A Hybrid Method of Fuzzy Simulation and Genetic Algorithm to Optimize Constrained Inventory Control Systems with Stochastic Replenishments and Fuzzy Demand

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Abstract

Multi-periodic inventory control problems are mainly studied by employing one of two assumptions. First, the continuous review, where depending on the inventory level, orders can happen at any time, and next the periodic review, where orders can only be placed at the beginning of each period. In this paper, we relax these assumptions and assume the times between two replenishments are independent random variables. For the problem at hand, the decision variables (the maximum inventory of several products) are of integer-type and there is a single space-constraint. While demands are treated as fuzzy numbers, a combination of back-ordering and lost-sales is considered for the shortages. We demonstrate the model of this problem is of an integer-nonlinear-programming type. A hybrid method of fuzzy simulation (FS) and genetic algorithm (GA) is proposed to solve this problem. The performance of the proposed method is then compared with the performance of an existing hybrid FS and simulated annealing (SA) algorithm through three numerical examples containing different numbers of products. Furthermore, the applicability of the proposed methodology along with a sensitivity analysis on its parameters is shown by numerical examples. The comparison results show that, at least for the numerical examples under consideration, the hybrid method of FS and GA shows better performance than the hybrid method of FS and SA.

Keywords: Multiproduct Inventory Control; Partial Back-ordering; Stochastic Replenishment; Integer Nonlinear Programming; Fuzzy Simulation; Genetic Algorithm

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1. Introduction and Literature Review

The continuous review and the periodic review are the main applied policies in multi-periodic inventory control models. However, the underlying assumptions of these models restrict their proper use in real-world environments. In continuous review policy, one has the freedom to act anytime and place orders based upon the available inventory level. While in the periodic review policy, the user is allowed to place orders only in specific and predetermined times.

Two of the widely employed periodic review policies are the so-called (R, T) and (R, nT) policies. In the first one, at fixed predetermined intervals, T, the inventory is reviewed and an order is placed accordingly. The order quantity is determined by subtracting the on hand inventory from a predetermined value R. If this policy is used in an n-echelon inventory system, it is called (R, nT) policy. Further, the economic order quantity (EOQ) model along with the (r, Q) policy are the two other major periodic review inventory systems, where in the former the purchaser desires to determine the optimal quantity of the order while in the latter the optimal values of the reorder point and order quantity are sought.

There is substantial research reported in the literature in the area of multi-periodic inventory control. Some of these works are summarized in Tables (1) and (2). Table (1) (except for Taleizadeh et al. [39]) shows the main research efforts in the stochastic environment dealing with (R, T) and (r, Q) systems in which demand and lead-time are considered stochastic variables. The main constraints shown in these works are service level [30, 39], order quantity [3], and joint order [13, 33]. Although the decision variables such as order quantity, inventory level, reorder point and period length are similar, various assumptions have been made and different models and procedures have been proposed. For example Chiang [6] and Bylka [3] considered emergency orders, Feng and

Table (2) shows the main research efforts in the fuzzy environment performed on EOQ and economic production quantity (EPQ) models. In this category, the main constraints are budget [8, 41], space [28, 34, 41], and service level [41]. While the decision variables are similar to the ones in the stochastic environment, the demand and inventory costs are considered fuzzy variables. In some of these research undertakings such as [19, 26 and 34] production rate, price, and deterioration rate are considered fuzzy variables as well.

Insert Table (1) about here

Insert Table (2) about here

A careful observation of the works listed in Tables (1) and (2) reveals that while separate emphasis has been devoted to the stochastic nature of demand and lead-time, some real-world constraints of the systems have not been investigated simultaneously. For example, no work is reported where both demand and lead-time are probabilistic. Furthermore, some constraints have been partially studied, the decision variables have been considered integer, and constraints such as budget and space have not been investigated.

In addition, many researchers have successfully used meta-heuristic methods to solve complicated optimization problems in different fields of science and engineering. Some of these meta-heuristic algorithms are: fuzzy simulation (Taleizadeh et al. [41, 44]), genetic algorithms (Al-Tabtabai and Alex [2]), harmony search (Lee and Geem
[23], Geem et al. [17], Taleizadeh et al. [40, 41, 42, 43]), simulating annealing (Aarts and Korst [1], Taleizadeh et al. [39, 44]), ant colony optimization (Dorigo and Stutzle [10]), neural networks (Gaidock et al. [15]), threshold accepting (Dueck and Scheuer [11]), Tabu search (Joo and Bong [20]), and evolutionary algorithm (Laumanns et al. [22], Taleizadeh et al. [41]).

This paper first extends the periodic review works in both stochastic and fuzzy environments such that the replenishment intervals become random and demands assume a fuzzy nature. Then, it presents a hybrid algorithm to solve the problem. To be more specific, the extended model assumes a stochastic replenishment, i.e., stochastic period length, multiple products that are stored in a single capacity-constrained warehouse, fuzzy customer demand, and integer decision variables. Since the time between two replenishments includes the time required to order, the time needed to provide or produce items and the transportation time, the stochastic replenishment assumption is closer to reality than the usual assumption of a deterministic period length. Furthermore, there are many situations in practice where the customer demand is fuzzy, especially in manufacturing where due to the machine breakdowns, shortage of raw materials, and fluctuating rate of nonconforming production, and so on, the demand for on hand inventories of parts and subassemblies can be considered fuzzy. The hybrid algorithm consists of a GA for inventory control optimization and a method for fuzzy simulation to evaluate different solutions in the genetic optimization process.

The models developed in this research are useful for companies and manufacturers who are faced with uncertain demands that do not follow a stochastic pattern. In other words, manufacturers who are unable to assume certain probability distributions for the uncertain demands of their products can use fuzzy set theory to find suitable patterns. Additionally, the proposed method is beneficial in situations where due to some limitations on the production capacity, the supply of the raw material, and the
like, the period length may be uncertain and the goods may not be delivered on time. As an example, when demand increases and production capacity is limited, in case of breakdowns or late receipts of imported raw materials (when delayed at customs) the lead-time and hence the cycle length increases. Another example involves sales representatives that randomly visit retailers offering them product replenishments. The stochastic nature of these factors causes the period length to be stochastic.

The rest of this paper is organized as follows. In Section 2, the problem along with its assumptions is defined. The problem formulation comes in Section 3 after the parameters and the variables are defined. In this section, the single product problem is first modeled and then it is extended to a multiproduct formulation. In the fourth section of the paper, a hybrid algorithm is proposed to solve and analyze the problem at hand under special conditions. By incorporating a numerical example, the solution method is investigated in Section 5. Section 6 contains a sensitivity analysis, and finally the conclusion and recommendations for future research come in Section 7.

2. Problem Definition and Modeling

Consider a periodic-review inventory control model for one provider in which the period lengths are stochastic in nature, i.e., the times between two replenishments are independent random variables following either Uniform or Exponential probability distribution. Triangular fuzzy variables are used to model the demands of several products, and the partial back-ordering policy is employed for shortages, i.e., a fraction of unsatisfied demands is lost and the rest is back-ordered. Moreover, when demands are higher than replenishment levels, the back-ordered quantities in the previous cycle are carried over to the next cycle. In situations in which demands are lower than replenishment levels, extra inventories are carried over to the following cycle and are assumed to have an insignificant impact on the independence of the two periods.
Assuming all the produced items are sold, the costs associated with the inventory control system consists of holding, back-order, lost-sale, and purchase costs. Furthermore, the warehouse space is considered a constraint and the decision variables are integer. We need to identify the maximum inventory levels in each cycle such that the expected profit is maximized.

For the problem at hand, since the times between two replenishments are independent random variables, in order to maximize the expected profit over the planning horizon one needs to consider only one period. Moreover, since the costs associated with the inventory control system are holding and shortage (back-order and lost-sale), we need to calculate the expected inventory level and the expected required storage space in each period. Before doing this, parameters and variables of the model are defined based on the works of Taleizadeh et al. [39, 41, 42]).

2.1. Defining Parameters and Variables of the Model

For \( i = 1, 2, ..., n \), define the parameters and the variables of the model as

\( R_i : \) Maximum inventory level of the \( i^{th} \) product

\( T_i : \) A random variable denoting the time between two replenishments (cycle length) of the \( i^{th} \) product

\( h_i : \) Holding cost per unit inventory of the \( i^{th} \) product in each period

\( \pi_i : \) Back-order cost per unit demand of the \( i^{th} \) product

\( W_i : \) Purchasing cost per unit of the \( i^{th} \) product

\( P_i : \) Selling price per unit of the \( i^{th} \) product

\( D_i : \) Constant demand rate of the \( i^{th} \) product

\( \beta_i : \) Percentage of unsatisfied demands of the \( i^{th} \) product that is back-ordered
\( I_i \): Expected \( i^{th} \) product inventory multiplied by the cycle time

\( L_i \): Expected \( i^{th} \) product lost-sale in each cycle

\( B_i \): Expected \( i^{th} \) product back-order in each cycle

\( Q_i \): Expected order size of the \( i^{th} \) product in each cycle

\( f_i \): Required warehouse space per unit of the \( i^{th} \) product

\( F \): Total available warehouse space

\( C_h \): Expected holding cost per cycle

\( C_b \): Expected shortage cost in back-order state

\( C_l \): Expected shortage cost in lost-sale state

\( C_p \): Expected purchase cost

\( r \): Expected revenue obtained from sales

\( Z \): Expected profit obtained in each cycle

The pictorial representation of the single-product problem is given in Section 2.2. In Section 2.3, we first consider a single-product problem, and then, the formulation to the multi-product modeling is extended in Section 2.4.

### 2.2. Inventory Diagram

According to Ertogal and Rahim [13] and considering the times between replenishments stochastic variables, two cases may occur. In the first case the time between replenishments is less than the time required for the inventory level to reach zero (see Figure 1), and in the second, it is greater (see Figure 2) (Taleizadeh et al. [39, 41, 42]). Figure (3) depicts the shortages in both cases. In the above figures, \( t_o \) denotes the time at which the inventory of the \( i^{th} \) product reaches zero.
2.3. Single Product Model with Constant Demand

In this section, we first model a single-product inventory problem with constant demand where stochastic replenishments, back-orders, and lost-sales are allowed. Then, the model is extended in Section 2.4 to contain several products with fuzzy demands.

2.3.1. Calculating the Costs and the Profit

In order to calculate the expected profit in each cycle, we need to evaluate all the terms in equation (1) [12].

\[ Z = r - C_p - C_h - C_c - C_l = PQ - WQ - hI - \pi B - (P - W)L \]

(1)

Based on Figure (3), \( L, B, I, \) and \( Q \) are evaluated by the following equations (Taleizadeh et al. [39, 41, 42]):

\[ L = \left(1 - \beta\right) \int_{t_D}^{T_{Max}} (DT - R) f_T(t) dt \quad t_D < T \leq T_{Max} \]

(2)

\[ B = \beta \int_{t_D}^{T_{Max}} (DT - R) f_T(t) dt \quad t_D < T \leq T_{Max} \]

(3)

\[ I = \int_{t_{Max}}^{T_{Max}} (RT - \frac{DT^2}{2}) f_T(t) dt + \int_{t_D}^{T_{Max}} \frac{R^2}{2D} f_T(t) dt \]

(4)

\[ Q = \int_{t_{Max}}^{T_{Max}} DT f_T(t) dt + \int_{t_D}^{T_{Max}} (R + \beta (DT - R)) f_T(t) dt \]

(5)

2.3.2. Presenting the Constraints

Since the total available warehouse space is \( F \), the space required for each unit of product is \( f \), and the upper limit for inventory is \( R \), the space constraint will be
In short, the complete mathematical model of the single-product inventory control problem with stochastic replenishments, back-orders, lost-sales, and constant demand is

$$
Max \ Z = (P - W) \left[ \int_{\tau_{\text{max}}}^{R} (DT) f_T(t) \, dt + \int_{\tau_{\text{max}}}^{R} (R + \beta(DT - R)) f_T(t) \, dt \right] \\
- h \left[ \int_{\tau_{\text{max}}}^{R} \left( RT - \frac{DT^2}{2} \right) f_T(t) \, dt + \int_{\tau_{\text{max}}}^{R} \frac{R^2}{2D} f_T(t) \, dt \right] \\
- \pi \beta \left[ \int_{\tau_{\text{max}}}^{R} (DT - R) f_T(t) \, dt \right] - (P - W)(1 - \beta) \left[ \int_{\tau_{\text{max}}}^{R} (DT - R) f_T(t) \, dt \right]$$

s.t.:

$$fR \leq F$$

$$R \geq 0, \ \text{and Integer}$$

### 2.4. Multiproduct Model with Fuzzy Demand

In the extending phase of the single-product model of Section 2.3 to the multiple product formulation of this section, the demands are assumed fuzzy and in addition to the two cases of back-order and lost-sales, their combination is considered as well.

Let $\tilde{D}_i$ denote the fuzzy demand for the $i^{th}$ product. Then, an extension of (7) to include $n$ products easily results in the multiproduct model as

$$
Max \ Z(R, \tilde{D}) = \sum_{i=1}^{n} [(P_i - W_i)Q_i - h_i I_i - \pi_i B_i - (P_i - W_i)L_i] = \\
\sum_{i=1}^{n} \left( P_i - W_i \right) \left[ \int_{\tau_{\text{max}}}^{R} \left( \tilde{D}_i T_i \right) f_{T_i}(t_i) \, dt_i + \int_{\tau_{\text{max}}}^{R} \left( R_i + \beta_i (\tilde{D}_i T_i - R_i) \right) f_{T_i}(t_i) \, dt_i \right] \\
- \sum_{i=1}^{n} h_i \left[ \int_{\tau_{\text{max}}}^{R} \left( RT_i - \frac{\tilde{D}_i T_i^2}{2} \right) f_{T_i}(t_i) \, dt_i + \int_{\tau_{\text{max}}}^{R} \frac{R_i^2}{2D_i} f_{T_i}(t_i) \, dt_i \right] \\
- \sum_{i=1}^{n} \pi_i \beta_i \left[ \int_{\tau_{\text{max}}}^{R} \left( \tilde{D}_i T_i - R_i \right) f_{T_i}(t_i) \, dt_i \right] - \sum_{i=1}^{n} (P_i - W_i)(1 - \beta_i) \left[ \int_{\tau_{\text{max}}}^{R} \left( \tilde{D}_i T_i - R_i \right) f_{T_i}(t_i) \, dt_i \right]
$$
\[ \sum_{i=1}^{n} f_i R_i \leq F \]

\[ R_i \geq 0, \text{ and Integer} \quad \forall \ i = 1, 2, \ldots, n \]

In what follows, two probability density functions for \( T_i \) are assumed and hence two models are developed. In the first model, \( T_i \) follows a uniform distribution, where the demands may occur in a finite and specific range (within an upper and a lower bound). In the second model, \( T_i \) follows an exponential distribution, where the demands may increase sharply. This model is suitable for seasonal or new products.

### 2.4.1. \( T_i \) Follows a Uniform Distribution

In this case \( T_i \) follows a uniform distribution in the interval \([T_{\text{Min}}, T_{\text{Max}}]\), i.e.,

\[ T_i \sim U[T_{\text{Min}}, T_{\text{Max}}] \]

and \( f_{T_i}(t_i) = \frac{1}{T_{\text{Max}} - T_{\text{Min}}} \). Accordingly, (8) is changed to

\[ \max Z(R, \bar{D}) = \sum_{i=1}^{n} \left[ \frac{h_i}{6\bar{D}^2(T_{\text{Max}} - T_{\text{Min}})} \right] R_i^3 - \sum_{i=1}^{n} \left[ \frac{2(P_i - W_i)(1 - \beta_i) + \pi_i \beta_i + h_i T_{\text{Max}}}{2\bar{D}_i(T_{\text{Max}} - T_{\text{Min}})} \right] R_i^2 \]

\[ + \sum_{i=1}^{n} \left[ \frac{4(P_i - W_i)(1 - \beta_i)T_{\text{Max}} + h_i T^2_{\text{Max}} + 2\pi_i \beta_i T_{\text{Max}}}{2(T_{\text{Max}} - T_{\text{Min}})} \right] R_i \]

\[ + \sum_{i=1}^{n} \left[ \frac{-h_i T_{\text{Max}}^3 \bar{D}_i + 3(P_i - W_i)(\beta_i T^2_{\text{Max}} - T^2_{\text{Min}})\bar{D}_i - 3T^3_{\text{Max}} \bar{D}_i(\pi_i \beta_i + (P_i - W_i)(1 - \beta_i))}{6(T_{\text{Max}} - T_{\text{Min}})} \right] \]

s.t.: (9)

\[ \sum_{i=1}^{n} f_i R_i \leq F \]

\[ R_i \geq 0, \text{Integer} \quad \forall \ i = 1, 2, \ldots, n \]
2.4.2. \( T_i \) Follows an Exponential Distribution

If \( T_i \) follows an exponential distribution with parameter \( \lambda_i \), then the probability density function of \( T_i \) is

\[
f_{T_i}(t_i) = \lambda_i e^{-\lambda_i t_i}.
\]

In this case, the model is derived as

\[
\text{Max } Z(R_i, D_i) = \sum_{i=1}^{n} \left\{ \frac{1}{\lambda_i} \left[ 2\hat{D}_i(1-\beta_i)(W_i - P_i) - \pi_i \beta_i \hat{D}_i \right] e^{-\left( \frac{R_i}{\lambda_i} \right)^{\beta_i}} + \frac{1}{\lambda_i} \left[ \hat{D}_i(P_i - W_i) - h_i R_i \right] \right. \\
\left. + \frac{h_i \hat{D}_i}{\lambda_i^2} \left( 1 - e^{-\left( \frac{R_i}{\lambda_i} \right)^{\beta_i}} \right) \right\}
\]

subject to:

\[
\sum_{i=1}^{n} f_i R_i \leq F
\]

\( R_i \geq 0 \), and Integer \( \forall i = 1, 2, \ldots, n \)

In the next section, a hybrid intelligent algorithm is introduced to find near optimum solutions of the formulated problems in (9) and (10).

3. A Hybrid Intelligent Algorithm

Since analytical solutions (if any) of the integer-nonlinear models in (9) and (10) are hard to obtain (Gen and [16]), a hybrid intelligent algorithm of fuzzy simulation and genetic algorithm is developed in this section. Some related research that have employed the fuzzy simulation approach along with other meta-heuristic algorithms include [20], [37], [41], and [44]. In the next subsection, a brief background in fuzzy simulation is given.

3.1. Some Definitions in Fuzzy Environment

In this paper, we adopt the concepts of the credibility theory including possibility, necessity, credibility of fuzzy events, and the expected value of a fuzzy variable as defined in [24, 47, 48].
**Definition 1**: Let \( \xi \) be a fuzzy variable with the membership function \( \mu(x) \). Then the possibility, necessity, and credibility measures of the fuzzy event \( \xi \geq r \) can be represented, respectively, by

\[
\begin{align*}
\text{Pos} \{ \xi \geq r \} &= \sup_{u \geq r} \mu(u) \\
\text{Nec} \{ \xi \geq r \} &= 1 - \sup_{u < r} \mu(u) \\
\text{Cr} \{ \xi \geq r \} &= \frac{1}{2} \left[ \text{Pos} \{ \xi \geq r \} + \text{Nec} \{ \xi \geq r \} \right]
\end{align*}
\] (11)

Definition 2: The expected value of a fuzzy variable is defined as

\[
E[\xi] = \int_{-\infty}^{\infty} \text{Cr} \{ \xi \geq r \} dr - \int_{-\infty}^{0} \text{Cr} \{ \xi \leq r \} dr
\] (14)

Definition 3: The optimistic function of \( \alpha \) is defined as

\[
\xi_{\sup}(\alpha) = \sup \left\{ r \mid \text{Cr} \{ \xi \geq r \} \geq \alpha \right\}, \quad \alpha \in (0, 1]
\] (15)

Definition 4: If \( \xi = (a, b, c) \) is a triangular fuzzy number with center \( b \), left width \( a > 0 \), and right width \( c > 0 \), then its membership function has the following form

\[
\mu(r) = \begin{cases} 
\frac{r - (b - a)}{a} & ; \quad b - a \leq r \leq b \\
\frac{(b + c) - r}{c} & ; \quad b \leq r \leq b + c \\
0 & ; \quad \text{elsewhere}
\end{cases}
\] (16)

Definition 5: For the fuzzy variable described in definition 4, the credibility of the event \( \text{Cr} \{ \xi \leq r \} \) is defined based on the definition in (13) as

\[
\mu(r) = \begin{cases} 
0 & ; \quad r \leq b - a \\
\frac{r - (b - a)}{2a} & ; \quad b - a \leq r \leq b \\
\frac{r - (b - c)}{c} & ; \quad b \leq r \leq b + c \\
1 & ; \quad \text{elsewhere}
\end{cases}
\] (17)
In this research, the triangular fuzzy variable is used to model the fuzzy demand.

### 3.2. Fuzzy Simulation

A fuzzy simulation technique is employed to estimate the fuzzy demands. Denoting \( \tilde{D}_i \) by \( \tilde{D}_i \), we randomly generate \( D^k_i \) from the \( \alpha \)-level sets of fuzzy variables \( \tilde{D}_i \), \( i = 1, 2, \ldots, n \) and \( k = 1, 2, \ldots, K \) as \( D^k = (D^k_1, D^k_2, \ldots, D^k_n) \) and \( \mu(D^k) = \mu_1(D^k_1) \land \mu_2(D^k_2) \land \cdots \land \mu_n(D^k_n) \), where \( \alpha \) is a sufficiently small positive number. Then, the expected value of the fuzzy variable is

\[
E[Z(R, \tilde{D})] = \int_0^\infty Cr\{Z(R, \tilde{D}) \geq r\} dr - \int_{-\infty}^0 Cr\{Z(R, \tilde{D}) \leq r\} dr
\] (18)

Provided that \( O \) is sufficiently large, for any number \( r \geq 0 \), \( Cr\{Z(R, D_k) \geq r\} \) can be estimated by

\[
Cr\{Z(R, D_k) \geq r\} = \frac{1}{2} \left( \max_{k=1,2,\ldots,O} \{\mu_k[Z(R, D_k) \geq r]\} + 1 - \max_{k=1,2,\ldots,O} \{\mu_k[Z(R, D_k) < r]\} \right)
\] (19)

And for any number \( r < 0 \), \( Cr\{Z(R, D_k) \leq r\} \) can be estimated by

\[
Cr\{Z(R, D_k) \leq r\} = \frac{1}{2} \left( \max_{k=1,2,\ldots,O} \{\mu_k[Z(R, D_k) \leq r]\} + 1 - \max_{k=1,2,\ldots,O} \{\mu_k[Z(R, D_k) > r]\} \right)
\] (20)

The procedure of estimating \( Z(R, \tilde{D}) \) in (19) and (20) is shown in the following algorithm.

1. Set \( E = 0 \) and initialize \( K \) and \( O \).
2. Randomly generate \( D^k_i \) from \( \alpha \)-level sets of fuzzy variables \( \tilde{D}_i \), and set \( D^k = (D^k_1, D^k_2, \ldots, D^k_n) \).
3. Set \( a = Z(R, D_1) \land Z(R, D_2) \land \cdots \land Z(R, D_O) \),
   \( b = Z(R, D_1) \lor Z(R, D_2) \lor \cdots \lor Z(R, D_O) \).
4. Randomly generate \( r \) from Uniform \([a,b]\).

5. If \( r \geq 0 \), then \( E \leftarrow E + C r \{ Z(R, \tilde{D}) \geq r \} \). Otherwise, \( E \leftarrow E - C r \{ Z(R, \tilde{D}) \leq r \} \).

6. Repeat 4 and 5 for \( O \) times.

7. Calculate \( E(Z(R, \tilde{D})) = a \lor 0 + b \land 0 + E \times \frac{b-a}{O} \).

Algorithm 1: Estimating \( Z(R, \tilde{D}) \)

### 3.3. Genetic Algorithm

In the usual form of genetic algorithm (GA), described by Goldberg [18], the best solution is the winner of the genetic game and any potential solution is assumed a creature determined by different parameters. Several authors have employed GA to solve complicated inventory control problems. A selection of these works is demonstrated in Table (3).

**Insert Table (3) about here**

In what follows the main characteristics of the genetic algorithm employed in this research are described.

#### 3.3.1. Chromosomes

A chromosome, an important part of GA, is a string or trail of genes that is considered the coded figure of a possible solution (proper or improper). In this paper, the chromosomes are strings of the maximum inventory levels of the products \( R_j \) that are integers (Taleizadeh et al. [39, 41, 42]). Therefore, integer numbers are randomly generated in the closed interval \([0,1000]\) to represent the genes. Moreover, infeasible chromosomes, the ones that do not satisfy the constraints of the models in (9) and (10),
are not considered. For an 8-product system, the chromosome structure is given in Figure (4).

**Insert Figure (4) about here**

### 3.3.2. Population

Each population or generation of chromosomes has the same size that is known as the population size denoted by $N$. Similar to Taleizadeh et al. [39, 41, 42], 50, 100, and 150 are chosen as different population sizes of the GA algorithm of the current research.

### 3.3.3. Crossover

In a crossover operation, mating pairs of chromosomes creates offspring. Crossover operates on the parents' chromosomes with the probability of $P_c$. If no crossover occurs, the offspring's chromosomes will be the same as their parents' (Ting and Liao [45]). Figure (5) depicts a single-point crossover operation in which $R_j$ shows the chromosome containing the maximum inventory levels of the products and the break point is chosen at $M=6$ (Taleizadeh et al. [39, 41, 42]).

In this research, a single point crossover with different probabilities ($P_c$) of 0.6, 0.7, and 0.8 is utilized. Note that infeasible chromosomes do not move to the new population.

**Insert Figure (5) about here**

### 3.3.4. Mutation

Mutation is the second operation in GA to explore new solutions by operating on each chromosome resulted from the crossover operation, where genes are replaced with
randomly selected numbers within the boundaries of the parameter [16]. To do this, a random number $RN$ between (0,1) is generated for each gene. If $RN$ is less than a predetermined mutation probability $P_m$, then the mutation occurs in the gene. Otherwise, it does not. The usual value of $P_m$ is 0.1 over the numbers of genes in a chromosome. In this research, 0.010, 0.015, and 0.020 are chosen as different values of $P_m$. Note that infeasible chromosomes resulted by this operation do not move to the new population (Taleizadeh et al. [39, 41, 42]). Figure (6) depicts a mutation operation in which $P_m$ is chosen to be 0.01 (Taleizadeh et al. [39, 41, 42]).

3.3.5. Objective Function Evaluation

In a maximization problem, the more adequate the solution, the greater the objective function (fitness value) will be. Therefore, the fittest chromosomes will take part in offspring generation with a larger probability. The fuzzy simulation of Section 3.2 is used to evaluate the objective function of this research (Taleizadeh et al. [39, 41, 42]).

3.3.6. Selection

Selection plays a central role in GAs by determining how individuals compete for survival. Selection weeds out the bad solutions and keeps the good ones. This can be performed by proportional fitness selection that assigns a selection probability in proportion to the fitness of the given individual. The tournament selection is the most commonly used method, in which a number of randomly picked individuals are compared to each other (Das et al. [9]). The fittest individual is then selected to be a part of the next generation. The tournament size determines how many individuals are to be compared per selected population. Because of the randomness of the selection method,
most techniques, including traditional recombination and mutation operators, cannot guarantee the survival of the current best solution. In this research, Elitism is used to provide guarantee by explicitly selecting the best individual or group of individuals. The implementation of these two techniques leads to duplicates of good individuals (Das et al. [9]).

In this paper, we move the five best solutions to the next generation as elites. After each generation, solutions are checked for feasibility in terms of satisfying the constraint. If the constraint is satisfied, the corresponding chromosome will immigrate to the next population, otherwise the solution will be removed, and the generation will continue until a sufficient number of chromosomes are produced.

3.3.7. Stopping Criterion

The last step in a GA is to check whether the algorithm has found a solution that is good enough to meet the user’s expectations. Stopping criterion is a set of conditions such that when satisfied, hints at a good solution.

In this research, since the population sizes of 50, 100, and 150 are used, it is better to stop the algorithm until a maximum number of 500 evaluations ($MN = 500$) are performed (Taleizadeh et al. [39, 41, 42]).

In short, the steps involved in the GA algorithm used in this research are

1. Set the parameters $P_c$, $P_m$, and $N$

2. Initialize the population randomly (the individuals should satisfy the constraints)

3. Evaluate the objective function for all chromosomes based on Flowchart (1)

4. Select an individual for mating pool by tournament selection method using elitisms

5. Apply crossover to each pair of chromosomes with probability $P_c$
6. Apply mutation to each chromosome with probability $P_m$

7. Replace the current population by the resulting new feasible population (before replacing the old population, the feasibility of the newly generated chromosomes is checked and reproduction will continue until a sufficient number of required chromosomes is obtained)

8. Evaluate the objective function

9. If the stopping criterion is met, stop. Otherwise, go to step 4.

In order to demonstrate and evaluate the performance of the proposed hybrid intelligent algorithm, in the next section we present three numerical examples that were originally used in Ertogal and Rahim [12]. In these examples, two cases of the uniform and the exponential distributions for the time between two replenishments are investigated. To validate the results obtained, an existing hybrid method of FS and SA [44] is employed as well.

4. Numerical Examples

Consider two multiproduct inventory control problems with different numbers of products. The first one has eight products, where its general data is given in Table (4). Tables (5) and (6) show the parameters of the uniform and the exponential distributions used for the times between two replenishments, respectively. The total available warehouse space is 4800, and Table (7) shows different values of the parameters of the GA method. In this research all different combinations of the parameters of GA ($P_c, P_m$ and $N$) given in Table (7) are employed and using the $Max (Max)$ criterion the best combination of the parameters has been selected. Moreover, $K = 15$ and $O = 100$ are
considered in the fuzzy simulation procedure. All runs are performed using MATLAB on a pentium4 computer with 2.2 GHZ coreduo2 processor.

In order to show the effectiveness of the proposed hybrid method of FS and GA in solving the complicated inventory problem of this research, Taleizadeh et al.'s [44] hybrid method of FS+SA is also employed to solve the numerical examples.

Tables (8) and (9) show the best results of the two approaches. The best combinations of the GA algorithms are shown in Table (10). Furthermore, the convergence paths of the objective-function values of the FS+GA and FS+SA algorithms for uniform and exponential distributions are shown in Figures (7) to (10).

The results in Tables (8) and (9) show that the hybrid FS+GA method provides a better near-optimal solution in terms of the objective-function value. Moreover, from Figures (7) to (10), one can observe that more generations and iterations are required to reach the best result in the case of uniform compared to exponential distribution.

In the first numerical example, to compare the performances of the two hybrid methods, while the number of runs in each example is set at 25 for both methods; the sample means of the CPU times in reaching the best solution in exponential distribution case are 9.35 and 9.59 seconds for FS+GA and FS+SA methods, respectively. The corresponding sample variances are 0.25 and 0.27. In the uniform case however, the sample means are 10.59 and 11.31 with the sample variances of 0.62 and 0.65, respectively. This shows that the proposed hybrid method has better performance in terms of the CPU time to reach the best result in both distributional cases.

Similar results are obtained for the next two numerical examples containing 20 and 40 products. The summarized CPU sample means in Table (11) show that, as expected, as the number of products increases, the required CPU time to reach the best solution increases as well. Further, the proposed hybrid FS+GA provides better results in
terms of the objective-function value and CPU time for both uniform and exponential cases of the two different problem sizes.

5. A Sensitivity Analysis

To study the effects of parameter changes on the best result obtained by the proposed method and the required CPU time, a sensitivity analysis is performed to investigate the effect of increase or decrease of the parameters, one at a time, by 20% and 40%. The parameters of the proposed method are the fuzzy demand ($D_i$), the parameters of the distribution of the period length ($T_{max}$ and $\lambda_i$), the crossover probability ($P_c$), the
mutation probability \( (P_m) \), the parameters of the fuzzy simulation algorithm \( (K \text{ and } O) \), the number of products \( (NP) \), and the maximum number of the population sizes \( (MN) \). Table (12) shows the results of the sensitivity analysis on the sample mean of the 25 best results obtained for the uniform and exponential distribution cases. The results in Table (12) show that there is a direct relationship between the objective-function value and the changes in \( D_i, T_{Max}, \text{ and } \lambda_i \), that is, increase or decrease of these parameters cause the objective function value to increase or decrease, respectively.

The numbers in Table (13) are the sample means of the 25 required CPU times to solve the problem. The relative percentages of increase or decrease in average CPU time compared to the ones required to achieve the results of Table (5) are also given. The results in Table (13) show that in all situations the average CPU time to solve the problem in a uniform case is larger than that of the exponential distribution. Furthermore, the fuzzy simulation parameters, \( K \text{ and } O \), do not have much impact on the CPU times. However, in both distributions, the CPU times are very sensitive to the changes in the number of decision variables. Finally, the parameters of the GA have relatively mild impact on the required CPU time.

Insert Table (12) about here

Insert Table (13) about here

6. Conclusion and Recommendation for Future Research

In this paper, a stochastic replenishment multiproduct inventory model was developed. Two integer-nonlinear programming models for two cases of uniform and exponential distribution of the time between two replenishments have been proposed. A
A hybrid method of FS and GA was developed to solve the problem and the results were validated by both a sensitivity analysis and a comparison with an existing hybrid method of FS and SA. The comparison results showed that at least for the selected numerical examples the proposed hybrid method of FS and GA had better performance in terms of objective-function values and required CPU time to obtain the best solution.

The models developed in this research can help the practitioners who are faced with uncertain demands that do not follow a probability distribution. Moreover, the models are helpful in situations in which due to some limitations on the production capacity, the supply of the raw material, and the like, the period length may be uncertain and the suppliers may not be able to deliver the goods on time.

Some avenues for future works follow.

1. The demand or other parameters of the problem may take uncertain forms (stochastic or rough) as well.
2. Some other probability density functions rather than uniform and exponential may be considered for the time between replenishments.
3. Some other meta-heuristic algorithms such as harmony search or particle swarm may be employed to solve the problem.
4. Fuzzy discount factor or fuzzy discrete delivery orders may be considered as well.
5. Differential Evolution can be considered as an effective technique to solve the problem.

7. Acknowledgements

The authors are thankful for constructive comments of reviewers that significantly improved the presentation of the article.
8. References


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Table (1): Literature review in stochastic environment

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<th>Author</th>
<th>Periodic Review</th>
<th>Continuous Review</th>
<th>Multi Products</th>
<th>Constraint</th>
<th>Discount</th>
<th>Fuzzy Environment</th>
<th>Stochastic Environment</th>
<th>Partial Back-ordering</th>
<th>Lost sale or Back-order</th>
<th>Solution Method</th>
<th>Decision Variable</th>
<th>Other Considerations</th>
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<td>Chiang [6]</td>
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<td>*</td>
<td>Demand</td>
<td>B</td>
<td>Dynamic Programming</td>
<td>Inventory level</td>
<td>Emergency Order</td>
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<td></td>
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<td>(r, Q)</td>
<td></td>
<td></td>
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<td>Heuristic</td>
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<td>Multiple replenishment</td>
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</tr>
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<td>(R, nT)</td>
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<td></td>
<td></td>
<td></td>
<td>Demand</td>
<td>B</td>
<td>Heuristic</td>
<td>Order Quantity, Reorder Point, Inventory level</td>
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<td>Non-zero lead time</td>
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<td></td>
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<td>B &amp; L</td>
<td>Dynamic Programming</td>
<td>Inventory Level &amp; Order Quantity</td>
<td>Two Models with back ordering and Lost Sale</td>
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<td></td>
<td></td>
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<tr>
<td>Eynan &amp; Kropp [13]</td>
<td>(EOQ)</td>
<td></td>
<td></td>
<td>*</td>
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<td>Demand</td>
<td>B</td>
<td>Heuristic</td>
<td>Period Length</td>
<td>Variable Shortage Cost</td>
<td></td>
<td></td>
</tr>
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<td>Mohebbi [29]</td>
<td>(r, Q)</td>
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<td></td>
<td></td>
<td></td>
<td>Demand</td>
<td>L</td>
<td>Software (MATLAB)</td>
<td>Reorder Point and Order Quantity</td>
<td>Discrete Demand &amp; Unreliable Supplier</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Qu et al. [33]</td>
<td>(R, T)</td>
<td></td>
<td></td>
<td>*</td>
<td>Joint Order</td>
<td></td>
<td>B</td>
<td>Heuristic</td>
<td>Rout, Order Quantity and Period length</td>
<td>Integrated Inventory-transportation System</td>
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<td>Taleizadeh et al. [39]</td>
<td>(R, T)</td>
<td></td>
<td></td>
<td>*</td>
<td>Service Level &amp; Space</td>
<td>Period Length</td>
<td>*</td>
<td>Simulated Annealing</td>
<td>Inventory level</td>
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30
Table (2): Literature review in fuzzy environment

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<th>Continuous Review</th>
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<th>Constraint</th>
<th>Fuzzy Environment</th>
<th>Lost sale or Back-order</th>
<th>Solution Method</th>
<th>Decision Variable</th>
<th>Other Considerations</th>
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<td>Demand</td>
<td>B &amp; L</td>
<td>Difuzzification &amp; Heuristic</td>
<td>Lead time &amp; Order Quantity</td>
<td>Variable lead-time</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Das et al. [8]</td>
<td>(EOQ)</td>
<td>* Budget &amp; Space</td>
<td>Demand</td>
<td>B</td>
<td>Difuzzification &amp; Heuristic</td>
<td>Order &amp; Shortage Quantities</td>
<td>Time varying demand and production rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hsieh [19]</td>
<td>(EPQ)</td>
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<td>All of the parameters are fuzzy.</td>
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<td>Batch Order</td>
<td>Demand, Inventory Costs</td>
<td>Possibility Theory &amp; Geometric Programming</td>
<td>Order Quantity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maiti &amp; Maiti [26]</td>
<td>(r, Q)</td>
<td>* Demand &amp; Price</td>
<td>Fuzzy Simulation &amp; Genetic Algorithm</td>
<td>Reorder Point, Order Quantity, Selling Price, Frequency of Advertisements</td>
<td>Two storages, advertisement, single and multi objective</td>
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<td></td>
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<tr>
<td>Mandal &amp; Roy [28]</td>
<td>(EOQ)</td>
<td>* Space</td>
<td>Constraint Goal, Inventory Costs</td>
<td>Fuzzy Geometric Programming</td>
<td>Order Quantity</td>
<td>Multi objective</td>
<td></td>
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<tr>
<td>Roy et al. [34]</td>
<td>(r, Q)</td>
<td>Space</td>
<td>Deterioration Rate</td>
<td>Fuzzy Simulation &amp; Genetic Algorithm</td>
<td>Order Quantity</td>
<td>Stochastic period length, two storage facilities, time varying demand</td>
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<td></td>
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<tr>
<td>Taleizadeh et al. [41]</td>
<td>(R,T)</td>
<td>* Budget, Space &amp; Service Level</td>
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<td>Fuzzy Simulation &amp; Genetic Algorithm</td>
<td>Inventory level</td>
<td>Partial back ordering &amp; incremental discount, stochastic period length</td>
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<td>Yao et al. [46]</td>
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<td>Heuristic</td>
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<td></td>
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<td>Chen et al. [5]</td>
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<td>Demand, Inventory Costs</td>
<td>B</td>
<td>Function Principle</td>
<td>Order &amp; Shortage Quantities</td>
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<tr>
<td>Author</td>
<td>Area</td>
<td>Variables</td>
<td>Gene Represents</td>
<td>Initialization</td>
<td>Mutation</td>
<td>Crossover</td>
<td>Stopping Criteria</td>
<td>Hybrid By</td>
<td>Other Considerations</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-------------------------------</td>
<td>-----------------------------------------------</td>
<td>-----------------</td>
<td>----------------</td>
<td>-----------------------------------------------</td>
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<td>------------------------</td>
<td>------------------------</td>
<td>----------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Roy et al. [34]</td>
<td>Integrated Production Inventory system</td>
<td>Cycle Length, Maximum Inventory level</td>
<td>Product Type</td>
<td>Random Generation</td>
<td>Randomly by using Mutation Probability</td>
<td>Single Point</td>
<td>Maximum Iteration Number</td>
<td>Fuzzy Logic</td>
<td>Fuzzy genetic algorithm is proposed.</td>
</tr>
<tr>
<td>Suer et al. [38]</td>
<td>Capacitated Lot Size Problem</td>
<td>Production Quantity, Human Resource Requirement, etc.</td>
<td>Product Type</td>
<td>Random Generation</td>
<td>Randomly by using Mutation Probability</td>
<td>Multiple Chromosome</td>
<td>Maximum Number of Generation</td>
<td>----------</td>
<td>Multiple chromosome crossover is proposed.</td>
</tr>
<tr>
<td>Shahabudeen &amp; Sivakumar [36]</td>
<td>Kanban System</td>
<td>Number of Kanban and Extra cards</td>
<td>Random Generation</td>
<td>Random Generation</td>
<td>Order Based Shift Mutation</td>
<td>Single Point</td>
<td>Maximum Number of Generation</td>
<td>----------</td>
<td>GA compared by simulated annealing and performed better.</td>
</tr>
<tr>
<td>Roy et al. [35]</td>
<td>Inventory Model with Deterioration Items</td>
<td>Order Quantity, Cycle Length</td>
<td>Product Type</td>
<td>Random Generation</td>
<td>Randomly by using Mutation Probability</td>
<td>Single Point</td>
<td>Maximum Number of Generation</td>
<td>Fuzzy Simulation</td>
<td>Necessity and possibility theories are considered.</td>
</tr>
<tr>
<td>Nichiapan &amp; Jawahar [31]</td>
<td>Vendor Managed Inventory</td>
<td>Sales Quantity of each Buyer Sales Quantity of Buyers</td>
<td>Random Generation</td>
<td>Random Generation</td>
<td>Randomly by using Mutation Probability</td>
<td>Single Point</td>
<td>Maximum Number of Generation</td>
<td>----------</td>
<td>GA based heuristic is proposed.</td>
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<tr>
<td>Taleizadeh et al. [42]</td>
<td>Inventory Model with Random Period Length</td>
<td>Maximum Inventory Level</td>
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<td>Randomly by using Mutation Probability</td>
<td>Two Point Crossover</td>
<td>Maximum Number of Generation</td>
<td>Pareto and TOPSIS Selections</td>
<td>TOPSIS is used to rank the Pareto.</td>
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<td>Newsboy Inventory System</td>
<td>Order Quantity</td>
<td>Product Type</td>
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<td>Randomly by using Mutation Probability</td>
<td>Two Point Crossover</td>
<td>Maximum Number of Generation</td>
<td>Goal Programming</td>
<td>GA solved a multi objectives problem</td>
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### Table (4): General data

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<th>Product</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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</thead>
<tbody>
<tr>
<td>( h_i )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \pi_i )</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<tr>
<td>( P_i )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
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<tr>
<td>( \beta_i )</td>
<td>0.5</td>
<td>0.9</td>
<td>0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>0.9</td>
<td>0.9</td>
<td>0.5</td>
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<td>( f_i )</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>6</td>
<td>6</td>
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<tr>
<td>( W_i )</td>
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<td>70</td>
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<tr>
<td>( D_i )</td>
<td>(7,10,13)</td>
<td>(7,10,13)</td>
<td>(7,10,13)</td>
<td>(18,20,22)</td>
<td>(18,20,22)</td>
<td>(18,20,22)</td>
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Table (5): Data for uniform distribution

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<th>3</th>
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Table (6): Data for exponential distribution

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<td>1/30</td>
<td>1/60</td>
<td>1/60</td>
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<td>1/30</td>
<td>1/60</td>
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</table>
Table (7): The parameters of the GA method

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<th>$P_m$</th>
<th>$N$</th>
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<td>0.7</td>
<td>0.015</td>
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<tr>
<td>0.9</td>
<td>0.020</td>
<td>150</td>
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Table (8): The best result for $R_i$ by FS+GA algorithm

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<th>Product</th>
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### Table (9): The best result for $R_i$ by FS+SA algorithm

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<tr>
<th>Distribution</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>$Z(R, \tilde{D})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>188</td>
<td>3</td>
<td>41</td>
<td>109</td>
<td>197</td>
<td>51</td>
<td>93</td>
<td>268</td>
<td>36366</td>
</tr>
<tr>
<td>Exponential</td>
<td>54</td>
<td>10</td>
<td>129</td>
<td>10</td>
<td>245</td>
<td>18</td>
<td>75</td>
<td>357</td>
<td>151550</td>
</tr>
</tbody>
</table>
Table (10): The best combination of the GA parameters

<table>
<thead>
<tr>
<th>Numerical Example with</th>
<th>$P_c$</th>
<th>$P_m$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Distribution</td>
<td>0.6</td>
<td>0.01</td>
<td>100</td>
</tr>
<tr>
<td>Exponential Distribution</td>
<td>0.6</td>
<td>0.015</td>
<td>100</td>
</tr>
</tbody>
</table>
Table (11): The summarized results of the second and the third numerical examples

<table>
<thead>
<tr>
<th>Example</th>
<th>Uniform</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FS+GA</td>
<td>FS+SA</td>
</tr>
<tr>
<td></td>
<td>Objective Function Value</td>
<td>CPU time (s)</td>
</tr>
<tr>
<td>Second Example</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(twenty products)</td>
<td>106,450</td>
<td>31.24</td>
</tr>
<tr>
<td>Third Example</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(forty products)</td>
<td>218,790</td>
<td>64.38</td>
</tr>
</tbody>
</table>
Table (12): The effects of the parameter changes on the objective-function value

<table>
<thead>
<tr>
<th>% Changes in Parameters</th>
<th>$D_i$</th>
<th>$T_{Max}$</th>
<th>$\lambda_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Changes in</td>
<td>Uniform Distribution</td>
<td>Exponential Distribution</td>
<td>Uniform Distribution</td>
</tr>
<tr>
<td></td>
<td>Objective Function</td>
<td>Objective Function</td>
<td></td>
</tr>
<tr>
<td>+40</td>
<td>+34.38</td>
<td>+29.49</td>
<td></td>
</tr>
<tr>
<td>+20</td>
<td>+19.45</td>
<td>+16.82</td>
<td></td>
</tr>
<tr>
<td>-20</td>
<td>-17.46</td>
<td>-21.37</td>
<td></td>
</tr>
<tr>
<td>-40</td>
<td>-37.39</td>
<td>-34.79</td>
<td></td>
</tr>
<tr>
<td>+40</td>
<td>+21.45</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>+20</td>
<td>+13.67</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>-20</td>
<td>-11.4</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>-40</td>
<td>-19.49</td>
<td>----</td>
<td></td>
</tr>
<tr>
<td>+40</td>
<td>----</td>
<td>+17.45</td>
<td></td>
</tr>
<tr>
<td>+20</td>
<td>----</td>
<td>+9.56</td>
<td></td>
</tr>
<tr>
<td>-20</td>
<td>----</td>
<td>-8.57</td>
<td></td>
</tr>
<tr>
<td>-40</td>
<td>----</td>
<td>-16.49</td>
<td></td>
</tr>
</tbody>
</table>
Table (13): The results of the sensitivity analysis on CPU time in FS+GA algorithm

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Uniform</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Difference</td>
</tr>
<tr>
<td>$NP$</td>
<td>20</td>
<td>+4%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>+9%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>+18%</td>
</tr>
<tr>
<td>$O$</td>
<td>10</td>
<td>-7%</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>-2%</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>+3%</td>
</tr>
<tr>
<td>$K$</td>
<td>5</td>
<td>-2%</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-0.5%</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>+1%</td>
</tr>
<tr>
<td>$P_c$</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>+1%</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>+6%</td>
</tr>
<tr>
<td>$P_m$</td>
<td>0.010</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.015</td>
<td>+3%</td>
</tr>
<tr>
<td></td>
<td>0.020</td>
<td>+6%</td>
</tr>
<tr>
<td>$MN$</td>
<td>750</td>
<td>+9%</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>+12%</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>+17%</td>
</tr>
</tbody>
</table>
Figure (1): Presenting the inventory cycle when $T_{\text{min}} \leq T_i \leq t_{D_i}$.
Figure (2): Presenting the inventory cycle when $t_{D_i} < T_i \leq T_{Max}$.
Figure (3): Presenting shortages with back-orders and lost-sales
Table: The structure of a chromosome

| $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $R_6$ | $R_7$ | $R_8$ |

Figure (4): The structure of a chromosome
Figure (5): The single-point crossover operation
Figure (6): A sample of mutation operation
Figure (7): The convergence path of the best result by FS+GA in the uniform example
Figure (8): The convergence path of the best result by FS+GA in the exponential example
Figure (9): The convergence path of the best result by FS+SA in the uniform example
Figure (10): The convergence path of the best result by FS+SA in the exponential example