A general equivalent circuit model is developed for a wireless energy transfer system composed of multiple coils via coupled magnetic resonances. To verify the developed model, four types of wireless energy transfer systems are fabricated, measured, and compared with simulation results. To model a system composed of $n$-coils, node equations are built in the form of an $n$-by-$n$ matrix, and the equivalent circuit model is established using an electric design automation tool. Using the model, we can simulate systems with multiple coils, power sources, and loads. Moreover, coupling constants are extracted as a function of the distance between two coils, and we can predict the characteristics of a system having coils at an arbitrary location. We fabricate four types of systems with relay coils, two operating frequencies, two power sources, and loads. The function of characteristic impedance conversion is measured. The characteristics of all systems are measured and compared with the simulation results. The flexibility of the developed model enables us to design and optimize a complicated system consisting of many coils.

Keywords: Energy transfer, wireless, magnetic resonance, model, multi-coils.

I. Introduction

In recent years, there has been an increasing interest in wireless power transfer technology. In particular, significant progress has been charted for inductively coupled systems [1]-[11]. Inductively coupled power transfer systems have been developed for a wide range of applications, including vehicle battery charging systems, and a very high end-to-end system efficiency of up to 80% has been documented [4], [8]. However, most studies have been restricted to close range, that is, typically shorter than 30% of the coil diameter. The transmission distance is generally close to 1 cm [4], and 15 cm is considered a fairly large distance [8]. Results at the mid-range (that is, more than twice the coil diameter) have not been reported.

Recently, MIT proposed a new scheme based on strongly coupled magnetic resonances, thus presenting a potential breakthrough for a mid-range wireless energy transfer [12], [13]. The fundamental principle is that resonant objects exchange energy efficiently, while non-resonant objects do not. Figure 1 shows the basic coil system composed of four coils: drive, transmit resonance, receive resonance, and load coils. The transmit resonance coil is coupled to the drive coil which is linked to a power amplifier that supplies energy to the system. The receive resonance coil is coupled with the load coil to provide the power to an external load. The scheme is carried with a power transfer of 60 W and has RF-to-RF coupling efficiency of 40% for a distance of 2 m, which is more than three times the coil diameter. We expect that coupled magnetic resonances will make possible the commercialization of a mid-range wireless power transfer.

Magnetic resonance coupling is a new concept in wireless energy transmission. The analyses used in early research were...
based on pure physical theory and failed to provide tangible findings for electrical engineers [12]-[18]. The reports [8]-[23] do not clearly present practical design methods. Reports [24]-[26] accurately present the characteristics of a coupled magnetic resonance system, and these reports can be practically applied to the design. Through the circuit analysis of an equivalent model, the system transfer function can be obtained as a function of frequency, and several key parameters of the system can be analyzed [24]. Also, the coupled mode theory [12] can be extended for an analysis of other systems [25]. The proposed methods are easily applied to a system composed of several coils and having negligible weak coupling. However, if the system is composed of many coils and weak coupling affects the system characteristics, it is very difficult to apply the methods for analysis.

RF engineering is also a candidate method for the analysis of a coupled magnetic resonance system. In [26], the equivalent circuit model with a compensation source is developed to take into account both weak coupling and loss. The model has been established in an RF simulator and the parameters for the model are extracted through a measured S-parameter. Simulation results show strong agreement with the measurement results [26].

Previous studies have analyzed the system consisting of four coils. In this paper, the node equation in the form of a matrix is written for a system composed of n-coils by expanding the node equation of [26]. The general model of the coupled magnetic resonance system is established using an electric design automation tool, and the model enables us to design a system composed of several coils, power sources, and loads.

$$\begin{align*}
\frac{j \omega L_1}{i_{L_1}} + \frac{j \omega M_{12}}{i_{L_2}} + \frac{j \omega M_{13}}{i_{L_3}} + \cdots + \frac{j \omega M_{1n}}{i_{L_n}} &= \begin{bmatrix} 1 / j \omega C_1 & 0 & 0 & 0 \\ 0 & 1 / j \omega C_2 & 0 & \cdots & 0 \\ 0 & 0 & 1 / j \omega C_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 / j \omega C_n \end{bmatrix} \times \begin{bmatrix} i_{C_1} \\ i_{C_2} \\ i_{C_3} \\ \vdots \\ i_{C_n} \end{bmatrix} 
\end{align*}$$

To verify the model, we fabricate four types of systems that have relay coils, two operating frequencies, two power sources, and the function of characteristic impedance conversion. We measure the characteristics of all systems and compare them with the simulation results.

$$\begin{align*}
\begin{bmatrix} i_{L_1} \\ i_{L_2} \\ \vdots \\ i_{L_n} \end{bmatrix} + \begin{bmatrix} i_{C_1} \\ i_{C_2} \\ \vdots \\ i_{C_n} \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.
\end{align*}$$

II. Modeling for System Composed of n-coils

1. Node Equation and Equivalent Circuit Model

In general, the wireless power transfer system has three types of coils: a coil with a power source, a floating coil, and a coil with a load. The resonance frequency of floating coils is an operating frequency. Figure 2 shows the proposed equivalent circuits of three such types of coils. The equivalent circuit of a system composed of n-coils has a number of resonant circuits, n, and each resonant circuit has a number of compensation sources of n−1. The compensation source represents the effect of mutual inductance related to coupling with other coils [26]. Here, $L_{ij}$ is a self-inductance, and $C_{ij}$ is a capacitance of the coil.

From the equivalent circuit of each coil, we can obtain (1) and (2) in the form of a matrix by writing the current in each branch at the node whose voltage is $V_{ijk}$. This matrix can be
established in Advanced Design System (ADS), which is design software widely used for RF and microwave applications [27].

2. Parameter Extraction for System Composed of $n$-Coils

To model a system composed of $n$-coils, we have to extract the following parameters. Each coil has the parameters of capacitance, self-inductance, and $R_{\text{Loss}}$. Also, to model the compensation sources, we need coupling constants for the possible combinations of coils. As a result, there is a total of $3n + n(n-1)/2$ parameters. For example, a system composed of 4 coils has 18 parameters, and a system composed of 6 coils has 33 parameters.

To simulate the system characteristics as the distance between coils, we need a function whose output is a coupling constant and whose input is the distance between coils. Figure 3 shows an extracted graph with such a function. We have measured the coupling constant according to the distance and extracted coefficients of (3) by fitting with the measurement results.

$$\kappa = A(x + C)^B,$$  \hspace{1cm} (3)

where $x$ is the distance between the coils and $\kappa$ is the coupling constant. This procedure is performed for the possible combinations of coils. Table 1 shows the extracted coefficients. Each coil used in Table 1 has a diameter of 15 cm, and the diameter of the conductor section is 1 mm. Coil-1 has twelve turns and a pitch of 3 mm and coil-A has one turn.

We can predict the characteristics of a system in which the coils have arbitrary locations by establishing (3) in the simulator. The completed model requires just the location of each coil for simulation.

III. Model Validation and Experiment Results

Using the developed model, we predict the characteristics of various systems, measure their characteristics, and compare them with the simulation results. Each system has relay coils, two operating frequencies, two power sources, and the function of characteristic impedance conversion. The system configuration is very useful when applying it to a commercial product of wireless power transfer.

1. Relay System

Although the technology based on strongly coupled magnetic resonances is able to transmit energy over a much longer distance than the traditional method can, degradation in efficiency is unavoidable in a system in which the transmitter is away from the receiver. In this case, we can improve the efficiency by adding coils to create a relay device. Figure 4(b) shows such a relay system. A system without relay coils, shown in Fig. 4(a), shows an efficiency of –18 dB (1.6%).

To improve the efficiency of the system in Fig. 3(a), we can use the relay coils, which are the same as coil-1. The number of the required coils and their location can be determined through simulation using the developed model. Because of the fast simulation time, we can easily obtain the transfer characteristics according to the position of the relay coils, and
the optimum position for maximum efficiency can be determined. Figure 5(a) shows the improved transfer characteristics achieved through such a process. After adding two relay coils, the efficiency is improved to –4.2 dB (38%). Figure 5(b) shows the measurement and simulation results, simultaneously. The agreement between the model and experiment data is fairly good. This example shows that we can predict and optimize the characteristics of the system through the developed model without experimentation.

The proposed relay scheme overcomes the mid-range limitation of the present wireless electric system, allowing more flexible power transmission without sacrificing efficiency.

2. Two-Tone System

Some applications of wireless energy transfer require two or more receiving devices, and magnetic resonance coupling is expected to support multiple receivers at the same time. However, if the resonance frequencies of receiving devices are the same, interference caused by a coupling of receiving coils makes the implementation very difficult. When multiple receivers are in close proximity to each other, there is a coupling interaction between the receivers, which makes the resonant peak split into separate peaks [28].

We overcome this difficulty through the use of resonant coils with different resonance frequencies. The coupling interaction occurs only when coils have the same resonant frequency. Thus, if the resonance frequencies of coils are different, the resonant peak frequency does not split and it can be maintained.

Figure 6 shows the experimental setup for a system with two receiving devices. The two coils of the transmit device in the middle of the system have different resonance frequencies, and the two receiving coils on both sides have corresponding resonance frequencies. A coil with a 3 mm pitch has a resonance frequency of 9.8 MHz, and a coil with a 4 mm pitch has a resonance frequency of 11.3 MHz. The number of turns of each coil is 11. Figure 7 shows the measurement and simulation results of power transferred to two receiving devices at both ends. The peak frequency of gain to each receiver is shifted slightly from the resonance frequency of the resonant coil, from 9.8 MHz to 10.9 MHz at the left receiver and from 11.2 MHz to 13.2 MHz at the right receiver. This means that the coupling interaction is small but still exists despite the different resonant frequencies of 9.8 MHz and 11.2 MHz.

In the proposed system, wireless power transmission to two
receivers can be done at the same time by generating a two-tone signal at the transmitter. However, the transmitter of the system needs as many resonant coils as receivers and a multi-tone signal should be generated, which makes it difficult to implement.

Ultimately, it is possible to simulate such a system using the developed model. As shown in Fig. 7, the predictions of the model are in agreement with the measurement results.

3. System with Two Transmitting Devices

As an alternative to including relay coils in the system to improve transmission efficiency, we can add more transmit coils to the system to improve transmission efficiency. Figure 8 shows a photograph of such a system. To improve the efficiency of the normal system in Fig. 8(a), we can place a transmit coil at the other side, as shown in Fig. 8(b). After adding the coil, we achieve an efficiency improvement of 2.6 dB. Figure 9(a) shows the measurement results, and Fig. 9(b) shows a comparison with the simulation. It also shows that we can predict the system characteristics through simulation using
Fig. 10. Photograph of system for charging-zone experiment. Receiving device is placed in an arbitrary location.

Fig. 11. Efficiency vs. location of receiving coil. Red line shows corresponding frequency.

Fig. 12. Photograph of system for impedance conversion experiment.

Fig. 13. Measured S-parameter of system in which impedance of input port is 50 Ω and output is 50 Ω: (a) $S_{11}$ and (b) $S_{21}$.

4. Conversion of Characteristic Impedance

Depending on the structure of the power source or value of the load impedance, we may wish to design the characteristic impedance of the system with a value other than 50 Ω. In this case, an inductor and capacitor can be used for the matching function. However, there may be a drawback, such as an increase in cost and heat loss at the lumped elements. We can solve these problems using a feeding coil and load coil with different inductance values. Equation (4) is an analytic result ignoring the weak coupling and heat loss in a conventional system composed of four coils [26]. From (4), input impedance from the power source is changed in proportion to the
inductance ratio of the feeding and load coils.

\[
Y_m = j\omega_o C_4 + \frac{L_4}{L_4} \left( Y_0 + j\omega_0 C_1 + \frac{1}{j\omega_0 L_1} \right) + j\omega_0 \frac{L_4}{L_1} C_4
\]

\[
= \frac{L_4}{L_1} Y_0. \tag{4}
\]

Figure 12 shows a photograph of the system used for the impedance change experiment. The feeding coil has one turn and an inductance of 0.467 \( \mu \)H. The load coil has two turns and an inductance of 1.505 \( \mu \)H. The inductance of the load coil is three times that of the feeding coil. Therefore, we can obtain maximum efficiency if the load impedance is three times the characteristic impedance of the power source.

Figures 13 and 14 are the measurement results of the system, in which the port impedance is 50 \( \Omega \) to 50 \( \Omega \) and 50 \( \Omega \) to 150 \( \Omega \), respectively. The port impedance of 50 \( \Omega \) to 150 \( \Omega \) shows well-matched results without reflected power (\( S_{11} \)) and higher gain (\( S_{21} \)). Figure 15 shows the measurement and simulation results for the system with a port impedance of 50 \( \Omega \) to 150 \( \Omega \). We can verify the usefulness of the developed model once again from a well-predicted value of \( S_{21} \).

IV. Conclusion

It is clear that a field simulation is the most obvious method for the analysis of wireless power transfer via coupled magnetic resonances, such as HFSS or CST. However, it takes a long time to perform a field simulation, and it is not possible to perform such a simulation when there is a variety of different coil configurations and characteristics according to the distance between the coils. For example, it takes about 10 min to solve one case with the MIT system using HFSS. Moreover, it shows different results depending on boundary conditions.

In this paper, a novel circuit model for a wireless power transfer system via coupled magnetic resonances was proposed. We built general node equations of the system, composed of \( n \) coils, and established an equivalent circuit model in ADS [27]. The model has the data of a coupling constant according to the
distance between the coils, and we can simulate a system in which coils are placed at arbitrary points. As a result, we can perform a very fast simulation for the different coil configuration composed of many coils.

To verify the model, we made four types of systems, measured their S-parameters, and made a comparison with the simulation results. The configuration of a system is very significant when developing a mid-range wireless power transfer system where the magnetic resonance is expected to be the most advantageous.

References


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