Steady State Analysis of QP-MIP for Location Management*

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Abstract - The paging signaling overhead is occurred in Paging extensions for Mobile IP(P-MIP) which was proposed to reduce registration signaling overhead of Mobile IP(MIP). And quick paging IP scheme using residence pattern of mobile user(QP-MIP) is proposed to overcome the above inefficiency. To the exact analysis of QP-MIP and P-MIP, the Mobile Node(MN) state transition behavior is modeled and the steady state probability of the MN states is derived using a semi-Markov process approach. The effect of residence timer on the steady state probability is investigated. And we compare the performance of QP-MIP with P-MIP based on session-to-mobility ratio.

Keywords: Steady state analysis, Location management, Mobile IP(MIP), P-MIP, QP-MIP.

1 Introduction

As a demand of mobility in the Internet increases, Mobile IP(MIP) [1] was proposed. MIP provides a global mobility solution that provides node mobility regardless of devices, access technologies. And according to the tendency of the many users, reducing the signaling overhead associated with mobility management will be more important to provide an efficient communication. So, Paging Extensions for Mobile IP(P-MIP) [2] is proposed to minimize the signaling overhead and optimize the mobility management performance.

In P-MIP, however, the paging signaling overhead is presented because of paging the whole paging area(PA). To improve this problem, quick paging IP scheme using residence pattern of mobile user(QP-MIP) [3] is proposed. They compared the performance of QP-MIP with P-MIP using the transmission cost and the processing cost. There is a difference of performance based on the value of residence timer. And residence timer value is changed based on the probability that an Mobile Node(MN) is in the idle(normal) or the idle(quick) state. Therefore, we derive the steady state probability of the MN states and compare the performance of QP-MIP with P-MIP based on session-to-mobility ratio(SMR).

This paper is composed as follows. We discuss related works in section 2. The analysis of the MN’s steady state probability is explained in section 3. Then the performance analysis of P-MIP and QP-MIP is described in section 4. Finally, we conclude the discussion in section 5.

2 Related works

Mobile IP supports a simple mobility mechanism. An MN’s location is tracked by its Home Agent(HA) which binds the care-of address or co-located care-of address used by the MN in a visited network to the MN’s home address. When an MN moves to a new network, it registers its new care-of address with its HA. In this case, Mobile IP uses protocol tunneling to hide an MN’s home address from intervening routers between its home network and its current location. The tunnel terminates at the MN’s care-of address. The care-of address must be an address to which datagrams can be delivered via conventional IP routing. At the care-of address, the original datagram is removed from the tunnel and delivered to the MN.

This basic mechanism presents a number of challenges to the widespread deployment of MIP with respect to handoff performance and scalability issues. Figure 1 illustrates the routing of data to and from an MN away from home, once the MN has registered with its HA. In Figure 1, the MN is using a foreign agent care-of address, not a co-located care-of address.

Zhang et al. proposed a protocol to reduce the registration signaling overhead by adding a set of simple paging extensions in Figure 2. In P-MIP, an MN does not register when it is in the idle state, as long as it stays in the same PA. A HA and a Foreign Agent(FA) become to know idle state are traced with their PA staying, and this is how P-
MIP can guarantee its scalability. However, a Correspondent Node (CN) could feel additional delay which was not introduced in MIP, which does not have the idle state for the MN. This delay is inevitable when the MN of the idle state needs to be located precisely by the HA or the CN.

Chung et al. have researched on steady state analysis of P-MIP mobility management [4]. He assumes that the active state is divided into two sub-states and proposes the modified MN state transition diagram in Figure 4. And he derives steady state probabilities of each state in P-MIP using a semi-Markov process approach and investigates the relative cost of P-MIP to that of MIP.

3 Analysis of mobile node state transitions

3.1 Derivation of steady state probability

The steady state probabilities of an MN’s states are derived. For mathematical simplicity, the idle state is divided into two sub-states; idle(normal) and idle(quick). In the idle(normal) state, the MN acts in the same manner as PMIP. If the residence timer expires, the MN moves to the idle(quick) state from the idle(normal). The MN in the idle(quick) state acts in the same manner as QP-MIP. Figure 5 shows the MN state transition diagram in QP-MIP.
We make the following assumptions regarding the density functions of random variables, referring to [4]:

- incoming and outgoing sessions at an MN occur according to a Poisson process with parameters $\lambda_i$ and $\lambda_o$, respectively;
- cell and PA residence durations follow an exponential distribution with parameters $\lambda_c$ and $\lambda_{pa}$, respectively.

The steady state probabilities of an embedded Markov chain (Figure 5) are obtained by solving the following equations:

\[
\pi_j = \sum_{k=1}^{4} \pi_k P_{kj}, \quad j = 1, 2, 3, 4 \tag{1}
\]

\[
1 = \sum_{k=1}^{4} \pi_k \tag{2}
\]

where $\pi_i$ is the stationary probability of state $i$, and $P_{kj}$ is the state transition probability from states $k$ to $j$. To find the state transition probability $P_{kj}$ of the embedded Markov chain shown in Figure 5, it is necessary to derive the distribution of time interval $T_{kj}$ from the moment an MN enters states $k$ until the first event corresponding to state $j$ occurs.

Exit from the active(off) state is caused by any of the following events:
- incoming or outgoing session arrival ($T_{12}$);
- active timer expiration ($T_{13}$).

The probability density function (PDF) of $T_{12}$ is obtained as:

\[
f_{T_{12}}(t) = \lambda_{sa} e^{-\lambda_{sa} t} \tag{3}
\]

where is the session arrival rate given by:

\[
\lambda_{sa} = \lambda_i + \lambda_o \tag{4}
\]

The PDF of $T_{13}$ is obtained as:

\[
f_{T_{13}}(t) = \delta(t - \xi) \tag{5}
\]

where $\delta(\cdot)$ is the unit step function and $\xi$ is the active timer value.

Exit from the active(on) state is caused by the completion of session processing ($T_{21}$). The length of a data session is modeled as a Pareto distribution and the PDF of $T_{21}$ is given by [6]

\[
f_{T_{21}}(t) = \frac{\alpha k_s^\alpha}{t^{\alpha + 1}} \tag{6}
\]

where $k_s$ is the minimum duration of a session transmission and $\alpha$ is the heavy-tailedness.

Exit from the idle(normal) state is caused by any of the following events:
- PA update ($T_{31}$);
- incoming or outgoing session arrival ($T_{32}$);
- residence timer expiration ($T_{34}$).

The PDF of $T_{31}$ and $T_{32}$ are derived as:

\[
f_{T_{31}}(t) = \lambda_{pa} e^{-\lambda_{pa} t} \tag{7}
\]

and

\[
f_{T_{32}}(t) = \lambda_{oa} e^{-\lambda_{oa} t} \tag{8}
\]

The PDF of $T_{34}$ is obtained as:

\[
f_{T_{34}}(t) = \delta(t - \tau) \tag{9}
\]

where $\tau$ is the residence timer value.

Exit from the idle(quick) state is caused by any of the following events:
- PA update ($T_{41}$);
incoming or outgoing session arrival (T_{42}).

The PDF of T_{41} and T_{42} are derived as:

\[ f_{T_{41}}(t) = \lambda_{pa} e^{-\lambda_{pa} t} \quad (10) \]

and

\[ f_{T_{42}}(t) = \lambda_{sa} e^{-\lambda_{sa} t} \quad (11) \]

The state transition probabilities are derived referring to the embedded Markov chain shown in Figure 5. \( P_{21} \) is given by 1. Based on independence of \( T_{12} \) and \( T_{13} \), the probability \( P_{12} \) is expressed as:

\[ P_{12} = \int_0^\infty f_{T_{12}}(t) Pr(T_{13} > t) dt \]

\[ = \int_0^{\xi} \lambda_{sa} e^{-\lambda_{sa} t} dt \]

\[ = 1 - e^{-\lambda_{sa} \xi} \quad (12) \]

and the probability \( P_{13} \) is obtained as:

\[ P_{13} = 1 - P_{12} = e^{-\lambda_{sa} \xi} \quad (13) \]

Based on independence between \( T_{31}, T_{32} \) and \( T_{34} \), the probabilities \( P_{31} \) is expressed as:

\[ P_{31} = \int_0^\infty f_{T_{31}}(t) Pr(T_{32} > t) Pr(T_{34} > t) dt \]

\[ = \int_0^{\tau} \lambda_{pa} e^{-\lambda_{pa} t} e^{-\lambda_{sa} \tau} dt \]

\[ = \frac{\lambda_{pa}}{\lambda_{pa} + \lambda_{sa}} (1 - e^{-(\lambda_{pa} + \lambda_{sa}) \tau}) \quad (14) \]

The value of \( P_{32} \) and \( P_{34} \) are obtained as:

\[ P_{32} = \int_0^\infty f_{T_{32}}(t) Pr(T_{31} > t) Pr(T_{34} > t) dt \]

\[ = \int_0^{\tau} \lambda_{sa} e^{-\lambda_{sa} t} e^{-\lambda_{sa} \tau} dt \]

\[ = \frac{\lambda_{sa}}{\lambda_{pa} + \lambda_{sa}} (1 - e^{-(\lambda_{pa} + \lambda_{sa}) \tau}) \quad (15) \]

and

\[ P_{34} = \int_0^\infty f_{T_{34}}(t) Pr(T_{31} > t) Pr(T_{32} > t) dt \]

\[ = \int_0^{\infty} \delta(t - \tau) e^{-\lambda_{pa} t} e^{-\lambda_{sa} \tau} dt \]

\[ = e^{-(\lambda_{pa} + \lambda_{sa}) \tau} \quad (16) \]

Based on independence between \( T_{41} \) and \( T_{42} \), the probabilities \( P_{41} \) is derived as:

\[ P_{41} = \int_0^\infty f_{T_{41}}(t) Pr(T_{42} > t) dt \]

\[ = \frac{\lambda_{pa}}{\lambda_{pa} + \lambda_{sa}} \quad (17) \]

The value of \( P_{42} \) is expressed as:

\[ P_{42} = \int_0^\infty f_{T_{42}}(t) Pr(T_{41} > t) dt \]

\[ = \frac{\lambda_{sa}}{\lambda_{pa} + \lambda_{sa}} \quad (18) \]

All the state transition probabilities have been derived. In order to find the steady state probability of a semi-Markov process, the mean residence time of the MN in each state is calculated. The mean residence time in the active(off) state is obtained as:

\[ \overline{t_1} = E[t_1] = E[\min\{T_{12}, T_{13}\}] \]

\[ = \int_0^\infty Pr(T_{12} > t) Pr(T_{13} > t) dt \]

\[ = \int_0^\infty e^{-\lambda_{sa} \tau} Pr(T_{13} > t) dt \]

\[ = \frac{P_{32}}{\lambda_{sa}} = \frac{1}{\lambda_{sa}} - \frac{e^{-\lambda_{sa} \tau}}{\lambda_{sa}} \quad (19) \]

The mean residence time in the active(on) state is given by [6]

\[ \overline{t_2} = E[t_2] \]

\[ = \frac{k_s \alpha}{\alpha - 1} \quad (20) \]
The mean residence time in the **idle(normal)** state is expressed as:

\[
\bar{t}_3 = E[t_3] = E[\min\{T_{31}, T_{32}, T_{34}\}] \\
= \int_0^\infty Pr(T_{31} > t)Pr(T_{32} > t)Pr(T_{34} > t)dt \\
= \int_0^\infty \int_0^\infty \int_0^\infty e^{-\lambda_{pa1}t_1} e^{-\lambda_{pa2}t_2} e^{-\lambda_{sa}t_3} dt_1 dt_2 dt_3 \\
= \frac{P_{31}}{\lambda_{pa}} = \frac{1 - e^{-(\lambda_{pa} + \lambda_{sa})\tau}}{\lambda_{pa} + \lambda_{sa}} \tag{21}
\]

The mean residence time in the **idle(quick)** state is obtained as:

\[
\bar{t}_4 = E[t_4] = E[\min\{T_{41}, T_{42}\}] \\
= \int_0^\infty Pr(T_{41} > t)Pr(T_{42} > t)dt \\
= \int_0^\infty e^{-\lambda_{pa}t} Pr(T_{42} > t)dt \\
= \frac{P_{41}}{\lambda_{pa}} = \frac{1}{\lambda_{pa} + \lambda_{sa}} \tag{22}
\]

Finally, the steady state probability of the semi-Markov process is written as:

\[
P_k = \frac{\pi_k \bar{t}_k}{\sum_{i=1}^4 \pi_i \bar{t}_i} \tag{23}
\]

where the values of \(\pi_k\) and \(\bar{t}_k\) are obtained from (1) – (2) and (19) – (22), respectively.

### 3.2 A numerical example of the steady state probability

The effect of residence timer on steady state probabilities is investigated. Values of parameters for a numerical example are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_i)</td>
<td>3 (\text{hour})</td>
</tr>
<tr>
<td>(\lambda_o)</td>
<td>3 (\text{hour})</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1.2</td>
</tr>
<tr>
<td>(k_i)</td>
<td>3 (sec)</td>
</tr>
<tr>
<td>(n)</td>
<td>5</td>
</tr>
<tr>
<td>(\text{PA (n^2)})</td>
<td>25</td>
</tr>
<tr>
<td>(\lambda_c)</td>
<td>60 (\text{hour})</td>
</tr>
<tr>
<td>(\lambda_{pa} (\lambda_{c}/n))</td>
<td>12 (\text{hour})</td>
</tr>
<tr>
<td>(\xi)</td>
<td>0.001 (\text{hour})</td>
</tr>
</tbody>
</table>

Figure 6 illustrates the effect of residence timer \(\tau(\text{hour})\) on the steady state probabilities. \(P_1\) depends in the distribution of PA update and active timer value. And \(P_2\) depends only in the distribution of session arrivals and service time of a session. Thus, probabilities \(P_1\) and \(P_2\) are constant for varying the values of \(\tau\). As the value of \(\tau\) increases, it is more likely that the MN stays in the **idle(normal)** state. Thus, the value of \(P_3\) increases. On the contrary, it is less likely that an MN stays in the **idle(quick)** state as the value of \(\tau\) increases, resulting in a decrease in the value of \(P_4\). If the value of \(\tau\) is larger than 0.3 (\text{hour}), the \(P_4\) and \(P_3\) converge to almost 0 and a constant, respectively. Thus, \(P_3\) and \(P_4\) are less sensitive for large values of \(\tau\).

**Figure 6.** Effect of \(\tau\) on the steady state probabilities

### 4 Performance analysis

If an MN receives a paging request and it is still in the same registered cell, the MN does not need to register and it just sends paging reply. Unless the MN stays in the same registered cell when it receives a paging request, the MN
updates its current location by performing registration and paging reply. If the MN is in the idle state and if it stays in the same registered cell when the MN tries to send data, there is no need to register. However, if the MN has changed a new cell, it performs registration before sending data. To consider these registrations $P_{stay}$ is defined [4] as the probability that the MN is still in the same registered cell when there is any incoming or outgoing data session in the idle state.

In P-MIP, Cost of registration is defined as the number of registration messages per unit time per MN at the radio interface and is obtained as:

$$C_{P-reg} = P_{active} \lambda_{c} + P_{idle} \lambda_{pa} + P_{idle} (1 - P_{stay}) \lambda_{sa}$$  \hspace{1cm} (24)$$

Cost of paging is defined as the number of paging messages at the radio interface per unit time per MN and is obtained as:

$$C_{P-pag} = P_{idle} \lambda_{c} n^2$$  \hspace{1cm} (25)$$

In QP-MIP, cost of registration is similar to that of P-MIP.

$$C_{QP-reg} = P_{active} \lambda_{c} + P_{idle} \lambda_{pa} + P_{idle} (1 - P_{stay}) \lambda_{sa}$$  \hspace{1cm} (26)$$

And cost of paging is obtained as:

$$C_{QP-pag} = P_{n} \lambda_{c} n^2 + P_{4} \lambda_{c} n_{opt}$$  \hspace{1cm} (27)$$

where $n_{opt}$ is the number of prestored FVFAs.

The total cost of registration and paging in P-MIP and QP-MIP is expressed by:

$$C_{P-MIP} = \omega_{reg} C_{P-reg} + \omega_{pag} C_{P-pag}$$  \hspace{1cm} (28)$$

and

$$C_{QP-MIP} = \omega_{reg} C_{QP-reg} + \omega_{pag} C_{QP-pag}$$  \hspace{1cm} (29)$$

where $\omega_{reg}$ and $\omega_{pag}$ are weight factors of registration and paging signaling. Although it is hard to calculate the precise value of these weight factors, it is generally assumed that $\omega_{reg}$ is larger than $\omega_{pag}$ [7].

Values of parameters for the total cost of each scheme, $C_{P-MIP}$ and $C_{QP-MIP}$, are presented in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{sa}$</td>
<td>6 (/hour)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.2</td>
</tr>
<tr>
<td>$k_s$</td>
<td>3 (sec)</td>
</tr>
<tr>
<td>$n$</td>
<td>5</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.001 (/hour)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.001 (/hour)</td>
</tr>
<tr>
<td>$\omega_{reg}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\omega_{pag}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$P_{stay}$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Figure 7 shows the effect of session-to-mobility ratio on the total cost for $n_{opt} = 3$. If the mobility is small, there are few registrations. Thus, costs of P-MIP and QP-MIP are decreased as SMR increases. For various values of SMR, the performance of QP-MIP is always better. This is because, QP-MIP executes paging to some PAs($n_{opt}$) when an MN is in the idle state although P-MIP performs paging to whole PA($n$).

Figure 7. Effect of SMR on the total cost

5 Conclusions

To the exact analysis of QP-MIP and P-MIP, we model the MN state transition in QP-MIP and derive the steady state probability of the MN states using a semi-Markov process approach. Based on state transition modeling the effects of residence timer on the steady state probabilities is analyzed and the total cost of QP-MIP to that of P-MIP is investigated based on SMR. Therefore, we can
prove that performance of QP-MIP is better than that of P-MIP through the above results.

6 References


