Performance Evaluation of 2-D DOA Estimation Algorithms in Noisy Channels

Shahriar Shirvani Moghaddam and Ali Janan

Abstract: In this paper, an overview of three well-known two-dimensional Direction Of Arrival (DOA) estimation algorithms, namely, Multiple Signal Classification (MUSIC), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) and Propagator Method (PM) is presented. In order to reduce the computational complexity of 2-D methods, azimuth and elevation estimations are extracted from two one-dimensional estimations. As the main objective of this investigation, considering 1-D realization of 2-D DOA estimation algorithms and simulation them in MATLAB software, the Root Mean Square Error (RMSE) performance of these methods is compared in three cases, uncorrelated, correlated and coherent signals in the presence of white Gaussian noise as well as colored noise. Simulation results show that for uncorrelated signals, MUSIC in low Signal to Noise Ratios (SNRs) and ESPRIT in high SNRs offer lower RMSE. In the case of correlated and correlated signals, ESPRIT is the best choice in all SNRs. Finally, for colored noise scenario, PM provides more accurate estimation for low SNRs, while ESPRIT has a lower RMSE for high SNRs compared to two other methods.

Keywords: DOA estimation, ESPRIT, L-shape array, MUSIC, propagator method, two-dimensional.

1. INTRODUCTION

Due to wide applications of Direction Of Arrival (DOA) estimation of impinging signal to array antenna such as radar, sonar, earthquake prediction device, mobile communication and so on, it has attracted attention of many researchers. The DOA estimation is used to identify the location of the signal sources. So far, several methods have been developed to estimate [1]. At first, the methods were used for one-dimensional estimation but as regards in many applications, signal sources are not at the same height of array. Hence, in addition to azimuth angle, it is necessary to estimate elevation angle. Therefore, the methods were extended to two-dimensional cases. Among the DOA estimation methods, Maximum Likelihood (ML) methods have shown excellent performance, however their very high computational load will limit the use of these methods. Subspace methods such as Multiple Signal Classification (MUSIC), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) and Propagator Method (PM) are able to estimate the DOA with high accuracy and less computational burden compared with ML methods [1-3]. Despite MUSIC is based on noise subspace and is applicable for different array geometries, but it needs spectral search over a parameter space to estimate the DOA. So, it is computationally intensive. The high and complex computation of searching method can be reduced using search-free methods which are able to avoid the aforementioned spectral search step (via replacing it by polynomial rooting). One of search-free approaches is ESPRIT which is based on signal subspace. It introduces less complexity compared to MUSIC but it needs special array structure. Another search-free method is PM that does not require EigenValue Decomposition (EVD) of array covariance matrix or Singular Value Decomposition (SVD) of received data matrix [4]. As a result, the computational load of PM is less than the MUSIC and ESPRIT algorithms but its accuracy is low.

Many efforts have been directed to generalize the well known DOA estimation algorithms such as MUSIC and ESPRIT from the 1D-case where a single DOA is estimated to the 2D-case where the true azimuth and elevation angles of the source are to be determined. The main idea for generalization is to use a two-dimensional array instead of the Uniform Linear Array (ULA). However, joint estimation of the azimuth and elevation angles by planar array increases the computational complexity of the algorithm. It means that, conventional techniques for solving the 2D-estimation problems should be replaced by two independent problems, and then a pair-matching algorithm is to be used to match correctly the solutions of the two problems. The main drawback of planar arrays is its large number of elements which is not applicable in some applications such as cellular systems. Different arbitrary 2D-array geometries that have reduced the number of elements than the planar arrays are studied in details in [5, 6]. The above mentioned algorithms, MUSIC, ESPRIT, and PM can be applied to various structures of the array such as rectangular, triangular, L-shape and so on. L-shape array has simpler configuration compared to the others which consists of two independent ULAs. It shows very
good performance in 2-D DOA estimation and also it is used in many literatures [7-10]. Therefore, this array is used in this research work to one-dimensional realization of 2-D DOA techniques.

The rest of the paper organized as follows. In section 2, signal model for L-shape array is presented. Section 3 presents three 2-D DOA estimation algorithms. As the main goal of this research, 1-D realizations of two-dimensional MUSIC, ESPRIT and PM methods are simulated in noisy channels, white as well as colored, for both uncorrelated and correlated/coherent signals. Simulation results are provided in section 4 to evaluate and compare the performance of these DOA estimation methods. Finally, section 5 concludes this investigation.

2. SIGNAL MODEL

Fig. (1) shows an L-shape array structure in x-z plane consists of two orthogonal ULAs. Each ULA has N sensors with inter-element spacing d and the element placed at the origin is the reference. Suppose there are k narrowband signal sources with wavelength \( \lambda \) impinging on the array from different directions \((\theta_k, \phi_k)\) for \( k = 1, 2, ..., K \) where \( \theta_k \) and \( \phi_k \) are elevation and azimuth angles of the \( k \)-th signal, respectively. It is assumed that the number of sources already estimated using some standard techniques [11] and they are in the far-field with respect to the array location. The received signal at the \( Z \) and \( X \) subarrays can be modeled as:

\[
Z(t) = A_Z s(t) + n_Z(t)
\]

\[
X(t) = A_X s(t) + n_X(t)
\]

where \( Z(t) = [z_1(t), z_2(t), ..., z_n(t)]^T \) and \( X(t) = [x_1(t), x_2(t), ..., x_n(t)]^T \) are \( N \times 1 \) received signal vectors, \((.)^T\) denotes the transpose, \( s(t) = [s_1(t), s_2(t), ..., s_k(t)]^T \) is \( K \times 1 \) source vector, \( n_Z(t) \) and \( n_X(t) \) are \( N \times 1 \) additive noise vectors assumed to be independent of signals. \( A_Z \) and \( A_X \) are \( N \times K \) array response matrices as:

\[
A_Z = [a(\theta_1), a(\theta_2), ..., a(\theta_K)]^T
\]

\[
A_X = [a(\theta_1, \phi_1), a(\theta_2, \phi_2), ..., a(\theta_K, \phi_K)]^T
\]

where \( N \times 1 \) steering vectors can be written as:

\[
a(\theta_k) = [1, e^{-j2\pi \cos \theta_k \lambda}, ..., e^{-j2\pi (N-1) \cos \theta_k \lambda}]^T \quad \text{for} \ k = 1, 2, ..., K
\]

\[
a(\theta_k, \phi_k) = [1, e^{-j2\pi \sin \theta_k \cos \phi_k \lambda}, ..., e^{-j2\pi (N-1) \sin \theta_k \cos \phi_k \lambda}]^T \quad \text{for} \ k = 1, 2, ..., K
\]

Under this assumption, \( N \times N \) array covariance matrix of each subarray can be expressed as:

\[
R_{ZZ} = E[Z(t)Z^H(t)] = A_Z A_Z^H + R_{nn}
\]

\[
R_{XX} = E[X(t)X^H(t)] = A_X A_X^H + R_{nn}
\]

where \( R_{SS} = E[S(t)S^H(t)] \) is \( K \times K \) source covariance matrix, \( R_{nn} \) is \( N \times N \) noise covariance matrix. \( E(.) \) and \((.)^H\) denote the statistical expectation and the Hermitian transpose, respectively. If additive noise is white Gaussian stochastic process with zero-mean and variance \( \sigma^2 \), \( R_{nn} = \sigma^2 I \) where \( I \) is identity matrix.

In practice, \( R_{XX} \) is not known perfectly but it can be estimated from the data matrix as:

\[
\hat{R}_{ZZ}(t) = \frac{1}{M} \sum_{l=1}^{M} Z(t)Z^H(t)
\]

\[
\hat{R}_{XX}(t) = \frac{1}{M} \sum_{l=1}^{M} X(t)X^H(t)
\]

Eigenvalue decomposition of the covariance matrix is given by:

\[
R_{ZZ} = V_Z \Lambda_Z V_Z^H
\]

\[
R_{XX} = V_X \Lambda_X V_X^H
\]

where \( V \) contains eigenvectors and \( \Lambda \) is a diagonal matrix where each diagonal element presents an eigenvalue. The first \( K \) eigenvectors of each covariance matrix span signal subspace and the rest ones span noise subspace.

2.1. Correlated or Coherent Signals

\( R_{SS} \) is nonsingular and covariance matrix rank is less than the number of signals if the received signals are correlated or coherent. Under these conditions, performance of the DOA estimation methods will be reduced considerably. To overcome this problem, decorrelation methods such as Forward-Backward Spatial Smoothing (FBSS) can be applied to eliminate the correlation between signals [12, 13]. In forward-backward spatial smoothing, at first, each ULA is divided into \( L \) maximum overlapping subarrays. Each subarray contains \( N_0 \) elements where \( N_0 = N-L+1 \). Then, the forward and backward covariance matrices are extracted as average of subarrays covariance matrices in forward and backward directions, respectively. Forward spatial smoothing covariance matrix can be defined as:

\[
R_f = \frac{1}{L} \sum_{l=1}^{L} R_l^f
\]

where \( R_l^f \) is covariance matrix of \( l \)-th forward subarray. Also, backward spatial smoothing covariance matrix can be expressed as:

\[
R_b = \frac{1}{L} \sum_{l=1}^{L} R_l^b
\]

Finally, forward-backward spatial smoothing covariance matrix is given by:

\[
R = \frac{R_f + R_b}{2}
\]
2.2. Colored Noise

In the case of white Gaussian noise, $R_{nn}$ is diagonal with constant element, but if noise realizations are correlated, $R_{nn}$ is not diagonal and also diagonal elements are no longer equal together [14]. Under these circumstances, performance of the DOA methods may be significantly degraded.

3. TWO-DIMENSIONAL DOA ESTIMATION METHODS

In this section, three well-known subspace DOA estimation algorithms are reviewed. In order to decrease the computational load, two-dimensional estimation is converted into two one-dimensional estimations. First we estimate $\theta$ angles using the ULA in $Z$ axis. Then, the estimated $\theta$ angles and information from the array elements in $X$ axis are used to estimate the $\phi$ angles.

3.1. MUSIC

MUSIC algorithm was proposed by Schmidt [15]. Due to its universality, it is widely used. The algorithm searches for direction which noise subspace is orthogonal to steering vector. The MUSIC spectrum function in $Z$ axis is defined as

$$ P_{\text{MUSIC}}(\theta) = \frac{1}{\alpha^H(\theta) V_{n,z} V_{n,z}^H \alpha(\theta)} \tag{16} $$

where $V_{n,z}$ is noise subspace of $R_{zz}$. $P_{\text{MUSIC}}$ will be maximized in $\theta$ equals to elevation angle of one of the received signals. MUSIC spectrum function in $X$ axis can be expressed as follow

$$ P_{\text{MUSIC}}(\bar{\theta}_k, \phi) = \frac{1}{\alpha^H(\bar{\theta}_k, \phi) V_{n,x} V_{n,x}^H \alpha(\bar{\theta}_k, \phi)} \text{ for } k = 1, 2, ..., K \tag{17} $$

where $\bar{\theta}_k$ is obtained from peak of (16). The positions of $K$ largest peaks in equation (17) indicate azimuth angles estimation of incident signals.

3.2. ESPRIT

ESPRIT was proposed by Roy [16] and further was studied in [17-19]. In this algorithm, unlike the MUSIC, signal subspace is used and array is divided into two similar subarrays such that the displacement between corresponding elements in two subarrays is equal. Fig. (2) shows an example of linear array divided into two similar subarrays. Received signal vector and array response matrix in both $Z$ and $X$ axis are also divided into two subvectors and submatrices.

![Fig. (2). Two overlapping subarrays](image)

Received signal vector of subarrays in $Z$ axis can be written as

$$ Z_1(t) = A_{Z1} S(t) + N_{Z1}(t) \tag{18} $$
$$ Z_2(t) = A_{Z1} \Phi_x S(t) + N_{Z2}(t) \tag{19} $$

where definitions of $Z_1(t), Z_2(t), A_{Z1}, S(t), N_{Z1}(t),$ and $N_{Z2}(t)$ are similar to what was described in section 2 with proper index and superscript and $\Phi_x = \text{diag}[e^{-j2\pi d \cos \phi_1 \sin \theta_1}, ..., e^{-j2\pi d \cos \phi_k \sin \theta_k}]$ is a diagonal matrix where its diagonal elements express signal phase shift between corresponding elements in two subarrays. It is clear that all elevation angles can be estimated by obtaining the $\Phi_x$.

The main idea of ESPRIT is that the steering vector of $A_x$ and signal subspace of $R_{xz}$ span the same subspace. Therefore, there is an unique nonsingular matrix $T$ that

$$ V_{x, z} = A_{x} T \tag{20} $$

Now, we divide $V_{x, z}$ into two submatrices $V_{1, z}$ and $V_{2, z}$ where are composed from first $N - 1$ and last $N - 1$ rows of $V_{x, z}$, respectively. According to (20) the following equations can be written

$$ V_{1, z} = A_{z1} T \tag{21} $$
$$ V_{2, z} = A_{z1} \Phi_x T \tag{22} $$

From (21) and (22), yields:

$$ V_{2, z} = V_{1, z} T^{-1} \Phi_x T = V_{1, z} \Psi \tag{23} $$

where

$$ \Psi = T^{-1} \Phi_x T \tag{24} $$

Equation (24) shows that eigenvalues of $\Psi$ are diagonal elements of $\Phi_x$. Therefore, by solving equation (23) and obtaining eigenvalues of $\Psi$, the elevation angles can be estimated as

$$ \hat{\theta}_k = \cos^{-1} \left( \frac{\text{arg}(\lambda_k)}{2\pi d} \right) \text{ for } k = 1, ..., K \tag{25} $$

where $\lambda_k$ is $k$-th eigenvalue of $\Psi$.

Equation (23) can be solved using Least Squares (LS) or Total LS (TLS) methods. For many cases TLS ESPRIT presents higher performance compared to another one as reported in [20].

To estimate the azimuth angles, received signal vectors of each subarray in $X$ axis can be written as

$$ X_1(t) = A_{x1} S(t) + N_{x1}(t) \tag{26} $$
$$ X_2(t) = A_{x1} \Phi_x S(t) + N_{x2}(t) \tag{27} $$

where $\Phi_x = \text{diag}[e^{-j2\pi d \cos \phi_1 \sin \bar{\theta}_1}, ..., e^{-j2\pi d \cos \phi_k \sin \bar{\theta}_k}]$ and $\bar{\theta}_k$ has been estimated from (25). With the same ESPRIT procedure used for obtaining $\Phi_x$, $\Phi_x$ can be estimated which contains information about elevation and azimuth angles. Then, the azimuth angles can be estimated as

$$ \hat{\phi}_k = \cos^{-1} \left( \frac{\text{arg}(\lambda_k)}{2\pi d \sin \bar{\theta}_k} \right) \text{ for } k = 1, ..., K \tag{28} $$

3.3. PM

MUSIC and ESPRIT methods offer high resolution DOA estimations but they need EVD of the covariance matrix or SVD of received data matrix to estimate signal and noise subspaces. Therefore, these algorithms are computationally intensive, especially when the number of array sensors or incident sources is large. Propagator method (PM) which was proposed by Macros [21] and was developed in [7, 8] is a subspace technique without any EVD or SVD. Therefore,
computational burden of PM is much less than MUSIC and ESPRIT.

Signal model for PM is the same as that discussed for ESPRIT in section 3.2, but in this method \( \Phi_z \) and \( \Phi_x \) are estimated using PM [21].

Let \( B \) be defined as

\[
B = \begin{bmatrix}
A_z \\
A_z \Phi_z
\end{bmatrix}
\]

(29)

Then, both of \( A_z \) and \( B \) are partitioned into

\[
A_z = \begin{bmatrix}
A_{z1} \\
A_{z2}
\end{bmatrix}
\]

(30)

\[
B = \begin{bmatrix}
A_{z1} \\
B_2
\end{bmatrix}
\]

(31)

where \( A_{z1} \) and \( A_{z2} \) are composed of first \( K \) and last \( (N-1-K) \) rows of \( A_z \), respectively; also \( B_2 \) is composed of last \( (2N-2-K) \) rows of \( B \) and can be written as

\[
B_2 = \begin{bmatrix}
A_{z2} \\
A_{z1} \Phi_z \\
A_{z2} \Phi_x
\end{bmatrix}
\]

(32)

Assuming that \( A_{z1} \) is nonsingular (which is common for all subspace techniques) the propagator \( P \) is a unique linear operator which is defined as

\[
P^H A_{z1} = B_2
\]

(33)

If we define \( 2(N-1) \times 1 \) data vector \( q(t) \) as

\[
q(t) = \begin{bmatrix}
Z_1(t) \\
Z_2(t)
\end{bmatrix}
\]

(34)

the propagator can be estimated from the Data Matrix (DM) denoted as \( Q = [q(1), q(2), \ldots, q(M)] \) or sample covariance matrix (SCM), \( R_{qq} = 1/M \sum_{t=1}^{M} q(t)q^H(t) \) as follow. The partition of \( Q \) and \( R_{qq} \) are given by

\[
Q = \begin{bmatrix}
Q_1 \\
Q_2
\end{bmatrix}
\]

(35)

\[
R_{qq} = [R_1 R_2]
\]

(36)

where \( Q_1 \) and \( Q_2 \) consist of first \( K \) and last \( (2N-2-K) \) rows of \( Q \) while \( R_1 \) and \( R_2 \) consist of first \( K \) and last \( (2N-2-K) \) columns of \( R_{qq} \). Propagator matrix can be estimated from (35) and (36) using LS method as follows

\[
P_{DM} = (Q_1 Q_1^H)^{-1} Q_1 Q_2^H
\]

(37)

\[
P_{SCM} = (R_1^H R_1)^{-1} R_1^H R_2
\]

(38)

It is easily seen that \( B_2 \) and \( P^H \) (Hermitian transpose of either \( P_{DM} \) or \( P_{SCM} \) have the same dimensions, then \( P^H \) is partitioned in the similar way as

\[
P^H = \begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix}
\]

(39)

where the dimensions of \( P_1, P_2 \) and \( P_3 \) are identical with dimensions of \( A_{z2}, A_{z1}, \Phi_z \) and \( A_{z2} \Phi_z \), respectively. According to (33) we can derive the following equations between each pair of them

\[
P_1 A_{z1} = A_{z2}
\]

(40)

\[
P_2 A_{z1} = A_{z2} \Phi_z
\]

(41)

\[
P_3 A_{z1} = A_{z2} \Phi_x
\]

(42)

From equations (40) and (41), yields

\[
P_3 P_3^H A_{z2} = A_{z2} \Phi_x
\]

(43)

where \( # \) denotes pseudoinverse. Then, using eigenvalue factorization

\[
P_3 P_3^H = \Phi_x \Phi_x^H
\]

(44)

This means that the diagonal elements of matrix \( \Phi_x \) can be estimated by obtaining the eigenvalues of \( P_3 P_3^H \) or eigenvalues of \( P_2 \) from (41). Then, elevation angles are given by equation (25).

With the same PM procedure used for obtaining \( \Phi_x \), we can estimate \( \Phi_z \). Then, azimuth angles can be estimated from equation (28).

4. SIMULATION RESULTS

This section compares performance of the three 2-D DOA estimation methods described in section 3 through several numerical experiments by simulation in MATLAB software.

The sensor array in Fig. (1) is used with 6 elements in each ULA (total number of L-shape array elements is 11) and \( d = \lambda / 2 \). The number of snapshots is \( M = 200 \).

We evaluate the performance in terms of joint Root Mean Square Error (RMSE) of the elevation and azimuth angles estimations under different Signal to Noise Ratios (SNRs). Joint RMSE is defined as

\[
RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^{T} \sum_{k=1}^{K} (\hat{\theta}_k - \theta_k)^2}
\]

(45)

where \( T \) is the number of independent Monte Carlo trials, \( \hat{\theta}_k \) and \( \theta_k \) are estimated elevation and azimuth angles of the \( t \)-th trial for \( k \)-th signal, while \( \hat{\theta}_k \) and \( \theta_k \) are exact values. SNR varies from 0 to 20 dB with step size 1 dB and for each SNR, 1000 trials are carried out.

In the first simulation, the DOA estimation performance of these methods for two uncorrelated equal power signals arriving from far-field in directions \((30^\circ, 120^\circ)\) and \((60^\circ, 20^\circ)\) is derived in the presence of white Gaussian noise with variance 1. Fig. (3) shows the joint RMSE of two signals versus SNR for 2-D MUSIC, ESPRIT and PM methods. It can be seen that the MUSIC algorithm provides lower RMSE in low SNRs, whereas ESPRIT offers the lowest RMSE in high SNRs and PM shows the weakest performance in all SNRs. In our simulations, it is observed that 2-D MUSIC method requires much more simulation time compared to 2-D ESPRIT and 2-D PM, while required simulation time for 2-D PM is less than two other methods. Unlike ESPRIT and PM, MUSIC requires an exhaustive search through all possible steering vectors; therefore it needs more computations compared to ESPRIT and PM which are search-free. In other hand, the computational load of ESPRIT and PM are in order of \( O(N^3 + 2N^2M) \) and \( O(2NMK) \), respectively [1]. It is clear that when the number of array elements is large, computational burden of PM is much less than ESPRIT.
In the second experiment, 2-D DOA estimation algorithms are compared for FBSS-smoothed coherent signals. In this simulation, 3 equal power coherent signals coming from (30°, 90°), (50°, 50°) and (70°, 30°) are considered. The complex coefficients of coherent signals are 1, (0.4 + 0.8i) and (0.7 – 0.2i), respectively and each ULA in Fig. (1) consists of 8 elements. Other specifications are the same as those given for first simulation. Joint RMSE versus SNR are depicted in Fig. (4). It is obvious that FBSS can effectively enhance 2-D DOA estimation of the coherent signals. Moreover, it is observed that the ESPRIT provides the best performance among three methods, even in low SNRs. This may be due to the fact that in ESPRIT, each ULA is divided into two subarrays, therefore pseudo spatial smoothing is carried out in algorithm procedure.

The third simulation considers the scenario that incident signals are correlated and FBSS as a pre-processing is used to decorrelate the signals. Correlated signals are generated by passing the signal through a first order Auto-Regressive (AR1) filter with transfer function

\[ T(z) = \frac{1}{1 - \rho z^{-1}} \]

(46)

where \( \rho \) is correlation coefficient. In this simulation, \( \rho = 0.9 \) is considered and other parameters are the same with second simulation. The joint RMSE against SNR is shown in Fig. (5). Comparing Fig. (4) and Fig. (5), it is clear that RMSE for correlated signals is lower than coherent signals. This implies that FBSS can better deal with correlated signals. In this case, ESPRIT exhibits the most accurate DOA estimation among three methods.

In the fourth simulation, it is assumed that the parameters are the same as the ones used in the first simulation but white noise is changed to colored noise. Fig. (6) depicts simulation results. It is observed that the PM provides the lowest RMSE in low SNRs, while ESPRIT offers higher performance in high SNRs.

It is worth noting that MUSIC gives poor performance in low SNRs, the reason is that the MUSIC is based on noise subspace and in colored noise scenario, accuracy of noise subspace estimation is reduced significantly.

**CONCLUSIONS**

In this paper, performance of three well-known two-dimensional DOA estimation algorithms, MUSIC, ESPRIT and PM was evaluated for L-shape array in the case of white and colored noisy channels considering both uncorrelated and correlated/coherent signals. Simulation results showed that for uncorrelated signals, MUSIC offers better performance in low SNRs, whereas ESPRIT has superior perfor-
performance in high SNRs. Table 1 briefly shows specification of these methods.

In the case of coherent as well as correlated signals, ESPRIT shows higher performance compared to MUSIC and PM in all SNRs. In colored noise scenario, PM provides more accurate estimation among three approaches for low SNRs, while ESPRIT has a higher performance in high SNRs.

**Table 1. Comparison of specifications of 3 subspace 2-D algorithms.**

<table>
<thead>
<tr>
<th>2-D Algorithm</th>
<th>Accuracy</th>
<th>Computational load</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUSIC</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>ESPRIT</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>PM</td>
<td>Medium</td>
<td>Low</td>
</tr>
</tbody>
</table>

**CONFLICT OF INTEREST**

The authors confirm that this article content has no conflicts of interest.

**ACKNOWLEDGEMENTS**

This work was supported by Research Institute for ICT (ITRC) under contract number 19248/500 (28.12.1390).

**LIST OF ABBREVIATIONS**

2D = Two Dimensional
DOA = Direction Of Arrival
ESPRIT = Estimation of Signal Parameters via Rotational Invariance Techniques
EVD = Eigen Value Decomposition
FBSS = Forward Backward Spatial Smoothing
LS = Least Squares
MATLAB = MATrix LABoratory
MUSIC = MUltiple SIgnal Classification
PM = Propagator Method
RMSE = Root Mean Square Error
SNR = Signal to Noise Ratio
SVD = Singular Value Decomposition
ULA = Uniform Linear Array

**REFERENCES**


