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Internal null stabilization for some diffusive models of population dynamics

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Abstract

We investigate the large-time behavior of the solutions to some Fisher-type models with nonlocal terms describing the dynamics of biological populations with diffusion, logistic term and migration. Two types of logistic terms are taken into account. A necessary condition and a sufficient condition for the internal null stabilizability of the solution to a Fisher model with nonlocal term are provided. In case of null stabilizability (with state constraints) a feedback stabilizing control of harvesting type is proposed. The rate of stabilization corresponding to the feedback stabilizing control is dictated by the principal eigenvalue to a certain linear but not selfadjoint operator. A large principal eigenvalue leads to a fast stabilization to zero.

Another goal is to approximate this principal eigenvalue using a method suggested by the theoretical result concerning the large time behavior of the solution to a certain Fisher model with a special logistic term. An iterative method to improve the position (by translations) of the support of the feedback stabilizing control in order to get a larger principal eigenvalue, and consequently a faster stabilization to zero is derived. Numerical tests illustrating the effectiveness of the theoretical results are given.

Keywords: Null stabilization, Diffusive models, Population dynamics, Principal eigenvalue, Feedback control, Numerical methods

1. Setting of the problems

After the pioneering work of Fisher [18] the mathematical modeling of spatially structured populations has been carefully analyzed, given rise to a flourishing literature on the diffusive biological models (see [14,27,28]). The local/nonlocal intra or interspecific interactions of one or several interacting populations species were taken into account by several authors (see e.g. [7,15,19,20]). The following Fisher-type model describes the dynamics of a single biological population species which is free to move in an isolated habitat $\Omega$:

$$\frac{\partial y(x, t)}{\partial t} - d\Delta y(x, t) = a(x)y(x, t) - k(x)y(x, t)^2, \quad x \in \Omega, \quad t > 0.$$ 

Here $\Omega \subset \mathbb{R}^N$ ($N \in \{2, 3\}$) is a bounded domain, with a smooth enough boundary $\partial \Omega$, $y(x, t)$ is the population density at position $x$ and moment $t$, $d > 0$ is the diffusion coefficient, $a(x)$ is the natural growth rate at position $x$, and $k(x)y^2$ is a logistic term.

The logistic term describes a local intraspecific competition for resources. If a migration phenomenon occurs then the population dynamics is described by

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(H1) \( d, k, \), are positive constants, \( c \in L^\infty(\Omega) \);
(H2) \( F \in L^\infty(\Omega \times \Omega), k, y_0 \in L^\infty(\Omega) \), and

\[
0 \leq F(x, x') \quad \text{a.e. \((x, x') \in \Omega \times \Omega)\), and}
\]

\[
k_0 \leq k(x), \quad 0 \leq y_0(x) \quad \text{a.e. \(x \in \Omega\),}
\]

where \( k_0 \) is a positive constant.

**Definition 1.1.** The population is internally null-stabilizable if for any \( y_0 \in L^\infty(\Omega) \), \( y_0(x) \geq 0 \) a.e. \( x \in \Omega \) there exists a control \( u \in L^\infty_{\text{loc}}(\bar{\Omega} \times [0, +\infty)) \) such that the solution \( \gamma^u \) to (1.2) satisfies

\[
0 \leq \gamma^u(x, t) \quad \text{a.e. \(x \in \Omega\), \(\forall t \geq 0\)} \tag{1.3}
\]

and

\[
\lim_{t \to +\infty} \gamma^u(t) = 0 \quad \text{in} \ L^\infty(\Omega). \tag{1.4}
\]
If such a control exists, we also say that (1.2) is internally null stabilizable. We are talking here of course about internal null stabilization and stabilizability with state constraints (the state of the system must be nonnegative), but for the sake of simplicity we prefer to simply call them internal null stabilization and internal null stabilizability.

The study of the internal null stabilizability is related to the magnitude of the principal eigenvalue $\lambda_1^{\omega}$ for
\[
\begin{aligned}
-d\Delta \varphi(x) - c(x)\varphi(x) - \int_{\Omega} F(x, x')\varphi(x')dx' &= \lambda \varphi(x), \quad x \in \Omega \setminus \overline{\omega}, \\
\varphi(x) &= 0, \quad x \in \partial \omega, \\
\partial_{\nu}\varphi(x) &= 0, \quad x \in \partial \Omega,
\end{aligned}
\] (1.5)
and to the magnitude of the principal eigenvalue $\lambda_{1\gamma}^{\omega}$ for
\[
\begin{aligned}
-d\Delta \varphi(x) - c(x)\varphi(x) - \int_{\Omega} F(x, x')\varphi(x')dx' + \gamma \varphi(x) &= \lambda \varphi(x), \quad x \in \Omega, \\
\varphi(x) &= 0, \quad x \in \partial \Omega.
\end{aligned}
\] (1.6)

The existence and properties of both $\lambda_1^{\omega}$ and $\lambda_{1\gamma}^{\omega}$, and of their corresponding eigenfunctions follow via Krein–Rutman’s theorem ([117]) - see [5–7]. An important property we shall use throughout this paper is that
\[
\lim_{\gamma \to +\infty} \lambda_{1\gamma}^{\omega} = \lambda_1^{\omega}.
\]

We provide a necessary condition and a sufficient condition for the internal null stabilizability of (1.2). In case of internal null-stabilizability, the feedback control $-\gamma \varphi$ (with $\gamma$ large enough) will be a stabilizing control. The rate of stabilization to 0 of the solution corresponding to this feedback control is $e^{-\gamma \varphi}$. So, a large $\lambda_{1\gamma}^{\omega}$ lead to a fast stabilization to 0 of the solution to (1.2) corresponding to the above mentioned feedback control.

For basic results concerning general reaction–diffusion systems we recommend the monograph [30], and for general results concerning the stabilization of PDE we refer to [25].

The relationship between the null stabilizability of one of the components of a reaction–diffusion system and the principal eigenvalue to the principal eigenvalue problems of some related problems which have particular forms of (1.5) and (1.6) has been investigated in [1–7,22]. The rate of stabilization to 0 of the solution corresponding to some simple feedback control is also dictated by $\lambda_{1\gamma}^{\omega}$. Hence, it is obvious the importance of deriving a method to maximize $\lambda_{1\gamma}^{\omega}$ on the set of its translations or at least to derive an iterative method to improve the position (by translations) of the support of the feedback stabilizing control in order to get a larger principal eigenvalue for (1.6), and consequently a faster stabilization. Remark that these are difficult problems due to the fact that the operator involved is not self-adjoint. So, variational techniques using min–max formula cannot be used. Thus, as it was stressed by Henrot, El Haj Laamri, Schmitt [22] the general methods for maximization problem of the principal eigenvalue, following the lines of [21,23] cannot be used. A certain method to maximize the principal eigenvalue for the sum of an elliptic and an integral operator is proposed in [22]. Another goal of the present paper is to approximate the principal eigenvalues for (1.5) and (1.6) using a method suggested by the study of the large time behaviour of the solution to (1.1)’ and to derive an iterative method to improve the position (by translations) of the support of the feedback stabilizing control. Numerical tests illustrating the effectiveness of the theoretical results are given.

Here is the plan of the paper. Section 2 concerns the large-time behavior for models Eqs. (1.1) and (1.1)’. In the next section we investigate the internal null-stabilization for (1.2). In Section 4 we give a method to approximate the principal eigenvalue to (1.6) by a certain value $\zeta - \Phi^\omega$, where $\zeta$ is a constant and $\Phi^\omega$ is a certain function of $\omega$ to be defined in Section 4. So, instead of finding a position of $\omega$ which provides a large value for $\lambda_{1\gamma}^{\omega}$, we could treat the approximating problem of finding the position of $\omega$ which provides a small value for $\Phi^\omega$. We calculate the derivative with respect to translations of $\omega \mapsto \Phi^\omega$. This allow to derive a conceptual iterative algorithm to improve at each step the position (by translation) of $\omega$ in order to get a smaller value for $\Phi^\omega$. Numerical tests are provided in Section 5 and some conclusions in Section 7.

2. Large-time behavior of the solutions to (1.1) and (1.1)’

Let us discuss for the beginning the asymptotic behavior for the following linear problem related to (1.1):
\[
\begin{aligned}
\partial_t y &= d\Delta y + c(x)y + \int_{\Omega} F(x, x')y(x', t)dx', \quad (x, t) \in Q, \\
\partial_y y(x, t) &= 0, \quad (x, t) \in \Sigma, \\
y(x, 0) &= y_0(x), \quad x \in \Omega.
\end{aligned}
\] (2.1)

For this purpose we need to refer to the principal eigenvalue $\lambda_1$ of the following problem:
\[
\begin{aligned}
-d\Delta \varphi(x) - c(x)\varphi(x) - \int_{\Omega} F(x, x')\varphi(x')dx' &= \lambda \varphi(x), \quad x \in \Omega, \\
\varphi(x) &= 0, \quad x \in \partial \Omega.
\end{aligned}
\] (2.2)

Problem (2.2) may be rewritten (in $L^2(\Omega)$) as
\[
A\varphi = \lambda \varphi,
\]
where
\[ D(A) = \{ \varphi \in H^2(\Omega); \partial_i \varphi(x) = 0 \text{ a.e. } x \in \partial\Omega \}, \]
\[ \langle A\varphi \rangle \cdot c = -2\Delta \varphi - \varphi \int_\Omega F(\cdot, \varphi(x)) \, dx, \quad \forall \varphi \in D(A). \]

The operator \( A \) is not necessarily self-adjoint. For existence and basic properties of \( \lambda_1 \) and of the corresponding eigenfunctions for \( A \) as well as for \( A^* \) (which are consequences of Krein–Rutman theorem) we refer to [5,6]. We have stated there the existence of a positive constant \( d_0 \) such that
\[ d_0 \leq \varphi_1(x), \quad \varphi_1(x) \text{ a.e. } x \in \Omega, \]
where \( \varphi_1 \) is the positive eigenfunction corresponding to \( A \) and to the eigenvalue \( \lambda_1 \), with the norm \( \|\varphi_1\|_{L^2(\Omega)} = 1 \), and \( \varphi_1^* \) is the positive eigenfunction corresponding to \( A^* \) and to the eigenvalue \( \lambda_1 \), with the norm \( \|\varphi_1^*\|_{L^2(\Omega)} = 1 \).

Problem (2.1) has a unique solution \( y \), which is nonnegative (see [5–7]). The following results has been proved in [3,6]:

**Lemma 2.1.**

(i) If \( \lambda_1 > 0 \), then
\[ \lim_{t \to -\infty} e^{t\lambda_1}y(t) = c_0\varphi_1 \text{ in } L^\infty(\Omega), \]
where \( c_0 = (\int_\Omega \varphi_1(x)\varphi_1^*(x) \, dx)^{-1} \int_\Omega y_0(x)\varphi_1^*(x) \, dx; \)
(ii) If \( \lambda_1 < 0 \), and \( y_0 \neq 0_{L^\infty(\Omega)} \), then
\[ \lim_{t \to -\infty} \int_\Omega y(x, t) = +\infty. \]

By (i) we get that for \( \lambda_1 > 0 \): \( \lim_{t \to -\infty} y(t) = 0 \text{ in } L^\infty(\Omega) \).

Let us turn back now to the large-time behavior of the solution to the population dynamics with diffusion, logistic term and migration (1.1). Namely, here is the main result of this section:

**Theorem 2.2.** Let \( y \) be the solution to (1.1).

(i) If \( \lambda_1 \geq 0 \), then
\[ \lim_{t \to -\infty} y(t) = 0 \text{ in } L^\infty(\Omega); \]
(ii) If \( \lambda_1 < 0 \), and \( y_0 \neq 0_{L^\infty(\Omega)} \), then we have persistence for \( y \).

If in addition, \( F(x, x') = F(\cdot, x) \text{ a.e. } (x, x') \in \Omega \times \Omega \), then
\[ \lim_{t \to -\infty} y(t) = y^* \text{ in } L^\infty(\Omega), \]
where \( y^* \) is the unique positive steady-state to (1.1).

**Proof.** We need to establish first a comparison result. For this purpose we set
\[ V = \{ q \in H^1(\Omega); \ q = 0 \text{ on } \Gamma \}, \]
where \( \Gamma \subset \partial\Omega \) is a measurable subset. Let \( y_j \in C([0, T]; L^2(\Omega)) \cap C((0, T]; V) \cap C^1((0, T]; L^2(\Omega)) (j \in \{1, 2\}), \) for any \( T \in (0, +\infty) \), be solutions to
\[
\begin{cases}
\partial_t y_j = d\Delta y_j + c(x)y_j - k(x)y_j^2, \\
+ \int_\Omega F_j(x, x')y_j(x', t) \, dx' + G_j(x, t), & (x, t) \in Q, \\
y_j(x, t) = 0, & (x, t) \in \Gamma \times (0, +\infty), \\
y_j(x, 0) = y_{0j}(x), & x \in \Omega,
\end{cases}
\]
respectively. \( \square \)

**Lemma 2.3** (comparison result). Assume in addition that for \( j \in \{1, 2\} \) we have \( y_{0j} \in L^\infty(\Omega), \ c_j \in L^\infty((0, +\infty); L^\infty(\Gamma)), \)
\( F_j \in L^\infty(\Omega \times \Omega), \ G_j \in L^\infty_{loc}((0, +\infty); L^2(\Omega)). \)
0 \leq y_{01}(x) \leq y_{02}(x) \text{ a.e. } x \in \Omega
0 \leq F_1(x,x') \leq F_2(x,x') \text{ a.e. } (x,x') \in \Omega \times \Omega
0 \leq G_1(x,t) \leq G_2(x,t) \text{ a.e. } (x,t) \in Q
0 \leq \zeta_1(x,t) \leq \zeta_2(x,t) \text{ a.e. } (x,t) \in \Gamma \times (0, +\infty).

Then,
0 \leq y_1(x,t) \leq y_2(x,t) \text{ a.e. } x \in \Omega, \forall t \geq 0.

Proof. In a standard manner it follows that \( y_1 \) and \( y_2 \) are nonnegative (see [29]). Set \( w = y_2 - y_1 \). Then, \( w \) is a solution to

\[
\begin{aligned}
\partial_t w &= d\Delta w + c(x)w - k(x)w(y_1 + y_2) + \int_{\Omega} F_1(x,x')w(x',t)dx', \\
&\quad + \int_{\Omega} (F_2(x,x') - F_1(x,x')) y_2(x',t)dx' + G_2(x,t) - G_1(x,t), \text{ (x,t) } \in Q,
\end{aligned}
\]

\[
\begin{aligned}
\partial_t w(x,t) &= 0, \quad (x,t) \in \Sigma \setminus \Gamma \times (0, +\infty),
\end{aligned}
\]

\[
\begin{aligned}
w(x,0) &= y_{02}(x) - y_{01}(x), \quad x \in \Omega.
\end{aligned}
\]

Multiplying the first equation in (2.3) by \(-w\) and integrating over \( \Omega \times (0, t) \) \((t > 0)\), one gets after an easy calculation that

\[
\frac{1}{2} \int_{\Omega} |w(x,t)|^2 dx \leq L \int_0^t \int_{\Omega} |w(x,s)|^2 dx ds
\]

for any \( t \geq 0 \), where \( L \) is a nonnegative constant. The conclusion of Lemma 2.3 follows via Bellman’s lemma. \( \square \)

Remark 2.1. The same conclusion holds if we take \( k \equiv 0 \).

Proof of Theorem 2.2 (continued).

(i) If \( \lambda_1 > 0 \), then we get by Lemma 2.3 that

0 \leq y(x,t) \leq \bar{y}(x,t) \text{ a.e. } x \in \Omega, \forall t \geq 0.

The solution \( y \) to (1.1) is a nonnegative function (and this follows in a standard manner; see [29]). Since, by Lemma 2.1 we have that \( \lim_{t \to +\infty} \bar{y}(t) = 0 \) in \( L^\infty(\Omega) \), then we immediately get that \( \lim_{t \to +\infty} y(t) = 0 \) in \( L^\infty(\Omega) \).

If \( \lambda_1 = 0 \), then multiplying the first equation in (1.1) by \( \varphi_1^\alpha \) and integrating over \( \Omega \) we infer that

\[
\begin{aligned}
\frac{d}{dt} \left( \int_{\Omega} y(x,t)\varphi_1^\alpha(x)dx \right) &= -\int_{\Omega} k(x)y(x,t)^2\varphi_1^\alpha(x)dx \leq -k_0 \int_{\Omega} y(x,t)^2\varphi_1^\alpha(x)dx
\end{aligned}
\]

for any \( t > 0 \). On the other hand

\[
\left( \int_{\Omega} y(x,t)\varphi_1^\alpha(x)dx \right)^2 \leq \left( \int_{\Omega} y(x,t)^2\varphi_1^\alpha(x)dx \right) \int_{\Omega} \varphi_1^\alpha(x)dx, \forall t \geq 0.
\]

We may conclude that

\[
\begin{aligned}
\frac{d}{dt} \left( \int_{\Omega} y(x,t)\varphi_1^\alpha(x)dx \right) &\leq -\frac{k_0}{\int_{\Omega} \varphi_1^\alpha(x)dx} \left( \int_{\Omega} y(x,t)^2\varphi_1^\alpha(x)dx \right)^\frac{1}{2}
\end{aligned}
\]

for any \( t > 0 \) and consequently that

\[
\int_{\Omega} y(x,t)\varphi_1^\alpha(x)dx \leq \frac{\frac{\int_{\Omega} y_0(x)\varphi_1^\alpha(x)dx}{1 + \frac{k_0c}{\int_{\Omega} \varphi_1^\alpha(x)dx} \int_{\Omega} y_0(x)\varphi_1^\alpha(x)dx}}}{1}
\]

for any \( t \geq 0 \). This last inequality implies that

\[
\lim_{t \to +\infty} \int_{\Omega} y(x,t)\varphi_1^\alpha(x)dx = 0
\]

and since

\[
0 \leq a_0 \int_{\Omega} y(x,t)dx \leq \int_{\Omega} y(x,t)\varphi_1^\alpha(x)dx
\]

for any \( t \geq 0 \), we may infer that \( \lim_{t \to +\infty} y(t) = 0 \) in \( L^1(\Omega) \) and in \( L^\infty(\Omega) \) as well (the convergence in \( L^\infty(\Omega) \) follows as in [5,6]).

(ii) If \( \lambda_1 < 0 \), then we shall treat for the beginning the case when

\[
c_1 \varphi_1^\alpha(x) \leq y_0(x) \leq c_2 \varphi_1^\alpha(x) \quad \text{a.e. } x \in \Omega
\]
where $0 < c_1 < c_2$, $c_1$ small enough and $c_2$ large enough. Let $p_j (j \in \{1, 2\})$ be the solutions to (1.1) corresponding to $y_0 := c_j \phi_j$, respectively. Since $c_1$ is small enough we get that $\partial_t p_1(x, t) > 0$ a.e. $x \in \Omega$, for any $t \in (0, t_1)$. The comparison result implies that $p_1$ is nondecreasing with respect to $t$.

For $c_2$ large enough we get in the same manner that $p_2$ is nonincreasing with respect to $t$. In addition

$$c_1 \phi_1(x) \leq p_1(x, t) \leq y(x, t) \leq p_2(x, t) \quad \text{a.e. } x \in \Omega, \quad \forall t \geq 0. \tag{2.4}$$

We deduce that $y$ is persistent.

On the other hand we get that $\lim_{t \to +\infty} p_1(t) = \bar{p}_1$ in $L^2(\Omega)$, and passing to the limit in (1.1) we get that $\bar{p}_1$ is a positive steady-state for (1.1).

In the same manner one gets that $\lim_{t \to +\infty} p_2(t) = \bar{p}_2$ in $L^2(\Omega)$, and that $\bar{p}_2$ is a positive steady-state for (1.1). It is obvious that

$$c_1 \phi_1(x) \leq \bar{p}_1(x) \leq \bar{p}_2(x) \leq c_2 \phi_2(x) \quad \text{a.e. } x \in \Omega.$$

Assume now that

$$F(x, x') = F(x, x') \quad \text{a.e. } (x, x') \in \Omega \times \Omega.$$

$\bar{p}_i$ are both positive solutions to

$$\begin{cases}
    d \Delta p + c(x)p - k(x)p^2 + \int_{\Omega} F(x, x')p(x')dx' = 0, & x \in \Omega,
    \\
    \partial_t p(x) = 0, & x \in \partial \Omega.
\end{cases} \tag{2.5}$$

Multiplying the first equation in (2.5) where $p := \bar{p}_1$, by $\bar{p}_2$ we get that

$$-d \int_\Omega \nabla \bar{p}_1(x) \cdot \nabla \bar{p}_2(x) dx + \int_\Omega c(x) \bar{p}_1(x) \bar{p}_2(x) dx$$

$$- \int_\Omega k(x) \bar{p}_1(x)^2 \bar{p}_2(x) dx + \int_\Omega \int_\Omega F(x, x') \bar{p}_1(x') dx' \bar{p}_2(x) dx = 0.$$

Multiplying the first equation in (2.5) where $p := \bar{p}_2$, by $\bar{p}_1$, we get that

$$-d \int_\Omega \nabla \bar{p}_1(x) \cdot \nabla \bar{p}_2(x) dx + \int_\Omega c(x) \bar{p}_1(x) \bar{p}_2(x) dx - \int_\Omega k(x) \bar{p}_2(x)^2 \bar{p}_1(x) dx + \int_\Omega \int_\Omega F(x, x') \bar{p}_2(x') dx' \bar{p}_1(x) dx = 0.$$

The symmetry of $F$ implies that

$$\int_\Omega \int_\Omega F(x, x') \bar{p}_1(x') dx' \bar{p}_2(x) dx = -\int_\Omega \int_\Omega F(x, x') \bar{p}_2(x') dx' \bar{p}_1(x) dx = \int_\Omega \int_\Omega F(x, x') \bar{p}_2(x') dx' \bar{p}_1(x) dx$$

and so we get that

$$\int_\Omega k(x) \bar{p}_1(x)^2 \bar{p}_2(x) dx = \int_\Omega k(x) \bar{p}_1(x)^2 \bar{p}_2(x) dx.$$ This implies that

$$\int_\Omega k(x) \bar{p}_1(x) \bar{p}_2(x) (\bar{p}_2(x) - \bar{p}_1(x)) dx = 0$$

and since $k, \bar{p}_j$ are positive functions and $\bar{p}_2 - \bar{p}_1$ is a nonnegative one, we finally conclude that $\bar{p}_1 = \bar{p}_2$ in $L^\infty(\Omega)$.

If we denote this element by $y'$, then by (2.4) we get that

$$\lim_{t \to +\infty} y(t) = y' \quad \text{in } L^2(\Omega)$$

and in $L^\infty(\Omega)$ (the convergence in $L^\infty(\Omega)$ follows as in [5]).

For a general $y_0 \in L^\infty(\Omega), y_0(x) \geq 0$ a.e. $x \in \Omega$, there exists $t_2 > 0$ such that

$$c_1 \phi_1(x) \leq y(x, t_2) \leq c_2 \phi_2(x) \quad \text{a.e. } x \in \Omega,$$

where $c_1$ and $c_2$ are positive constants. The conclusion in this general case follows from the previous particular case.

Let now consider the model (1.1)' (with the nonlocal logistic term). Remark that model (1.1)' is a separable one, i.e., the solution $y$ to (1.1)' may be written as

$$y(x, t) = y_1(x, t) z(t),$$

where $y_1$ is the solution to

$$\begin{cases}
    \partial_t y = d \Delta y + c(x) y + \int_\Omega F(x, x') y(x', t) dx' + \lambda_1 y, & (x, t) \in Q, \\
    \partial_t y(x, t) = 0, & (x, t) \in \Sigma, \\
    y(x, 0) = y_0(x), & x \in \Omega.
\end{cases}$$

and $z$ is the solution to

$$\begin{cases}
    \partial_t z = k(x) z^2, & (x, t) \in Q, \\
    \partial_t z(x, t) = 0, & (x, t) \in \Sigma, \\
    z(x, 0) = \bar{z}(x), & x \in \Omega.
\end{cases}$$
This particular structure of the solution allowed us to establish in [3] the following results:

**Theorem 2.4.**

(i) If \( \lambda_1 > 0 \), then
\[
\lim_{t \to +\infty} y(t) = 0 \quad \text{in } L^\infty(\Omega);
\]

(ii) If \( \lambda_1 < 0 \), and \( y_0 \neq 0 \text{ in } \Omega \), then
\[
\lim_{t \to +\infty} y(t) = c_0 \varphi_1 \quad \text{in } L^\infty(\Omega),
\]
where
\[
c_0 = -\frac{\hat{\lambda}_1}{k_1 \int_{\Omega} \varphi_1(x)dx}.
\]

Remark that the limit in (ii) does not depend on \( y_0 \). Starting from this result a method to approximate \( \lambda_1 \) will be derived in Section 4.

### 3. Internal null stabilization for (1.2)

Here is the main result of this section:

**Theorem 3.1.** If (1.2) is internally null-stabilizable then the principal eigenvalue to (1.5) satisfies \( \lambda^0_1 > 0 \).

Conversely, if \( \lambda^0_1 > 0 \), then (1.2) is internally null-stabilizable and for large enough \( \gamma > 0 \), the feedback control \( u := -\gamma y \) realizes (1.3) and (1.4).

**Proof.** If (1.2) is internally null-stabilizable, then for any \( y_0 \) satisfying the hypotheses and for any stabilizing control \( u \in L^\infty(\Omega \times [0, +\infty)) \), the solution to (1.2) satisfies (1.3) and (1.4). So for any \( \alpha > 0 \), there exists \( T_\alpha > 0 \) such that
\[
0 \leq k(x) y^\alpha(x, t) < \epsilon \quad \text{a.e. } x \in \Omega, \forall t > T_\alpha.
\]

By the comparison result established in Section 2 we get that
\[
0 \leq z(x, t) < y^\alpha(x, t) \quad \text{a.e. } x \in \Omega \setminus \overline{\Omega}, \forall t > T_\alpha,
\]
where \( z \) is the solution to the following problem:

\[
\begin{aligned}
\partial_t z &= d \Delta z + c(x) z - \epsilon z + \int_{\Omega} F(x, x') z(x', t) dx', \quad x \in \Omega \setminus \overline{\Omega}, \ t > T_\alpha, \\
z(x, t) &= 0, \quad x \in \partial \Omega, \ t > T_\alpha, \\
\partial_t z(x, t) &= 0, \quad x \in \partial \Omega, \ t > T_\alpha, \\
z(x, T_\alpha) &= y^\alpha(x, T_\alpha) = z_0(x), \quad x \in \Omega \setminus \overline{\Omega}.
\end{aligned}
\]

By (3.1) we conclude that
\[
\lim_{t \to +\infty} z(t) = 0 \quad \text{in } L^2(\Omega \setminus \overline{\Omega}).
\]

On the other hand, for any \( y_0 \in L^\infty(\Omega), y_0(x) \geq 0 \text{ a.e. } x \in \Omega, y_0 \neq 0 \text{ in } \Omega \), we have that \( y^\alpha(x, T_\alpha) > 0 \text{ a.e. } x \in \Omega \setminus \overline{\Omega} \).

Let \( \varphi^*_1 \) be the eigenfunction for
\[
-\Delta \varphi(x) - c(x) \varphi(x) - \int_{\Omega} F(x, x') \varphi(x') dx' = \lambda^0_1 \varphi(x), \quad x \in \Omega \setminus \overline{\Omega},
\]
\[
\varphi(x) = 0, \quad x \in \partial \Omega,
\]
\[
\varphi(x) = 0, \quad x \in \partial \Omega.
\]

(\( \varphi^*_1 \) is an eigenfunction for \( A^* \) corresponding to the eigenvalue \( \lambda^0_1 \) satisfying \( \| \varphi^*_1 \|_{L^2(\Omega \setminus \overline{\Omega})} = 1 \), \( \varphi^*_1(x) > 0 \text{ a.e. } x \in \Omega \setminus \overline{\Omega} \).

Multiplying the first equation in (3.2) by \( \varphi^*_1 \) and integrating on \( \Omega \setminus \overline{\Omega} \) we get that
\[
\frac{d}{dt} \int_{\Omega \setminus \overline{\Omega}} z(x, t) \varphi^*_1(x) dx = -(\lambda^0_1 + \epsilon) \int_{\Omega \setminus \overline{\Omega}} z(x, t) \varphi^*_1(x) dx.
\]
a.e. \( t > T_e \), and consequently
\[
\int_{\Omega} z(x, t) \phi_i(x) dx = e^{-(\lambda_{i}^0 + \epsilon)(t - T_e)} \int_{\Omega} z_0(x) \phi_i(x) dx
\]
for any \( t \geq T_e \).

Since \( \int_{\Omega} z_0(x) \phi_1(x) dx > 0 \) and taking into account (3.3), we may infer that \( \lambda_{1}^0 + \epsilon > 0 \) for any \( \epsilon > 0 \), and consequently \( \lambda_{1}^0 \geq 0 \).

Conversely, assume that \( \lambda_{1}^0 > 0 \). By Lemma 2.1 in [5] we have that \( \lim_{T \to +\infty} \lambda_{1}^0 = \lambda_{1}^0 \), where \( \lambda_{1}^0 \) is the principal eigenvalue for
\[
\begin{cases}
-\Delta \phi(x) - c(x) \phi(x) - \int_{\Omega} F(x, x') \phi(x') dx' + \gamma \chi_{\omega} \phi(x) = \lambda \phi(x), & x \in \Omega, \\
\partial_n \phi(x) = 0, & x \in \partial \Omega.
\end{cases}
\]

(3.4)

So, for large enough \( \gamma > 0 \) we have that the principal eigenvalue to (3.4) satisfies \( \lambda_{1}^0 > 0 \). For such a \( \gamma \) we consider the feedback control \( u := -\gamma y \). Problem (1.2) becomes
\[
\begin{cases}
\partial_y y = d \Delta y + c(x) y - k(x) y^2 + \int_{\Omega} F(x, x') y(x', t) dx' - \gamma \chi_{\omega} y, & (x, t) \in Q, \\
\partial_n y(x, t) = 0, & (x, t) \in \Sigma, \\
y(x, 0) = y_0(x), & x \in \Omega.
\end{cases}
\]

(3.5)

By Theorem 2.2 (applied for \( c := c - \gamma \chi_{\omega} \)) we get that the solution \( y \) to (3.5) satisfies
\[
\lim_{t \to +\infty} y(t) = 0 \quad \text{in} \quad L^\infty(\Omega),
\]
at the rate of \( e^{-\lambda_{1}^0 t} \). \( \square \)

Remark 3.1. It is obvious that for a larger value of \( \lambda_{1}^0 > 0 \) we get a faster stabilization to 0 of the solution to (1.2) corresponding to the feedback control \( u := -\gamma y \).

It is of great importance that for a given positive harvesting effort \( \gamma \) (which corresponds to an affordable expense) to find a position of \( \omega \) such that \( \lambda_{1}^0 > 0 \) to be large. Hence, even if for a given positive \( \gamma \), the feedback control do not diminish to 0 the size of the population species (the stabilizability condition is not fulfilled), we however get an important diminishment (but with persistence) of it.

4. Evaluation of the principal eigenvalues of (2.2) and (1.6)

In this section we consider a large \( \gamma > 0 \) and a constant \( \zeta > \lambda_{1} \). Consider the following particular population dynamics model:
\[
\begin{cases}
\partial_y y = d \Delta y + c(x) y - k(x) y^2 + \int_{\Omega} F(x, x') y(x', t) dx', & (x, t) \in Q, \\
\partial_n y(x, t) = 0, & (x, t) \in \Sigma, \\
y(x, 0) = 1, & x \in \Omega.
\end{cases}
\]

(4.1)

Theorem 4.1. The solution \( y \) to (4.1) satisfies
\[
\lim_{t \to +\infty} \int_{\Omega} y(x, t) dx = \zeta - \lambda_{1}.
\]

Proof. Indeed, by Theorem 2.4 we have that
\[
\lim_{t \to +\infty} y(t) = \frac{\zeta - \lambda_{1}}{\int_{\Omega} \phi_1(x) dx} \phi_1 \quad \text{in} \quad L^\infty(\Omega)
\]
and consequently we get the conclusion. \( \square \)

Remark 4.1. Theorem 4.1 provides an approximating method for \( \lambda_{1} \). Namely, for \( T \) large enough, \( \zeta - \int_{\Omega} y(x, T) dx \) approximates \( \lambda_{1} \). Moreover, for any initial datum \( y_0 \in L^\infty(\Omega), y_0(x) \geq 0 \) a.e. \( x \in \Omega, y_0 \equiv 0_{L^\infty(\Omega)} \) the same conclusion holds.
Since our interest (related to the stabilization problem) will be to find a position for $\omega$ which provides a large value of $\lambda^o_x$, it is obvious the importance of investigating the problem of finding a position for $\omega$ which gives a small value for

$$\Phi^\omega = \int_\Omega y^\omega(x,t)dx,$$

where $y^\omega$ is the solution to (4.2):

$$\begin{cases}
\delta_y \gamma = d\Delta y + c(x)y + \gamma y - y(x,t) \int_\Omega y(x,t)dx + \int_\Omega F(x,x')y(x',t)dx' - \gamma \lambda^o_x y, & (x,t) \in \Omega_T, \\
\delta_y y(x,t) = 0, & (x,t) \in \Sigma_T, \\
y(x,0) = 1, & x \in \Omega,
\end{cases}$$

(4.2)

(here $\gamma > \lambda^o_x$, $\Omega = \Omega \times (0,T)$, $\Sigma_T = \partial \Omega \times (0,T)$).

Let $\omega_0$ be a nonempty open subset of $\Omega$, with a smooth enough boundary and such that $\Omega \cap \overline{\omega_0}$ is a domain. Consider $\Omega$ the set of all translations of $\omega_0$, satisfying $\omega \subset \Omega$. For any $\omega \in \Omega$ and $V \in \mathbb{R}^N$ we define the derivative

$$d\Phi^\omega(V) = \lim_{\varepsilon \to 0} \frac{\Phi^{\omega + \varepsilon V} - \Phi^\omega}{\varepsilon}.$$ 

For basic results and methods in the optimal shape design theory we refer to [21,23]. Here is a theoretical result (see also [3]) which gives the derivative of $\Phi^\omega$ with respect to translations:

**Theorem 4.2.** For any $\omega \in \Omega$ and $V \in \mathbb{R}^N$ we have that

$$d\Phi^\omega(V) = \gamma \int_0^T \int_{\omega_0} y^\omega(x,t)p^\omega(x,t)v(x) \cdot Vd\sigma dt,$$

(4.3)

where $v(x)$ is the normal inward versor at $x \in \partial \omega$, inward with respect to $\omega$, where $p^\omega$ is the solution to the adjoint problem

$$\begin{cases}
\delta_t \gamma + d\Delta p + c(x)p + \int_\Omega F(x,x')p(x',t)dx' = \gamma \lambda^o_x p + \gamma \zeta p - (\int_\Omega y^\omega(x,t)dx)p - \int_\Omega y^\omega(x,t)p(x,t)dx = 0, & (x,t) \in \Omega_T, \\
\delta_t p(x,t) = 0, & (x,t) \in \Sigma_T, \\
p(x,T) = 1, & x \in \Omega.
\end{cases}$$

We multiply the first equation in (4.2) by $p^\omega$ and integrate on $(0,T) \times \Omega$. We obtain that

$$\int_\Omega z(x,T)p^\omega(x,T)dx - \int_\Omega z(x,0)p^\omega(x,0)dx - \int_0^T \int_\Omega z(\delta_t p^\omega + d\Delta p^\omega + c(x)p^\omega + \int_\Omega F(x',x)p^\omega(x',t)dx' + \gamma \lambda^o_x p^\omega)dx dt$$

$$+ \int_0^T \int_\Omega y^\omega(x,t)dx \left( \int_\Omega z(x,t)p^\omega(x,t)dx \right) dt + \int_0^T \int_\Omega y^\omega(x,t)dx \left( \int_\Omega y^\omega(x,t)p^\omega(x,t)dx \right) dt + \int_0^T \int_\Omega \gamma \lambda^o_x zp^\omega dx dt$$

$$- \gamma \int_0^T \int_{\omega_0} y^\omega(x,t)p^\omega(x,t)v(x) \cdot Vd\sigma dt = 0.$$
5. A descent method for maximizing the principal eigenvalue $\lambda_1^{\omega^*}$

Let us show for the beginning that if $\Omega = (-a,a) \times (-b,b)$ ($a, b \in (0, +\infty)$) and $\omega_0 = (x_1, x_2) \times (\beta_1, \beta_2)$ are rectangles with $\omega_0 \subset \Omega$, $F \equiv 0$, $c$ a constant function and $\gamma$ a positive constant, then

$$\max \{ \lambda_1^{\omega^*} : \omega \in \mathcal{O} \}$$

is attained for $\omega$ with the same center as $\Omega$.

This will be in accordance to the numerical test we are going to provide later in this section.

Let $\omega^*$ be the translation of $\omega_0$ which has the center $(0,0)$, $\lambda_1^{\omega^*}$ be the principal eigenvalue for

$$\begin{cases}
-d\Delta \psi(x) - c\psi(x) + \gamma \chi_{\omega^*}(x)\psi(x) = \lambda \psi(x), & x = (x_1, x_2) \in \Omega, \\
\partial_\nu \psi(x) = 0, & x = (x_1, x_2) \in \partial \Omega
\end{cases}$$

(5.1)

and $\psi^*_1$ be the unique eigenfunction corresponding to $\lambda_1^{\omega^*}$ (and (5.1)) and satisfying $\psi^*_1(x_1,x_2) > 0$ in $\Omega$, $\|\psi^*_1\|_{L^2(\Omega)} = 1$. The eigenfunction $\psi^*_1$ belongs to $H^2(\Omega)$ and consequently to $C(\Omega)$ (actually, $\psi^*_1$ has much better regularity properties, we however do not need in what follows).

Consider now the following three functions: $\psi_2(x_1,x_2) = \psi^*_1(-x_1,x_2)$, $\psi_3(x_1,x_2) = \psi^*_1(-x_1,x_2)$, $\psi_4(x_1,x_2) = \psi^*_1(x_1,-x_2)$, for $x = (x_1, x_2) \in \Omega$. It is obvious that $\psi_2, \psi_3, \psi_4$ are also positive eigenfunctions for (5.1) corresponding to $\lambda_1^{\omega^*}$ and having the $L^2(\Omega)$-norm equal to 1.

The uniqueness of such an eigenfunction for (5.1) (positive, with the $L^2$-norm 1) implies that $\psi_2 \equiv \psi_3 \equiv \psi_4 \equiv \psi^*_1$, i.e.

$$\psi^*_1(x_1,x_2) = \psi^*_1(-x_1,x_2) = \psi^*_1(-x_1,-x_2) = \psi^*_1(x_1,-x_2)$$

(5.2)

for any $(x_1,x_2) \in \Omega$.

Let us prove that $\lambda_1^{\omega^*} < \lambda_1^{\omega_0}$, for any $V \in \mathbb{R}^2 \setminus \{(0,0)\}$ such that $V + \omega^* \subset \Omega$. This is equivalent to the fact that

$$\lambda_1^{\omega^*} < \lambda_1^{\omega_0}$$

where $\lambda_1^{\omega_0}$ is the principal eigenvalue for

$$\begin{cases}
-d\Delta \psi(x) - c\psi(x) + \gamma \chi_{\omega_0}(x)\psi(x) = \lambda \psi(x), & x \in \Omega, \\
\partial_\nu \psi(x) = 0, & x \in \partial \Omega
\end{cases}$$

(5.3)

where $\Omega = -V + \Omega$.
We shall use for this purpose the technique of rearrangements (see [26]). Let \( V = (a - a_1, b - b_1) (a_1, b_1 \in \mathbb{R}) \). We shall treat only the case when \( a_1 > a, b_1 \in (0, b) \); the other cases follow in the same manner.

Let \( \Omega_1 = (a_1 - 2a, a) \times (-b, b_1), \Omega_2 = (a_1 - 2a, a) \times (b_1, b), \Omega_3 = (-a_1 - 2a) \times (b_1, b), \Omega_4 = (-a_1 - 2a) \times (-b, b_1), \) and \( D_2 = (a_1 - 2a, a) \times (b_1 - 2b, -b), D_3 = (a_1) \times (b_1 - 2b, -b), D_4 = (a_1) \times (-b, b_1) \) (see Fig. 1).

Consider the following function

\[
\psi_i(x_1, x_2) = \begin{cases} 
\psi_i^1(x_1, x_2), & (x_1, x_2) \in \Omega_1, \\
\psi_i^1(x_1, x_2 + 2b), & (x_1, x_2) \in \Omega_2, \\
\psi_i^1(x_1 - 2a, x_2 + 2b), & (x_1, x_2) \in \Omega_3, \\
\psi_i^1(x_1 - 2a, x_2), & (x_1, x_2) \in \Omega_4.
\end{cases}
\]

Since \( \psi_i^1 \in C(\overline{\Omega}) \) and taking into account (5.2), we may extend \( \psi_i \) by continuity to \( \overline{\Omega} \). The definition of \( \psi_i \) implies that \( \psi_i \in H^1(\overline{\Omega}) \) and that

\[
\int_{\Omega} |\nabla \psi_i(x_1, x_2)|^2 \, dx_1 \, dx_2 = \int_{\Omega} |\nabla \psi_i^1(x_1, x_2)|^2 \, dx_1 \, dx_2.
\]

Let us prove that the last inequality is strict. Assume by contradiction that we have equality. This implies that

\[
\psi_i \equiv \psi_i^1 = \psi_i^1(x_1, x_2) \quad \text{for } (x_1, x_2) \in \overline{\Omega_1}.
\]

Since \( \psi_i^1 \in C(\overline{\Omega}) \) and taking into account (5.2), we may extend \( \psi_i \) by continuity to \( \overline{\Omega} \). The definition of \( \psi_i \) implies that \( \psi_i \in H^1(\overline{\Omega}) \) and that

\[
\int_{\Omega} |\nabla \psi_i(x_1, x_2)|^2 \, dx_1 \, dx_2 = \int_{\Omega} |\nabla \psi_i^1(x_1, x_2)|^2 \, dx_1 \, dx_2.
\]

So, \( ||\psi_i||^2_{L^2(\Omega)} = ||\psi_i^1||^2_{L^2(\Omega)} = 1 \) by Rayleigh’s principle. We get that

\[
\lambda_{1V}^{\omega} = d \int_{\Omega} |\nabla \psi_i^1(x_1, x_2)|^2 \, dx_1 \, dx_2 - c \int_{\Omega} |\psi_i^1(x_1, x_2)|^2 \, dx_1 \, dx_2 + \gamma \int_{\partial \Omega} |\psi_i^1(x_1, x_2)|^2 \, dx_1 \, dx_2
\]

Let us prove that the last inequality is strict. Assume by contradiction that we have equality. This implies that \( \psi_i \) is a positive eigenfunction for (5.3) corresponding to \( \lambda_{1V}^{\omega} \), and so its normal derivative on \( \partial \Omega \) is 0. Taking into account the definition of \( \psi_i \), we conclude that \( \lambda_{1V}^{\omega} \) is the principal eigenvalue for

\[
\begin{cases}
-d\Delta \psi(x) - c \psi(x) = \lambda \psi(x), & x \in \Omega_1, \\
\partial_n \psi(x) = 0, & x \in \partial \Omega_1,
\end{cases}
\]

as well and \( \psi_i^1 \) is a positive corresponding eigenfunction. It is obvious that the principal eigenvalue for (5.4) is in fact \(-c\).

By Rayleigh’s principle applied to (5.1) we get that

\[
d \int_{\Omega} |\nabla \psi_i^1(x_1, x_2)|^2 \, dx_1 \, dx_2 + \gamma \int_{\partial \Omega} |\psi_i^1(x_1, x_2)|^2 \, dx_1 \, dx_2 = 0
\]

and so \( \psi_i^1 \equiv 0 \) on \( \Omega \), which is absurd.

So, we get the conclusion

\[
\lambda_{1V}^{\omega} < \lambda_{1V}^{\omega}.
\]

Remark 5.1. Based on Theorem 4.2 we derive the following conceptual iterative algorithm to improve the position (by translation) of \( \omega \in \mathcal{O} \) (in order to obtain a smaller value for \( \Phi^{\omega} \)).

**Step 0**: Choose \( \omega^{(0)} \) the initial position of \( \omega \);

set \( \Phi^{(0)} := 10^6, j := 0 \);

**Step 1**: compute \( y^{(j+1)} \) the solution of Problem (4.2) corresponding to \( \omega := \omega^{(j)} \);

compute \( \Phi^{(j+1)} = \int_{\Omega} y^{(j+1)}(\chi, T) \, dx \);
Step 2: if $|\Phi^{(j+1)} - \Phi^{(j)}| < \varepsilon_1$ or $\Phi^{(j+1)} \geq \Phi^{(j)}$
then stop ($x^{(j)}$ is the support of the control)
else go to Step 3;

Step 3: compute $p^{(j+1)}$, the solution of Problem (4.3) corresponding to $y^{\omega} := y^{(j+1)}$;

Step 4: compute

$$V := -\int_0^T \int_{\Omega} y^{(j+1)}(x,t)p^{(j+1)}(x,t) \, d\sigma \, dt;$$

if $|V| < \varepsilon_2$
then stop ($x^{(j)}$ is the support of the control)
else go to Step 5

Step 5: compute the new position of $\omega$,

$$\omega^{(j+1)} := \rho V + \omega^{(0)};$$

Step 6: if $\omega^{(j+1)} = \omega^{(0)}$
then stop ($x^{(j+1)}$ is the support of the control)
else $j := j + 1$; go to Step 1.

In Step 5, $\rho > 0$ is a given parameter, and $\varepsilon_1 > 0$ in Step 2 and $\varepsilon_2 > 0$ in Step 4 are prescribed convergence parameters.

6. Numerical experiments

We introduce equidistant discretization nodes for both axes corresponding to $\Omega \subset \mathbb{R}^2$. The interval $[0,T]$ is also discretized by equidistant nodes. The parabolic system from Step 1 is approximated by a finite difference method, ascending with respect to time levels. An (almost) implicit scheme is used. The resulting algebraic linear system is solved by Gaussian elimination. The parabolic system from Step 3 is treated in a similar way, but descending with respect to time levels.

For more information about gradient (descent) methods, e.g. [9, Section 2.3]. The steplength $\rho$ from Step 5 is variable from an iteration to the next one. To fit it we have used Armijo method (see [8]). To simplify the discretization formulae for the numerical tests we consider $\Omega$ and $\omega$ to be squares. Namely, $\Omega = (0,1) \times (0,1)$ and the length side of $\omega$ to be 0.2.

We also consider $d = 1$ and $c - \zeta$ a positive constant.

Fig. 2. The first and the sixth positions of $\omega$. 
Example. We take the space discretization step $\Delta x_1 = \Delta x_2 = 0.05$, $T = 1$ and the time discretization step $\Delta t = 0.05$ since the finite difference method used is implicit. The space variable is $x = (x_1, x_2)$ and the nodes along both axes $Ox_1$ and $Ox_2$ are numbered from 1 to 20. We take $c - \zeta = 30, \gamma = 120$. For the convergence tests we consider $\varepsilon_1 = \varepsilon_2 = 0.001$. Moreover $F(x, x') = F(x_1, x_2, x'_1, x'_2) = x_1 + x_2 + x'_1 + x'_2 + 3$.

We start with $\omega^{(0)}$ having the left-down corner at node $(1,1)$ and the MATLAB program corresponding to the above algorithm gives after 6 iterations the (sub) optimal $\omega$ having the left-down corner at node $(9,9)$, that is in a central position of $\Omega$. This is in accordance with the theoretical result established at the beginning of this section. The convergence was obtained by the test in Step 4. The evolution of the position of $\omega$ is reported in Fig. 2. The corresponding graph of the (sub) optimal state $y(x_1, x_2, t)$ for $t = 0.5$ is given in Fig. 3.

For non-symmetric $F$, the numerical tests lead us to a final position of $\omega$ that is not in the center of $\Omega$. However we get the same fast convergence.

7. Conclusions

It is well known that any biological population living in a habitat interact with other population species. Hence it is of great importance to be able to precise if the population will persist or will extinct. Sometimes it is imperative for the ecosystem to introduce a control in order to eradicate the population or to diminish its size (with persistence) to a level which do not endanger other population species in the habitat.

The asymptotic behavior of the solutions to Fisher type models for population dynamics with nonlocal term modeling migration is the first problem investigated in the present paper. Two types of logistic terms are taken into account: one describing a local competition for resources and one describing a nonlocal competition. Each of these models is suitable to certain specific situations. Another important problem related to the population dynamics is the internal null-stabilizability with state constraints (the eradicability of the population). The control is considered to act in a subdomain $\omega$ of the whole habitat $\Omega$.

The relationship between the magnitudes of the principal eigenvalues to (1.5) and (1.6) ($\lambda^\alpha_{\omega}$ and $\lambda^\alpha_{\Omega}$, respectively) and the internal null-stabilizability with state constraints for the Fisher model with migration and local logistic term has been established following the ideas in [3,6]. In case of null-stabilizability (with state constraints) a feedback stabilizing control of harvesting type acting on $\omega$ ($u := -\gamma y$, with large enough $\gamma > 0$) is provided. However, the asymptotic behavior results we have established in the present paper show that for a given positive harvesting effort $\gamma$ (which corresponds to an affordable expense) such that $\lambda^\alpha_{\Omega} < 0$ we however get a diminishment of the population size but also its persistence. In fact, the persistence of the population species is often important for the ecosystem.
The asymptotic behavior corresponding to the model with nonlocal logistic term suggests an alternative method to approximate the principal eigenvalue $\lambda_{10}^\infty$ of (1.6). The rate of stabilization corresponding to this feedback stabilizing control is dictated by the magnitude of principal eigenvalue $\lambda_{10}^\infty$. A large principal eigenvalue leads to a fast stabilization to zero or to an important diminishment (but with persistence) of the population size. In order to get an important diminishment of the population size it is imperative to find a position of $\omega$ such that $\lambda_{10}^\infty$ to be large.

An iterative method to improve the position (by translations) of the support of the feedback stabilizing control in order to get a larger principal eigenvalue $\lambda_{10}^\infty$ and so a faster stabilization to zero is derived next. When $F$ is symmetric and both $\Omega$ and $\omega$ are squares with horizontal and vertical sides we have shown here that the optimal position of $\omega$ is a central one. The numerical tests have shown a fast convergence in both situations: when $F$ is symmetric and when $F$ is nonsymmetric (5 or 6 iterations). In case of symmetric $F$ the tests lead us to the optimal position for $\omega$ (central position) showing accordance with the theoretical results.

Acknowledgement

This work was supported by the CNCSIS (Romanian National Research Council) grant PN-II-ID-PCE-2012-4: “Optimal Control and Stabilization of Nonlinear Parabolic Systems with State Constraints. Applications in Life Sciences and Economics”.

The authors are indebted to the referees for their valuable comments and suggestions to improve the paper.

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