Imaging performance in differential phase contrast CT compared with the conventional CT–Noise equivalent quanta $NEQ(k)$

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ABSTRACT

The grating-based x-ray differential phase contrast (DPC) CT is emerging as a new technology with the potential for extensive preclinical and clinical applications. In general, the performance of an imaging system is jointly determined by its signal property (modulation transfer function–$MTF(k)$) and noise property (noise power spectrum–$NPS(k)$), which is characterized by its spectrum of noise equivalent quanta. As reported by us previously, owing to an adoption of the Hilbert filtering for image reconstruction in the fashion of filtered backprojection (FBP), the noise property of DPC-CT characterized by its $NPS(k)$ differs drastically from that of the conventional attenuation-based CT ($1/|k|$ trait vs. $|k|$ trait). In this work, via system analysis, modeling and simulated phantom study, we initially investigate the signal property of DPC-CT characterized by its $MTF(k)$ and compare it with that of the conventional CT. In addition, we investigate the DPC-CT’s spectrum of noise equivalent quanta $NEQ(k)$ – the most important figure of merit (FOM) in the assessment of an imaging system’s performance – by taking the $MTF(k)$ and $NPS(k)$ jointly into account. Through such a thorough investigation into both the signal and noise properties, the imaging performance of DPC-CT and its potential over the conventional attenuation-based CT can be fully understood and appreciated.

Keywords: CT, x-ray phase CT, x-ray differential phase contrast CT, MTF, noise equivalent quanta, NEQ

1. INTRODUCTION

Recognizing its potential of substantially improving the subject contrast in soft tissues, increasing effort is being devoted to exploring the potential of x-ray tube and grating based differential phase contrast CT (DPC-CT) for early detection of cancer and other diseases1-3. The initial exploration was quite qualitative, in which significant improvement in the contrast of soft tissues in human specimens or very small animals was demonstrated1,2. Recently, the investigation is becoming quantitative, especially on the noise property of DPC-CT4-9. It is a fundamental understanding that the subject contrast of soft tissues in the DPC-CT imaging is intrinsically determined by their refractive interaction with x-ray beam10-12, while the performance of DPC-CT is determined by its signal and noise transfer properties8,9,13-33. The signal transfer property of DPC-CT is dependent on its modulation transfer function $MTF(k)$13-26, while the noise transfer property can only be thoroughly characterized by its noise power spectrum $NPS(k)$, i.e., the variation of noise intensity as a function over spatial frequency $1/|k|$, whereas that of the latter with $|k|$. It also has been indicated8,9 that the root cause for such a radical difference is the adoption of the Hilbert filter kernel, rather than the ramp filter kernel that is notorious in noise, for image reconstruction in the DPC-CT using the filtered backprojection algorithms.

An imaging system can in general be cascaded into two stages – image formation (or record or detection as termed in the literature) and image presentation or display16,23,24. Such decomposition is straightforward in digital imaging modalities, e.g., the DPC-CT and the conventional CT to be investigated, but quite intricate in the early days when only analog imaging modalities, e.g., the x-ray screen-film radiography, were available. Quite a few factors may degrade the performance of an imaging system, which include but are not limited to (i) the ubiquitous random fluctuation – noise – in data acquisition, (ii) the anatomic and physiologic variation in patient population and (iii) the intra- and inter-observer variability when a diagnosis is made based on the images presented21,24. These factors entangle one another and thus make the performance assessment of an imaging system extremely challenging. With recourse to the Bayesian statistical
decision theory and information theory, a theoretical framework for the performance assessment of an imaging system has been established through the tremendous and successful effort of leading scientists in this field\textsuperscript{13-24}. In a way analogous to the signal detection in Radar and telecommunication, the detection of pathological lesion with a medical imaging system is treated mathematically as a decision making process. An observer makes the decision between two hypotheses – $H_1$: lesion present (abnormal or positive); $H_2$: lesion absent (normal or negative) – according to whether the value of a decision function exceeds a threshold or not. If the decision function is the Bayesian likelihood, the decision maker becomes an ideal observer, who minimizes the risks while making the decision, i.e., maximizing the area under the Receiver Operating Characteristics (ROC) curve\textsuperscript{16,23,24}. This means that the figure of merit (FOM) obtained by an ideal observer sets the upper bound of an imaging system’s performance\textsuperscript{16,23,24}.

Assume the imaging system under study is linear and shift-invariant, and the noise contaminating the system is Gaussian and stationary. Given a signal specified in the spatial frequency domain as $\Delta S(k)$, i.e., the signal known exactly (SKE)\textsuperscript{23,24}, the squared signal-to-noise-ratio or the detectability index defined by

$$SNR^2 = \int |\Delta S(k)|^2 NEQ(k) dk$$

(1)

is an ideal observer FOM to rank the imaging system’s performance\textsuperscript{23}, wherein the spectrum of noise equivalent quanta $NEQ(k)$ may be in different functional forms over medical imaging modalities (see e.g., references\textsuperscript{16,17,22,24} for detail). An ideal observer is assumed to have prior knowledge of the task and the statistical properties of the noise. The squared signal-to-noise ratio or detectability index of a Bayesian ideal observer defined in eq. (1) is actually a measure of the overlap between the probability density distributions of the decision variable corresponding to the two hypotheses\textsuperscript{23,24}, which sums up all the intrinsic factors influencing the imaging system’s performance, but excludes the factors related to image presentation and the neurological and psychological characteristics of the observer’s visual system. Eq. (1) is an integration of a neat factorization and implies that, given a specific task $|\Delta S(k)|^2$, the performance of an imaging system can be optimized by maximizing its spectrum of noise equivalent quanta $NEQ(k)$. In practice, eq. (1) can be extended to deal with more complicated situations, wherein the noise observes the Poisson distribution and is not stationary, the imaging system is non-linear and shift-variant, and even the signal is superimposed on a random background\textsuperscript{16,23,24,26}.

Owing to the important role played by the spectrum of noise equivalent quanta $NEQ(k)$ in the FOM to assess an imaging system’s performance\textsuperscript{16,23,24,26,28-30}, we investigate the $NEQ(k)$ of DPC-CT in this manuscript. Furthermore, due to the fact that Gaussian noise approaches Poisson noise if the detected number of photons is large, we assume the noise in the DPC-CT is Gaussian. Under the framework of Bayesian ideal observer, we derive, analyze, evaluate and verify the $NEQ(k)$ of DPC-CT and compare it with the conventional CT.

2. MATERIALS AND METHODS

We conduct computer simulation study only, thereby the possible systematic and random errors that may exist in a physical DPC-CT and compromise the accuracy and precision of evaluation and verification can be excluded.

2.1 Imaging mechanism of DPC-CT implemented with x-ray tube and gratings

The architecture of x-ray tube and grating based DPC-CT\textsuperscript{2} is shown in Fig. 1 (a). $G_1$ is a phase grating and $G_2$ an absorption grating, which can be fabricated with photolithography, deep chemical etching and electroplating\textsuperscript{34}. $G_1$ and $G_2$ work together as a shearing interferometer\textsuperscript{35-37} to detect the wavefront alteration caused by the object in x-ray beam. A CCD x-ray detector is employed for data acquisition, in which the tube irradiates the specimen that rotates along its own axis by a range satisfying the data sufficiency condition. The key component of the imaging chain is grating $G_1$, a diffraction interferometer based on the Talbot effect\textsuperscript{35-37}. Fig. 1 (b) shows how $G_1$ works by virtually decomposing it into absorption gratings $A$ and $B$. The extra optical path of grating $B$ relative to grating $A$ is half wavelength, which is equivalent to a $180^\circ$ or $\pi$ phase shift. The beams corresponding to gratings $A$ and $B$ undergo different optical paths before they reach the gratings, recombine after they pass through and generate interference fringes.

The imaging mechanism of x-ray tube and grating based DPC-CT is elaborated in references\textsuperscript{8,35-37}, and a concise review is given here for completeness. According to Fresnel analysis\textsuperscript{38,39}, the irradiance $I(x, z)$ at the CCD detector is

$$I_{AB}(x, z) \sim \phi(x + \Delta x/2, y) - \phi(x - \Delta x/2, y) \equiv \frac{\partial \phi(x, y)}{\partial x} \Delta x,$$

(2)
where $\Delta x$ is a displacement in $x$-direction, and $\phi(x, y)$ is the phase corresponding to $\Delta x$, which is the projection of the refractive coefficient along the x-ray path $Z$

$$\phi(x, y) = \frac{2\pi}{\lambda} \int_z Z \delta(x, y, z)dz .$$  

Eq. (2) shows that the irradiance depends on the derivative of the phase variation along the x-axis. After the x-ray passes $G_2$, the irradiance at detector $D$ is

$$I_{d_x, d_y}(x) = a_0(d_x, d_y) + \sum_{m=1}^{\infty} a_m(d_x, d_y) \cos \left( \frac{2\pi m x}{g_2} + \phi_m(d_x, d_y) \right),$$

where $(d_x, d_y)$ is the coordinate in the detector and $g_2$ the period of grating $G_2$. If the object in the beam is not pure phase, i.e., it attenuates the x-ray beam, one needs to linearly shift $G_2$ along the x-axis. Via Fourier Analysis, one can determine $a_0(d_x, d_y), a_1(d_x, d_y)$ and $\phi_1(d_x, d_y)$ from eq. (4). In fact, one has

$$\frac{\partial \phi(x, y)}{\partial x} = \frac{\partial \phi_1(x, y)}{\partial x} \frac{\lambda_x}{g_2},$$

where $a_0(d_x, d_y)$ and $\partial \phi(x, y)/\partial x$ are the foundation of x-ray attenuation and phase CT imaging, respectively. Substituting the $\phi(x, y)$ defined in eq. (3) into eq. (5), we further get

$$\phi_1(d_x, d_y) = \frac{\lambda_x Z}{g_2} \frac{\partial \phi(x, y)}{\partial x} = \frac{\lambda_x Z}{g_2} \frac{\partial}{\partial x} \left( \int_z Z \delta(x, y, z)dz \right) \frac{\partial \phi(x, y)}{\partial x}.$$ 

**Figure 1.** The diagrams showing (a) the schematic of an x-ray tube and grating based DPC-CT and (b) the schematic of virtual grating decomposition, in which the upper and lower Talbot patterns associates with gratings A and B, respectively.

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**Figure 1.** The diagrams showing (a) the schematic of an x-ray tube and grating based DPC-CT and (b) the schematic of virtual grating decomposition, in which the upper and lower Talbot patterns associates with gratings A and B, respectively.
This means that the phase retrieved through Fourier Analysis of eq. (4) is the projection of the refractive coefficient’s derivative, and this is the underlying reason that the phase CT implemented with x-ray tube and gratings is called differential phase contrast CT. Once $\frac{\partial \phi(x, y)}{\partial x}$ data are acquired, tomographic images can be reconstructed using the filtered backprojection (FBP) algorithms\textsuperscript{40-43}.

The modeling of data acquisition in the x-ray tube and grating based DPC-CT through the schematic of Fig. 1 (b) and eqs. (2) – (6) has been evaluated and verified in reference\textsuperscript{8}, and presented in Fig. 2 are the results of a contrast-detail (C-D) phantom\textsuperscript{25} (see section III for its definition). Using the C-D phantom that is good at evaluating the overall performance of imaging method, the correctness and accuracy in modeling and simulating data acquisition and image reconstruction of the DPC-CT implemented with x-ray tube and gratings are evaluated and verified.

![Figure 2](image)

**Figure 2.** Transverse images of the C-D phantom generated by the x-ray tube and grating based DPC-CT CT (a) and the conventional CT (b) (x-ray exposure $10^7$ photon/cm$^2$ projection, detector cell 48×48 μm$^2$).

## 2.2 The spectra of noise equivalent quanta $NEQ(k)$ of the DPC-CT and conventional CT

The morphologic difference in the noise of conventional CT images against the white noise suggested that there existed an inter-pixel correlation\textsuperscript{14-17} and this can be confirmed by the noise power spectrum $NPS(k)$, even though there is no inter-cell correlation in the noise of detector cells during projection data acquisition. Since then, the groundwork of using noise power spectrum $NPS(k)$ and spectrum of noise equivalent quanta $NEQ(k)$ to analyze the signal and noise behavior of a CT system or CT imaging method has been laid out by the researchers in the field\textsuperscript{13-21}. In this section, we follow the convention to derive the functional form of $NEQ(k)$ of the DPC-CT implemented with x-ray tube and gratings and compare it with that of the conventional CT.

### 2.2.1 The noise equivalent quanta spectrum $NEQ(k)$ of conventional CT

A number of strategies, e.g., Barrett’s method\textsuperscript{14}, the central slice theorem\textsuperscript{15}, statistical detection theory\textsuperscript{17} and information theory\textsuperscript{16}, have been exercised to obtain the noise power spectrum $NPS(k)$ of the conventional CT, all leading to the same functional form

$$NPS_a(k) = \frac{\pi}{m\bar{N}} |k| |MTF(k)|^2, \quad (7)$$

where subscript $a$ represents “attenuation” or “amplitude” in the conventional CT, and $k$ the radial frequency

$$k = \sqrt{k_x^2 + k_y^2}. \quad (8)$$

$m$ is the number of projections, and $\bar{N}$ is the mean number of x-ray photons detected at a detector cell. Since no object should be placed in x-ray beam in the investigation of noise property, $\bar{N}$ is assumed equal across all the detector cells.
$MTF(k)$ is the overall algorithmic contribution, including windowing and/or boosting in frequency domain for trade-off between noise and spatial resolution, to the modulation transfer function (MTF) of a CT system.

The multiplication of $m$ and $\overline{N}$ has been defined as the noise equivalent quanta$^{16,17}$, and accordingly eq. (7) can be expressed as

$$NPS_a(k) = \frac{\pi}{NEQ_a} |k|M_{TF}(k)|^2.$$  \hspace{1cm} (9)

Consequently, the spectrum of noise equivalent quanta $NEQ_a(k)$ of the conventional CT is$^{14-17}$

$$NEQ_a(k) = \frac{\pi}{NPS_a(k)} |k|M_{TF}(k)|^2.$$  \hspace{1cm} (10)

### 2.2.2 The spectrum of noise equivalent quanta $NEQ(k)$ of DPC-CT

As derived in the appendix and Ref.$^{44}$, the noise power spectrum $NPS_p(k)$ and spectrum of noise equivalent quanta $NEQ_p(k)$ of the x-ray tube and grating based DPC-CT are

$$NPS_p(k) = \left(\frac{g_2}{\lambda z_T}\right)^2 \frac{a(1-\varepsilon_2)}{2\pi \varepsilon^2 |k|N_{p}M_{N}} MTF^2(k),$$  \hspace{1cm} (11)

$$NEQ_p(k) = \left(\frac{g_2}{\lambda z_T}\right)^2 \frac{a(1-\varepsilon_2)}{2\pi \varepsilon^2 |k|N_{p}M_{N}} MTF^2(k),$$  \hspace{1cm} (12)

where subscript $p$ denotes “phase” in the x-ray tube and grating based DPC-CT. $\lambda$ is the wavelength of x-ray, $z_T$ the fractional Talbot distance and $g_2$ the period of analyzer grating $G_2$. It should be pointed out that, owing to the replacement of the ramp filter in the conventional CT with the Hilbert filter in the DPC-CT, the factor $|k|$ in the denominators of eqs. (9) and (10) moves to the denominators of eqs. (11) and (12). This is the root cause that accounts for all the radial difference in the noise characteristics between the DPC-CT and the conventional CT. Readers are referred to the appendix and Ref.$^{44}$ for the detail in deriving eqs. (10) and (12).

### 2.3 Measurement of modulation transfer function $MTF(k)$

It has been an established practice to measure MTF of a conventional CT system with a wire phantom$^{25,45}$. The wire is usually made of metal, such as tungsten, and placed in either air or water, as long as the attenuation of the thin wire does not exceed the dynamic range of the CT detector. Moreover, it has been well verified that, as long as its diameter is substantially smaller than the detector cell dimension, the influence of the wire’s thickness on the MTF measurement can be ignored$^{45}$. This approach is adopted for the measurement of MTF in x-ray tube and grating based DPC-CT with one caution – the dynamic range of phase retrieval in DPC-CT is just $2\pi$, which is substantially smaller than its counterpart in the conventional CT. Hence, the wire phantom in the simulation study conducted in this work is assumed as a thin cylinder with the refraction coefficient of water placed in air, so that the resultant phase variation is within the $2\pi$ range, i.e., avoiding the phase warping phenomena. Once the transverse image of the thin wire is acquired, a 2-D projection along either the x- or y-axis is carried out, followed by 1-D Fourier transform to obtain the $MTF(k)$.

### 2.4 Quantitative evaluation of the spectrum of noise equivalent quanta $NEQ(k)$

According to eqs. (10) and (12), the spectrum of noise equivalent quanta is jointly determined by the noise power spectrum and modulation transfer function. The noise power spectrum can be calculated by taking the Fourier Transform of the autocorrelation function that is obtained using a large number of noise images. An alternative approach that is more efficient in computation is to take the average of the squared Fourier Transform of a large number of noise images or regions within images containing noise only, i.e., no object in x-ray beam$^{17}$. The obtained 2D noise power spectrum is circular symmetric about its origin, as predicted in eqs. (9) and (11). The modulation transfer functions are acquired using the methods depicted in subsection 2.3 and are also in circular symmetry, and thereby, the spectrum of noise equivalent quanta of the conventional CT and DPC-CT are in circular symmetry too.
3. EVALUATIONS

We constrain ourselves to investigating the DPC-CT’s $NEQ_p(k)$ via simulation studies in the parallel beam geometry, mainly because of the following reasons: (i) the x-ray beam in the differential phase contrast CT satisfies the paraxial condition and thus the beam is almost parallel, (ii) the image reconstruction algorithms in the parallel beam geometry outperform those in the fan beam geometry from the perspective of noise uniformity significantly and thus almost all the clinical CT scanners based on the third generation geometry (fan beam or cone beam) adopt the parallel beam reconstruction algorithms via fan-to-parallel rebinning\textsuperscript{42}, and (iii) most of the simulation studies to investigate the noise power spectrum of CT imaging thus far have been carried out in the parallel beam geometry to exclude the influence of rebinning and weighting schemes. To avoid any interference caused by scatter and beam hardening, a monochromatic x-ray source (30 keV) is assumed, which irradiates an object by 360° at 1° steps so that no weighting effect can be induced to degrade the noise uniformity. At 30 keV, a 20-fold up-sampling is assumed to simulate the x-ray beam’s propagation through the gratings $G_1$ and $G_2$ with periods 8 and 4 μm, respectively, while the distance between these two gratings is 193.6 mm, i.e., 1/16 of the Talbot distance. The size of each detector cell for data acquisition is 32×32 μm$^2$, and 1280 detector cells constitute a detector array to make a 40.96×40.96 mm$^2$ field of view (FOV) for data acquisition. Grating $G_1$ shifts 10 times at step 0.4 μm along the x-axis to retrieve the phase information corresponding to the refractive property of the object to be imaged.

![Figure 3](image3.png)

**Figure 3.** Transverse image of the air phantom generated by: (a) the x-ray tube and grating based DPC-CT and (b) the conventional CT (x-ray exposure $5\times10^6$ photon/cm$^2$·projection, detector cell size 32×32 μm$^2$).

![Figure 4](image4.png)

**Figure 4.** The modulation transfer function $MTF(k)$ of the x-ray tube and grating based DPC-CT compared with the conventional CT at detector cell dimension: 32×32 μm$^2$ (a) and 48×48 μm$^2$ (b) (x-ray exposure $5\times10^6$ photon/cm$^2$·projection).

The data $\partial \phi(x, y)/\partial x$ specified in eq. (5) for DPC-CT are acquired with gratings $G_1$ and $G_2$ in the x-ray beam, while that for the conventional CT images are acquired without the gratings. The FBP reconstruction algorithm in the parallel
geometry$^{40}$ is used, wherein the finite Hilbert transform$^{41,42}$ is used in reconstruction for DPC-CT and the ramp filter$^{40}$ for conventional CT. To investigate the potential imaging performance of the DPC-CT and compare it with that of the conventional CT, no windowing or boosting techniques$^{40}$ are adopted for image reconstruction.

Prior to analyzing the spectrum of noise equivalent quanta $NEQ(k)$, the well-known contrast-detail (C-D) phantom$^{25}$ at outer dimension $37.68 \times 28.26 \text{mm}^2$ is employed to evaluate and verify the simulation accuracy of x-ray propagation, data acquisition and tomographic image generation of the differential phase contrast CT implemented with x-ray tube and gratings. The simulation conditions of the C-D phantom are: $30 \text{keV}$; complex refraction coefficient $n = 1 - \delta + i\beta = 1 - 5.37 \times 10^{-7} + i2.64 \times 10^{-10}$, consistent with that specified in reference$^{14}$; rod size (left to right): 16, 32, 64, 96, 128, 192, 256, 384, 512 and 1024 $\mu\text{m}$; rod contrast (bottom to top): 5% ~ 50% at step 5% against the background.

A thin cylindrical water phantom (diameter: 1/10 detector cell size) with its complex refraction coefficient $n = 1 - \delta + i\beta = 1 - 2.5604 \times 10^{-7} + i1.2353 \times 10^{-10}$ is placed in air to measure the spatial resolution – modulation transfer function. A uniform air phantom, i.e., nothing placed in the x-ray beam is employed to study the noise power spectrum and resultant spectrum of noise equivalent quanta. The x-ray flux observing the Poisson distribution is set at $5 \times 10^6 \text{ photon/cm}^2 \cdot \text{projection}$ in the simulation study, which is consistent with that of x-ray micro-CT in preclinical applications. The image matrix of reconstructed water phantom is $1280 \times 1280$, and a total of $6 \times 6 = 36$ regions at $128 \times 128$ matrix dimension within three uniform water phantoms (and thus a total of 108 regions at $128 \times 128$ matrix dimension) are used for noise power spectrum analysis via 2D Fourier Transform, in which adequate zero padding to convert the data matrix from $128 \times 128$ to $256 \times 256$ is implemented to avoid aliasing effect.

4. RESULTS

4.1 Noise morphology in the DPC-CT compared with that of conventional CT

Transaxial images of the air phantom generated by the DPC-CT implemented with x-ray tube and gratings and the conventional attenuation-based CT at the x-ray exposure $10^7 \text{ photon/cm}^2 \cdot \text{projection}$ are presented in Fig. 3 (a) and (b) respectively. It is observed that the morphology or granularity of the noise in the image generated by the DPC-CT implemented with x-ray tube and gratings is substantially different from that in the image generated by the conventional CT. The noise in the former looks clumpy, while that in latter appears granular. As we reported in our previous publication, the primary reason underlying this substantial difference is the difference in their noise power spectrum, because of the Hilbert filtering kernel used in image reconstruction$^{41,42}$.

4.2 MTF of the DPC-CT compared with that of conventional CT

The MTFs of the DPC-CT and conventional CT have been thoroughly evaluated in our quantitative investigation. Plotted in Fig. 4 (a) and (b) are the results at detector cell size $32 \times 32 \mu\text{m}^2$ and $48 \times 48 \mu\text{m}^2$, respectively. An inspection of

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**Figure 5.** The spectra of noise equivalent quanta $NEQ(k)$ of the DPC-CT corresponding to detector cell dimension: (a) $32 \times 32 \mu\text{m}^2$, (b) $64 \times 64 \mu\text{m}^2$ (x-ray exposure $5 \times 10^6 \text{ photon/cm}^2 \cdot \text{projection}$).
Fig. 4 shows that, even though the noise texture/granularity is substantially different between the DPC-CT and conventional CT as demonstrated in Fig. 3, their spatial resolution measured by the modulation transfer function is almost identical. We have experienced in the conventional CT that a difference in the noise texture/granularity in general implies a difference in the spatial resolution. Fortunately, however, this is not the case between the DPC-CT and the conventional CT, as reinforced by a joint inspection of Fig. 3 and 4.

4.3 Spectra of noise equivalent quanta of the DPC-CT compared with that of conventional CT

The profiles along the radial line at 45° cross the $\Delta S(f)$ of both the DPC-CT and conventional CT are plotted in Fig. 5 (a) – (d). The detector cell dimension is 32×32 μm$^2$ to 48×48 μm$^2$, respectively. As such, the variation of $\Delta S(f)$ of the DPC-CT as a function over the detector cell dimension can be evaluated. Note that the fluctuation in the profiles of the spectra of noise equivalent quanta is quite severe, because only 108 regions at matrix dimension 128×128 in the air phantom are engaged in the measurement and calculation. With an increasing number of ensemble samples (images or regions within the images of uniform phantom), smoother profiles corresponding to the noise power spectrums can be obtained. Actually, as demonstrated in Fig. 5, we carried out a 5th-order polynomial fitting on the measured spectrum of noise equivalent quanta, which asymptotically predicts their trend if a larger number of ensemble samples, e.g., more than 1,000, are available for the quantitative analysis.

5. DISCUSSIONS AND CONCLUSIONS

It has been reported$^{8,9}$ that the x-ray tube and grating based differential phase contrast CT is of the $I/|k|$ trait in its noise power spectrum in contrast to the $|k|$ trait in the conventional CT, which leads to its substantial advantage over the conventional CT in its noise property with decreasing detector cell dimension. With a rigorously derivation of the noise equivalent quanta spectrum $NEQ(k)$, an experimental evaluation to verify the derived $NEQ(k)$, and a comparison with that of the conventional CT, we investigate the DPC-CT’s signal and noise properties, especially its noise equivalent quanta spectrum $NEQ(k)$, in this work. In general, the noise equivalent quanta spectrum of an imaging system is in the functional form$^{16,17,22-26}$

$$NEQ(k) = G^2 \frac{MTF^2(k)}{NPS(k)}.$$  \hfill (13)

By definition, $G$, $MTF(k)$ and $NPS(k)$ represent the imaging system’s transfer characteristics of large area gain, signal and noise, respectively. In various imaging modalities, eq. (13) can be in specific expression. For instance, the $NEQ(k)$ of x-ray radiography and fluoroscopy is expressed in exactly the same analytic form as eq. (13), whereas that of the conventional CT is expressed in eq. (10). Note that the term $G$ is absent from eq. (10), since the gain of a tomographic imaging system is unity, i.e., $G \equiv 1$. It is interesting and important to note that factor $|k|$ is in the denominator of DPC-CT’s $NEQ(k)$ (see eq. (12)), whereas it is in the numerator of that of the conventional CT (see eq. (10)). The root cause underling such a drastic difference is that, the noise power spectrum of DPC-CT is of $I/|k|$ trait, whereas that of the conventional CT is of $|k|$ trait, as reported in Ref$^{8,9}$.

The experimental evaluation shows that, given a detector cell dimension, the $MTF(k)$ of the DPC-CT is virtually the same as that of the conventional CT (see Fig. 4). This means that, despite of the difference in the imaging mechanisms between the DPC-CT and conventional CT, their $MTF(k)$ is primarily determined by detector cell’s dimension, which warrants comparable signal transfer characteristics and spatial resolution between these two imaging methods. This is important in practice, especially in the scenarios wherein the complementary information drawn from the DPC-CT and the conventional CT are jointly of relevance for preclinical and clinical applications.

The significant difference in the $NEQ(k)$ between the DPC-CT and the conventional CT is experimentally verified by the results presented in Fig. 5, which demonstrates that, at low (below 10% Nyquest frequency determined by detector cell size) and high (beyond 50% Nyquest frequency), the DPC-CT outperforms the conventional CT substantially, while their difference at mid-spatial frequency (between 10% and 30% Nyquest frequency) is modest. If the signal $\Delta S(f)$ (see eq. (1)) fed into the DPC-CT and conventional CT is identical and distributes uniformly from zero to the Nyquest frequency, i.e., the objects to be imaged are in various sizes, the total detectability index of the DPC-CT is approximately two-fold that of the conventional CT. However, if the signal $\Delta S(f)$ is only of high spatial frequency, i.e., the objects are small and approach twice the detector cell size, the signal and noise properties or detectability index of...
the DPC-CT is substantially larger than the conventional CT. Note that the conventional CT using image reconstruction algorithm in the fashion of filtered backprojection has been suffering from high frequency noise since the advent of x-ray CT, as evidenced by the significant drop in its \( \text{NEQ}(k) \) with increasing spatial frequency. In practice, a tomographic imaging method suffering less from noise at high frequency is definitely desirable. The DPC-CT implemented with x-ray tube and grating is just such an imaging modality with its \( \text{NEQ}(k) \) quite uniform from zero to the Nyquest frequency and thus does not suffer from high frequency noise. It should be emphasized that such a feature is of profound significance in early detection of tumor or other diseases because pathophysiological lesions usually start out at small sizes.

By referring to eq. (13), it is not hard to understand that, since their signal transfer properties expressed in the MTF \((k)\) are almost identical at given detector cell dimension, the significant difference in the \( \text{NEQ}(k) \) between DPC-CT and the conventional attenuation-based CT is mainly due to their radical difference in noise power spectrum \( \text{NPS}(k) \). Furthermore, as reported in the literature, the subject contrast among the refraction coefficient of soft tissues for the DPC-CT can be substantially larger than the attenuation counterpart for the conventional CT\(^{46,47} \). This means that the signal in the DPC-CT \( \Delta S(k)_p \) can be significantly larger than its counterpart \( \Delta S(k)_a \) in the conventional CT. Hence, in addition to the DPC-CT’s advantage in the \( \text{NEQ}(k) \), the significantly larger signal \( \Delta S(k)_p \) may ultimately enable the DPC-CT to outperform the conventional CT substantially in soft tissue differentiation.

The goal of this work is to carry out an ideal observer study of the upper bound of DPC-CT’s imaging performance. To be consistent with this goal, the imaging chain of both the DPC-CT and conventional CT are assumed ideal in system modeling, analysis and evaluation. It is advised to understand that the imaging performance, including the spectrum of noise equivalent quanta \( \text{NEQ}(k) \) and detectability index, of a physical DPC-CT system would be degraded by the imperfection in its imaging chain in practical situations. Finally, prior to ending this manuscript, we would like to indicate two important points: (1) The DPC-CT implemented with x-ray tube and grating, in which the refraction coefficient is used for tomographic imaging, is just one way to detect the projection of a physical parameter’s derivative for tomographic imaging. If other physical parameter can be utilized, the spectrum of noise equivalent quanta \( \text{NEQ}(k) \) of the resultant tomographic imaging system would be very similar to that of the DPC-CT unveiled in this investigation; (2) similar to the scenario in the noise power spectrum \( \text{NPS}(k) \), the root cause underlying the significant difference in the DPC-CT’s spectrum of noise equivalent quanta \( \text{NEQ}(k) \) is the substitution of the ramp filtering in image reconstruction with the Hilbert filtering.

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APPENDIX: Noise in x-ray tube and grating based DPC-CT

In an x-ray tube and grating based DPC-CT, we have the relationship between the phase \( \phi(u, v) \) and \( \delta(x, y, z) \) – the three-dimensional refraction function of the object in x-ray beam

\[
\phi(u, v) = -\frac{2\pi\lambda}{g_2} \frac{\partial}{\partial z} \int dz \delta(x, y, z) = -\frac{\lambda z_T}{g_2} D(x, y),
\]

where \( \lambda \) is wavelength, \( z_T \) the fractional Talbot distance and \( g_2 \) the period of grating \( G_2 \). In practice, phase \( \phi(u, v) \) can be retrieved by stepping grating \( G_1 \) linearly along x-axis (Fig. 1). At each step

\[
x_k = \frac{k}{M} g_2, \quad k = 1, 2, \ldots, M,
\]

we have the measured x-ray irradiance at \((u, v)\) in the detector

\[
N^{(k)}(u, v) = \sum_{l=\frac{M}{2}}^{M-1} N_l \left( 1 + \frac{\delta_{lk}}{2} \exp \left[ i\phi_l(u, v) + 2\pi i \frac{lk}{M} \right] \right),
\]

\( N^{(k)}(u, v) \) is the DPC-CT's noise equivalent quanta at \((u, v)\) in the detector.

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where \( N_l \) are non-negative real number and \( \delta_{l0} \) is the Kronecker’s delta \( \delta_{lm} \) by setting \( m = 0 \), i.e.,

\[
\delta_{l0} = \begin{cases} 
1 & l = 0 \\
0 & l \neq 0 \end{cases}.
\] (A4)

Note that \( \phi_1(u, v) \) is the phase we want to retrieve for reconstruction of the 3-D refraction distribution using eq. (A1).

The discrete Fourier transform of both sides of eq. (A3) gives

\[
\frac{1}{2} MN_1 \exp[i \phi_1(u, v)] = \sum_{k=1}^{M} N^{(k)}(u, v) \exp \left( -2\pi i \frac{k}{M} \right).
\] (A5)

Noting the fact that \( N_1 \) is non-negative real number, eq. (A5) further gives

\[
\exp[i \phi_1(u, v)] = \frac{\sum_{k=1}^{M} N^{(k)}(u, v) \exp \left( -2\pi i \frac{k}{M} \right)}{\left| \sum_{k=1}^{M} N^{(k)}(u, v) \exp \left( -2\pi i \frac{k}{M} \right) \right|^{1/2}}.
\] (A6)

or equivalently

\[
\exp[i \phi(u, v)] = \frac{\sum_{k=1}^{M} N^{(k)}(u, v) \exp \left( -2\pi i \frac{k}{M} \right)}{\left| \sum_{k=1}^{M} N^{(k)}(u, v) \exp \left( -2\pi i \frac{k}{M} \right) \right|^{1/2}}.
\] (A7)

Here we have substituted \( \phi_1(u, v) \) with \( \phi(u, v) \). From eq. (A7) we have

\[
i \exp[i \phi(u, v)] \Delta \phi(u, v) = \frac{\sum_{k=1}^{M} \Delta N^{(k)}(u, v) \exp \left( -2\pi i \frac{k}{M} \right)}{\left[ \sum_{k=1}^{M} N^{(k)}(u, v) \exp \left( 2\pi i \frac{k}{M} \right) \right] \left[ \sum_{k=1}^{M} N^{(k)}(u, v) \exp \left( -2\pi i \frac{k}{M} \right) \right]^{3/2}}.
\] (A8)

Substituting eq. (A5) into eq. (A8) yields
\[ i \exp[i \phi(u, v)] \Delta \phi(u, v) = \frac{\sum_{k=1}^{M} \Delta N^{(k)}(u, v) \exp \left(-2\pi i \frac{k}{M} \right)}{\frac{1}{2} MN_1} \]

\[ - \frac{1}{2} \exp[2i \phi(u, v)] 2^{-2} M^2 N_1^2 \left[ \sum_{k=1}^{M} \Delta N^{(k)}(u, v) \exp \left(-2\pi i \frac{k}{M} \right) \right] \]

\[ - \frac{1}{2} \frac{2^{-2} M^2 N_1^2}{2} \left[ \sum_{k=1}^{M} \Delta N^{(k)}(u, v) \exp \left(-2\pi i \frac{k}{M} \right) \right] \]

\[ \frac{2^{-2} M^2 N_1^2}{2} M^3 N_1^3 \]

which can be concisely rewritten as

\[ \Delta \phi(u, v) = -i \sum_{k=1}^{M} \left\{ \frac{\exp \left(-2\pi i \frac{k}{M} \right)}{MN_1 \exp[i \phi(u, v)]} - \frac{\exp \left(2\pi i \frac{k}{M} \right)}{MN_1 \exp[-i \phi(u, v)]} \right\} \Delta N^{(k)}(u, v). \]  

(A10)

Consequently

\[ \langle \Delta \phi(u, v) \Delta \phi(u_1, v_1) \rangle = -\sum_{k, k_1=1}^{M} \left\{ \frac{\exp \left(-2\pi i \frac{k}{M} \right)}{MN_1 \exp[i \phi(u, v)]} - \frac{\exp \left(2\pi i \frac{k}{M} \right)}{MN_1 \exp[-i \phi(u, v)]} \right\} \left\{ \Delta N^{(k)}(u, v) \Delta N^{(k_1)}(u_1, v_1) \right\}. \]

\[ \langle \Delta \phi(u, v) \Delta \phi(u_1, v_1) \rangle = -\sum_{k, k_1=1}^{M} \left\{ \frac{\exp \left(-2\pi i \frac{k}{M} \right)}{MN_1 \exp[i \phi(u, v)]} - \frac{\exp \left(2\pi i \frac{k}{M} \right)}{MN_1 \exp[-i \phi(u, v)]} \right\} \left\{ \Delta N^{(k)}(u, v) \Delta N^{(k_1)}(u_1, v_1) \right\}. \]  

(A11)

Since the measured x-ray photons satisfy the Poisson distribution and are spatially uncorrelated, we have

\[ \langle \Delta N^{(k)}(u, v) \Delta N^{(k)}(u_1, v_1) \rangle = N^{(k)}(u, v) \delta_{\delta, \delta} \delta(u - u_1) \delta(v - v_1). \]  

(A12)

Inserting eq. (A12) into eq. (A11), we get

\[ \langle \Delta \phi(u, v) \Delta \phi(u_1, v_1) \rangle = -\sum_{k, k_1=1}^{M} \left\{ \frac{\exp \left(-2\pi i \frac{k}{M} \right)}{MN_1 \exp[i \phi(u, v)]} - \frac{\exp \left(2\pi i \frac{k}{M} \right)}{MN_1 \exp[-i \phi(u, v)]} \right\} N^{(k)}(u, v) \delta(u - u_1) \delta(v - v_1) \]

\[ \langle \Delta \phi(u, v) \Delta \phi(u_1, v_1) \rangle = -\sum_{k, k_1=1}^{M} \left\{ \frac{\exp \left(-2\pi i \frac{k}{M} \right)}{MN_1 \exp[i \phi(u, v)]} - \frac{\exp \left(2\pi i \frac{k}{M} \right)}{MN_1 \exp[-i \phi(u, v)]} \right\} N^{(k)}(u, v) \delta(u - u_1) \delta(v - v_1) \]
\[
\begin{align*}
\frac{2}{M^2 N_1^2} & \left\{ \sum_{k=1}^{M} N^{(k)}(u,v) - \frac{1}{2} \sum_{k=1}^{M} \left\{ \exp\left( -4\pi i \frac{k}{M} \right) \frac{\exp\left( 2i\phi(u,v) \right)}{\exp\left( -2i\phi(u,v) \right)} \right\} \right\} \\
& = \frac{2}{M^2 N_1^2} \left\{ MN_0 - \frac{M}{4} \left\{ \frac{N_2 \exp[i\phi_2(u,v)]}{\exp[2i\phi(u,v)]} + \frac{N_{-2} \exp[-i\phi_2(u,v)]}{\exp[-2i\phi(u,v)]} \right\} \right\} \delta(u-u_1)\delta(v-v_1),
\end{align*}
\]

in which eq. (A3) is used.

Defining

\[
\varepsilon = \frac{N_1}{N_0},
\]

\[
\varepsilon_2 = \frac{N_2}{4N_0} \left[ \frac{\exp(i\phi_2)}{\exp(2i\phi)} + \frac{\exp(-i\phi_2)}{\exp(-2i\phi)} \right],
\]

eq. (A13) can be concisely rewritten as

\[
\langle \Delta\phi(u,v)\Delta\phi(u_1,v_1) \rangle = \frac{2(1-\varepsilon_2)}{M N_0 \varepsilon^2} \delta(u-u_1)\delta(v-v_1).
\]

Note that the random variable \( N_0, N_1 \) and \( N_2 \) have been substituted with their corresponding mean value\(^7\). Consequently, we get

\[
\sigma^2_\phi = \frac{2(1-\varepsilon_2)}{M N_0 \varepsilon^2},
\]

Subsequently, according to eq. (A1), we have

\[
\frac{N_\theta}{\sigma^2_D} = \left( \frac{\lambda z_T}{g_2} \right)^2 \frac{N_\theta}{\sigma^2_\phi} = \left( \frac{z_T}{g_2} \right)^2 \frac{\varepsilon^2}{2(1-\varepsilon_2)} N_\theta M N_0 = \left( \frac{z_T}{g_2} \right)^2 \frac{\varepsilon^2}{2(1-\varepsilon_2)} NEQ
\]

where \( N_\theta \) is the total number of projection in data acquisition and the definition

\[
NEQ = N_\theta M N_0
\]

is consistent with those of the conventional CT\(^{15,16,18}\).

Finally, it should be noted that, if assume \( N_2 \) and \( N_{-2} \) be zero, we have \( \varepsilon_2 = 0 \) and eq. (A17) becomes exactly the same as that derived in reference\(^7\), in which only \( N_0, N_1 \) and \( N_{-1} \) are considered. In general, however, \( N_2 \) and \( N_{-2} \) are small but not equal to zero. Hence, the derivation given in this manuscript is more general.

**REFERENCES**


