NEW DIRECTION IN DEGREE CENTRALITY MEASURE: TOWARDS A TIME-VARIANT APPROACH

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Degree centrality is considered to be one of the most basic measures of social network analysis, which has been used extensively in diverse research domains for measuring network positions of actors in respect of the connections with their immediate neighbors. In network analysis, it emphasises the number of connections that an actor has with others. However, it does not accommodate the value of the duration of relations with other actors in a network; and therefore, this traditional degree centrality approach regards only the presence or absence of links. Here, we introduce a time-variant approach to the degree centrality measure - time scale degree centrality (TSDC), which considers both presence and duration of links among actors within a network. We illustrate the difference between traditional and time scale degree centrality measure by applying these two approaches to explore the impact of degree attributes of a patient-physician network evolving during patient hospitalisation periods on the hospital length of stay (LOS) both at a macro- and a micro-level. At a macro-level, both the traditional and time-scale approaches to degree centrality can explain the relationship between the degree attribute of the patient-physician network and LOS. However, at a micro-level or small cluster level, TSDC provides better explanation while the traditional degree centrality approach is found to be inadequate in explaining its relationship with LOS. Our proposed TSDC measure can explore time-variant relations that evolve among actors in a given social network.

Keywords: centrality; degree centrality; and time scale degree centrality.

1. Introduction

A social network is a collection of individuals called actors, each of whom is acquainted with some other subset of others by one or more different types of relations such as
friendship, kinship, common interest, sexual relationship and financial exchange. The study on social network has gained significant interest in recent research efforts as this topic persists and emerges almost everywhere where individuals gather to achieve their collaborative goals. To map and measure relationships among actors in a social networks, social network analysis (SNA) is used. SNA has been successfully applied to understand networks and their participants by evaluating locations of actors in the network. SNA views relationships among actors in terms of network theory consisting of nodes and ties (also called edges, links or connections). In the language of SNA, nodes are the individual actors within networks, and ties are the relationships between actors. Both actors and ties can be defined in different ways depending on the question of interest and context of the study.

Measures of SNA are found very useful in investigating network properties such as which actor is the most influential, what kind of relationships exist among actors and the nature of evolutionary dynamics of the network itself. One of the basic SNA measures is centrality which is a structural attributes of nodes in a network and is mainly used to quantify network locations of actors. Degree centrality is one of the three network centrality measures. The other two centrality measures are betweenness centrality and closeness centrality. The number of direct links connecting a node determines degree centrality of that node. Degree centrality is mainly relevant in the studies of popularity and activity of actors because it is primarily concerned with local point centrality. In a directed network, degree centrality can be assessed for in-degree and out-degree, where in represents other actors connecting to a particular actor and out represents that particular actor connecting other actors in the network. The application of degree centrality has a long history capturing structural properties of networks, both at the actor-level and network-level, in diverse research contexts, such as, gene-disease association, virtual research and development (R&D) groups, inter-organisational collaboration networks, finance and economic forecasting, knowledge sharing, scientific research collaboration networks, and email communication networks. On the other hand, degree centrality cannot capture the complete scope of node level relations in many situations. For example, if an academic joins a school of a university then she may immediately develop many relations with her colleagues. This will put her in a well-connected position in the academic-staff network of that school. However, for knowledge sharing and scientific collaboration this type of network position does not give her any advantage. Duration of a relationship, which matures over time and strengthens scientific relations, is very important in this context. To give another example, if a hospitalised patient is visited five times by different physicians then she has a degree centrality of five in the corresponding patient-physician network, although it does not reflect anything about how long she has been receiving treatment from those doctors, which is very important for her immediate disease recovery.

In this study, we deploy the concept of degree centrality for analysing the impact of the relations emerging within hospital or healthcare organisations between physician and patient on hospital outcome. In doing so, we propose a new measure, time scale degree...
centrality (TSDC), as the traditional degree centrality does not value the duration of links among actors. Our proposed TSDC measure, which is a time variant approach to traditional degree centrality measure, regards both the presence and duration of links among actors. This contribution is organised as follows. In the next section, we describe an empirical context (i.e. healthcare organisation) where traditional degree centrality cannot capture complete node level relation of a patient-physician network. In section three, we propose our time variant approach to the degree centrality measure (i.e. TSDC). In the context of patient-physician network we compare TSDC and traditional degree centrality in terms of their capabilities to explain hospital LOS in section four. This is followed by a discussion and conclusion of this study in section five.

Fig.1. Illustration of patient-physician interactions during hospitalised period of patient

2. Background of the Study

This study explores degree centrality in the context of healthcare organisations or hospitals where informal networks evolve between physician and patient for every hospitalised patient. It can be conceptualised that patients are being visited by hospital physicians during their inpatient hospital stay. Moreover, they might need to be seen by outside specialist physicians depending on the availability of hospital physicians and severity of their medical conditions. During their visits to patients, physicians examine patients’ medical conditions and their responses to previous medication. They can access all medical information about any patient from the medical log book, which is the depository of all previous medication history and suggested medical guidelines about that patient. At the end of their visit, physicians make further medical suggestions and update the medical log book accordingly. This is the common practice for hospitalised patients, which, over the time, forms a patient-physician network as illustrated in Fig.1. Every patient admission to a hospital creates such a patient-physician network where the network actors are the patient herself and all physicians who visit that patient at least once during her hospitalisation period. This network is evolutionary by nature because the number of links between physician actor and patient actor, and the entrances of new
physician actors to this network increase with the increase of the patient’s stay within a hospital. The evolution of such a network for a particular patient from our research dataset is given in Fig.2. The traditional approach to the degree centrality measure is not capable in capturing this evolutionary dynamics of patient-physician network as it considers only the presence and absence of links. It does not value the duration of links among actors. For example, in an inter-organisational network structure, organisation A has relation with organisation B for 40 years and with C for 14 years. A network analysis for node A will show a value of 2 for its degree centrality measure. This value, which simply shows that organisation A has relation with organisation B and C, cannot tell anything about the duration of their organisational relationship. However, in terms of inter-organisational collaboration, information and knowledge sharing, and organisational support, organisation B has more importance and trust than C to organisation A.

The ultimate goal of any healthcare provider is to provide low-price healthcare services with high level of patient satisfaction. Hospital expenses and hospital LOS are greatly influenced by the number of times patients are being visited by physicians during their inpatient hospital stay. During their visits to hospitalised patients, physicians give medical suggestions, which are stored in patients’ medical log books. For instance, if a patient is visited three times by physician Ph1 and two times by physician Ph2 then that patient receives medical suggestions five times and there are five different entries in that patient’s medical log book. Arguably, for the patient-physician network we can define...
Time variant approach to degree centrality measure

Degree centrality as the number of times a patient is being visited by physicians during her inpatient hospital stay (i.e. number of entries in that patient’s medical log book or the number of times this patient received medical advice from physicians). Then the following proposition can be proposed based on current research findings from the healthcare literature\textsuperscript{29,30}:

**Proposition (P1):** Degree centrality of the patient-physician network has positive correlation with LOS.

We use a health insurance claim dataset from a very large non-profit Australian health insurance organisation. It includes members’ claim data from January 2005 to February 2009. This dataset contains information about three different categories of claims: ancillary claim, medical claim and hospital claim. Ancillary claims are auxiliary claims for medical services, such as, dental, optical, physiotherapy, dietician and pharmaceutical services. All claims coming from specialist physicians except of the ancillary type are medical claims. The claims for the services provided to hospitalised patients are considered as hospital claims. In general, patients have medical claims, hospital claims and very few ancillary claims for their admissions to hospitals. Each physician visit is associated with one medical claim in the insurance claim dataset. For each hospital admission there is information about admission date and discharge date, which are used to measure LOS. This dataset is very large (about 14.87 million ancillary, 8.98 million medical and 3.1 million hospital claims) and contains medical claims for different types of patients (e.g. knee surgery, chronic patient and heart attack). From this large health insurance claim dataset, we only consider a total of 2553 inpatient hospital admission dataset of total hip replacement procedures for patients over 60 years old and who have had admissions only to private hospitals to test proposition P1.

As the dataset is large (i.e. sample size of 2553) and degree centrality values show a normal distribution, we apply the Pearson correlation method to test the relation between degree centrality and LOS. The result shows a strong positive relation (Pearson $\rho=0.396$, $p=0.001$, 2 tailed) between them. However, in a further level of investigation we find that many small clusters of patients show a weak correlation or no correlation between degree centrality and LOS. Each of these clusters is characterised by having the same LOS for its all member patients. For instance, we find a cluster of 14 patients, with LOS of 20 days and degree centrality values ranging from 9 to 52, show no significant correlation (Pearson $\rho=0.017$, $p=0.324$, 2 tailed) between degree centrality and LOS. For this cluster, our investigation to the visiting patterns of physicians shows that each patient has different pattern of physician visit over time (see Fig.3) although there is common structural similarities for physicians’ visiting patterns observed in all such small clusters. Fig.3 also shows the corresponding TSDC, which is explained in the next section, for degree centrality of each of four patients. We observe the following three common properties or trends for the visiting patterns of physicians to hospitalised patients under all such clusters, where all patients of a cluster have the same LOS:
(a) Patients have more physicians’ visits at the beginning of their hospital stay time if degree centrality value is low.
(b) Patients have more physicians’ visits at the end of their hospital stay period if degree centrality value is high.
(c) For average degree centrality value, patients have more physicians’ visits in the middle of their hospital stay period.

Based on the finding of P1, it can be argued that degree centrality of the patient-physician network has impact on hospital LOS. Further, for small clusters of patients who have same LOS we notice three different physicians’ visit patterns to hospitalised patients. Differences in visiting patterns indicate that medial suggestions are given to patients in different times as physicians give suggestions to patients only during their visits. That means persistency of medical advice is different for different patterns of physician visits. The way traditional degree centrality is defined (i.e. the number of entries in patient’s medical log book or number of physicians’ visits) cannot capture this persistency of medical suggestions. For this reason, traditional degree centrality fails to explain these micro-level patterns (i.e. small cluster) of patient and physician relationships although it can explain this well at the macro-level (i.e. for our research dataset of 2553 patients). To overcome this shortcoming of the traditional approach to degree centrality, a new variation to degree centrality is proposed in this study (i.e. TSDC) where both the presence and duration of links among nodes are taken into account to measure degree centrality for each node in a given network.

3. Time Scale Degree Centrality (TSDC)
In order to construct equations and develop an empirical basics for TSDC we start with two variations of traditional degree centrality (i.e. in_degree and out_degree) that can be defined by following two formulas\(^1\):

\[
d_i(\text{out\_degree}) = \sum_j x_{ij} \quad (1.1)
\]

\[
d_i(\text{in\_degree}) = \sum_j x_{ji} \quad (1.2)
\]

Where, \(x_{ij}\) represents the single cell value of the adjacency matrix \(X\) for network data.

To make comparisons of degree centrality of different nodes in different networks with different sizes, right hand sides of Eq. (1.1) and Eq. (1.2) are divided by \((N-1)\), where \(N\) is the number of nodes in the network representing the size of the network under study. The value \((N-1)\) is chosen as any node can at most be adjacent to \((N-1)\) nodes in a network of size \(N\). This makes a generic (i.e. regardless of network size) measure for degree centrality, \(D\). Therefore,

\[
D_i(\text{out\_degree}) = \frac{\sum_j x_{ij}}{N-1} \quad (1.3)
\]

\[
D_i(\text{in\_degree}) = \frac{\sum_j x_{ji}}{N-1} \quad (1.4)
\]

Where, \(x_{ij}\) represents the single cell value of the adjacency matrix \(X\) for network data.

Fig.4. Illustration of degree relation among nodes in a arbitrary network- Fig.4 (a) does not show time scale of relationship among nodes. Fig.4 (b), Fig.4 (c) and Fig.4 (d) show the relationship among nodes at times \(t_0\), \(t_1\) and \(t_2\) respectively (where \(t_2 > t_1 > t_0\)).
In order to overcome the shortcoming of the traditional approach to degree centrality, a new variation to degree centrality is proposed in this study. This proposed measure (i.e. TSDC) considers both the presence and duration of links among nodes to measure the degree centrality for each node in a given network. For example, the traditional in-degree and out-degree values for all nodes of Fig.4 (a) are illustrated in Table 1(a). Table 1(b) shows the degree centrality values (both in-degree and out-degree) for the network in Fig.4 (b), Fig.4 (c) and Fig.4 (d), which are observed at three different times, i.e. $t_0$, $t_1$ and $t_2$ (where $t_2>t_1>t_0$) respectively. If the network has been stable between $t_2$ and $T$ (where $T> t_2>t_1>t_0$) then TSDC for node $C$, for example, will be $\{T + (T-t_2)\}$ and $\{(T-t_2) + (T-t_1)\}$ for out-degree centrality and in-degree centrality respectively at time $T$.

<table>
<thead>
<tr>
<th>Node</th>
<th>In-degree Link</th>
<th>Out-degree Link</th>
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<tbody>
<tr>
<td>A</td>
<td>C</td>
<td>B, D</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>E</td>
</tr>
<tr>
<td>C</td>
<td>E, F</td>
<td>A, D</td>
</tr>
<tr>
<td>D</td>
<td>A, C, F</td>
<td>F</td>
</tr>
<tr>
<td>E</td>
<td>B, F</td>
<td>C</td>
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<tr>
<td>F</td>
<td>D</td>
<td>C, D, E</td>
</tr>
</tbody>
</table>

Table 1(a). Connectivity (in-degree and out-degree) table for the network of Fig. 4(a)

<table>
<thead>
<tr>
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<th>Out-degree Link</th>
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<tbody>
<tr>
<td>A</td>
<td>C</td>
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<tr>
<td>B</td>
<td>A</td>
<td>E</td>
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<tr>
<td>C</td>
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<tr>
<td>D</td>
<td>F</td>
<td>C</td>
</tr>
<tr>
<td>E</td>
<td>D</td>
<td>C, D, E</td>
</tr>
</tbody>
</table>

Table 1(b). Connectivity (in-degree and out-degree) table for the networks in Fig.4 (b), Fig.4 (c) and Fig.4 (d) that are observed at time $t_0$, $t_1$ and $t_2$ respectively.

In general for any network time scale degree centrality can be represented by:

\[
TSDC_{out\_degree} = \sum_{j} x_j(T - t_1) \tag{2.1}
\]

\[
TSDC_{in\_degree} = \sum_{j} x_j(T - t_1) \tag{2.2}
\]
Time variant approach to degree centrality measure

Where, $T$ is the total life time of the network and $t_i$ is the time when node $i$ joined in the network under consideration; and $x_{ij}$ represents the single cell value of the adjacency matrix $X$ for network data.

To make comparisons of TSDC for different nodes in different networks with different sizes and life time, the right hand sides of the Eq. (2.1) and Eq. (2.2) are divided by $(N-1)$, where $N$ is the number of nodes in the network representing the size of the network under study and $T$ is the life time of the network. The value $(N-1)$ is chosen as any node can at most be adjacent to $(N-1)$ nodes in a network of size $N$. This makes a generic (i.e. independent of network size) measure for time-based degree centrality, TSDC:

$$TSDC_{i}(\text{out degree})=\frac{\sum_{j} x_{ji}(T-t_j)}{(N-1)*T}$$ (2.3)

$$TSDC_{i}(\text{in degree})=\frac{\sum_{j} x_{ij}(T-t_i)}{(N-1)*T}$$ (2.4)

Where, $T$ is the total life time of the network and $t_i$ is the time when node $i$ joined in the network under consideration; and $x_{ij}$ represents the single cell value of the adjacency matrix $X$ for network data.

This proposed TSDC measure can be utilised to conduct a node-level network analysis in many contexts. For instance, it can be applied to analyse the positional strength of individual organisation in inter-organisational collaboration networks. To quantify the position of an organisation in an inter-organisational collaborative network, the proposed TSDC measure is very important as this measure emphasises both the time length and the number of inter-organisational relationships for an organisation.

According to inter-organisational collaboration theory, for any organisation both the duration of inter-organisational relationships and the number of relations are crucial for its successful and supportive collaboration with other organisations within the organisational network\textsuperscript{31-33}. The traditional degree centrality measure might produce inappropriate results regarding the organisational position in a given inter-organisational collaboration network because this measure considers only the presence of the number of relations with other organisations. Therefore, to evaluate the organisational collaboration ability for a single organisation in an inter-organisational network, the TSDC measure provides more insights than the traditional degree centrality measure.

4. TSDC versus Traditional Degree Centrality: Patient-Physician Network

We then apply the proposed TSDC to quantify the degree centrality of patients in a patient-physician network. This new measure (i.e. TSDC) shows positive association with LOS for all small clusters (i.e. Pearson rho=0.461, p=0.01, 2 tailed) although these
clusters show different relations (i.e. no significant correlation) between degree centrality and LOS when the traditional degree centrality measure was applied. For instance, the cluster of 20 patients who have LOS of 20 days shows significant positive correlation between TSDC and LOS (i.e. Pearson rho= 0.578, p=0.042, 2 tailed) whereas it shows different correlation output (i.e. Pearson rho=0.017, p=0.324, 2 tailed), as illustrated in Table 2, when tradition degree centrality was exercised. Further, the TSDC approach shows stronger correlation with LOS (Pearson rho=0.673, p=0.001 and 2-tailed) for our complete research dataset (i.e. for the entire 2553 patients dataset). This time variant of degree centrality (i.e. TSDC) can also explain why there is no significant correlation between LOS and degree centrality at a small cluster level. For example, it can explain the variability of degree centrality values for the four patients of the cluster that has the LOS of 20 days for all its member patients. The over time degree distributions of these four patients are illustrated in Fig.3. The degree values are largely different ranging from 52 to 93, although there is small variance among the corresponding TSDC values for these four patients as illustrated in this figure.

Table 2. Correlation coefficient (i.e. Pearson rho) between different variations of degree centrality and LOS

<table>
<thead>
<tr>
<th>Variable Names</th>
<th>Correlation Coefficient</th>
</tr>
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<tbody>
<tr>
<td>Traditional degree centrality and LOS (for complete dataset)</td>
<td>0.396**</td>
</tr>
<tr>
<td>Traditional degree centrality and LOS (for the cluster having LOS of 20 days)</td>
<td>0.017</td>
</tr>
<tr>
<td>TSDC and LOS (for complete dataset)</td>
<td>0.673**</td>
</tr>
<tr>
<td>TSDC and LOS (for the cluster having LOS of 20 days)</td>
<td>0.578*</td>
</tr>
<tr>
<td>TSDC and LOS (for all small clusters)</td>
<td>0.461*</td>
</tr>
</tbody>
</table>

* Correlation is significant at 0.05 level (2-tailed)
** Correlation is significant at 0.01 level (2-tailed)

Our proposed TSDC measure not only considers the tally of links but also regards the duration of links, which makes it capable to capture both the network-level and small cluster-level dynamics of relations between patient and physician in a patient-physician network. On the other hand, traditional degree centrality is error-prone to encapsulate cluster-level relations as it only regards the presence and absence of links. Thus, proposed TSDC of this article overcomes flaws of the traditional degree centrality measure and can provide more insights about node-level relations among actors in a given network.

TSDC is not a context-sensitive approach as a degree centrality measure. In this paper, we apply this to the patient-physician network. It can be applied to other contexts of network relations, such as longitudinal analysis of inter-firm relations or inter-bank relationship. Not only that, TSDC can be used as a substitute for the traditional degree centrality measure. If we set \((T-t)\) to 1 in Eq. (2.1) and Eq. (2.2), or set \((T-t)\) to \(T\) in Eq. (2.3) and Eq. (2.4) then our proposed TSDC will work just the same as the traditional degree centrality measure.
5. Discussion and Conclusion

In this paper, we propose a time-variant approach to traditional degree centrality, which takes into account both the number of relations and duration of those relations among actors in a given social network. Also, we develop equations to quantify and to make comparisons of TSDC of different nodes in different networks with different network sizes and life times. We further apply this new measure to the context of patient-physician networks. Our findings reveal that TSDC can provide better explanation compared to the traditional degree centrality in the analysis of patient-physician network to understand the effect of degree centrality on LOS. The equations developed in this study (i.e. Eq. (2.1) to Eq. (2.4)) for TSDC can be applied to any network although we apply here TSDC to patient-physician networks, which is a star-like network. These equations are developed in a generalised way and consider all links by which an actor is connected with other actors in a given network, regardless of the global structure of that network. Therefore, they can be applied to any network, which provides over time information about actor-level interactions, for measuring TSDC.

For the health insurance claim dataset of hospitalised patients, TSDC provides a better explanation of the relation between degree centrality of the patient-physician network and LOS. This can be explained by the fact that physicians’ suggestions propagate from the patient admission date to the discharge date through the patient’s medical log book. When physicians visit a patient, they give medical suggestions to that patient based on her health condition and previous medication history, which are being deposited in her medical log book. All suggestions by any physician to a patient at any day of her hospitalisation period are taken into consideration during any subsequent physician visit to that patient. Eventually, this expands the patient-physician network in a way such that if any physician is connected to this network by visiting a patient, he or she never leaves the network as the patient’s medical log book keeps his or her medical suggestions. An early physician’s suggestion lasts longer than any other subsequent physician’s suggestions through the medical log book. TSDC considers this time length of network relationship along with, like the traditional degree centrality, the number of links among actors.

The proposed TSDC can potentially be applied for weighted graphs. For that, the adjacency matrix (i.e. $X$) needs to be replaced by a corresponding weighted adjacency matrix (i.e. $W$) in Eq. (2.1) to Eq. (2.4). Theoretically, any kind of network relation, including our proposed time-based relation, can be represented by weighted graphs. If the patient-physician network is presented as weighted graph then the link weight (i.e. $w_{ij}$) could be the function of many factors, in addition to TSDC, including how much a patient is paying, the professional qualification and experience of her physician and so on. Based on this universal applicability of weighted graphs, it can be argued that our proposed TSDC, similar to the tie strength measure proposed by Granovetter\cite{2}, is a subclass of generic weighted degree centrality. However, representing the link weight (i.e. $w_{ij}$) as a function of more than one variable will demand to manage further computational complexities. Because of its numerous existences in different contexts of network
research, time-based network relations are considered separately in this study to propose a new variation (i.e. TSDC) of degree centrality measure. This new measure could potentially be used to analyse network relations characterised by instability and inconsistency; that is, breaking and reestablishing network relations among actors frequently.

Researchers have proposed models (e.g. time dependent network model by Berger et al.\textsuperscript{35}) and develop measures (e.g. centrality overlap by Braha and Bar-Yam\textsuperscript{36}) to study dynamics of time variant relations among actors in evolving networks\textsuperscript{35-38}. In this article, we introduce a time-variant approach to the degree centrality measure, which considers both the presence and duration of links among actors within a network. Moreover, generalisations or variations of node centrality measures have been proposed by researchers in the literature of network science. Opsahl et al.\textsuperscript{39} propose a generalised centrality measure for weighted networks based on weighted ties and links. A variation of the centrality metric for dynamic networks is proposed by Lerman et al.\textsuperscript{40}. They consider the number of paths existing over time in a network. Unlike them, TSDC developed in this study is based on both duration and number of links over time in a network. However, TSDC has few limitations. It cannot be applied for networks that have single information for their actor-level interactions (e.g. static networks). Further, TSDC requires more computational memory for adjacency matrices to store the time points when actors join the network. The adjacency matrix has to keep information about the presence or absence of all links between any pair of nodes as well as the time a link is being established.

In conclusion, we provide a new approach to the degree centrality measure in this article. In the context of patient-physician networks, we show that the TSDC concept can provide better explanations, compared to the traditional measure of degree centrality, in regards to the impact of the duration of patient-physician interactions on LOS. Our TSDC approach could contribute to the centrality era of social network analysis as this measure could be applied in other network domains such as inter-organisational collaboration networks.

References


