KINEMATICS AND WORKSPACE ANALYSIS OF 3 DOF COMPLIANT MICRO PARALLEL ROBOTS

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ABSTRACT

The paper presents the kinematics, control and workspace analysis of the 3 DOF compliant micro parallel robots. The paper handles with a class of micro parallel robots where because of the small size the classical joints of kinematic elements are replaced with compliant mechanisms (flexure hinges). One can obtain compact kinematic robots with a high accuracy. By using this type of articulations totally eliminates backlash and friction. Firstly, the kinematics of the micro parallel robots is investigated. Secondly, the workspace of the micro parallel robots is obtained.

1. INTRODUCTION

The need of high accuracy robots is increasing. These robotic systems are used in wide applications, such as precision machine tools, optical devices, and semiconductor manufacturing machines.

Parallel micro robots offer rigid structures, and parallel micro robots with flexure hinges have no error due to backlash or friction and there is no need for lubrication. They are also easy to manufacture and have simple structure. The most important disadvantage of this technology is the low range of motion but also from the material strength of the flexure hinge. The flexure hinges (Fig. 1), replace a conventional revolute joint with the aim of producing a limited angular motion about one axis.

Regarding the control of a micro parallel robot based on compliant mechanisms, due to the fact that the rotation motion of the link may be achieved by deflecting part of the hinge instead of the joint, the analysis and control is more complicated. That's why, taking into account all the above remarks, it can be deduced that mechanisms with flexure hinges are suitable only for the high accuracy positioning robotic systems with small workspace.

Our motivation for the research is to analyze and to enlarge the workspace of the micro compliant parallel robots. Compliant mechanisms are made as thin profiled straight or curved sections, beams that create elastic joints or arms. The most common shapes of the flexure hinges can be of type: circular, elliptical or corner-filled (Fig. 1).

Figure 1. Some shapes of the flexure hinges used for micro parallel robots.

\textbf{a)}
2. KINEMATICS ANALYSIS OF MICRO-PARALLEL ROBOTS

2.1. Discretization method

The workspace is one of the most important kinematic properties of manipulators, even by practical viewpoint because of its impact on manipulator design and location in a workcell [17]. A general numerical evaluation of the workspace can be deduced by formulating a suitable binary representation of a cross-section in the taskspace.

A cross-section can be obtained with a suitable scan of the computed reachable positions and orientations p, once the forward kinematic problem has been solved to give p as function of the kinematic input joint variables q. A binary matrix $P_{il}$ can be defined in the cross-section plane for a cross-section of the workspace as follows: if the $(i,j)$ grid pixel includes a reachable point, then $P_{il} = 1$; otherwise $P_{il} = 0$, as shown in Fig. 1.

Equations (1)-(4) for determining the workspace of a micro parallel robot by discretization method can be found in Ref. [18].

$$ i = \left[ \frac{x + \Delta x}{x} \right] ; \quad j = \left[ \frac{y + \Delta y}{y} \right] \quad (1) $$

where $i$ and $j$ are computed as integer numbers. Therefore, the binary mapping for a workspace of a micro parallel robot cross-section can be given as:

$$ P_{il} = \begin{cases} 0 & \text{if } P_{il} \notin W(H) \\ 1 & \text{if } P_{il} \in W(H) \end{cases} \quad (2) $$

where $W(H)$ indicates workspace region; $\in$ stands for "belonging to" and $\notin$ is for "not belonging to".

In addition, the proposed binary representation is useful for a numerical evaluation of the position workspace of the micro parallel robot by computing the sections areas $A$ as:

$$ A = \sum_{i=1}^{\text{num}} \sum_{j=1}^{\text{num}} \left( P_{ij} \Delta x \Delta y \right) \quad (3) $$

The workspace optimization problem can be also subject to design constraints such as:

$$ \left| x_{max} - x_{max}' \right| \leq 0; \quad \left| y_{max} - y_{max}' \right| \leq 0 \quad (4) $$

where the left-hand values correspond to the computed area $A$ and prime values describe the prescribed extreme reaches.

2.2. Kinematics analysis of 2 DOF micro-parallel robot

The kinematics relation between $x$ and $q$ of this 2 DOF micro parallel robot can be expressed solving the:

$$ \beta(x, q) = 0 \quad (5) $$

Then the inverse kinematics problem of the micro parallel robot can be found in Ref. [14].

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2.3. Kinematics analysis of 3 DOF micro-parallel robot

Since mobility of this micro parallel robot is three, three actuators are required to control this robot. The 3-BRR robot (Fig. 2b) is a three degree of freedom parallel mechanism.

This structure is also known as 3-BRR robot which is made from 3 kinematic chains, every chain is made up from two elements, the first element is the motor element.

It is connected at one end to the motor and the other end is connected by a passive rotation joint to the second element of the kinematic chain named the passive element.

On the other end of the second element is connected the end-effector by a passive rotation joint.

Knowing the position of point B of the end-effector the problem is reduced to an inverse kinematics problem of a serial robot with 2 arms.

\[ K_1 = 2(x_D - x_O)L_1 \]  
\[ K_2 = 2(y_D - y_O)L_1 \]  
\[ K_3 = l_2^2 - l_1^2 - (x_D - x_O)^2 - (y_D - y_O)^2 \]
\[ q_1' = 2[\alpha \tan 2(-K_2 + \sqrt{K_1^2 + K_2^2 + K_3^2 + K_2 - K_3})] \]

To control the movement of the end-effector of the robot the program uses the inverse kinematics problem, the user will enter the position \((P_x, P_y)\) and the orientation of the end-effector \(\varphi\) and the program will compute the angles of all three actuators.

The compute the angles the mechanism is decomposed into three serial manipulators. In the individual serial manipulator (Fig. 6) the end-effector is align by the offset angle with the relationships:

\[ \varphi_1 = \varphi + 210^\circ \]
\[ \varphi_2 = \varphi + 330^\circ \]
\[ \varphi_3 = \varphi + 90^\circ \]

Knowing the position and the rotation of the end-effector we can fix the point B.

\[ B_x = P_x - L_2 \cos(\varphi_1) \]
\[ B_y = P_y - L_2 \sin(\varphi_1) \]

The angle \(q_1\) have two solutions (Eq. 11-12), this is possible because the mechanism can have the point B in the same position in two situations when the elbow is up and when the elbow is down.

2.4. Kinematics analysis of 3 DOF micro-parallel robot

The passive joint values may also be determined for use in velocity, acceleration, and dynamics. The inverse position problem may be solved for each serial chain independently. The solution is not dependent on joint actuation. The inverse kinematics problem of the parallel robot can be solved by writing the following equations:

\[ q_1 = \sqrt{(x - l_1 \cos \varphi)^2 + (y - l_1 \sin \varphi)^2} \]
\[ q_2 = \sqrt{(-a + x - l_1 \cos \varphi)^2 + (y - l_1 \sin \varphi)^2} \]
\[ q_3 = \sqrt{(-a + b + x + l_1 \cos \varphi)^2 + (y + l_1 \sin \varphi)^2} \]

Then, we can obtain the inverse Jacobi matrix. From Eq. 15, yields that:

1. \(y - l_1 \sin \varphi = 0\), Link 1 and link 2 stay in straight line.
2. \( \cos(\phi) y + (a + b - x) \sin(\phi) = 0 \) Link 3 and platform stay straight in line.

\[(1) \text{Inverse kinematic singularities — when the determinant of } J_q \text{ goes to zero} \]
\[\det(J_q) = 0 \quad (14)\]

**Singular configurations of micro parallel robots**

\[
J^{-1}(x) = \begin{bmatrix}
\frac{x - (\cos(\phi))}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} & \frac{y - \sin(\phi)}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} & \frac{-\sin(\phi) x - \cos(\phi) y}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} \\
\frac{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} & \frac{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} & \frac{-\sin(\phi) x - \cos(\phi) y}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} \\
\frac{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} & \frac{-\sin(\phi) x - \cos(\phi) y}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} & \frac{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}}
\end{bmatrix} \quad (15)
\]

Singular configurations can be found by computing the determinant of the Jacobian matrix:

\[
\det(J^{-1}(x)) = \frac{2 \cos(\phi) y + (a + b - x) \sin(\phi)}{\sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2} \sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2} \sqrt{(x - a - b + \cos(\phi))^2 + (y + \sin(\phi))^2}} \quad (16)
\]

It means there are some nonzero \( \dot{q} \) vectors that result in zero \( \dot{\mathbf{x}} \) vectors. Infinitesimal motion of the moving platform along certain directions cannot be accomplished.

The micro parallel robot loses one or more degrees of freedom. On the other side, an inverse kinematic singular configuration the micro parallel robot can resist forces or moments in some directions with zero actuator forces or torques.

\[(2) \text{Direct Kinematic Singularities — when the determinant of } J_q \text{ goes to zero} \]
\[\det(J_q) = 0 \quad (17)\]

In this condition, there exist some nonzero \( \dot{\mathbf{x}} \) that result in zero \( \dot{q} \) vector.

This is, the moving platform can possess infinitesimal motion in some directions while all the actuators are completely locked. The moving platform gains 1 or more degrees of freedom. In other words, at a direct kinematic singular configuration, the micro parallel robot cannot resist forces or moments in some directions.

### 2.5. Workspace analysis of 3 DOF micro-parallel robot

Here, we propose an approach to compute and visualize the workspace of a 3 DOF micro parallel robot. Micro parallel robots are good candidates for microminiaturization into a micro-device.

**Case I.**

Workspace can be determined by using discretization method. There were identified several cases of workspace.

\[i_1 = i_2 \quad (18)\]

The main disadvantage of parallel robots is their small workspace in comparison to serial arms of similar size. Despite the advantages of parallel manipulators there are certain disadvantages to be encountered such as complicated kinematics and dynamics, many singular configurations, and poor workspace availability. It is very important to analyze the area and the shape of workspace for parameters given robot in the context of industrial application.

**Case II.**

\[i_1 \neq i_2, \quad i_2 > i_1, \quad i_2 + i_1 < A_1 A_2 \quad (19)\]

Singular configurations were identified as it is presented in the following figures.

**Figure 7.** Workspace of the 3-RRR parallel micro robot.
2.6. Workspace analysis of 3 DOF micro-parallel robot

Here, we propose an approach to compute and visualize the workspace of a 3 DOF micro-parallel robot. For determining the workspace of the micro parallel robot it was made a graphical user interface in Matlab.

The main drawbacks of these micro parallel robots are related to the difficult direct kinematic analysis and to the low ratio between workspace area and external dimensions.

The second problem can be fixed by means of an application based geometrical design: the efficiency of a parallel robot dramatically increases if its structure is studied considering carefully the task definition.

These micro parallel robots tend to be more accurate. Furthermore, the speed of displacement is often greater as the actuators are generally positioned on the fixed base.

Workspace problems, such as singularities, make the application of new trajectories more difficult. Frequently, the task must be adapted to take robot workspace constraints into account. In the future, it will be implemented a tracking controller for the dynamic model of the micro parallel robot, using a control law such that the robot velocities reach the given velocity inputs, and a fuzzy logic controller such that provided the required torques for the actual robot and new research will rely on intelligent control approaches such as fuzzy logic control and neural networks.

3. CONCLUSIONS

This paper presented the kinematics modeling of 3 DOF parallel robots with flexure hinges. In this paper are compared based on their experimental workspaces. These compliant micro-motion systems have a closed-loop parallel structure configuration which provides better stiffness and accuracy compared to serial structures. Micro parallel robots are known as the kind of robot whose workspace is limited and complex-shaped, and, due to that, whose movement is too constrained. One important reason for this problem is the fact that these robots have singularities inside its workspace and it is normally a complex task to determine the conditions that lead to singular configurations. For this type of micro-motion systems the possible applications include biological cell manipulation and micro-component assembly. The results of this paper can be used for the design, simulation, and control of any 3-DOF planar parallel robot. We conclude that accurate modeling enhances the position accuracy in the design of micro-parallel robots with flexure hinges. It is expected that these devices can be usefully employed for scanning Electron Microscopy (SEM), x-ray
lithography, mask alignment, micro-machining, and other similar technologies.

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