THE TRAGEDY OF MATHEMATICS IN RUSSIA

S. S. KUTATELADZE

Abstract. This is a brief overview of the so-called "case of Academician Luzin."

Tsunami swept over the Russian mathematical community in 1999 after publication of the book *The Case of Academician Nikolaĭ Nikolaevich Luzin* [1]. For the first time it revealed the complete shorthand notes of the meetings of a notorious special Commission of the Academy of Sciences of the USSR.

N. N. Luzin (1883–1950) was one of the founding fathers of the Moscow mathematical school. The list of his students contains Full Members of the Academy P. S. Aleksandroff (1886–1982), A. N. Kolmogorov (1903–1987), M. A. Lavrentiev (1900–1980), P. S. Novikov (1901–1975); Corresponding Members L. A. Lyusternik (1899–1981), A. A. Lyapunov (1911–1973), D. E. Menshov (1892–1988), A. Ya. Khinchin (1894–1959), L. G. Shnirelman (1905–1938); and many other mathematicians.

The Commission was convened after the article "Enemies under the Mask of a Soviet Citizen" in the *Pravda* newspaper on July 3, 1936. Luzin was accused of all theoretically possible instances of misconduct in science and depicted as an enemy that combined "moral unscrupulousness and scientific dishonesty with deeply concealed enmity and hatred to every bit of the Soviet life." It was alleged that he publishes "would-be scientific papers," "feels no shame in declaring the discoveries of his students to be his own achievements," stands close to the ideology of the "black hundred", orthodoxy, and monarchy "fascist-type modernized but slightly." All Russian scientists of the elder generation knew about the *Pravda* editorial and the savage dissolution of "luzinism." The newly-published archive files open to the public that some students of Luzin were the active participants of the political assault on their teacher. The key role was played by P. S. Aleksandroff who headed the Moscow topological school. Also active at the meetings of the Commission were A. N. Kolmogorov, L. A. Lyusternik, A. Ya. Khinchin, and L. G. Shnirelman. The political attacks on Luzin were vigorously supported by members of the Commission S. L. Sobolev (1907–1989) and O. Yu. Schmidt (1891–1956). A. N. Krylov (1863–1945) and S. N. Bernstein revealed valor in the vigorous defence of Luzin. The final clause of the official Resolution of the Commission read as follows: "Everything of the above, summarizing the overwhelming material evidence in possession of

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I am very grateful to W. A. J. Luxemburg for attracting my attention to the inadvertent omission in the preprint of this article of a reference to the revealing article [2] by G. G. Lorentz (1910–2006). Also, I acknowledge that this essay summarizes my position at the informal lobby discussions at the Mini-Simposium on Convex Analysis which was held at Lomonosov Moscow State University, February 2-4 (2007). I am especially grateful to Professor V.M. Tikhomirov, Chairman of the Mini-Simposium, for endurance, friendliness, and hospitality.

the Academy of Sciences, completely ascertains the characteristics of Luzin in the *Pravda* newspaper."

All participants of the events of 1936 we discuss had left this world. They seemingly failed to know that the files of the Commission are all safe and intact. Today we are aware in precise detail of what happened at the meetings of the Commission and around the whole case. The mathematical community painfully reconsiders the events and rethinks the role of the students of Luzin in his political execution.

P. S. Novikov and M. A. Lavrentiev were not listed as participants of the public persecution of Luzin (despite the fact that both were mentioned at the meetings of the Commission among the persons robbed by Luzin). It transpires now why M. A. Lavrentiev was the sole author of a memorial article in *Russian Mathematical Surveys* on the occasion of the 90th anniversary of the birth of Luzin. He also included this article in the collection of his papers on the general issues of science and life [3, 4]. M. A. Lavrentiev was the chairman of the editorial board of the selected works of Luzin which were published by the decision of the Academy of Sciences of the USSR after the death of Luzin on the occasion of the 70th anniversary of the birth of Luzin. P. S. Aleksandroff and A. N. Kolmogorov were absent from the editorial board.

Practically the same are the comments on their relationship with Luzin which were left by P. S. Aleksandroff and A. N. Kolmogorov. Their statements are still shared to some extent by their numerous students. It is customary to emphasize that Luzin was not so great a mathematician as his students that had persecuted him. Some moral fault is persistently incriminated to Luzin in the untimely death of M. Ya. Suslin (1894–1919) from typhus fever. Luzin is often blamed for all his disasters at least partly. He is said to deserve all punishments and if not all then it is not his students' fault but stalinism and the curse of the epoch. These arguments reside in the minds of not only the elders but also the youngsters. The best of them view the Luzin case as the mutual tragedy of all participants.

However, we should distinguish the personal tragedy of Luzin from the tragedy of the Moscow school and the tragedy of the national mathematical community. The students of Luzin who participated in the persecution of the teacher never considered their own fates tragical.

P. S. Aleksandroff wrote in his reminiscences [6]:

"Knowing Luzin in his green creative years, I got acquaintance with a truly inspired teacher and scholar who lived only by science and in the name of science. I met a person who resided in the sphere of the sublime human treasures which is forbidden for any rotten ghost or spirit. When a human being leaves this sphere (and Luzin had left it once), he is doomed to surrender to the forces that were described by Goethe as follows:

> Ihr führt in's Leben uns hinein,
> Ihr lasst den Armen Schuldig werden
> Dann überlasst Ihr ihn der Pein,
> Denn jede Schuld rächt sich auf Erden.
> Into our life you lead us in,
> The wretch's guilt you bring to birth,
> Then bring affliction down on sin,
> For all guilt takes revenge on Earth.

In his terminal years Luzin saw the bottom of the sour bowl of the revenge that was described by Goethe."[1]

It is worth observing that Khinchin, hostile to Luzin, commented on the accusations that Luzin drove Suslin to death [5]: "Suslin is called the student perished by N. N. Luzin. Why, when a man dies from typhus fever this is a rather exaggerated expression. In fact Suslin could possibly get typhus fever in Ivanovo. Furthermore, in the common opinion it was N. N. who tried and expelled Suslin from Ivanovo. However, the transfer from Moscow to Ivanovo I view as a favor to Suslin who was not hostile to Luzin in those days."

Narrating his reminiscences of P. S. Aleksandroff, A. N. Kolmogorov told in 1982 [7]: "My entire life as a whole was full of happiness." Neither he nor Aleksandroff nor other participants of the persecution of Luzin had ever treated the "case of Luzin" as a common tragedy with Luzin. They were correct in this judgement but on the grounds completely different from those they declared.

If Luzin were guilty then his fault would belong to the sphere of the personal mathematical relations between a teacher and a student. No convincing evidence of Luzin's plagiarism was ever submitted. The alleged accusations that he ascribed to H. Lebesgue (1875–1941) or kept a grip of Suslin's results are poorly disguised and baseless. To prove the scientific misconduct of Luzin it was alleged that Luzin played the underdog and flattered Lebesgue by attributing to Lebesgue his sieve method. On the other hand, Lebesgue wrote in his preface to the Luzin book on analytic sets as follows: "Anyone will be astonished to find out from Luzin's book that I had incidentally invented the sieve method and was the first to construct an analytic set. However, nobody could be more amazed than me. Mr. Luzin feels himself happy only when he has managed to ascribe his own discoveries to someone else" [8]. The students were "more pious than the Pope."

It is easy to assume the genuine or imaginary injustice and prejudice of Luzin in citing his students as well as the genuine or imaginary feebleness of Luzin in overcoming mathematical obstacles. We may agree to see hypocrisy in Luzin's decision to vote against P. S. Aleksandroff in the elections to a vacancy of an academician despite his personal letter of support of Aleksandroff to A. N. Kolmogorov. Well, there is nothing untypical of the academic manners or extraordinary in Luzin's conduct, is there? It is the true background of the "case of Luzin," isn't it?

Available is the following testimony of W. Sierpinski (1882–1969), a famous Polish mathematician who was declared to be a "blatant black hundredist" at the meetings of the Commission of the Academy of Sciences of the USSR on the "Case of Luzin": "When I was in Moscow in September, 1935, Mr. Aleksandroff assured me that the apprehensions of Luzin are purely imaginary and that he respects Luzin, his former teacher. In my presence Aleksandroff shook hands with Luzin and declared that he would always be a friend of Luzin" [9].

The pretentious reconciliation of P. S. Aleksandroff with Luzin which was described by Sierpinski and which was later publicly refuted by P. S. Aleksandroff is in no way similar to the refusal of Luzin to support the election of P. S. Aleksandroff as an academician, isn't it? It is in general belief that this refusal was the reason for A. N. Kolmogorov to slap the face of Luzin publicly in 1946. Luzin

[1] Aleksandroff cited the poem *Harfenspieler* dated as of 1795 by Johann Wolfgang von Goethe (1749–1832) and gave a rough translation into Russian. The lines in English here belong to Vernon Watkins (1906–1967).

was twenty years older than A. N. Kolmogorov. Luzin was a teacher of A. N. Kolmogorov and carried the heavy burden of political accusations that were imposed on Luzin with participation of P. S. Aleksandroff and A. N. Kolmogorov. Luzin was granted "mercy" and accepted at the country house of A. N. Kolmogorov and P. S. Aleksandroff in Komarovka before the elections.[2] Everyone at the meeting remembered the most important matter that Luzin was victimized and must surrender to the noble victors, didn't he? It transpires now, doesn't it? We can compare the internal academic matters, say Luzin's misconduct and even plagiarism, with the accusations of subversive activities against the Soviet life, can't we?

These grave and vexed questions...

I must emphasize explicitly that in my opinion all moral accusations against Luzin are absolutely inconvincible. That which was submitted as proofs was inadequate even in the times of the Commission neither for P. L. Kapitsa (1894–1984), nor V. I. Vernadsky (1863–1945), nor A. Denjoy (1884–1974), nor Lebesgue, nor many other elder persons.

The objection of Kapitsa was expressed on July 6 in his letter to V. M. Molotov who was the Chairmen of the Council of the People's Commissars of the USSR. Vernadsky wrote in his diary on the next day "Letters to Luzin, Chaplygin, and Fersman about him. Majority treats as demonstrated the slander and insinuations.

[2] V. M. Tikhomirov wrote about the meeting in Komarovka: "The correspondence of L. S. Pontryagin and his student and friend I. I. Gordon reveals that Luzin was accepted and served a meal in Komarovka" [7, p. 83]. The relevant excerpt of a letter of Pontryagin of December 24, 1946 [13, Letter No. 49] reads as follows: "You are interested in a joint work of Kolmogorov and Luzin. This should be narrated rather than written since the voice is needed to express everything fully. Kolmogorov told me in summer that his only inconvenience as regards the election of Aleksandroff is the fact that Aleksandroff had become an indisputable candidate four months before the voting. Pusics [= the collective nickname of Kolmogorov and Aleksandroff (**S. K.**)] made an enormous preliminary work in the sense of entering into various agreements with academicians. For instance, there was a promise to Vinogradov to support Lavrentiev in reward for Vinogradov's support of Aleksandroff. It seemed indeed that everyone will vote for Aleksandroff. For example, Bernstein himself nominated Aleksandroff at a meeting of the institute; well, in actuality, he nominated Chebotarev too. Kolmogorov had reached an agreement with the bosses that he would be nominated to the expert commission. The first glimpse of disappointment was the fact that he was not nominated to the commission. However, he hoped that this was not very important. After the session of the expert commission there were a few closed meetings of academicians in which they discussed all candidates. It was at this stage that Kolmogorov became aware that none of the members of the expert commission supported Aleksandroff. Moreover, Bernstein vehemently objected and said that Aleksandroff had a harmful area of research. The behavior of Bernstein seems far from comprehensible to me by now; maybe, he simply had a quarrel with Pusics. All the rest is rather clear. Lavrentiev turned out somehow to be an indisputable candidate and needed no support from Kolmogorov who was out off the commission at that. Therefore, Vinogradov needed neither Kolmogorov nor Pusic. As regards Sobolev and Khristianovich, the former hates Pusic [= the nickname of Aleksandroff (**S. K.**)] for Sobolev's dismissal from the directorship; while the latter is Sobolev's friend and crony. In these circumstances, there was no hope of success. The only possibility remained that some mathematicians among academicians would support Aleksandroff; physicists wanted to support him, but they surely could never try to confront all mathematicians. Luzin became the hope of Pusics. He was invited to Komarovka and promised his support. However, he spoke against Aleksandroff at the final closed meeting. Departing from this meeting, Kolmogorov was absolutely upset and stung. He came to Luzin and said that he would have nothing in common with Luzin ever since. Luzin pretended that he did not understand anything and began to talk as follows: 'Dear me, calm down. Forget it. You are ill. Relax.' This is what must be narrated with expression. Kolmogorov then answered him: 'So what shall I do to you: spit at your physiognomy or slap your mug?' After a short thought, he dared the latter."

M[ay] b[e], he [is needed] abroad but not at home. I am afraid that this disgusting article will affect him much. Many conversations and many impressions." On the same day he sent a letter to Academician A. E. Fersman (1883–1945), a member of the Commission. Vernadsky wrote: "I think that such an episode would eventually be perilous to the Academy were it led to the expulsion of N. N. [Luzin] from the Academy or any similar actions. We would slide down the slippery slope" [10].

Lebesgue's letter of August 5, 1936 is in order now. I remind that Lebesgue was elected in 1929 to the Academy of Sciences of the USSR for his outstanding contribution to mathematics. The great Lebesgue, the author of that very "Lebesgue integral" which is indispensable in modern mathematics, was in the state of utmost indignation and anger. He wrote: "You will see that it was not yesterday when the attacks on Luzin began with the aim of firing him and emptying place for Aleksandroff. You will see there that I was already mixed in this by contrasting 'my' science, which is bourgeoise and useless, to *analysis situs* [topology], a proletarian and useful science. Since the former was the science of Luzin; whereas the latter, the science of Aleksandroff. What is curious is that he begins as Urysohn whose papers he inherited at the same starting point that was mine. With the only difference that Urysohn cited me whereas Aleksandroff has never cited me anymore since he must now speak badly of me in his struggle against Luzin!" [9].

Another evidence of Sierpinski: "I share the opinion and the same opinion is shared by my Polish colleagues that the presence of Aleksandroff, Khinchin, Kolmogorov, and Shnirelman who confronted their former teacher in the most dishonest manner and slanderously accused him is intolerable at any meeting of decent persons" [9].

The method of political insinuations and slander was used against the old Muscovite professorship many years before the article in *Pravda*. The declaration of November 21, 1930 of the "initiative group" of the Moscow Mathematical Society which consisted of L. A. Lyusternik, L. G. Shnirelman, A. O. Gelfond (1906–1968), and L. S. Pontryagin (1908–1988) claimed that "there appeared active counterrevolutionaries among mathematicians" [5]. Some of these were pointed out, namely, D. F. Egorov (1869–1931), a teacher of Luzin. Shortly before Egorov had been arrested, and Luzin decided it wise to leave the university (he was later accused of this removal by his students). In his life's-description, dated as of the late 1970s, Academician Pontryagin wrote [11]: "The two public actions, in 1936 as regards Luzin and in 1939 as regards elections, were the important stages of my uprising as a public person. In my opinion both were the struggle for rightful ends."

This is inconsistent with the position of Luzin who wrote in his letter of 1934 to L. V. Kantorovich (1912–1986) after the ugly declaration signed by Gelfond that his choice in Moscow for the forthcoming election of corresponding members of the Academy "will be Gelfond who has recently made a discovery worthy of a genius" [12].

A broad campaign against Luzin and "luzinism" waged over this country in 1936. Fortunately, Luzin was not repressed nor expelled from the Academy. Some historians opine that there was a relevant oral direction of Joseph Stalin.[3]

[3]It was disclosed recently that the above-mentioned letter of Kapitsa to Molotov was multiplied in 16 copies for the members of the Political Bureau of the All-Union Communist Party (Bolsheviks) and discussed over with other letters in support of Luzin.

However, the badge of an enemy under the mask of a Soviet citizen was pinpointed to Luzin during 14 years up to his death. The monstrosity over Luzin is absolutely incomparable with the alleged accusations of moral misconduct.

The human passions and follies behind the 1930s tragedy of mathematics in Russia are obvious. But was there a mathematical background? Some roots are visible.

We are granted the blissful world that has the indisputable property of unicity. The solitude of reality was perceived by our ancestors as the ultimate proof of unicity. This argument resided behind the incessant attempts at proving the fifth postulate of Euclid. The same gives grounds for the common search of the unique best solution of any human problem.

Mathematics has never liberated itself from the tethers of experimentation. The reason is not the simple fact that we still complete proofs by declaring "obvious." Alive and rather popular are the views of mathematics as a toolkit for natural sciences. These stances may be expressed by the slogan "mathematics is experimental theoretical physics." Not less popular is the dual claim "theoretical physics is experimental mathematics." This short digression is intended to point to the interconnections of the trains of thought in mathematics and natural sciences.

It is worth observing that the dogmata of faith and the principles of theology are also well reflected in the history of mathematical theories. Variational calculus was invented in search of better understanding of the principles of mechanics, resting on the religious views of the universal beauty and harmony of the act of creation.

The twentieth century marked an important twist in the content of mathematics. Mathematical ideas imbued the humanitarian sphere and, primarily, politics, sociology, and economics. Social events are principally volatile and possess a high degree of uncertainty. Economic processes utilize a wide range of the admissible ways of production, organization, and management. The nature of nonunicity in economics transpires: The genuine interests of human beings cannot fail to be contradictory. The unique solution is an oxymoron in any nontrivial problem of economics which refers to the distribution of goods between a few agents. It is not by chance that the social sciences and instances of humanitarian mentality invoke the numerous hypotheses of the best organization of production and consumption, the most just and equitable social structure, the codices of rational behavior and moral conduct, etc.

The twentieth century became the age of freedom. Plurality and unicity were confronted as collectivism and individualism. Many particular phenomena of life and culture reflect their distinction. The dissolution of monarchism and tyranny were accompanied by the rise of parliamentarism and democracy. Quantum mechanics and Heisenberg's uncertainty incorporated plurality in physics. The waves of modernism in poetry and artistry should be also listed. Mankind had changed all valleys of residence and dream.

In mathematics the quest for plurality led to the abandonment of the overwhelming pressure of unicity and categoricity. The latter ideas were practically absent, at least minor, in Ancient Greece and sprang to life in the epoch of absolutism and Christianity. Cantor was a harbinger of mighty changes, claiming that "das *Wesen* der *Mathematik* liegt gerade in ihrer *Freiheit*." Paradoxically, the resurrection of freedom expelled mathematicians from Cantor's paradise.

Nowadays we are accustomed to the unsolvability and undecidability of many problems. We see only minor difficulties in accepting nonstandard models and modal logics. We do not worry that the problem of the continuum is undecidable within Zermelo–Fraenkel set theory. However simple nowadays, these stances of thought seemed opportunistic and controversial at the times of Luzin. The successful breakthroughs of the great students of Luzin were based on the rejection of his mathematical ideas. This is a psychological partly Freudian background of the case of Luzin. His gifted students smelled the necessity of liberation from description and the pertinent blissful dreams of Luzin which were proved to be undecidable in favor of freedom for mathematics. His students were misled and consciously or unconsciously transformed the noble desire for freedom into the primitive hatred and monstrosity. This transformation is a popular fixation and hobby horse of the human beings through the ages.

Terrible and unbearable is the lightheaded universal fun of putting blame entirely on Luzin for the crimes in mathematics in which he was hardly guilty with the barely concealed intention to revenge his genuine and would-be private and personal sins. We should try and understand that the ideas of description, finitism, intuitionism, and similar heroic attempts at the turn of the twentieth century in search of the sole genuine and ultimate foundation were unavoidable by way of liberating mathematics from the illusionary dreams of categoricity. The collapse of the eternal unicity and absolutism was a triumph and tragedy of the mathematical ideas of the first two decades of the last century. The blossom of the creative ideas of Luzin's students stemmed partly from his mathematical illusions in description.

The struggle against Luzin had some mathematical roots that were impossible to extract and explicate those days. We see clearly now that the epoch of probability, functional analysis, distributions, and topology began when the idea of the ultimate unique foundation was ruined for ever. Gödel had explained some trains of thought behind the phenomenon, but the mathematicians par excellence felt them with inborn intuition and challenge of mind.

It is the tragedy of mathematics in Russia that the noble endeavor for freedom had launched the political monstrosity of the scientific giants disguised into the cassocks of Torquemada.

History and decedents are out of the courts of justice. Scientists and ordinary persons must see and collect facts. Never accuse the passed away, but calmly and openly point out that which was in reality. Explain the difference between moral accusations and political insinuations to the youth. Demonstrate the difficulty and necessity of the repairing of mistakes and repentance. Show how easy it is to forgive oneself and accuse the others.

We must work out and transfer to the next generations the objective views of the past. Of its successes and tragedies. With love and doubts, with the understanding of our unfortunate fate and the honor of objectivity. It is the personal faults and failures that we are to accuse and repair first of all. They knew even in Ancient Rome that we should tell nothing or good about the dead. Facts did never pass away. Luzin was accused by the Moscow Mathematical Society and the Academy of Sciences. These scientific institutions are alive.

Any attempt at discerning morality in the past immorality is dangerous since it feeds this immorality by creating the comfortable environment of immorality in the present and future. The stamina of a scientist by belief is a discontinuous function.

Evil and genius coexist from time to time. Mathematics does not inoculate morality. Manuscripts do not burn. . . .

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Sobolev Institute of Mathematics
4 Koptyug Avenue
Novosibirsk, 630090 RUSSIA
E-mail address: sskut@member.ams.org