A Multi-Objective Optimization Approach to Water Management.

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Abstract: Many optimization models exist for water management systems but there is a knowledge gap in linking bio-economic objectives with the optimum use of all water resources under conflicting demands. The efficient operation and management of a network of nodes comprising storages, canals, river reaches and irrigation districts under environmental flow constraints is very challenging. Minimization of risks associated with agricultural production requires accounting for uncertainty involved with climate, environmental policy and markets. Due to conflicts between multiple goal requirements and the competing water demands of different sectors, a multi-criteria decision-making (MCDM) framework was developed to analyze production targets under physical, biological, economic and environmental constraints. This approach is described by analyzing the conflicts between profitability, variable costs of production and pumping of groundwater for a hypothetical irrigation district.

Keywords: Optimization; Minimization; Multi-criteria decision-making

1. INTRODUCTION

Many decision support systems in agricultural enterprises use the conventional linear programming approach to optimize a single objective function such as total gross margin. However, as agricultural systems become more complex, multiple objectives that are in conflict need to be addressed. Competition for scarce resources by different enterprises is a major concern in many agricultural production systems. Competition occurs at the farm level e.g. between different crops as well as at a regional level, where utilization of scarce water resources for agricultural purposes often comes into conflict with the requirement for in stream ecosystem services. For example, in bio-economic systems conflict may arise from maximizing economic returns (i.e. net revenue) as opposed to minimizing the use of resources such as water, fertilizer applications etc. On the other hand minimizing costs rather than maximizing net revenue may also be important in some water management systems. Under these conditions, multiple criteria decision-making techniques are useful tools to explore different management options. These techniques permit optimization of several objectives in many different logical formulations (Piech and Reyman, 1993). A multi-criteria approach has been used extensively to solve diverse decision problems including risk assessment in agricultural systems (Berbel, 1993).

Mendoza et al., (1993) used Fuzzy Multiple Objective Linear Programming (FMOLP) techniques in forest planning where imprecise objective function coefficients are involved. Furthermore, Tecle, (1998) used Compromise Programming (CP) to develop a multi-objective decision support system for analyzing multi-resource forest management problem. The use of these techniques enables the decision maker to study the trade-offs and conflicts between, for example, profitability (measured by economic returns) and risk (measured by Partial Absolute Deviation (PAD)). This paper demonstrates the application of a MCDM technique called Goal Programming (GP) to water resource allocation problems with conflicts between irrigation water demand and in stream environmental flow requirements. First, a solution is sought for the single objective function formulation and compared to a solution of three objective functions (Net Revenue (NR), Variable Costs (VC) and Total Groundwater Pumping from the irrigation areas (TP)) using Goal Programming.

2. MODEL FORMULATION

2.1. Objective functions, decision variables and constraints

The multi-objective problem described in this paper consists of three objective functions: maximizing net returns (NR), minimizing variable
cost (VC) and minimizing total supplementary groundwater pumping requirements to meet crop demand from the irrigated areas. Conceptually, NR and VC may represent the view of resource economists while minimizing total pumping may be the desired goal to avoid groundwater mining and pollution of aquifers. The management options to achieve the above objectives consist of selection of an appropriate mix of crops, optimum level of groundwater pumping and appropriate allocation of water for irrigation and environment. Constraints imposed on the system include seasonal environmental flows targets. In addition, water allocation rules and pumping targets for each month are constraints imposed on the system.

The three objective functions are formulated as follows:

\[
\begin{align*}
\text{Max} & \quad NR = \sum_c CGM(c) \times X(c) - \sum_{c} \sum_{m} (WREQ(c,m) \times X(c) \times C_w) - P \times \sum_{c} \sum_{m} P(c,m) \\
\text{Min} & \quad VC = \sum_c \sum_{m} (X(c) \times WREQ(c,m) \times C_w) + \sum_c \times X(c) \times V\text{cost}(c) \\
\text{Min} & \quad TP = \sum_{c} \sum_{m} P(c,m)
\end{align*}
\]

where \(X(c)\) = area of crop \(c\) (Ha), \(CGM(c)\) = gross margin for crop \(c\) ($), \(WREQ(c,m)\) = water requirement for crop \(c\) in month \(m\) (ML), \(C_w\) = total cost of water per unit volume ($/ML), \(P(c,m)\) = cost of groundwater pumping and delivery ($/ML), \(V\text{cost} = \) variable cost (such as fertilizer and pesticides applications) per hectare other than water cost for crop \(c\) and \(P(c,m) = \) volume of ground water pumped from irrigation areas for crop \(c\) in month \(m\) (ML).

The model consists of a network of nodes that connect supply nodes to irrigation or urban areas (demand nodes). The links connecting the nodes include river reaches that may carry environmental flows as well as irrigation canals. The continuity equation for each node \((i)\) assuming no storage at the node is given by the following:

\[
\sum_{j} Q(i,j) = \sum_{k} Q(k,i)
\]

where \(Q(i,j) = \) flow of water from node \(i\) to node \(j\), \(Q(k,i) = \) flow of water from node \(k\) to node \(i\).

The physical and environmental constraints imposed on the model are given by the following:

Total water use in the irrigation areas should not exceed total allocation in a given month:

\[
\sum_{c} \left( X(c) \times WREQ(c,m) \right) \leq \text{Allocation}(m)
\]

\(m = 1,...,12\) (5)

The sum of all crop areas is equal to the total farm area:

\[
\sum_{c} X(c) = \text{TArea}
\]

(6)

Environmental flows in each month should equal or exceed target flows:

\[
\text{Env}_f(m) \geq \text{Environmental flow}(m)
\]

\(m = 1,...,12\) (7)

Total pumping (TP) from the irrigation area in any month should be less than or equal to allowable pumping.

\[
\sum_{c} P(c,m) \leq \text{Pump}(m)
\]

\(m = 1,...,12\) (8)

where \(\text{Allocation}(m) = \) monthly water allocation for irrigation areas (ML), \(\text{TArea} = \) Total irrigable farm area (Ha), \(\text{Env}_f(m) = \) environmental flow (ML) in month \(m\), \(\text{Environmental flow}(m) = \) target environmental flow in month \(m\) and \(\text{Pump}(m) = \) allowable pumping in the irrigated areas for month \(m\).

Two auxiliary equations were used to restrict the minimum cropped area to a given value when the crop area becomes a basic variable in the solution vector:

\[
- X(c) + mArea \leq \text{TArea} \times Y(c)
\]

\(X(c) \leq \text{TArea} \times (1 - Y(c))\) (9)

where \(mArea = \) minimum crop area (Ha) and \(Y(c) = \) binary variable for crop \(c\). For the illustration problem given in the next section the minimum crop area was assigned a value of 1000 Ha for all calculations.

3. **THE EXAMPLE PROBLEM**

In order to demonstrate model application a hypothetical 3-node network is chosen to illustrate the concepts outlined above. The network consists of a supply node (e.g. reservoir), a demand node (e.g. irrigation area), a distribution node and an environmental flow link. The network is schematically illustrated below (see Figure 1). The supply schedule and environmental flow targets are usually stipulated by water sharing plans and flow rules of a river system. These flows may be dependent on climate, aesthetics, social, economic and environmental factors.
The demand node (node 3) is made up of a total irrigable land of 30,000 Ha with a potential for growing six crops (maize, wheat, canola, rice, oats and barley). Groundwater pumping from the irrigable area is permitted to satisfy crop water demand if surface water supplies are not sufficient. Each of these crops is characterized by crop water requirements and growing period within the year. The continuity equation (4) was recast for the example problem as:

\[ IRR_{\text{node3}(m)} + \text{pump}(m) = \sum_{c} X(c) \times WREQ(c,m) \]
\[ m = 1,...,12 \]

\[ w_{\text{node2}(m)} = S_{\text{node1}(m)} - \text{Chn\_losses(link(1,2))} \times \text{Chn\_l(link(1,2))} \]

\[ w_{\text{node2}(m)} - \text{Env\_f(m)} = IRR_{\text{node3}(m)} \]
\[ + \text{Chn\_losses(link(2,3))} \times \text{Chn\_l(link(2,3))} \]

where \( IRR_{\text{node3}(m)} \) = surface water available at node 3 for the irrigation area in month \( m \), \( w_{\text{node2}(m)} \) = surface water available at the distribution node 2 in month \( m \), \( S_{\text{node1}} \) = reservoir supply in month \( m \), \( \text{Env\_f(m)} \) = environmental flow in month \( m \), \( \text{Chn\_losses} \) = channel seepage per unit length, \( \text{Chn\_l} = \) channel length(m) and \( \text{link}(i,j) = \) link joining node \( i \) to \( j \).

Reference evapo-transpiration (ET) and rainfall data for the example problem are shown in Tables 1 and 2. The pay-off matrix and the corresponding crop mix was determined using the three objective functions given by (1) to (3) and constraint equations (5) to (12) for the dry, average and wet seasons and shown in Tables 3 to 5. The elements of the pay-off matrix were obtained by optimizing each of the objectives (1), (2) and (3) individually and then calculating the values of the remaining objectives using the solution vector of the decision variables. For example, the first row of Table 3 shows results from maximizing NR. When net revenue is maximized, its maximum value is $34,348,685 and the cost associated with it is $20,180,526 and total groundwater pumping from the irrigated area was 16,632 ML. The crop mix obtained by maximizing net revenue is Rice (4789 Ha), Canola (19518 Ha), Oats (1000 Ha) and Maize (4693 Ha). The diagonal elements of the pay-off matrix in Table 3 are the optimum values for each individual goal. The results clearly indicate the degree of conflict between the three objectives. However, the pay-off matrix in Table 3 indicates that for the dry season there is not much difference between minimizing total cost and minimizing total pumping. Obviously, the decision maker is very likely to be interested in a combination of maximum NR, minimum cost and minimum total pumping. However, because the objectives are in conflict, some sort of compromise solution must be found. Several MCDM methods are used to obtain solutions including Multi-Objective Programming (MOP), Compromise Programming (CP) and Goal Programming (GP). MOP methods generate a set of efficient solutions sometimes called Pareto optimal solutions and can be very difficult to

**Table 1.** Rainfall (ML/Ha or x100 mm) for dry, average and wet seasons.

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>0.15</td>
<td>0.03</td>
<td>0.14</td>
<td>0.11</td>
<td>0.3</td>
<td>0.21</td>
<td>0.14</td>
<td>0.39</td>
<td>0.17</td>
<td>0.26</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>Average</td>
<td>0.22</td>
<td>0.12</td>
<td>0.28</td>
<td>0.27</td>
<td>0.27</td>
<td>0.29</td>
<td>0.4</td>
<td>0.3</td>
<td>0.35</td>
<td>0.37</td>
<td>0.26</td>
<td>0.28</td>
</tr>
<tr>
<td>Wet</td>
<td>0.49</td>
<td>0.18</td>
<td>0.33</td>
<td>0.32</td>
<td>0.73</td>
<td>0.49</td>
<td>0.42</td>
<td>0.42</td>
<td>0.45</td>
<td>0.48</td>
<td>0.32</td>
<td>0.36</td>
</tr>
</tbody>
</table>

**Table 2.** Reference Evapo-transpiration (ET, ML/Ha) for dry, average and wet seasons.

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>2.92</td>
<td>2.41</td>
<td>1.94</td>
<td>1.22</td>
<td>0.69</td>
<td>0.47</td>
<td>0.54</td>
<td>0.83</td>
<td>1.27</td>
<td>1.91</td>
<td>2.49</td>
<td>2.94</td>
</tr>
<tr>
<td>Average</td>
<td>2.72</td>
<td>2.24</td>
<td>1.87</td>
<td>1.12</td>
<td>0.67</td>
<td>0.46</td>
<td>0.52</td>
<td>0.74</td>
<td>1.11</td>
<td>1.72</td>
<td>2.24</td>
<td>2.63</td>
</tr>
<tr>
<td>Wet</td>
<td>2.65</td>
<td>2.16</td>
<td>1.84</td>
<td>1.08</td>
<td>0.59</td>
<td>0.41</td>
<td>0.43</td>
<td>0.7</td>
<td>1.02</td>
<td>1.67</td>
<td>2.16</td>
<td>2.58</td>
</tr>
</tbody>
</table>
Table 3. Pay-off matrix and crop-mix for dry season

<table>
<thead>
<tr>
<th>Optimization Goal</th>
<th>Net Revenue ($)</th>
<th>Total Cost ($)</th>
<th>Total Pumping (ML)</th>
<th>Crop-Mix (Ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Revenue</td>
<td>34348685</td>
<td>20180526</td>
<td>16632</td>
<td>Rice 4789</td>
</tr>
<tr>
<td>Total Cost</td>
<td>26107450</td>
<td>14873443</td>
<td>0</td>
<td>Wheat 13803</td>
</tr>
<tr>
<td>Total Pumping</td>
<td>27032049</td>
<td>15569178</td>
<td>0</td>
<td>Barley 1000</td>
</tr>
</tbody>
</table>

Table 4. Pay-off matrix and crop-mix for average rainfall season

<table>
<thead>
<tr>
<th>Optimization Goal</th>
<th>Net Revenue ($)</th>
<th>Total Cost ($)</th>
<th>Total Pumping (ML)</th>
<th>Crop-Mix (Ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Revenue</td>
<td>37811126</td>
<td>1961400</td>
<td>7345</td>
<td>Rice 9845</td>
</tr>
<tr>
<td>Total Cost</td>
<td>20417517</td>
<td>12639862</td>
<td>3998</td>
<td>Wheat 13828</td>
</tr>
<tr>
<td>Total Pumping</td>
<td>28486042</td>
<td>15977407</td>
<td>0</td>
<td>Barley 1000</td>
</tr>
</tbody>
</table>

Table 5. Pay-off matrix and crop-mix for wet rainfall season

<table>
<thead>
<tr>
<th>Optimization Goal</th>
<th>Net Revenue ($)</th>
<th>Total Cost ($)</th>
<th>Total Pumping (ML)</th>
<th>Crop-Mix (Ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Revenue</td>
<td>39407159</td>
<td>19865143</td>
<td>0</td>
<td>Rice 11488</td>
</tr>
<tr>
<td>Total Cost</td>
<td>17582653</td>
<td>12437679</td>
<td>0</td>
<td>Wheat 23123</td>
</tr>
<tr>
<td>Total Pumping</td>
<td>28896551</td>
<td>15490436</td>
<td>0</td>
<td>Barley 1000</td>
</tr>
</tbody>
</table>

3.1. The Goal Programming Model

GP solves the multiple objective problem by introducing the objectives into the problem as constraints and setting targets to be achieved. The objectives are included in the problem by adding positive \( p_i \) and negative \( n_i \) deviation variables that describe over-achievement and under-achievement of each goal.

The weighted version of goal programming model (WGP) was used in this example. The model is defined to minimize only the undesirable deviations from defined targets:

implement when the number of objectives is large. On the other hand, CP looks for a solution as close as possible to the “Ideal Point”. This point is normally taken as the individual optimal solutions.
Table 7. Positive (p) and negative (n) deviational variables for dry, average and wet seasons ($ for indices 1 and 2 and ML for index 3).

<table>
<thead>
<tr>
<th>index</th>
<th>n</th>
<th>p</th>
<th>n</th>
<th>p</th>
<th>n</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12375000</td>
<td>0</td>
<td>12301000</td>
<td>0</td>
<td>11099000</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2635600</td>
<td>0</td>
<td>1884000</td>
<td>0</td>
<td>1672700</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The net revenue was under-achieved by more than one million dollars in all seasons while cost exceeded target by over one half million dollars. The pumping target was achieved.

The sensitivity analysis of applying different weights is demonstrated below. If we apply twice as much weight to the NR goal (i.e. $\alpha_1 = 2$, $\alpha_2 = \alpha_3 = 1$) the following results were obtained.
4. CONCLUSIONS

Most water management systems are concerned with satisfying conflicting demands of various groups and MCDM techniques provide the mechanism for resolving these conflicts. They provide better results than simple linear programming (LP) solutions because they integrate the effect of all the objectives simultaneously. There are an increasing number of highly sophisticated LP solvers that could easily be adapted to solve MCDM problems using Goal Programming (GP) or Weighted Goal Programming (WGP) as illustrated with the example problem. The application of MCDM techniques to the simple nodal-network example problem demonstrates its ability to provide solutions that integrate different goals and trade-offs. The pay-off matrix for the three goals illustrates the degree of conflict between the different goals and trade-offs. The effect of different ET and rainfall (dry, average and wet) on NR, crop areas, environmental flows and water allocated to the irrigation areas was clearly demonstrated. Furthermore, the sensitivity of the weights assigned to the different goals was shown to have marked impact on optimal crop areas and the degree of under- and over-achievement of the selected targets for all the three goals. Maximization of NR was almost equivalent to minimization of total pumping under dry climatic conditions. By attaching different weights to goals, sets of decision variables (crop area, water allocations) could be formulated for different seasons that could aid in policy formulations and decision-making. Although goal programming is a useful tool to analyze MCDM problems, there is a difficulty of selecting the target values and weights for the different goals.

5. REFERENCES


