A hybrid algorithm to solve a capacitated location allocation problem in fuzzy and stochastic environment

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Abstract: In this article, a capacitated location-allocation problem is considered in which customer’s demands and locations are uncertain. While the demands are fuzzy, the locations are stochastic and follow a normal distribution. The distance between the locations and the customers are Euclidean and squared Euclidean. Two closed-form expressions based on Euclidean and Euclidean square are used to evaluate the expected distance between the customers and the facilities. Moreover, three types of programming (fuzzy expected cost, fuzzy $\beta$-cost minimization, and credibility maximization) are developed to model the problem. In order to solve the models, a hybrid intelligent algorithm is applied where the simplex algorithm, fuzzy simulation and a modified genetic algorithm are integrated. Finally, to illustrate the efficiency of the proposed algorithm some numerical examples are presented.

Keywords: Location allocation problem, Fuzzy programming, Fuzzy simulation, hybrid intelligent algorithm

1. INTRODUCTION

The location-allocation (LA) problem is the one in which a certain number of new facilities are placed in between a number of specific customers in a feasible area such that the transportation cost from the facilities to the customers is minimized. This problem was first introduced by Cooper [1] and a many researchers such as Hakimi [2], Gen and Cheng [3], Badri [4], and Lee [5] extended it thereafter. Furthermore, Logenderan and Terrell [6], Sherali and Rizzo [7], Carrizosa et al. [8], Carrizosa et al. [9], and Zhou [10] are the ones who considered stochastic LA problems.

In un-capacitated LA problems, each facility has an unconstrained capacity and the customers’ demands are satisfied by the nearest facility. However, in capacitated location allocation problems the customers’ demands do not have to be supplied by the nearest facility, making the problem more complicated, especially when a fuzzy capacitated problem is involved.

Zhou and Liu [11] studied a stochastic capacitated LA problem in which customers’ demands were stochastic and the locations were deterministic. They used three types of stochastic programming to model their problem. Zhou and Liu [12] considered a fuzzy capacitated location allocation problem with fuzzy demands in which location of customers was deterministic. Wen and Iwamura [13] proposed a fuzzy facility LA problem under the Hurwicz criterion. Wen and Iwamura [14] utilized a LA problem in random fuzzy environment. They used $(\alpha,\beta)$-cost minimization model under the Hurwicz criterion to formulate the problem. Abiri and Yusefli [15] proposed an application of probabilistic programming to the fuzzy LA problem where demands were fuzzy and locations were deterministic.

In real world problems, the customers’ locations are usually not deterministic. In these situations, it is difficult to find a distance function for stochastic location of customers. Durmaz et al. [16] showed a capacitated location allocation problem with stochastic customer locations and deterministic customer demands. They applied some heuristic algorithms to solve the problem and employed used two methods, exact and approximate, to evaluate the expected distances between the facilities and the customers.

This paper considers a capacitated location allocation problem with probabilistic customers’ locations, in which two exact methods are proposed to evaluate the expected distances. The probability distribution of the locations is assumed normal and the distances are Euclidean and squared Euclidean. Meanwhile, since it is usually not easy to estimate a probability distribution for the demands, they are assumed fuzzy.

Megiddo and Supowit [17] proved that the LA problems are strongly NP-hard. As a result, a large number of meta-heuristic algorithms are proposed to solve LA models. Zhou and Liu [11, 12] and Wen and Iwamura [13, 14] used a hybrid intelligent algorithm for a capacitated LA in which they employed the simplex algorithm, simulation and genetic algorithm. Abiri and Yusefli [15] proposed a GA, and Liu and Zhou [18] developed a neural network and a simulated annealing to solve a fuzzy capacitated location allocation problem with minimum risk criteria.
Genetic algorithm (GA) has been applied to solve different optimization models in recent decades. Further, a large number of research works in capacitated LA models used GA to obtain near-optimum solutions. In this paper, in order to solve the models, the simplex algorithm, fuzzy simulation and a modified genetic algorithm are integrated to make a hybrid intelligent algorithm.

2. THE MODELS

In this article, in order to model the capacitated LA problem at hand, three types of fuzzy programming models (fuzzy expected cost, fuzzy β-cost minimization, and credibility maximization) are proposed. They are based on the following definitions on the fuzzy event \( \xi \geq r \), respectively [12].

\[
\begin{align*}
\text{Pos}(\xi \geq r) &= \sup_{h \in \Theta} \mu(h), \\
\text{Nec}(\xi \geq r) &= 1 - \sup_{h \in \Theta} \mu(h), \\
Cr(\xi \geq r) &= \frac{1}{2}(\text{Pos}(\xi \geq r) + \text{Nec}(\xi \geq r)).
\end{align*}
\]

2.1. Fuzzy expected cost model

Since the customer demand is assumed fuzzy, its expected value, \( E(C(x_1, x_2 | \theta)) \), is fuzzy as well. In the fuzzy expected cost model, the expected value of the customer demand is evaluated by the following expression.

\[
E(C(x_1, x_2 | \theta)) = \int_{r}^{\infty} Cr(\theta \in \Theta | C(x_1, x_2 | \theta) \geq r) dr
\]

Then, in order to find the stochastic location \( (x_1, x_2) \), the following expected cost minimization model is proposed.

\[
\begin{align*}
\min_{\theta} & \int_{0}^{\infty} Cr(\theta \in \Theta | C(x_1, x_2 | \theta) \geq r) dr \\
\text{Subject to:} & \\
g_{r}(x_1, x_2) & \leq 0, k = 1, 2, \ldots, p
\end{align*}
\]

where \( g_{r}(x_1, x_2) \leq 0, k = 1, 2, \ldots, p \) are potential regions to locate new facilities. The model is different from traditional fuzzy models because there is a sub-optimal in it, i.e.,

\[
\begin{align*}
\min z &= \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} E(d(x_i, a_j)) \\
\text{s.t.} & \\
\sum_{j=1}^{m} w_{ij} &= \xi_j(\theta), \quad j = 1, 2, \ldots, m \\
\sum_{i=1}^{n} w_{ij} &\leq S_i, \quad i = 1, 2, \ldots, n \\
\sum_{j=1}^{m} w_{ij} &\geq 0, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m
\end{align*}
\]

2.2. Fuzzy β-cost minimization model

The second type of fuzzy programming model is fuzzy β-cost minimization model [12], where \( \beta \) is the predetermined confidence level. The proposed model of minimization the β-cost of LA problem is defined as:

\[
\begin{align*}
\min_{\theta} & D^{\beta} \\
\text{Subject to:} & \\
Cr(C(x_1, x_2 | \theta) \leq D^{\beta}) & \geq \beta \\
g_{r}(x_1, x_2) & \leq 0, k = 1, 2, \ldots, p
\end{align*}
\]

In which \( D^{\beta} \) is the β-cost.

2.3. Credibility maximization model

The third type of the fuzzy programming model is the dependent-chance programming [12]. That is the credibility of fuzzy event \( C(x_1, x_2 | \theta) \leq F \) should be maximized. Based on Liu [12], the credibility maximization model for the LA problem at hand is presented as follows,

\[
\begin{align*}
\max & \int_{r}^{\infty} Cr(\theta \in \Theta | C(x_1, x_2 | \theta) \leq F) dr \\
\text{Subject to:} & \\
g_{r}(x_1, x_2) & \leq 0, k = 1, 2, \ldots, p
\end{align*}
\]

where \( F \) is a given level of the total transportation cost. Interested readers are referred to [12] for more explanations.

In order to solve the proposed models, a hybrid intelligent algorithm is used in this research where fuzzy simulation, simplex algorithm, and GA are integrated. The results of employing the algorithm on some generated problems are encouraging.

References

Periodicals:

