training vector at the AFB output of the associated ideal (channel- and noise-free) OFDM/OQAM system.

\[
\left\| \hat{\mathbf{y}}_{\mathrm{QAM}}^{\ell} \right\|^2 = \sum_{k=0}^{L_h-1} \sum_{m=0}^{N_h-1} a_{\ell,m} g_{\ell,m}(k) \hat{g}_{\ell,m}(k)
\]

\[
= \sum_{m=0}^{L_h-1} \sum_{l=0}^{g_{\ell,m}(k)} g_{\ell,m}(k) \hat{g}_{\ell,m}(k)
\]

\[
= \sum_{m=0}^{L_h-1} \sum_{l=0}^{g_{\ell,m}(k)} a_{\ell,m} g_{\ell,m}(k) \hat{g}_{\ell,m}(k)
\]

\[
+ \sum_{m=0}^{L_h-1} \sum_{m \neq k} a_{\ell,m} g_{\ell,m}(k) \hat{g}_{\ell,m}(k)
\]

Obviously, \[\sum_{m=0}^{L_h-1} g_{\ell,m}(k) \hat{g}_{\ell,m}(k) = 1 \text{ and } \sum_{m=0}^{L_h-1} g_{\ell,m}(k) \hat{g}_{\ell,m}(k) = 0, m \neq k \].

The (time domain) MSE expression for the sparse preamble with \( L_h \) nonzero pilot tones in the OFDM/OQAM system is the same as in the CP-OFDM system, i.e., \( \mathrm{MSE}_{L_h} = \| \mathbf{C} \|_2^2 \left( \mathbf{F}_{L_h \times L_h} \right)^{-1} \). Our optimization problem can therefore be stated as follows:

\[
\min_{a_{\ell,m}, 0 \leq 1 \leq L_h} \text{MSE}_{L_h}
\]

s.t.

\[
\sum_{m=0}^{L_h-1} a_{\ell,m}^2 \leq \mathcal{E}
\]

But the solution to this problem is known. It is the class of equipowered and equispaced pilot tones \([15],[3] \).


REFERENCES


Novel Low-Complexity SLM Schemes for PAPR Reduction in OFDM Systems

Chih-Peng Li, Sen-Hung Wang, and Chin-Liang Wang

Abstract—Selected mapping (SLM) schemes are commonly employed to reduce the peak-to-average power ratio (PAPR) in orthogonal frequency-division multiplexing (OFDM) systems. It has been shown that the computational complexity of the traditional SLM scheme can be substantially reduced by adopting conversion vectors obtained by using the inverse fast Fourier transform (IFFT) of the phase rotation vectors in place of the conventional IFFT operations [C.-L. Wang and Y. Ouyang, “Low-Complexity Selected Mapping Schemes for Peak-to-Average Power Ratio Reduction in OFDM Systems,” IEEE Trans. Signal Process., vol. 53, no. 12, pp. 4652–4660, Dec. 2005]. To ensure that the elements of these phase rotation vectors have an equal magnitude, conversion vectors should have the form of a perfect sequence. This paper presents three novel classes of perfect sequence, each of which comprises certain base vectors and their cyclically shifted versions. Three novel low-complexity SLM schemes are then proposed based upon the unique structures of these perfect sequences. It is shown that while the PAPR reduction performances of the proposed schemes are marginally poorer than that of the traditional SLM scheme, the three schemes achieve a substantially lower computational complexity.

Index Terms—Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR).

I. INTRODUCTION

Orthogonal-frequency-division multiplexing (OFDM) is a promising technique for high data rate transmission due to its high spectral efficiency and robustness against the interference inherent in multi-path channels. However, OFDM systems suffer the drawback of a high peak-to-average power ratio (PAPR) of the transmitted signal. The literature contains various methods for PAPR reduction in

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OFDM systems, including clipping [1], coding [2]–[4], selected mapping (SLM) [5], [6], partial transmit sequences (PTS) [7]–[10], and tone reservation (TR) [11], [12]. Although the TR method provides the lowest complexity of all distortionless methods so far, it is achieved at the expense of bandwidth efficiency. Traditional SLM schemes have a better bandwidth efficiency, but require a bank of inverse fast Fourier transforms (IFFTs) to produce candidate signals, resulting in a dramatic increase in computational complexity. To overcome this drawback, Wang and Ouyang [5] presented a low-complexity scheme in which the IFFTs were replaced by conversion vectors obtained by taking the IFFT of the phase rotation vectors. Unfortunately, for most of the conversion vectors proposed in [5], the elements of the corresponding phase rotation vectors do not have the same magnitude, leading to significant degradation in bit error rate (BER) performance.

In this paper, three novel low-complexity SLM schemes are proposed, where the IFFT blocks were replaced by conversion vectors. We first claim that conversion vectors should be specified in the form of perfect sequences. Three novel classes of perfect sequences are then introduced, each comprising certain base vectors and their cyclically shifted equivalents. These sequences are then utilized as the basis for three low-complexity SLM schemes. It is shown analytically that the computational complexities of the three schemes are substantially lower than that of the traditional SLM scheme, while their PAPR reduction performance differs by no more than 0.64 dB than that obtained using the traditional SLM scheme.

II. SYSTEM MODEL AND REVIEW OF THE SLM SCHEME BY WANG AND OUYANG

In general, obtaining an improved approximation of the true PAPR in the discrete-time case requires the time domain candidate signals to be oversampled. For an OFDM system with \( N \) subcarriers, an oversampling rate of \( L \) can be achieved by inserting \( (L - 1) \cdot N \) zeros in the middle of the modulated symbol vector to form a \( 1 \times LN \) frequency domain data vector \( \mathbf{X} \), i.e.,

\[
\mathbf{X} = \begin{bmatrix} X_0 X_1 \cdots X_{N/2-1} & 0 \cdots 0 & X_{N/2} X_{N/2+1} \cdots X_{LN-1} \end{bmatrix} \tag{1}
\]

where \( X_i \) is the modulated symbol of the \( i \)th subcarrier. Then, an \( LN \)-point IFFT process is performed to generate the oversampled time-domain signal vector \( \mathbf{x} \), where the \( n \)th element of \( \mathbf{x} \) is given by

\[
x[n] = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X[k] \cdot \exp \left( \frac{j2\pi nk}{LN} \right),
\]

\( n = 0, 1, \ldots, LN - 1 \). \tag{2}

\( X[k] \) is the \( k \)th element of \( \mathbf{X} \). The PAPR of the discrete-time OFDM signal is defined as

\[
PAPR = \max_{0 \leq n \leq LN-1} \frac{|x[n]|^2}{E[|x[n]|^2]}.
\] \tag{3}

Note it was derived in [13] that the true PAPR is at most \( C_L = 1/(\cos(\pi/2L))^2 \) times the estimated PAPR from \( L \) times oversampling. A better estimate for \( L = 3/2 \) was given in [14].

To overcome the drawbacks of the traditional SLM scheme, Wang and Ouyang [5] presented a low-complexity SLM scheme (denoted as W&O scheme) in which the IFFT blocks were replaced by conversion vectors obtained by taking the IFFT of the phase rotation vectors. Candidate signals were generated by performing a circular convolution of the IFFT of the data sequence with these conversion vectors. Unfortunately, the elements of most of the phase rotation vectors adopted in [5] have different magnitudes. As a result, the signals of different subcarriers have different gains and the signal power of some of the subcarriers is attenuated, leading to a serious degradation in the BER performance.

III. STRUCTURES OF PERFECT SEQUENCES/CONVERSION VECTORS ADOPTED IN CURRENT STUDY

Note that for the case in which the elements of the phase rotation vectors all have the same magnitude, the periodic autocorrelation function (PACF) of the corresponding conversion vectors has the form

\[
\sum_{m=0}^{N-1} g[m] \cdot g^*[m-n] = E \cdot \delta[n], \quad 0 \leq n \leq N - 1
\] \tag{4}

where \( g[m] \) is the \( m \)th element of the conversion vector, \( * \) is the complex conjugate operation, \( (\cdot)_N \) denotes the modulo \( N \) operation, \( E \) is a constant, and \( \delta[n] \) is a delta function. The sequences which satisfy the above described condition are defined as perfect sequences in this study [15].

The perfect sequences adopted in the current study are compositions of certain base vectors and their cyclically shifted equivalents. In our proposed schemes, candidate signals are generated by applying an \( LN \)-point circular convolution of the time domain signal vector \( \mathbf{x} \) with the base vectors. Note that to reduce the computational complexity of the conversion process, the following constraints are imposed:

1) the maximum number of non-zero elements in the base vectors is limited to 4;
2) the non-zero elements in the base vectors must belong to the set \( \{ \pm 1, \pm j, \pm 1 \pm j \} \).

Having performed an exhaustive search, three classes of perfect sequence of length \( LN \) were identified. If the above constraints are removed, more perfect sequences can be obtained. However, the computational complexity of the conversion process will be significantly increased.

A. Class I Perfect Sequences/Conversion Vectors

The Class I conversion vector \( \mathbf{G}_a \) is a \( 1 \times LN \) vector of the form

\[
\mathbf{G}_a = [g_a[0], g_a[1], \ldots, g_a[LN-1]].
\]

In particular, \( \mathbf{G}_a \) is a composite of two \( 1 \times LN \) base vectors:

\[
\mathbf{G}_{a1} = \begin{bmatrix} -1, 0, \ldots, 0, 1, 0, \ldots, 0 \\ \underbrace{\ldots}_{LN-1} \end{bmatrix}
\]

and

\[
\mathbf{G}_{a2} = \begin{bmatrix} 1, 0, \ldots, 0, 1, 0, \ldots, 0 \\ \underbrace{\ldots}_{LN-1} \end{bmatrix}
\] \tag{5}

where \( LN/2 \geq 1 \). The Class I conversion vectors can be expressed in the general form \( \mathbf{G}_a(c, w, m) = c \cdot \{ \mathbf{G}_{a1} + w \cdot \mathbf{G}_{a2} \} \), where \( c \) is any complex constant, \( w \in \{ \pm 1, \pm j \} \), \( \mathbf{G}_{a1} \) denotes the \( m \)th right cyclic shift of \( \mathbf{G}_{a2} \), and \( m = 0, 1, \ldots, LN/2 - 1 \). Therefore, the \( n \)th element of \( \mathbf{G}_{a2} \) is given by \( g_{a2}^m[n] = g_{a2}^m[(n - m) \cdot LN] \). It can be shown that different values of \( c \) result in the same PAPR. Therefore, a value of \( c = 1 \) is adopted throughout the rest of this study.
B. Class II Perfect Sequences/Conversion Vectors

The Class II conversion vector, \( \mathbf{G}_k \), is a composite of two \( 1 \times L \) base vectors, \( \mathbf{G}_{i1} \) and \( \mathbf{G}_{i2}^0 \):

\[
\mathbf{G}_{i1} = \begin{bmatrix} g_{i1}[0], 0, \ldots, 0, g_{i1} \left[ \frac{L}{4} \right], 0, \ldots, 0, g_{i1} \left[ \frac{L}{2} \right], 0, \ldots, 0 \end{bmatrix}
\]

(6)

\[
\mathbf{G}_{i2}^0 = \begin{bmatrix} g_{i2}^0[0], 0, \ldots, 0, g_{i2}^0 \left[ \frac{L}{4} \right], 0, \ldots, 0, g_{i2}^0 \left[ \frac{L}{2} \right], 0, \ldots, 0 \end{bmatrix}
\]

(7)

where \( LN/4 \geq 1 \), \( g_{i1}[0] \in \{1, -1\} \), and \( g_{i1}[LN/4], g_{i2}^0[LN/4] \in \{1 \pm j, -1 \pm j\} \). Note that \( g_{i1}[LN/4] = v \cdot g_{i2}^0[LN/4] \), where \( v \in \{1, \pm j\} \). In addition, \( g_{i2}^0[0], g_{i1}[LN/2], \) and \( g_{i2}^0[LN/2] \) are given respectively by \( v \cdot g_{i2}^0[0] = (j \cdot g_{i1}[L/4]) \cdot \text{Re}\{g_{i1}[L/4]\} \), \( v \cdot g_{i2}^0[L/4] = (j \cdot g_{i1}[L/4]) \cdot \text{Im}\{g_{i1}[L/4]\} \), and \( v \cdot g_{i2}^0[L/4] = (j \cdot g_{i1}[L/4]) \cdot \text{Im}\{g_{i1}[L/4]\} \). As discussed above, a value of \( c = 1 \) is adopted in this study.

C. Class III Perfect Sequences/Conversion Vectors

The Class III conversion vector, \( \mathbf{G}_c \), is a composite of four \( 1 \times L \) base vectors, namely

\[
\mathbf{G}_{i1}^0 = \begin{bmatrix} 1, 0, \ldots, 0, 1, 0, \ldots, 0, 1, 0, \ldots, 0 \end{bmatrix}
\]

(8)

\[
\mathbf{G}_{i2}^0 = \begin{bmatrix} 1, 0, \ldots, 0, j, 0, \ldots, 0, -1, 0, \ldots, 0 \end{bmatrix}
\]

(9)

\[
\mathbf{G}_{i3}^0 = \begin{bmatrix} 1, 0, \ldots, 0, -1, 0, \ldots, 0, 1, 0, \ldots, 0 \end{bmatrix}
\]

(10)

\[
\mathbf{G}_{i4}^0 = \begin{bmatrix} 1, 0, \ldots, 0, -j, 0, \ldots, 0, -1, 0, \ldots, 0 \end{bmatrix}
\]

(11)

The Class III conversion vectors have the following general form

\[
\mathbf{G}_c = p_1 \cdot \mathbf{G}_{i1}^0 + p_2 \cdot \mathbf{G}_{i2}^0 + p_3 \cdot \mathbf{G}_{i3}^0 + p_4 \cdot \mathbf{G}_{i4}^0
\]

(12)

where \( p_1, p_2, p_3, \) and \( p_4 \) are complex constants of equal magnitude, and \( m_1, m_2, m_3, \) and \( m_4 \) denote the number of right cyclic shifts, \( 0 < m_i \leq \lfloor LN/4 \rfloor - 1, i = 1, 2, 3, \) and 4. Since it can be shown that the magnitudes of \( p_1, p_2, p_3, \) and \( p_4 \) have no effect on the PAPR, values of \( p_1 = 1 \) and \( p_2, p_3, p_4 \in \{1, \pm j\} \) are adopted in this study.

IV. PROPOSED LOW-COMPLEXITY SLM SCHEMES

Based on the three conversion vector classes and their unique structures demonstrated above, this section presents three novel low-complexity SLM schemes.

A. Proposed Scheme I (PS I)

The first scheme presented in this study (denoted hereafter as Proposed Scheme I and denoted as PS I) adopts both the Class I and Class II conversion vectors. Since the Class I conversion vectors are composites of two different base vectors and the circular convolution process is a linear operation, the candidate signals generated using the Class I conversion vectors with \( c = 1 \) can be written as

\[
y = x \oplus_{LN} \mathbf{G}_c = x \oplus_{LN} \mathbf{G}_{a1} + w \cdot \left\{ x \oplus_{LN} \mathbf{G}_{a2}^0 \right\}
\]

where \( \oplus_{LN} \) denotes the \( LN \)-point circular convolution operation, \( \mathbf{A}_{a1} \equiv x \oplus_{LN} \mathbf{G}_{a1}, \) and \( \mathbf{A}_{a2}^0 \equiv x \oplus_{LN} \mathbf{G}_{a2}^0 \). The \( m \)-th element of \( \mathbf{A}_{a2}^0 \) can be expressed as

\[
A_{a2}^0[n] = \sum_{q=0}^{LN-1} g_{a2}^0[q] \cdot x \left[ (n - q) \mod LN \right] \equiv A_{a2}^0 \left[ (n - m) \mod LN \right]
\]

(13)

In other words, \( \mathbf{A}_{a2}^0 \) is the \( m \)-th right cyclic shift of \( \mathbf{A}_{a2}^0 \). Note that a similar result is also obtained for the Class II conversion vectors. Fig. 1 presents the architecture of Proposed Scheme I, in which the candidate signals, \( y(m_a) \) and \( y(m_b) \), are generated using various combinations of cyclic shifts of \( m_a \) and \( m_b \). The possible values of \( m_a \) and \( m_b \) are given in Section III-A and III-B, respectively. Therefore, the maximum numbers of candidate signals generated using Class I and Class II conversion vectors are \( LN/2 \) and \( LN \), respectively. Note that by performing a series of simulations, it was found that the PAPR value is insensitive to the value of \( w \), and thus \( w \) is assigned a value of one throughout the remainder of this study.

B. Proposed Scheme II (PS II)

To enhance the PAPR reduction performance, we combine Proposed Scheme I with the traditional SLM scheme to form the second proposed scheme (denoted hereafter as Proposed Scheme II and denoted as PS II) as shown in [6], where two parallel IFFTs were adopted and a random phase rotation vector was applied before the second IFFT operation. Since two IFFTs are required in Proposed Scheme II, the computational complexity is substantially increased. However, the random phase rotation vector before the second IFFT operation increases the randomness of the corresponding phase rotation vector of the time-domain transmitted signal, resulting in an improvement of the PAPR reduction performance.
where are generated by performing an IEEE TRANSACTIONS ON SIGNAL PROCESSING, Vol. 58, No. 5, May 2010 2919 base vectors and has values of (15) where . Since the base vector is given by , where

\[ g_i^{m_1}[n] = \begin{cases} 1, & n = m_1, m_1 + LN/4, m_1 + LN/2, m_1 + 3LN/4 \\ 0, & \text{otherwise} \end{cases} \] (16)

the \( n \)th element of can be written as

\[ A_{m_1}^{m_2} [n] = \sum_{p=0}^{LN-1} g_{m_p}^{m_1}[(q)_L] \cdot x[(n - q)_L] = A_{m_1}^{m_2}[(m - n)_L]. \] (17)

In addition, since \( g_0^{m_1}[n] \) has only four non-zero elements, we can obtain

\[ A_{m_1}^{0} [n] = \sum_{p=0}^{LN-1} g_0^{m_1}[(q)_L] \cdot x[(n - q)_L] = x[(n)_L] + x[(n - LN/4)_L] + x[(n - LN/2)_L] + x[(n - 3LN/4)_L]. \] (18)

\[ A_{m_1}^{0} [n + LN/4] = \sum_{p=0}^{LN-1} g_0^{m_1}[(q)_L] \cdot x[(n + LN/4 - q)_L] = A_{m_1}^{0} [n]. \] (19)

where \( 0 \leq n \leq LN/4 - 1 \). Applying a similar derivation process, it can be shown that

\[ A_{m_1}^{0} [n] = A_{m_1}^{0} [n + LN/4] = A_{m_1}^{0} [n + LN/2] = A_{m_1}^{0} [n + 3LN/4], \quad 0 \leq n \leq LN/4 - 1. \] (20)

Equation (20) implies that the computation of \( A_{m_1}^{0} \) consists of four identical parts. A similar result is obtained for the other three base vectors in (15). In general, given \( A_{m_1}^{0} = [a_1 \cdot a_2 \cdot a_3 \cdot a_4] \), where \( i \) is the index of the base vector, and \( a_1, a_2, a_3, \) and \( a_4 \) are \( 1 \times LN/4 \) vectors, then \( a_1 = u_1 \cdot a_1, a_3 = u_3 \cdot a_1, \) and \( a_4 = u_4 \cdot a_1 \), where \( u_1, u_3, \) and \( u_4 \) are complex factors of unit magnitude with the values shown in Table I. As a result, only the first quarter of the circular convolution is required since the remaining three quarters can be obtained directly from the results of the first quarter.

![Architecture of Proposed Scheme III.](image)

**TABLE I**

<table>
<thead>
<tr>
<th>( m_1 )</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( u_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>( j )</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**C. Proposed Scheme III (PS III)**

In the third scheme presented in this study (denoted hereafter as Proposed Scheme III and annotated as PS III), the candidate signals are generated by performing an \( LN \)-point circular convolution of the time-domain signal vector \( \mathbf{x} \) with the Class III conversion vector \( \mathbf{G}_r \):

\[ \mathbf{y}_m = \mathbf{x} \otimes \mathbf{G}_r \] (15)

where \( \mathbf{A}_d^{m_1} \) represents the index of the base vectors and has values of \( i = 1, 2, 3, \) and 4. In (15), \( \mathbf{A}_d^{m_1} = \{ A_{m_1}^{0}, A_{m_1}^{1}, A_{m_1}^{2}, A_{m_1}^{3} \} \). Since the base vector was given by \( \mathbf{G}_r = \{ g_{r1}^{0}[n], g_{r1}^{1}[n], \ldots, g_{r1}^{LN-1}[n] \} \), where

\[ g_{r1}^{m_1}[n] = \begin{cases} 1, & n = m_1, m_1 + LN/4, m_1 + LN/2, m_1 + 3LN/4 \\ 0, & \text{otherwise} \end{cases} \] (16)

the \( n \)th element of \( \mathbf{A}_d^{m_1} \) can be written as

\[ A_{m_1}^{m_2} [n] = \sum_{p=0}^{LN-1} g_{m_p}^{m_1}[(q)_L] \cdot x[(n - q)_L] = A_{m_1}^{m_2}[(m - n)_L]. \] (17)

In addition, since \( g_0^{m_1}[n] \) has only four non-zero elements, we can obtain

\[ A_{m_1}^{0} [n] = \sum_{p=0}^{LN-1} g_0^{m_1}[(q)_L] \cdot x[(n - q)_L] = x[(n)_L] + x[(n - LN/4)_L] + x[(n - LN/2)_L] + x[(n - 3LN/4)_L]. \] (18)

\[ A_{m_1}^{0} [n + LN/4] = \sum_{p=0}^{LN-1} g_0^{m_1}[(q)_L] \cdot x[(n + LN/4 - q)_L] = A_{m_1}^{0} [n]. \] (19)

where \( 0 \leq n \leq LN/4 - 1 \). Applying a similar derivation process, it can be shown that

\[ A_{m_1}^{0} [n] = A_{m_1}^{0} [n + LN/4] = A_{m_1}^{0} [n + LN/2] = A_{m_1}^{0} [n + 3LN/4], \quad 0 \leq n \leq LN/4 - 1. \] (20)
Fig. 3. Number of complex additions as a function of the number of candidate signals ($LN = 2048$).

**TABLE II**

**COMPUTATIONAL COMPLEXITY OF VARIOUS SCHEMES**

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Number of Complex Multiplications</th>
<th>Number of Complex Additions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional SLM Scheme</td>
<td>$(MLN/2) \cdot \log_2(LN)$</td>
<td>$MLN \cdot \log_2(LN)$</td>
</tr>
<tr>
<td>W&amp;O Scheme</td>
<td>$(LN/2) \cdot \log_2(LN)$</td>
<td>$LN \cdot \log_2(LN) + 3(M-1)LN$</td>
</tr>
<tr>
<td>Proposed Scheme I</td>
<td>$(LN/2) \cdot \log_2(LN)$</td>
<td>$LN \cdot \log_2(LN) + (M+7)LN$</td>
</tr>
<tr>
<td>Proposed Scheme II</td>
<td>$LN \cdot \log_2(LN)$</td>
<td>$2LN \cdot \log_2(LN) + (M+14)LN$</td>
</tr>
<tr>
<td>Proposed Scheme III</td>
<td>$(LN/2) \cdot \log_2(LN)$</td>
<td>$LN \cdot \log_2(LN) + 3MLN$</td>
</tr>
</tbody>
</table>

required in W&O scheme I for $M > 5$. In contrast, Proposed Scheme III involves the largest number of complex additions for $M \geq 12$, but yields the best PAPR reduction performance, as will be demonstrated in the following section. In addition, Proposed Scheme II requires two IFFTs and results in a similar PAPR reduction performance to that of Proposed Scheme III.

VI. SIMULATION RESULTS

Simulations were performed to evaluate the PAPR characteristic of the proposed SLM schemes. The simulations assumed that the data were 16-QAM modulated and that the system contained $N = 64$ subcarriers. To estimate the PAPR, the OFDM signal was oversampled by a factor of $L = 4$.

Fig. 4 compares the PAPR performance of Proposed Schemes I and II with those of W&O scheme and the traditional SLM scheme. It is seen that for a given number of candidate signals ($M$), the PAPR reduction performance of the Proposed Scheme I is very similar to that of the scheme presented by Wang and Ouyang [5]. From a detailed inspection, the performance loss of Proposed Scheme I relative to that of the traditional SLM scheme is found to be $0.64$ dB for $M = 32$ and $\text{Prob}[\text{PAPR} > \gamma] = 10^{-4}$. It can also been that Proposed Scheme II has a better performance than Proposed Scheme I. However, this improvement is achieved at the expense of an extra IFFT operation. Fig. 5 shows the PAPR reduction performance of Proposed Scheme III, where the maximum performance loss of Proposed Scheme III relative to the traditional SLM scheme is $0.2$ dB for $M = 32$ and $\text{Prob}[\text{PAPR} > \gamma] = 10^{-4}$.

It is worth noting that the SLM schemes can be classified as probabilistic methods. The basic idea behind which is to lower the probability of occurrence of a peak and these methods use the limited redundancy not to eliminate the peaks but to make them less frequent. Therefore, the SLM scheme cannot give a guarantee on PAPR. On the other hand, coding methods have been adopted for PAPR reduction [2]–[4]. These coding schemes are deterministic algorithms that de-randomize the search for the sign vector and achieve a guaranteed PAPR. In particular, Sharif and Hassibi have proposed an algorithm [2] (denoted as S&H scheme) to design signs that guarantee deterministically that the PAPR is less than $c \log n$ for any number of subcarriers $n$, where $c$ is a
The constant independent of $n$. In addition, a greedy algorithm proposed in [3] has lower complexity compared to that of [2] while achieving similar PAPR reduction. Fig. 6 compares the PAPR reduction performance of various schemes. It can be seen that S&H scheme has a better performance when $\gamma$ is small. However, the PAPR reduction performance of S&H scheme is similar to that of Proposed Scheme II and III for $M = 8$ and $\text{Prob}(\text{PAPR} > \gamma) < 2 \times 10^{-4}$.

VII. Conclusion

Three novel perfect sequence classes have been proposed in this paper and then been used to construct three low-complexity SLM schemes. Although the PAPR reduction performance of the proposed schemes is marginally poorer than that of the traditional SLM scheme, the three schemes have a substantially lower computational complexity. Specifically, Proposed Scheme III yields the best PAPR results, with a maximum performance loss of no more than 0.2 dB compared to the result obtained using the traditional SLM scheme. For $L_N = 2048$ and $M = 32$, the number of complex multiplications required in Proposed Scheme I and III is just 3.13% of that required in the traditional SLM scheme.

Appendix

Proof of the Perfect Sequences

Both the Class I and the Class II perfect sequences are particular cases of the Class III perfect sequences. Therefore, the proof of perfect sequences is only presented for the Class III sequences in this appendix.

A sequence $g[n]$ is defined as a perfect sequence in this study if and only if the periodic autocorrelation function (PACF) has the form of (4). Performing the discrete Fourier transform (DFT) to both sides of (4) yields $|G[u]|^2 = N \cdot E, s = 0, 1, \ldots, N - 1$, where $G[u]$ is the DFT of $g[n]$. Therefore, the outcome of the DFT of a perfect sequence have equal magnitude.

Applying the DFT to (8)–(11) results in the following equation:

$$G_s[u] = \frac{N}{\sqrt{2 \pi}} \cdot \sum_{u=1}^{N} p_u \cdot e^{-j 2 \pi s m u / N}, \quad s = 0, 1, \ldots, N - 1$$

where $G_s[u]$ is the $s$th element of the DFT of $G^u$. Since $G_s[u]$ has non-zero values when $u = (s) + 1$, substituting (21) into (22) yields

$$G_s[u] = \frac{N}{\sqrt{2 \pi}} \cdot \sum_{u=1}^{N} p_u \cdot e^{-j 2 \pi s m u / N}, \quad u = (s) + 1.$$ 

Let the magnitude of $p_1, p_2, p_3, p_4$ be equal to $\sqrt{P}$. Since the exponential term has a unit magnitude, the square of the absolute value of $G_s[u]$ is given by $|G_s[u]|^2 = N^2 \cdot P, s = 0, 1, \ldots, N - 1$. Since $G_s[u]$ have equal magnitude, the PACF of $G_s$ has the form of a delta function and $G_s$ is proven to be a perfect sequence.

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