiebert (1999) termed this element "cultural scripts". They defined cultural scripts as the "generalized knowledge about an event that resides in the heads of participants. These scripts guide behaviour and also tell participants what to expect" (p. 85). Through discovering the cultural script of teaching and having prior knowledge of it before a career in education commences, teachers and all stakeholders can gain an improved understanding and increase the likelihood of change in the classroom.

Obtaining a deeper understanding of classroom activities from this point of view also means that we should take seriously the role and impact of cultural scripts in determining the effectiveness of logic of a lesson. Likewise, it is equally important to realize that many of these implicit rules underlying classroom practices are yet to be made explicit. Benedict (1946) uses a metaphor to describe this situation: "It is hard to be conscious of the eyes through which one looks" (p. 14). Sarkar Arani et al. (2017) proposed the use of a different "lens". The term "lens" in this context can be defined as the cultural perspective of one particular group of people, which may be different from those of another.

Why cross-cultural analysis?

Analyzing a lesson, or a teaching scenario in general, through the lens of practitioner and researcher with different sociocultural or professional backgrounds is believed to produce two elementary results. Firstly, it leads to the discovery of cultural scripts of a particular lesson, and secondly, it enables you to be aware of the existence of your own otherwise obscure lens that you may have in your subconscious.

Through the analysis of a lesson in this way, researchers can obtain a deeper understanding of logic and the cultural script of teaching. This is because practitioners, through their different lenses, arguments, and interpretation, may be more familiar with the epistemology that they hold themselves. This can also be seen in the day-to-day decisions made in the classroom and the process of improving praxis (Biesta, 2010). Cross-cultural analysis also provides an alternative perspective for us to know how to recognize the "tacit" theory of knowledge underlying practice (Dreher et al., 2021). As Friedman enlightens, "research *in*

practice, not research *on* practice" (2006, p. 132) may be a key element to induce change. Knowing this may alter our image, values, and comprehension as researchers, encouraging us to change our landscape of inquiry for a new science of education (Thomas, 2012). This could inevitably lead to more of a focus on evidence-based and specific sequence analysis of a lesson to understand the culture and logic of the teaching and process of praxis, which, as Elliott (2016, p. 279) argued, "teachers are largely unaware of". In applying cross-cultural analysis, our study plans to make apparent the structure of meanings that are hidden in lesson practice, rather than looking for the best practice and transferring it fundamentally, helping teachers to become aware of several key areas of their lesson for an improved pedagogy.

Methodology

This study utilizes a qualitative data collection methodology based on a cross-cultural lesson analysis. This process involves observing lessons, and recording and transcribing them and all other discussions thereafter. Transcripts of the lesson are then distributed to researchers and educators prior to further discussions. Within each transcript discourse, participants are numbered and referred to by number in subsequent discussions. Transcript as a text provides empirical evidence of a lesson study like this one.

Participants of this research included the teacher of the mathematics lesson; the 19 eight-year-old students; and a selection of researchers, Malaysian Participants and Japanese Participants (MPs and JPs) who took part in discussions of the lesson at different stages after completion. During the class observation session, a total of 14 participants were present, including three Japanese researchers, eight locally based Malaysian teachers, and three MP researchers (Table 1). The teachers who observed the lesson had a range of experience from 10 to over 20 years. Four of these teachers had about 20 years of experience, while the remaining six had about 10. The teacher, Ms. Zulai, was a veteran teacher of the second grade with 10 years of experience and had taught the students of that class for one year prior to this study.

Three researchers based in Japan attended the lesson to operate all the equipment to record the lesson, including three digital voice recorders and three video cameras. Through their presence, discussions consequently were easier to lead, as they not only observed the lesson in person but also operated the equipment and could affirm any discrepancy of viewpoints that may appear in discussions if participants relied on transcripts alone.

Data collection procedures

The case-based study reported in this paper was implemented by the teachers of Sungai Jai Elementary School, in the Kampung Sungai Jai, Negeri Sembilan region of Malaysia. A second-grade mathematics lesson (eight-year-old students) in a public elementary school in this school was observed on February 24, 2017. The school had six classes, one for each grade (one to six), with 15 to 25 students in each class.

This school was chosen for three reasons. Firstly, the school had a collaborative link with the MP researchers of this study. Secondly, this school had expressed an interest in improving the pedagogy with the premise of redesigning their mathematics teaching scripts and transforming the student learning environment. Finally, the school teachers tried to provide

Table 1 Data collection procedures

Stage	Date	Event	Location	Participants
1	Feb. 24, 2017	Lesson observation	Sungai Jai Elementary School, in the Kampung Sungai Jai, Negeri Sembilan, Malaysia (same location)	19 students (MPs)
				1 class teacher (MP)
				1 principal (MP)
				3 researchers (JPs)
				3 researchers (MPs)
				8 teachers (MPs)
				1 principal (MP)
2	Feb. 24, 2017	Initial post-lesson discussion		1 class teacher (MP)
				3 researchers (JPs)
				3 researchers (MPs)
				8 teachers (MPs)
				6 researchers (JPs)
3	Jul. 13, 2017	Transcript-based lesson analysis & discussion	Nagoya University, Nagoya, Japan	8 educators (JPs)
				4 post-graduate students (JPs)
				3 researchers (JPs)
4	Jul. 29, 2017	Transcript-based lesson analysis & discussion	National University of Malaysia	26 local educators
				7 post-graduate students (MPs)
				4 researchers (JPs)
5	Nov. 4, 2017	Transcript-based lesson analysis & discussion	National University of Malaysia	6 educators (MPs)
				3 post-graduate students (MPs)

a learning environment which encourages students to actively solve problems by focusing on the process of "doing", "understanding", and fully comprehending mathematics.

There were four further phases of data collection following the lesson observation on February 24, 2017 (Table 1). The first phase involved discussions immediately after the class on the same day. Discussions were recorded for these sessions and translated, first from Malay to English, and then into Japanese, for the benefit of all participants. The objective of having observers from different sociocultural backgrounds is to ensure a rich variation of perspectives of each lesson. Each observer should be able to provide a different account of the lesson through their cultural lens.

The second data collection phase was held in Nagoya University, Japan, on July 13, 2017, and was attended by 18 people. This group included six researchers, eight educators, and four graduate students. This discussion was conducted and recorded in Japanese and then translated into an English transcript.

The third data collection phase was held on July 29, 2017, at the National University of Malaysia, Bangi, Selangor, Malaysia. This session was attended by 36 people, including the teacher of the class that was observed; the three co-authors; and local educators, professors, and graduate students from the area. Discussions at this session were held in Malay language. One of the co-authors also helped to provide Malay-Japanese interpretations of main points discussed on the day. All the discussions were recorded, then transcribed and translated into English by the MP of this study for the benefit of readers of this paper. Therefore, any misunderstandings in linguistical nuance between expressions or lexicon used to express classrooms in the educational context were minimized.

A further discussion meeting on the lesson was held with the collaborative participation of 13 locally based Malaysian teachers, university professors, and several graduate students. This discussion, lasting for three hours, was held primarily in Malay and translated for the purpose of this paper into English. This group met at the National University of Malaysia on Nov. 4, 2017.

The lesson

The mathematics lesson started with the teacher distributing small pizzas, with a diameter of 15 cm, to all students. The teacher then instructed students to cut the pizza into equal-size parts to be shared with their family members, regardless of how many family members they had. Next, the teacher proceeded to ask students to represent the appearance of the pizza in writing, and then to put it away to be taken home later. This task was an introductory procedure before progressing to the focus of the lesson: fractions. After the introduction of the learning task, the teacher emphasized that fractions must be the same size to be fair. With that, the teacher began a series of activities such as using figures connected with mathematics and math kits to check and consolidate students' understanding. At the end of activity 4 (Table 2), the teacher walked around the class to assess students' worksheets and help students self-evaluate, rewarding their work with stars.

The teacher of this lesson participated in two discussion meetings after the mathematics lesson in Malaysia and had the opportunity to explain the lesson process and her choices of teaching materials. She was also interviewed and wrote reflection notes on her teaching.

Table 2 The process of the mathematics lesson

Category	Teaching and learning activities
Introduction	(T1-Ca197)
	(1): (T1-C92) Teacher distributes pizzas around the class
	(2): (T93-Cs148) Teacher instructs students how to cut pizza in half
	(3): (T149-Cs180) Teacher asks students how to write in a clear and simple way
	(4): (T181-Ca197) Teacher confirms students are learning
Development	(T198-T621)
	(1): (T198-Ca213) Teacher confirms that "Fractions have to be the same size to be equal"
	(2): (T214-Ca322) Activity 1: Placing multiple-sized slices into a circle
	(3): (T323-T458) Activity 2: Distributing group tasks; how to cut a ribbon into equal lengths
	(4): (T486-C621) Activity 3: Bridge map; matching the correct figure with the correct fraction
Turn	(T622-C733)
	(1): (T622-C733) Evaluated Activity 4: Making fractions through shading multiple-sized shapes
Conclusion	(T734-T801)
	(1): (T734-T801) Identifying the fraction from the diagram

T denotes speech by the teacher; C, an unidentified student; Ca, most students in the class; and Cs, several students in the class. The number suffix refers to the utterance number of the lesson transcript.

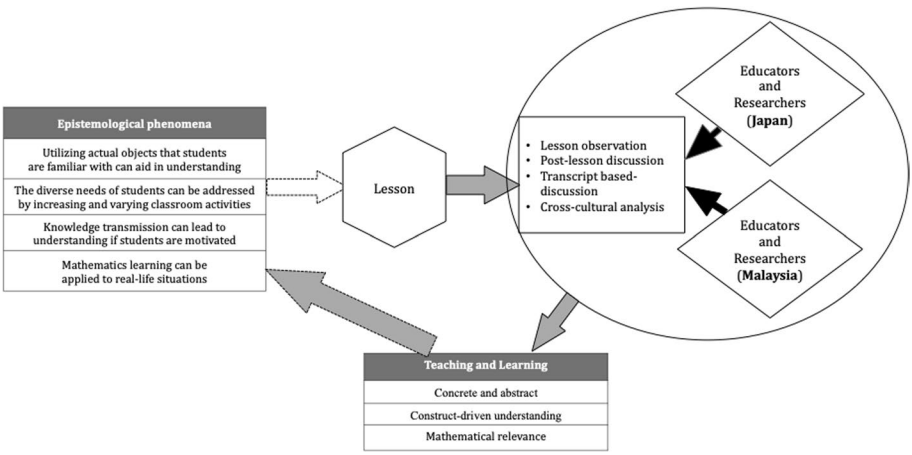

Figure 1 Cross-cultural analysis framework

Cross-cultural analysis framework

The study relied on qualitative data analysis, "a research method for the subjective interpretation of the content of text data through the systematic classification process of coding and identifying themes or patterns" (Hsieh & Shannon, 2005, p. 1278) to interpret the data collected. Notably, we employed the deductive approach, which moves from the general to

the specific, aiming to classify a wide range of data into small content categories (Weber, 1990) (Figure 1).

Here, the process of classifying the data is divided into three main phases: preparation, organization, and reporting. These phases are comparable to Elo and Kyngäs's research (2007). In the preparation phase, researchers familiarized themselves with the three elementary data sets: lesson observation, lesson transcript, and transcripts of all the discussion meetings after the lesson in both Malaysia and Japan. The data was then reviewed against the video recordings and teaching materials such as the textbook, worksheets, math kits, and learning resources, which included a counting gadget, math blocks, linking cubes, and a circle map.

This step provided a sense of the whole data that functioned as an insight into the next phase: organization. In the organization phase, the lesson was divided into several segments, or parts, to understand the segments' relationships better. Researchers logically divided the lesson process into specific parts (introduction, development, turn, and conclusion) as the logic of the lesson structure (Sarkar Arani et al., 2017). Within the process of a lesson, it seems that the most important stage is "turn". "Turn" refers to a scenario in which students' ideas and opinions are given life through conversation, dialogue, and discussion. Based on this, the lesson takes a turn away from its previous course and acts as a new source of learning for participants.

In the next phase, reporting, researchers read the lesson transcripts and discussion meetings held after the lesson to discuss a more in-depth review. The researchers cross-referenced each other's interpretation of themes that emerged. Something that was equally important was ensuring that themes effectively represented the concepts and accurately captured what was intended (Schreier, 2012). Finally, the results of the analysis were described using the themes identified.

Findings

This section will examine the themes determined from the content of the cross-cultural discussion meetings in Japan and Malaysia, particularly through the eyes of the learner and the quality of learning. For each theme, we present the opinions from both JPs and MPs. Then we discuss similarities and differences between these thoughts.

Our findings illustrated the shared uneasiness when the concept of fairness was employed to define fractions. However, the MPs seemed to emphasize the positive consequences of connecting mathematics to real-life situations, mainly through how pizza was used to represent the abstract concept of fractions and values of fairness. The JPs, on the other hand, debated the risk of bringing overly realistic situations into mathematics learning and then deliberately overlooking the reality when it was inconvenient for the teacher. The summary of the findings is presented (Table 3) below.

Classroom activities: Concrete and abstract

One of the most cited aspects in the discussion meetings was the number of teaching activities that were observed. Interestingly, JPs and MPs perceived this aspect somewhat differently. JPs expressed their concern about the number of activities, which they perceived to be overwhelming for the students. The comments shared regarding this point included, "*I think students were in quite a rush*", "*[the lesson comprised a] lot of activities*", "*A variety*

Table 3 Summary of findings

Teaching and Learning	Malaysian participants (MPs)	Japanese participants (JPs)
Classroom activities	Less is a bore	Less is more
Pupils' learning motivation	Can be motivated through appropriate extrinsic rewards	Should be driven by the learning task itself
	Pupils' passiveness roots from the teacher's questioning skill	Pupils' passiveness roots from a lack of drive in learning
Teaching materials	Choosing pizza as teaching material is appropriate	Choosing pizza as teaching material is appropriate
Mathematical relevance	Integration of value (fairness) in mathematics teaching and reasoning for the mathematics concept is encouraged	Integration of value (fairness) in mathematics teaching and reasoning for the mathematical concept is not encouraged
	Good to establish connections between mathematics and real-life situations	Risky to bring the overly realistic situation into mathematics teaching

of things including pizzas, waffles, biscuit shapes emerged one after another". Particularly, as a schoolteacher, one JP mentioned a concrete amount of time needed to conduct a similar lesson in Japan, and claimed, "*I was surprised that this lesson was only one hour*".

Due to the number of teaching activities, JPs were uncertain about how well the students received the content. For instance, another JP and retired school principal said, "*I understand the intention of wanting to use different things, but while I was reading the lesson transcript, I wonder if students could truly follow and understand the lesson*". The seemingly overwhelming activities are thought to be why students could not take time to think and discuss the lesson content. For example, another JP claimed, "*I think murmurs among students when they talk to peers next to them to discuss the content of the lesson are important. I believe there wasn't sufficient opportunity for students to be able to do much talking or thinking time for students [as seatwork]*". This comment pointed to the number of teaching activities that put excessive strain on students' development and learning.

In addition, the teacher of the lesson appeared to emphasize the number and diversity of activities because she perceived it as a feature of an effective lesson. According to another JP, "*There was an overemphasis by the teacher on making the lesson fun*". This point was well-summarized by a Japanese professor who concluded by saying, "*…from the beginning, the teacher was enthusiastic about including a variety of activities. …I feel that the teacher was more concerned about filling in the lesson with activities as it made the lesson feel more substantial*". This observation was something that was reflected on by other participants.

Malaysian participants involved in the discussion meetings also agreed with the view that there was a large number of activities. The MPs, however, felt that the teacher's strategy of varying the teaching activities was beneficial for the students. From the students' learning point of view, the participants advocated the choice of teaching activities. Among the reasons given by an MP, one said the activities were "*student-centered because based on most of the activities, students worked it out by themselves*". It was also noted by the same discussant that "*students construct their own understanding*", a comment which verifies the value of collective learning whereby students of that age may in fact learn more from their peers than from the teacher.

Participants also provided an elaborate analysis of the types of activities observed. For instance, an MP observed how the Concrete-Pictorial-Abstract (CPA) approach was employed in the lesson: "*In terms of tangible aids, the teacher started with the use of pizza. Secondly, an explanation of the related concept was then provided. The concept of the fraction was introduced to students through written words, through sentences. Consequently, the teacher proceeds to use stickers and circle maps to emphasize her point. Henceforth, the lesson returned back to a Concrete element, with the use of ribbons and papers*". The MPs presented more observations related to the types of learning taught through the teaching activities. These were concrete types of descriptions, including remarks such as "*the lesson applied scientific and mathematical skills, namely the use of a non-standard system such as big and small*", "*Active learning was also applied in this lesson",* and *"It also involved psychomotor skills such as coloring and cutting*". Participants in Malaysia appeared to be familiar with the terms and approaches of teaching. The variety of teaching activities made possible by increasing the number of activities was also recognized for the positive effect it has on students.

An analysis of the data revealed that the participants from Japan and Malaysia agreed on the number and diversity of the teaching activities. The values attributed to it were nonetheless different. While the JPs had some reservations about the number and diversity of

the teaching activities, the MPs appreciated the teacher's efforts and highlighted the positive effects of such diverse teaching activities.

From the Japanese researchers' lens, although students appeared to be a little restless, it could be observed that students admirably listened to the teacher's instructions and answered her questions. Specifically, the students could be seen helping each other, while others briefly worked individually, with those around looking on intently to see what they were doing. While the teacher tried emphatically to create group work opportunities for her students, the students did in general seem a little confused and did not know how to react at several stages during this period. A noteworthy point here is when the teacher asked students to repeat in a parroting manner. Through a thorough inspection of the data, and through the lens of one Japan-based researcher, it is clear to see that the result of this was that the students, rather than learning passively, were learning kinesthetically.

Mathematical relevance: Fraction and fairness

The concept of "fairness" was also a recurring theme in post-discussions. This notion occurred during the lesson segment when the students were instructed to cut the pizza into two parts. The teacher stressed that if the pizza was not cut into two equal parts, it would not be fair and therefore not a fraction. The relationship between fairness and fractions was debated by the JPs. Most of the participants held some doubts about the relevance and appropriateness of equating fairness with fractions, with particular mention to the teacher's utterance (T212), "*If it is not fair, it is not a fraction*".

One JP said, "*I wonder about the appropriateness of connecting mathematical thinking to the concept of fairness*". A different JP also noticed how the teacher used "fairness" as a keyword to teach fractions: "*I was wondering how appropriate it is to use the concept of 'fairness' as a way of mathematical teaching*". In a similar vein, a JP mathematics teacher responded, "*I wonder if students have given the concept of 'fairness' and 'equality' a proper thought*". All the comments seemed to question the correctness of defining fraction through fairness. At the end of the discussion, a Japanese researcher presented his view on equating fairness with fractions: "*…my concern is, when an action has become relevant, the power to control the relevance has reverted to the teacher's side. In some way, the teacher was trying to break away from the teacher-controlled lesson, and thus, the teacher put much effort into achieving that*". The participant viewed the issue of relating fractions to fairness from the perspective of real-life context versus a mathematical equation. The teacher attempted to use pizza to make students feel closer to their daily life practices, and what's more, she asked the students to cut the pizza to share it with their siblings. Nevertheless, a contradiction was observed concerning those students who have more than two siblings. The teacher (T107) said, "*For today, we are not cutting it into many parts because this pizza is small. We will cut it into parts of the same size*". In this scene, instead of making the learning situation close to the students' real life, the teacher requested the students to deliberately move away from reality.

The participants from Malaysia shared some views with the JPs with regards to the link between fairness and fractions. Some MPs expressed their disagreement with the teacher's action of equating fairness to sameness, which in turn affects the definition of a fraction. Remarkably, they said, "*Same does not always mean fair. Sometimes we introduce the wrong concept to the students. Fair does not always mean the same*", "*Fairness does not have to be equal*", and "*Fairness has a different definition in the eyes of the students, whereas fraction, as we know, is about having equal parts*". While some participants questioned the accuracy of fairness, sameness, and fractions, some researchers recognized the

importance of integrating the value of fairness in mathematics teaching. Comments that imply this included, "*Value integration is important in every subject. If we look here, the first value is fairness*" and "*we can inculcate values, when we talk about the concept of fairness*". Apart from that, many of the MPs responded positively to the decision of using pizza to teach fractions. Firstly, the teacher who consulted with the teacher during lesson planning acknowledged, "*I suggested using objects that are close to students' daily life, environment. That was when the idea of pizza appeared. Now, we are trying to facilitate students to be able to make the transition from something concrete to something abstract*". It is clear that pizza was chosen because the teacher thought it could facilitate the understanding of abstract concepts. Other participants who concurred with this opinion stated, "*The teacher used pizza maybe because it would attract students' attention and is easy to understand*". Different from the concern raised by the JPs, the MPs applauded the connection between mathematics and real-life context because "*In the past, we tend to focus more on procedural knowledge. So, we can try to challenge students more, to make them aware that learning about fractions is practical*". Another MP mentioned that this is a good example of contextualized learning.

Findings of this study illustrate the shared uneasiness when concepts such as fairness are employed to define fractions. However, the MPs seemed to emphasize the positive consequences of connecting mathematics to real-life situations, mainly how pizza was used to represent the abstract concept of fractions and integrate values of fairness. Several JPs, on the other hand, debated the risk of bringing overly realistic situations into mathematics learning and then deliberately overlooking the reality when it was inconvenient to the teacher.

The topic of "fairness" was brought up 13 times in class, with particular emphasis during the section of the lesson when students were instructed to cut pizza into sections. In T40, the teacher referred to the aspect of sharing something, most probably an item of food with a parent, stating that "*I am an adult, so I eat more, but you are [a child, and therefore] smaller, so you eat less*", and whether this point was fair or not. Students responded by saying they would agree with that decision (if it was their parents), but that the idea of splitting something in an unequal manner to compensate for body size or age was unfair. One JP claimed that this point may not be true, as a bigger person would inevitably eat more than a smaller person, which in turn would be fair. This notion again sheds light on varying concepts of fairness both as a cultural concept and as an understanding of what it is. One impression of this idea is that in some Western countries the phrase "*you cut, I choose*" or "*I cut, you choose*" is a fair way of splitting something in two, especially for children. It seems the theme of fairness and equality was a recurring theme observed in this class.

Discussion and implications

Pedagogical reasoning: Less is more or bore?

Participants from Japan and Malaysia seemed to perceive the relationship between the number and diversity of teaching activities. During discussions, one interaction between Malaysian and Japanese researchers asked a question that expressed hesitation. "*Don't students in Japan get bored when most of the time the whole lesson only deals with one or two activities?*" This doubt is most likely held by many Malaysian teachers upon their

observations of a Japanese lesson. Comparatively, this question reflects the importance of various teaching activities for the teachers in a Malaysian classroom for at least two reasons.

First, each activity is deemed to play its role. The teacher, therefore, aimed to distribute information over several representations and make connections between them. For instance, the first activity of cutting the pizza was a means to represent fractions in concrete material (pizza). Also of relevance was the task of cutting ribbon, a different concrete material, into equal lengths. In this activity, the students could verify the concept of equal parts, which they learned about in the pizza activity, through the folding action of the ribbon. The verification step was not possible in the pizza activity. Next, in the bridge map activity, the students were expected to link the illustrative representation (diagrams) to the symbolic representation (fractions) using the interpretations provided by the teacher. In the last activity, the students were required to create illustrative representations (shading the diagrams) based on the symbolic ones (fractions) given. Interestingly, in some of the activities, students were divided into groups and each group was given a different task.

The second reason could be attributed to the teacher's intention to accommodate different learning styles. In Malaysia, a learner's style is one of the most emphasized areas during teacher training. In discussions, the participants in Malaysia used plenty of terms to describe teaching strategies, such as the Concrete-Pictorial-Abstract, conceptual mastery using the reasoning approach, collaborative learning, and psychomotor learning. It could be said that schoolteachers, researchers, and teacher-educators were relatively familiar with the teaching strategies.

Teachers in general most likely believe that all individuals learn in different ways and thus should be taught differently, as advocated by researchers such as Dunn (1984) and Moran (1991). Hence, the teacher may have planned for the lesson to contain different activities to accommodate as many learner styles as possible. As pointed out by one JP, "*Students could cut the pizza before moving on to the task of writing fractions. I think that is something vital to be considered for slow learners*". Another participant even suggested to "*freeze students' conceptual mastery by emphasizing reasoning, using different types of concrete objects in addition to pizza*". That is to say, contrary to the Japanese participants' opinions that "*less is more*", the teacher and the Malaysian participants believe that "*less is a bore*".

Fairness in the teaching of fractions: Risky or effective?

The theme that recurred most in the discussion meetings was the concept of fairness in teaching fractions. Participants from both Malaysia and Japan expressed their interpretation of the teacher's statement of defining fractions through fairness:

T212: If it is not fair, it is not a fraction. Got it?

The appropriateness of the definition was a topic of discussion among JPs. There are likely two reasons why the concept of fairness was perceived as problematic if used to define a fraction. Firstly, the concept of fairness may be different between children and adults. The exchanges in the lesson, as illustrated below, demonstrated how the adult and student viewed fairness differently. At the beginning of the lesson, the teacher attempted to explain to the students that one's physical size should not be a factor in splitting something unequally. The students responded with agreement to this explanation.

T36: Say we want to share something. One person gets the smaller portion, and I get the bigger portion. Is that OK?

C37: No.
T38: Is that OK?
Ca39: No.
T40: Why not? I am an adult; I eat more. You eat less. You are smaller, aren't you? Is that OK?
Ca41: Yes.
T42: Is that OK?
Cs43: Yes.
T44: Is that fair?
C45: No.

However, later in the lesson, it was observed that some of the students cut their pizzas into parts of different sizes. When asked if it was because they wanted their siblings/parents to have the bigger parts, they agreed. When probed if this was fair, the students replied yes, it was fair. The teacher repeated her question, and this time, the students responded it would not be fair to give the bigger parts to an adult.

T133: There is an extra part here. (teacher points to section) Whom do you want to give it to? To your mother because she is an adult? Your mom gets the bigger part while you get the smaller part. Is that OK?

C134: OK.
T135: Is that OK? Is it fair?
S136: It's fair.
T137: Is it fair?
Cs138: No…
T139: Is that fair? Well, you see, we are going to eat the pizza together, aren't we? So, we have to share the same size. If one gets a small part, the others should get a small part too. If one gets a big part, the others should get a…?
C140: Big part too.

The exchanges above shed light on how adults and students viewed fairness differently, a JP practitioner said. To the teacher (adult), "*equality is always fair*". The students, on the other hand, believe that "*equality is not always fair*". The students inadvertently adopted feelings of affection towards their family when conducting their tasks in this class. Therefore, their interpretation of fairness might not be the same as the teacher's. This raises the question of the appropriateness of defining the objective concept of fractions with a subjective interpretation of fairness. Apart from the adults-versus-children perspective in fraction-fairness, the discrepancy between school mathematics and real-life situations is also deemed to be a reason why defining fractions with fairness is tricky. One JP researcher cautioned about the ways of integration between mathematics and real life beyond the school and classroom. Specifically, in this lesson, the teacher attempted to "wrap" the concept of fractions in the form of a pizza-sharing question. The wrapping was working rather well at the beginning of the lesson until the teacher asked the students to ignore the real-life context; i.e., the number of siblings they may have at home.

However, from the viewpoint of MPs, the teacher's decision to liken fairness to a mathematical fraction could be perceived as an endeavor to integrate values into the mathematics classroom. While this action was a positive effort to humanize mathematics, MPs viewed

it with caution, because the interpretation of fairness could differ between adults and children. In addition, the mathematical relevance of the situation of pizza sharing seemed to be manipulated by the teacher, thus causing a real-life situation to be less realistic. This ambiguity between mathematics curriculum and real-life situation beyond school also affected the students' decision in the pizza-sharing context.

To adults, two equal portions should be the same size, but to a student who may have younger siblings, the sentiment could be different. This raises the question of what fairness and equality are, and how they are interpreted by each student. As such, we can understand from this cross-cultural analysis that students may have a different sense of fairness than adults.

Using pizza as a teaching material in this context seems to have been an effective way to teach the mathematical concept of fractions, an MP researcher claimed. Nonetheless, from the point of view of one JP, it would be a good idea to discuss and reconsider why the teacher in this lesson decided to adopt the method of using pizza to teach fractions in this risky way. From a close examination and interpretation of the lesson transcript, we see that the students inadvertently seemed to adopt feelings of affection towards their family when learning tasks were set to them.

Conclusions

This study provided a cross-cultural lesson analysis that focused on pedagogical reasons and decisions in a mathematics classroom in Malaysia, through the eyes of Malaysian and Japanese researchers. Based on the analysis of discussion data, we identified the main aspects of teaching and learning and underlying epistemological beliefs. Particularly, we find a deeper level of reasoning behind student learning through the teacher pedagogical reasoning of using teaching materials such as small pizzas, stickers, and circle maps in class. This study also provides a trans-national learning outlook among Malaysia and Japan to better understand the logic behind teaching and realizing the underlying epistemology in mathematics pedagogy. For instance, from the MPs' perspective, students may be inspired by being involved in the learning task and can be motivated through appropriate extrinsic rewards. However, from the Japanese educators' lens, their learning was sustained through motivation, particularly through rewards such as pizza. Therefore, students were still passive. Students' learning in Japan, nonetheless, is built based on their interest in the learning task; i.e., students should be driven to learn by the learning, and learning task, itself.

This cross-cultural analysis illustrates that the underlying epistemology in pedagogical decisions is culturally based. For example, while MPs see the root of students' passiveness as the teacher's questioning skills, JPs may see it in a different way. They thought that the task should be the main drive for learning, not the teaching or teacher. Comparatively, by analyzing the point a Japanese mathematics educator articulated, it may be a good idea to discuss and reconsider why the teacher in this lesson decided to adopt the method of using pizza to teach fractions in this way. From this study, we can see that the students inadvertently adopted feelings of affection towards their family when conducting tasks set to them. It means the connection of fractions with fairness, and the integration of cutting pizza and eating it in a mathematics class, is encouraged, and to MPs it made a good connection between mathematics and social values. However, through the eyes of JP researchers, such specific integration between mathematics and social values is sometimes risky, and reasoning for this decision could be discussed further among teachers and researchers.

In addition, this cross-cultural analysis accentuates the Japanese perception of the so-called "less is more", when the Malaysian mindset may believe that "less is bore". After being introduced to fundamental theories in class, students in Japan are given more time to think and reflect on the meaning behind each theory. Whereas in Malaysia, teachers are encouraged to allow more time for reflection on a variety of notions that emerge in class. This also shows differences in the cultural script of teaching between Japan and Malaysia that can be learned and reconsidered as a research question in other nations. From this study we have come to realize that mathematics equations may also be influenced by external cultural factors unique to the student individual. Explicitly, why and how do some abstract concepts, such as "a + b = c", sometimes culturally become "a + b = c + something else" in practice?

For future studies, it may be interesting to see how this cross-cultural analysis supports teachers, schools, and curriculum developers to understand and revise their cultural script of teaching to encourage further "change within improvement". In addition, we hope findings of this study may support future research in making explicit the values that underpin and shape teaching mathematics, as well as the integration between real life, social values, curriculum, and the way of schooling and teaching culturally. Such knowledge is crucial as it can lead to a deeper understanding of one's cultural scripts, which can further facilitate the improvement of pedagogical practices worldwide.

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References

Baldry, F., Mann, J., Horsman, R., et al. (2023). The use of carefully planned board work to support the productive discussion of multiple student responses in a Japanese problem-solving lesson. *Journal of Mathematics Teacher Education, 26,* 129–153.

Benedict, R. (1946). *The chrysanthemum and the sword: Patterns of Japanese culture*. Houghton Mifflin Company.

Biesta, G. J. J. (2010). *Good education in an age of measurement: Ethics, politics, democracy*. Paradigm Publisher.

Bruner, J. (2016). *The culture of education*. Harvard University Press.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2003). *Classroom discussions: Using math talk to help students learn*. Math Solutions Publications.

Cuban, L. (2013). *Inside the black box of classroom practice: Change without reform in American education*. Harvard Education Press.

Dreher, A., Lindmeier, A., Feltes, P., Wang, T.-Y., & Hsieh, F.-J. (2021). Do cultural norms influence how teacher noticing is studied in different cultural contexts? A focus on expert norms of responding to students' mathematical thinking. *ZDM—Mathematics Education, 53*(1), 165–179. https://doi.org/10.1007/s11858-020-01197-z.

Dunn, R. (1984). Learning style: State of the science. *Theory into Practice, 23*(1), 10–19.

Elliott, J. (2016). Significant themes in developing the theory and practice of lesson study. *International Journal for Lesson and Learning Studies, 5*(4), 274–280.

Elo, S., & Kyngäs, H. (2007). The qualitative content analysis process. *Journal of Advanced Nursing, 62*(1), 107–115.

Fey, J. (1979). Mathematics teaching today: Perspectives from three national surveys. *Mathematics Teacher, 72,* 490–504.
Friedman, V. J. (2006). Action science: Creating communities of inquiry in communities of practice. In P. Reason & H. Bradbury (Eds.), *Handbook of action research* (pp. 131–143). Sage Publications.
Fullan, M. (2015). *The new meaning of educational change*. Fifth edition. Teachers College Press.
Gallimore, R. (1996). Classrooms are just another cultural activity. In B. K. Speece & D. L. Keogh (Eds.), *Research on classroom ecologies: Implications for inclusion of children with learning disabilities* (pp. 229–250). Erlbaum.
Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1–27). Lawrence Erlbaum Associates Inc.
Hiebert, J., & Stigler, J. W. (2017). Teaching versus teachers as a lever for change: Comparing a Japanese and a U.S. perspective on improving instruction. *Educational Researcher, 46*(4), 169–176.
Hsieh, H. F., & Shannon, S. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research, 15,* 1277–1288.
Jacobs, J. K., & Morita, E. (2002). Japanese and American teachers' evaluations of videotaped mathematics lessons. *Journal for Research in Mathematics Education, 33*(3), 154–175.
Kaur, B., Anthony, G., Ohtani, M., and Clarke, D. (2013). *Student voice in mathematics classrooms around the world*. Sense Publishers.
Mesiti, C., Artigue, M., Hollingsworth, H., Cao, Y., & Clarke, D. (2021). *Teachers talking about their classrooms: Learning from the professional lexicons of mathematics teachers around the world*. Routledge.
Moran, A. (1991). What can learning styles research learn from cognitive psychology? *Educational Psychology, 11*(3–4), 239–245.
Putnam, R. T., Lampert, M., & Peterson, P. L. (1990). Alternative perspectives on knowing mathematics in elementary schools. *Review of Research in Education, 16,* 57–150.
Ravitch, D. (2020). *Slaying Goliath: The passionate resistance to privatization and the fight to save America's public schools*. Knopf.
Ryle, G. (2000). *The concept of mind*. University of Chicago Press.
Sarkar Arani, M. R., Shibata, Y., Sakamoto, M., Iksan, Z., Amirullah, A. H., & Lander, B. (2017). How teachers respond to students' mistakes in lessons: A cross-cultural analysis of a mathematics lesson. *International Journal for Lesson and Learning Studies, 6*(3), 249–267.
Schreier, M. (2012). *Qualitative content analysis in practice*. Thousand Oaks.
Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. Free Press.
Thomas, G. (2012). Changing our landscape of inquiry for a new science of education. *Harvard Educational Review, 82*(1), 26–51.
Tyack, D., & Cuban, L. (1997). *Tinkering towards utopia: A century of public school reform*. Revised edition. Harvard University Press.
Weber, R. P. (1990). *Basic content analysis.* 2nd edition. Sage Publications.

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