Power and Subcarrier Allocation Scheme for Uplink OFDMA

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Abstract—In this contribution, we propose a power and subcarrier allocation scheme for uplink orthogonal frequency division multiple access (OFDMA) based on the capacity difference between different users. The scheme works iteratively and performs three major tasks. In the first step, the subcarrier with largest capacity difference of two active users is found. Then, the subcarrier selection of these two users is altered, with subsequent power allocation over their active carriers. During the last step, the subcarrier of the user which maximizes the sum-rate is kept active. As one user is deselected per subcarrier during each iteration, the algorithm will stop after \( M \cdot (K - 1) \) iterations. Furthermore, we will compare our scheme with some known schemes from the literature.

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) is one major access technique in wireless communication. It is adopted in several standardized technologies, such as WiMAX or 3GPP LTE. Some of the advantages of OFDMA are flexibility, for example in assigning subcarriers to users in combination with power allocation, improved diversity and simple equalization in the frequency domain. In OFDMA the access of the channel is either in the downlink (broadcast channel) or uplink (multiple-access channel) direction. As opposed to downlink OFDMA, optimal subcarrier and power allocation in uplink OFDMA is not solvable in polynomial time, due to the distributive power constraints of the different users. Therefore all combinatorial subcarrier assignments, including the corresponding optimal power allocations needs to be carried out, in order to find the maximum sum-rate.

In [1], the authors address the sum-rate optimality of uplink OFDMA. Here, the conditions and the probability of OFDMA being sum-rate optimal is derived and the performance gap of OFDMA being suboptimal versus optimal multicarrier multiple access solution is investigated. This is preconditioned by allowing the users sharing the same subcarriers and under the constraint of superposition coding and successive interference decoding. The authors in [2], [3] and [4] propose a greedy subcarrier allocation scheme in combination with an iterative power allocation algorithm based on water-filling. Here, the subcarriers are selected according to the marginal rate function. Basically the same approach is taken in [5], where the decision is taken according to a utility function to achieve max-min fairness. In [6] the allocation problem is solved by relaxing the integer constraint resulting from choosing only one user per subcarrier. Besides all these heuristic approaches, the authors in [7] and [8] tackle the allocation problem with the help of game theory.

In this contribution, we address the problem of subcarrier and power allocation in an uplink OFDMA system. As the exhaustive search (ES) for finding the optimal solution is only feasible for a small number of users and subcarriers, we introduce a simple, yet powerful algorithm which performs close to the optimum.

The main difference between the proposed algorithm in this contribution and the known ones from literature is the idea to select the subcarrier which has the maximal difference in capacity between all users and then deselecting the user on that specific subcarrier with the least sum-rate. Here, the probability that this user will achieve a higher capacity on that specific subcarrier during the optimization process is low, as the user has to spend more power for the subcarrier, which has to be taken from the others. Still the probability of a suboptimal selection increases as the capacity differences decreases and users on a specific subcarrier have nearly the same capacities.

The paper is organized as follows. Section II introduces the system model and states the optimization problems for optimal subcarrier and power allocation. Then, in Section III, the proposed suboptimal allocation scheme is illustrated, followed by the briefly description of some other allocation schemes taken from literature in Section IV. Afterwards, the complexity analysis is performed in Section V. The paper finishes with the conclusions in Section VII, after showing some simulation results in Section VI.

Throughout this paper, italic upper case letters denote elements of matrices, which are given by bold upper case letters. Vectors and its elements are characterized by bold and italic lower case letters. The transpose of a vector is specified by \((\cdot)^T\). Furthermore, blackboard bold letters are used for describing sets.

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II. SYSTEM MODEL

The uplink OFDMA system under investigation consists of \( K \) users \( k \in \mathbb{K} = \{1, \ldots, K\} \), which are simultaneously accessing the channel to communicate with the receiver. In order to achieve orthogonal access and to avoid superposition of different users, each subcarrier \( m \in \mathbb{M} = \{1, \ldots, M\} \) is occupied by only one user. Thus, the subcarrier location for each user is stored in a user specific set \( \mathbb{M}(k) \), which have to fulfill the following condition of \( \mathbb{M} = \bigcup_{k \in \mathbb{K}} \mathbb{M}(k) \) and \( \mathbb{M}(k) \cap \mathbb{M}(l) = \{\} \) for \( k \neq l \) and \( \forall k, l \in \mathbb{K} \).

Given the Gaussian distributed transmit information \( x(k) \) of user \( k \) with size \( (|\mathbb{M}(k)| \times 1) \) and the user specific binary subcarrier allocation matrix \( W(k) \) of size \( (M \times |\mathbb{M}(k)|) \), the received signal \( y \) in the frequency domain is calculated by

\[
y = \sum_{k \in \mathbb{K}} H(k)^{\top} P(k) W(k) x(k) + n ,
\]

assuming all users are perfectly synchronized in time and frequency. In (1), \( n \) has size \( (M \times 1) \) and denotes the additive white Gaussian noise (AWGN) term with \( n_m \sim \mathcal{CN}(0, \sigma_N^2) \). Both, the user specific channel transfer function \( H(k) = \text{diag}\{h(k)\} \) and the power coefficient matrix \( P(k) = \text{diag}\{p(k)\} \) are diagonal matrices of size \( (M \times M) \) with elements \( h_m(k) \) and \( p_m(k) \) on the main diagonal. For the later mentioned, the condition \( p_m(k) = 0 \) for \( \forall m \notin \mathbb{M}(k) \) and \( \forall k \in \mathbb{K} \) holds.

The elements of the complete power allocation matrix \( P = [(p^{(1)})^T, \ldots, (p^{(K)})^T] \) are constrained by the maximum available power for a specific user, which is given by

\[
\sum_{m \in \mathbb{M}(k)} p_m(k) \leq p_{\text{total}}(k) .
\]

Assuming a normalized symbol alphabet with \( \mathbb{E}[x_m(k)^2] = 1 \), the resulting receive signal-to-noise ratio (SNR) at the destination consist of the individuals SNRs

\[
\gamma_m(k) = \frac{p_m(k) |h_m(k)|^2}{\sigma_N^2} ,
\]

of user \( k \) on subcarrier \( m \).

In order to gain the complete subcarrier assignment matrix \( W = [(w^{(1)})^T, \ldots, (w^{(K)})^T] \) of size \( (M \times K) \), the user specific binary subcarrier assignment vectors \( w_m(k) \) are created according to \( w_m(k) = \sum_{l=1}^{L_m} W_{m,l} \), \( \forall m \in \mathbb{M} \).

Therewith, each column of \( W \) contains the subcarrier assignment vector of a single user. Furthermore, the condition \( W_{m,k} = 1, \forall m \in \mathbb{M}(k), \forall k \in \mathbb{K} \) holds, i.e. \( W \) contains the subcarrier selection \( \{0, 1\} \) (off/on), with each row only occupied by a single (on)-element.

A. General optimization problem

In this subsection we recapitulate the general optimization problem for uplink OFDMA, for example as described in [3]:

\[
R^{\text{max}}(W, P) = \sum_{k \in \mathbb{K}} \sum_{m \in \mathbb{M}} w_{m,k}^{(k)} \log_2 \left( 1 + \frac{p_{m,k}^{(k)} |h_{m,k}^{(k)}|^2}{\sigma_N^2} \right)
\]

subject to

\[
\sum_{k \in \mathbb{K}} w_{m,k}^{(k)} = 1, \forall m \in \mathbb{M} \quad (5)
\]

\[
\sum_{m \in \mathbb{M}(k)} p_{m,k}^{(k)} \leq p_{\text{total}}, \forall k \in \mathbb{K} \quad (6)
\]

\[
p_{m,k}^{(k)} \geq 0, \forall k \in \mathbb{K}, \forall m \in \mathbb{M} \quad (7)
\]

\[
w_{m,k}^{(k)} \in \{0, 1\}, \forall k \in \mathbb{K}, \forall m \in \mathbb{M} . \quad (9)
\]

Since each user has its own power constraint, the allocation problems intertwine [3] each other. In order to solve the optimization problem of (4), all \( K^M \) subcarrier assignment (SCA) matrices \( W \) fulfilling the constrains (6) and (9) have to be evaluated together with optimal power allocation (PA) per assignment. Afterwards, the combination \( (W, P) \) is chosen which maximizes the sum-rate. As the complexity of the exhaustive search (ES) grows exponentially with the number of users and subcarriers, other suboptimal but simpler algorithms have to be derived for solving the uplink OFDMA power and subcarrier allocation problem.

B. Optimal and uniform power allocation

For a given subcarrier allocation for user \( k \), the optimal power allocation can be explicitly found with the water-filling (WF) algorithm, as the objective function is jointly concave [9]. Thus, the optimal power allocation is solved utilizing the Lagrangian method

\[
\mathcal{L}(\lambda, P) = \sum_{m \in \mathbb{M}(k)} w_{m,k}^{(k)} \log_2 \left( 1 + \gamma_{m,k}^{(k)} \right) - \lambda \sum_{m \in \mathbb{M}(k)} w_{m,k}^{(k)} p_{m,k}^{(k)} , \quad (10)
\]

with the Lagrangian multiplier \( \lambda \) for constraint (7) and (8). Choosing the Lagrange multiplier \( \lambda \) such that the power constraint is met, the following solution is obtained

\[
p_{m,k}^{(k)} = \max \left( 1 - \frac{\sigma_N^2}{|h_{m,k}^{(k)}|^2}, 0 \right) . \quad (11)
\]

As the above optimal power allocation solution is a time intensive task, simpler but non-optimal allocation schemes, such as the uniform power allocation, might be worth for consideration. Here, the power is uniformly distributed among the active carriers of a user, according to

\[
p_{m,k}^{(k)} = \frac{1}{|\mathbb{M}(k)|} p_{\text{total}} . \quad (12)
\]
After the power allocation is performed, the sum-rate for user $k$ is given as
\[
R^{(k)}(W, P) = \sum_{m \in \mathcal{M}^{(k)}} w_m^{(k)} \log_2 \left(1 + \gamma_m^{(k)}\right),
\] (13)
and the overall sum-rate results to
\[
R^{\text{sum}} = \sum_{k \in \mathbb{K}} R^{(k)}(W, P).
\] (14)

C. Subcarrier selection

For a given power allocation, user $k$ obtains subcarrier $m$ if it maximizes
\[
k = \arg\max_l \left\{\log_2 \left(1 + \gamma_m^{(k)}\right)\right\}.
\] (15)
This solution is then stored in the user specific column of the subcarrier allocation matrix $W$.

III. PROPOSED ALGORITHM

The proposed subcarrier and power allocation (PA) scheme is depicted in Algorithm 1. During the initialization phase, all subcarriers are assigned to every user. Then water-filling (10), for optimal power allocation, or uniform power allocation (12) is performed per user for the given set of utilized subcarriers. After the power allocation, the algorithm searches for the subcarrier $m$ with the highest capacity difference between two users $k_1, k_2 \in \mathbb{K}$. Then, the algorithm assigns subcarrier $m$ to user $k_1$, i.e. $w_m^{(k_1)} = 1$ and disables $m$ for user $k_2$ by setting $w_m^{(k_2)} = 0$. Afterwards, power allocation is performed for user $k_1$ and user $k_2$ in order to calculate the sum-rate $R^{\text{sum}} = R^{(k_1)} + R^{(k_2)}$ with (13) for both users. Once, this is done, the assignment of $w_m^{(k_1)}$ and $w_m^{(k_2)}$ is toggled and the resulting sum-rate after a new PA is compared with $R^{\text{sum}}$. In the last step, the algorithm keeps the subcarrier assignment (SCA) and the corresponding PA, which maximizes the sum-rate. The procedure continues by searching the next largest capacity difference and stops until all subcarriers have been assigned to exactly one user.

In the beginning of the allocation scheme, the capacity difference between two selected users is large and deselecting the user with less sum-rate would be the best choice. With each deselected subcarrier per user, the available power concentrates on less subcarriers, which increases their capacities. Therefore, as the algorithm advances, the capacity differences decrease and intuitively the probability of deselecting the subcarrier which would eventually maximize the sum-rate increases. To lower the probability of a suboptimal decision, subcarrier switching between two users is carried out with subsequent power allocation.

IV. OTHER SUBOPTIMAL ALLOCATION SCHEMES

In order to evaluate the performance of the proposed algorithm, we compare it with different schemes taken from literature [10] and [2].

Two basic approaches with low complexity are SCA before PA and SCA after PA. In case of SCA before PA, the subcarriers are selected using (3) with uniform PA at the beginning. Then, $w_m^{(l)} = 1$ is set for user $k$ with the highest SNR on subcarrier $m$, disabling all other users on that subcarrier with $w_m^{(l)} = 0$, $l \in \mathbb{K}$, $l \neq k$. Afterwards, optimal or uniform power allocation is carried out.

For SCA after PA, the optimal power allocation with the water-filling algorithm is performed with all subcarriers enabled for every user. Afterwards, the subcarrier $m$ for user $k$ with the highest capacity is selected and deselecting all other users for $m$.

The second approach is also taken for the improved greedy algorithm [10] during the initialization phase. Next, the algorithm sets $w_m^{(k)} = 1$, $k \in \mathbb{K}$ with $w_m^{(l)} = 0$, $l \in \mathbb{K}$, $l \neq k$ for every subcarrier $m$. So, each user is enabled once per subcarrier (disabling all others) with subsequent sum-rate (14) calculation for the corresponding selection. The user-selection, which achieves maximum sum-rate is kept. This process is iteratively performed until no further improvement in $R^{\text{sum}}$ is obtained.

Another greedy algorithm, called MaxRT+WF/EQ (maximum marginal rate subcarrier allocation and water-filling or equal PA), is developed by the authors in [2]. All subcarriers are deselected during the initialization. Then, uniform or optimal power allocation is carried out for each unallocated subcarrier $m$ and each user $k$, before selecting the user and subcarrier which maximizes (15). The algorithm performs iteratively until all subcarriers have been assigned.

V. COMPLEXITY ANALYSIS

The worst-case complexity of the proposed algorithm is affected by two parts, i.e. the search operations and subsequently the power allocation. For the initialization phase in line 1 of Algorithm 1, the worst-case complexity of $O(KM)$ results from the power allocation and the
calculation of the subcarrier capacities. After the initialization, the while-loop is performed with \( M \cdot (K - 1) \) iterations. This comes from the fact, that during each iteration exactly one subcarrier is deselected for only one user. At line 3, every subcarrier is searched for the strongest and weakest active user in terms of capacity. This has complexity \( O(M \cdot 2K) \). Furthermore, the largest difference in capacity is searched for, which usually has complexity \( O(M) \) but can be directly performed during the search operation in line 4. The last step, relevant for the complexity analysis, is the power allocation in line 4, which is performed 4-times with complexity \( O(M) \) each time. The overall worst-case complexity results to \( O(K M^2) \) and will be dominated by \( M \), since one can assume that there are more subcarriers than users, i.e. \( M \gg K \).

In Table I, the complexity of different allocation schemes, which have been described in Section IV, are depicted for comparison.

<table>
<thead>
<tr>
<th>Method</th>
<th>Complexity</th>
<th>Lit.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exhaustive Search</td>
<td>( O(K^M) )</td>
<td>-</td>
</tr>
<tr>
<td>Proposed alg.</td>
<td>( O(K^2 M^2) )</td>
<td>-</td>
</tr>
<tr>
<td>Improved greedy</td>
<td>( O(K^2 M^2) )</td>
<td>[10]</td>
</tr>
<tr>
<td>MaxRT + WF</td>
<td>( O(K M^2) )</td>
<td>[2]</td>
</tr>
<tr>
<td>SCA after PA</td>
<td>( O(K M) )</td>
<td></td>
</tr>
<tr>
<td>SCA before PA</td>
<td>( O(K M) )</td>
<td></td>
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</tbody>
</table>

**TABLE I**

**COMPLEXITY FOR DIFFERENT ALLOCATION SCHEMES WITH OPTIMAL PA (WATER-FILLING)**

In the first two figures, we compare the proposed algorithm together with the other approaches from literature with the optimal solution found by exhaustive search (ES). Due to the complexity of \( O(K^M) \) for ES, only \( K = 4 \) users and \( M = 6 \) subcarriers have been considered, leading to 4096 possible SCA’s with corresponding optimal power allocation per SCA.

In Figure 1, the sum-rates \( R_{sum} \) for a simulation environment of \( K = 4 \) users and \( M = 6 \) subcarriers are compared. For the simulation, the implementation [11] of the 3GPP spatial channel model [12], utilizing the suburban macro channel scenario with macrocell path-loss, based on the modified COST231 Hata urban propagation model was used. The different users have been randomly placed and evenly distributed in a radius of 500m around the base station. Furthermore, the average path loss is normalized to unity, such that the mean received SNR at the base station is equal for all users. All 1000 simulated channels are frequency selective and uncorrelated among the users. As the result in figure 1 shows, all allocation algorithms perform equally well with a minor performance gain for the proposed allocation scheme. Here, the corresponding sum-rate \( R_{sum} \) is very close to \( R_{max} \) from the ES solution.

In Figure 2 the normalized number of differently assigned subcarriers in the SCA matrix and the difference

**Fig. 1.** Sum-rates for the different allocation schemes, including the exhaustive search solution

**Fig. 2.** Subcarrier assignment and relative sum-rate difference for the different schemes, compared with the exhaustive search solution
in relative sum-rate compared with the ES solution are depicted. The first mentioned is calculated by comparing the complete subcarrier assignment $W^{ES}$, resulting from the exhaustive search solution, with $W$, taken from the different algorithms. Thus, the normalized SCA difference yields to

$$P_{SCA\text{-diff.}} = \frac{1}{M} \sum_{m \in M} b_m,$$

(16)
given vector $b$ with elements

$$b_m = \begin{cases} 1, & \text{if } W^{ES}_m (w_m)^T = 0 \\ 0, & \text{else} \end{cases}$$

and $W^{ES} = [W^{ES}_{m,1}, \ldots, W^{ES}_{m,K}]$, $w_m = [W_{m,1}, \ldots, W_{m,K}]$.

Although the proposed algorithm starts with a high difference in the SCA at the low SNR regime, the sum-rate is closer to the ES solution as compared with the other algorithms. Therefore, a SCA solution close to the optimum does not guarantee a better rate performance. This can be interpreted as follows: There exist subsets $M^{(k)}$ for which an assignment can be changed with only minor effect on the sum-rate $R_{\text{sum}}$.

Figure 3 gives an indication of why deselecting the user with less sum-rate is a good choice in the proposed scheme. As the algorithm advances, the capacity difference between two selected users decreases and eventually the number of suboptimal selections, compared with the exhaustive search solution, will increase.

Figure 4 shows the sum-rate for a higher number of users and subcarriers, i.e. $K = 20$ and $M = 64$, but for the same channel parameters as for figure 1. Again, the proposed method achieves a slightly better performance compared to the improved greedy algorithm, followed by MaxRT+WF. The sum-rate difference for the SCA before and after optimal power allocation schemes decreases as the SNR increases. Furthermore, the sum-rates for all schemes diverge as compared to figure 1. This is due to the increasing number of subcarrier allocation possibilities and hence an increasing number of suboptimal decisions.

Figure 5 depicts the normalized difference in sum-rate for all sub-optimal allocation schemes between optimal power allocation (10) and uniform PA (12). It is shown, that the difference in the sum-rate decreases as the SNR increases, which coincides with [13]. For example, carrier allocation after PA has a loss in $R_{\text{sum}}$ of approx. 42% at $-10$dB, whereas the improved greedy algorithm already achieves nearly the same performance with uniform PA as with optimal PA, even in the lower SNR regime. So, for high SNR, uniform power allocation should be considered as it nearly achieves the same performance with less computational complexity.

VII. Conclusion

In this paper, we proposed a new sub-optimal subcarrier and power allocation scheme for the uplink OFDMA system. Unlike other proposals, the subcarriers selection is based on the capacity difference. Furthermore, we compared our method with the optimal solution, found by exhaustive search and other solutions taken from literature. It is shown that our sub-optimal approach performs well compared to these allocation schemes. Besides showing results for optimal power allocation and uniform PA, the complexity is also investigated.
Proposed algorithm
Improved Greedy
Carrier allocation after PA
Carrier allocation before PA
MaxRT+WF

\[
\frac{R_{\text{sum}}^{\text{opt}} - R_{\text{sum}}^{\text{equ}}}{R_{\text{sum}}^{\text{opt}}}
\]

\( \text{SNR}(\text{dB}) \rightarrow \)

Fig. 5. Comparison between optimal power allocation with uniform PA for the different allocation schemes, normalized to the optimal PA

REFERENCES