J. Nonlinear Funct. Anal. 2014 (2014), Article ID 6 

MATHRES

Journal of Nonlinear Functional Analysis

Available online at http://jnfa.mathres.org

# SET-VALUED GENERALIZED CONTRACTIONS IN 0-COMPLETE PARTIAL METRIC SPACES

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**Abstract.** In this paper, we prove some common fixed point theorems for a set-valued mapping and a self mapping of a 0-complete partial metric space. Some new fixed point result for generalized contractions in 0-complete partial metric spaces equipped with partial orders are also proved. Our results generalize some recent results in metric and partial metric spaces. Some examples are provided which show that the generalizations are proper.

**Keywords.**: Set-valued mapping; 0-complete partial metric space; Fixed point; Common fixed point.

## 1. Introduction-Preliminaries

Banach contraction principle ensures the existence of fixed points of a self contraction of a complete metric space. Due to simplicity and applications, it is generalized by several authors by changing the contraction condition on self maps; see [30] and the references therein. Nadler [27] generalized it for set-valued mappings.

Let $(X,d)$ be a metric space and $CB(X)$ denotes the collection of all nonempty, closed and bounded subsets of $X$. For $A,B \in CB(X)$ we define

$$H(A,B) = \max\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\},$$

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Received February 6, 2014

where $d(x,A) = \inf\{d(x,a) \colon a \in A\}$ is the distance of point $x$ from set $A$. It is known that $H$ is a metric on $CB(X)$, called the Hausdorff metric induced by the metric $d$.

The first was Nadler [27] who proved the fixed point theorems for the mappings defined from $X$ into $CB(X)$ and satisfying a particular contractive condition and this result is known as the Nadler's fixed point theorem.

**Theorem 1.1.** [27] *Let $(X,d)$ be a complete metric space and let $T$ be a mapping from $X$ into $CB(X)$ such that for all $x,y \in X$,*

$$H(Tx,Ty) \leq \lambda d(x,y),$$

*where $0 \leq \lambda < 1$. Then $T$ has a fixed point.*

After the work of Nadler, several authors proved fixed point results for set-valued mappings; see, for example, [1, 9, 8, 11, 13, 14, 19, 20, 21, 24, 26, 29, 37, 38, 39] and the references therein.

On the other hand, Matthews [25] introduced the notion of a partial metric space as a part of the study of denotational semantics of dataflow networks, with the interesting property "non-zero self distance" in spaces. He showed that the Banach contraction mapping theorem can be generalized to the partial metric context for applications in program verification. Subsequently, several authors; see, for example, [1, 2, 3, 4, 6, 8, 10, 12, 15, 17, 18, 22, 23, 24, 28, 31, 32, 35], derived fixed point theorems in partial metric spaces. See also the presentation by Bukatin *et al.* [12], where the motivation for introducing non-zero self distances is explained, which is also leading to interesting research in foundations of topology. Romaguera [31] introduced the notion of 0-Cauchy sequence, 0-complete partial metric spaces and proved some characterizations of partial metric spaces in terms of completeness and 0-completeness.

Recently, Aydi *et al.* [8] introduced the notion of partial Hausdorff metrics and extended the Nadler's theorem in partial metric spaces. The result of Aydi *et al.* [8] is generalized by several authors; see, for example, [5, 7, 16, 34, 36]. In this paper, we generalize the results of Aydi *et al.* [8] by proving some common fixed point theorems for a set-valued mapping and a single-valued mapping in 0-complete partial metric spaces. We use a more general contractive condition than used by Aydi *et al.* [8], as well as we prove our results in a more general setting of 0-complete partial metric spaces. A fixed point result for a set-valued generalized contraction in 0-complete partial metric spaces equipped with partial order is also proved. Some examples are provided which show that the generalizations are proper.

Consistent with [6, 8, 17, 23, 25, 31, 33], the following definitions and results will be needed in the sequel.

**Definition 1.2.** A partial metric on a nonempty set $X$ is a function $p : X \times X \to \mathbb{R}^+$ ($\mathbb{R}^+$ stands for nonnegative reals) such that for all $x, y, z \in X$ :

(P1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(P2) $p(x,x) \leq p(x,y)$,

(P3) $p(x,y) = p(y,x)$,

(P4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair $(X, p)$ such that $X$ is a nonempty set and $p$ is a partial metric on $X$.

It is clear that, if $p(x,y) = 0$, then from (P1) and (P2) $x = y$. But if $x = y$, $p(x,y)$ may not be 0. Also every metric space is a partial metric space, with zero self distance.

**Example 1.3.** If $p : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is defined by $p(x,y) = max\{x,y\}$, for all $x, y \in \mathbb{R}^+$, then $(\mathbb{R}^+, p)$ is a partial metric space.

Some more examples of partial metric space can be seen in [8, 23, 25].

Each partial metric on $X$ generates a $T_0$ topology $\tau_p$ on $X$ which has a base the family of open $p-$balls $\{B_p(x,\varepsilon) : x \in X,\ \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X :\ p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

**Theorem 1.4.** [25] *For each partial metric* $p$: $X \times X \to \mathbb{R}^+$ *the pair* $(X,d)$ *where,* $d(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ *for all* $x, y \in X$, *is a metric space.*

Here $(X,d)$ is called induced metric space and $d$ is the induced metric. In further discussion until unless specified $(X,d)$ will represent induced metric space.

Let $(X, p)$ be a partial metric space.

(1) A sequence $\{x_n\}$ in $(X, p)$ converges to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x_n,x)$.

(2) A sequence $\{x_n\}$ in $(X,p)$ is called Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n,x_m)$.

(3) $(X,p)$ is said to be complete if every Cauchy sequence $\{x_n\}$ in $X$ converges with respect to $\tau_p$ to a point $x\in X$ such that $p(x,x)=\lim_{n,m\to\infty} p(x_n,x_m)$.

(4) A sequence $\{x_n\}$ in $(X,p)$ is called 0-Cauchy sequence if $\lim_{n,m\to\infty} p(x_n,x_m)=0$. The space $(X,p)$ is said to be 0-complete if every 0-Cauchy sequence in $X$ converges with respect to $\tau_p$ to a point $x\in X$ such that $p(x,x)=0$.

**Lemma 1.5.** [25, 31, 33] *Let $(X,p)$ be a partial metric space and $\{x_n\}$ be any sequence in $X$.*

*(i) $\{x_n\}$ is a Cauchy sequence in $(X,p)$ if and only if it is a Cauchy sequence in metric space $(X,d)$.*

*(ii) $(X,p)$ is complete if and only if the metric space $(X,d)$ is complete. Furthermore, $\lim_{n\to\infty} d(x_n,x)=0$ if and only if $p(x,x)=\lim_{n\to\infty} p(x_n,x)=\lim_{n,m\to\infty} p(x_n,x_m)$.*

*(iii) Every 0-Cauchy sequence in $(X,p)$ is Cauchy in $(X,d)$.*

*(iv) If $(X,p)$ is complete then it is 0-complete.*

The converse assertions of (iii) and (iv) do not hold. Indeed the partial metric space $(\mathbb{Q}\cap[0,\infty),p)$, where $\mathbb{Q}$ denotes the set of rational numbers and the partial metric $p$ is given by $p(x,y)=max\{x,y\}$, provides an easy example of a 0-complete partial metric space which is not complete. It is easy to see that every closed subset of a 0-complete partial metric space is 0-complete.

Let $(X,p)$ be a partial metric space. Let $CB^p(X)$ be the family of all nonempty, closed and bounded subsets of the partial metric space $(X,p)$, induced by the partial metric $p$. Note that closedness is taken from $(X,\tau_p)$ ($\tau_p$ is the topology induced by $p$) and boundedness is given as follows: A is a bounded subset in $(X,p)$ if there exist $x_0\in X$ and $M\geq 0$ such that for all $a\in A$, we have $a\in B_p(x_0,M)$, that is, $p(x_0,a)<p(a,a)+M$.

For $A,B\in CB^p(X)$ and $x\in X$, define

$$p(x,A)=\inf\{p(x,a)\colon a\in A\},\qquad \delta_p(A,B)=\sup\{p(a,B)\colon a\in A\}.$$

**Lemma 1.6.** [6] *Let $(X,p)$ be a partial metric space, $A \subset X$. Then $a \in \overline{A}$ if and only if $p(a,A) = p(a,a)$.*

**Proposition 1.7.** [8] Let $(X,p)$ be a partial metric space. For any $A,B,C \in CB^p(X)$, we have the following:

(i) $\delta_p(A,A) = \sup\{p(a,a)\colon a \in A\}$;

(ii) $\delta_p(A,A) \leq \delta_p(A,B)$;

(iii) $\delta_p(A,A) = 0$ implies that $A \subseteq B$;

(iv) $\delta_p(A,B) \leq \delta_p(A,C) + \delta_p(C,B) - \inf_{c \in C} p(c,c)$.

Let $(X,p)$ be a partial metric spaces. For $A,B \in CB^p(X)$, define

$$H_p(A,B) = \max\{\delta_p(A,B), \delta_p(B,A)\}.$$

**Proposition 1.8.** [8] Let $(X,p)$ be a partial metric spaces. For $A,B,C \in CB^p(X)$, we have

(h1) $H_p(A,A) \leq H_p(A,B)$;

(h2) $H_p(A,B) = H_p(B,A)$;

(h3) $H_p(A,B) \leq H_p(A,C) + H_p(C,B) - \inf_{c \in C} p(c,c)$.

**Corollary 1.9.** [8] *Let $(X,p)$ be a partial metric space. For $A,B \in CB^p(X)$ the following holds*

$$H_p(A,B) = 0 \text{ implies that } A = B.$$

In view of Proposition 1.8 and Corollary 1.9, we call the mapping $H_p\colon CB^p(X) \times CB^p(X) \to [0,\infty)$, a partial Hausdorff metric induced by $p$.

**Lemma 1.10.** [8] *Let $(X,p)$ be a partial metric space, $A,B \in CB^p(X)$ and $h > 1$. For any $a \in A$, there exists $b = b(a) \in B$ such that*

$$p(a,b) \leq hH_p(A,B).$$

The following Lemma is crucial for the proof of our main result and its proof is similar as the proof of Lemma 1.10.

**Lemma 1.11.** *Let $(X,p)$ be a partial metric space and $A,B \in CB^p(X)$, $a \in A$. Let $\varepsilon > 0$ be arbitrary then there exists $b \in B$ such that*

$$p(a,b) \leq H_p(A,B) + \varepsilon.$$

**Definition 1.12.** Let $X$ be any nonempty set, $F\colon X \to 2^X, g\colon X \to X$ be mappings.

(a) An element $x \in X$ is called a fixed point of $F$ if $x \in Fx$.

(b) An element $x \in X$ is called a common fixed point of $F$ and $g$ if $x = gx \in Fx$.

(c) An element $x \in X$ is called a coincidence point of $F$ and $g$ if $gx \in Fx$.

(d) An element $y \in X$ is called a point of coincidence $F$ and $g$ if there exists $x \in X$ such that $y = gx \in Fx$.

(e) Mapping $F$ and $g$ are said to be commuting if $gFx = Fgx$ for all $x \in X$.

(f) Mappings $F$ and $g$ are said to be weakly compatible if $gx \in Fx$ implies $gFx \subseteq Fgx$.

## 2. Fixed point theorems on partial metric spaces

**Theorem 2.1.** *Let $(X,p)$ be a 0-complete partial metric space, $F\colon X \to CB^p(X)$ and $g\colon X \to X$ be two mappings such that $Fx \subset g(X)$ for all $x \in X$ and $g(X)$ is a closed subset of $X$. Suppose the following condition holds:*

(1) $$H_p(Fx,Fy) \leq \lambda \max\{p(gx,gy), p(gx,Fx), p(gy,Fy), \frac{p(gx,Fy)+p(gy,Fx)}{2}\}$$

*for all $x,y \in X$, where $\lambda \in [0,1)$. Then $F$ and $g$ have a point of coincidence $v \in X$.*

**Proof.** Let $x_0 \in X$ be arbitrary. As, $Fx_0 \in CB^p(X)$ and $Fx_0 \subset g(X)$, let $y_1 = gx_1 \in Fx_0$ for some $x_1 \in X$, so by Lemma 1.11, there exists $y_2 = gx_2 \in Fx_1, x_2 \in X$ such that

$$p(gx_1,gx_2) \leq H_p(Fx_0,Fx_1) + \theta_1,$$

where $\theta_1 > 0$ is arbitrary. Similarly, there exists $y_3 = gx_3 \in Fx_2, x_3 \in X$ such that

$$p(gx_2,gx_3) \leq H_p(Fx_1,Fx_2) + \theta_2,$$

where $\theta_2 > 0$ arbitrary. Continuing this procedure we obtain $y_{n+1} = gx_{n+1} \in Fx_n$ and

(2) $$p(gx_n,gx_{n+1}) \leq H_p(Fx_{n-1},Fx_n) + \theta_n$$

for all $n \in \mathbb{N}$ and arbitrary $\theta_n > 0$. For any $n \in \mathbb{N}$ it follows from (1) and (2) that

$$\begin{aligned} p(y_n, y_{n+1}) &= p(gx_n, gx_{n+1}) \\ &\leq H_p(Fx_{n-1}, Fx_n) + \theta_n \\ &\leq \lambda \max\{p(gx_{n-1}, gx_n), p(gx_{n-1}, Fx_{n-1}), p(gx_n, Fx_n), \\ &\quad \frac{p(gx_{n-1}, Fx_n) + p(gx_n, Fx_{n-1})}{2}\} + \theta_n, \end{aligned}$$

as $y_n = gx_n \in Fx_{n-1}$ for all $n \in \mathbb{N}$, it follows from the above inequality that

$$\begin{aligned} p(y_n, y_{n+1}) &\leq \lambda \max\{p(y_{n-1}, y_n), p(y_{n-1}, y_n), p(y_n, y_{n+1}), \\ &\quad \frac{p(y_{n-1}, y_{n+1}) + p(y_n, y_n)}{2}\} + \theta_n \\ &\leq \lambda \max\{p(y_{n-1}, y_n), p(y_n, y_{n+1}), \frac{p(y_{n-1}, y_n) + p(y_n, y_{n+1})}{2}\} + \theta_n. \end{aligned}$$

As, $\max\{a, b, \frac{a+b}{2}\} = \max\{a, b\}$ for all $a, b \in \mathbb{R}^+$, therefore, it follows from the above inequality that

$$p(y_n, y_{n+1}) \leq \lambda \max\{p(y_{n-1}, y_n), p(y_n, y_{n+1})\} + \theta_n. \tag{3}$$

As, $\theta_n > 0$ is arbitrary and $\lambda \in [0, 1)$, we can choose $\delta > 0$ such that

$$\lambda + \delta = \alpha(\text{say}) < 1 \ \text{ and } \ \theta_n = \delta \max\{p(y_{n-1}, y_n), p(y_n, y_{n+1})\}.$$

Therefore, we obtain from (3) that

$$p(y_n, y_{n+1}) \leq \alpha \max\{p(y_{n-1}, y_n), p(y_n, y_{n+1})\}. \tag{4}$$

If $\max\{p(y_{n-1}, y_n), p(y_n, y_{n+1})\} = p(y_n, y_{n+1})$ then it follows from (4) that

$$p(y_n, y_{n+1}) \leq \alpha p(y_n, y_{n+1}) < p(y_n, y_{n+1}) \ \text{ (as } \alpha < 1\text{)},$$

a contradiction. Therefore, we must have $\max\{p(y_{n-1}, y_n), p(y_n, y_{n+1})\} = p(y_{n-1}, y_n)$ and then from (4) we have

$$p(y_n, y_{n+1}) \leq \alpha p(y_{n-1}, y_n).$$

It follows from successive applications of the above inequality that

$$p(y_n, y_{n+1}) \leq \alpha^n p(y_0, y_1). \tag{5}$$

For $m,n \in \mathbb{N}$ with $m > n$, using (5) we obtain

$$\begin{aligned} p(y_n,y_m) &\leq p(y_n,y_{n+1}) + p(y_{n+1},y_{n+2}) + \cdots + p(y_{m-1},y_m) \\ &\quad -[p(y_{n+1},y_{n+1}) + p(y_{n+2},y_{n+2}) + \cdots + p(y_{m-1},y_{m-1})] \\ &\leq \alpha^n p(y_0,y_1) + \alpha^{n+1} p(y_0,y_1) + \cdots + \alpha^{m-1} p(y_0,y_1) \\ &\leq \alpha^n [1 + \alpha + \alpha^2 + \cdots] p(y_0,y_1) \\ &= \frac{\alpha^n}{1-\alpha} p(y_0,y_1). \end{aligned}$$

As, $0 < \alpha < 1$ we obtain from the above inequality that

$$\lim_{n,m\to\infty} p(y_n,y_m) = 0.$$

Therefore, $\{y_n\} = \{gx_n\}$ is a 0-Cauchy sequence in $g(X)$. As, $(X,p)$ is 0-complete and $g(X)$ is closed, there exists $u,v \in X$ such that $v = gu$ and

$$\lim_{n\to\infty} p(y_n,v) = \lim_{n\to\infty} p(gx_n,gu) = \lim_{n,m\to\infty} p(y_n,y_m) = p(v,v) = 0. \tag{6}$$

We shall show that $v$ is a point of coincidence of $F$ and $g$. As, $y_n = gx_n \in Fx_{n-1}$ for all $n \in \mathbb{N}$ we have

$$\begin{aligned} p(v,Fu) &\leq p(v,y_{n+1}) + p(y_{n+1},Fu) \\ &\leq p(v,y_{n+1}) + H_p(Fx_n,Fu) \\ &\leq p(v,y_{n+1}) + \lambda \max\{p(gx_n,gu), p(gx_n,Fx_n), p(gu,Fu), \\ &\quad \frac{p(gx_n,Fu) + p(gu,Fx_n)}{2}\} \\ &\leq p(v,y_{n+1}) + \lambda \max\{p(y_n,v), p(y_n,y_{n+1}), p(v,Fu), \\ &\quad \frac{p(y_n,Fu) + p(v,y_{n+1})}{2}\} \\ &\leq p(v,y_{n+1}) + \lambda \max\{p(y_n,v), p(y_n,y_{n+1}), p(v,Fu), \\ &\quad \frac{p(y_n,v) + p(v,Fu) + p(v,y_{n+1})}{2}\}. \end{aligned} \tag{7}$$

In view of (6) for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$\max\{p(y_n,v), p(y_n,y_{n+1}), p(v,Fu), \frac{p(y_n,v) + p(v,Fu) + p(v,y_{n+1})}{2}\} = p(v,Fu)$$

and $p(v,y_{n+1}) < \varepsilon$ for all $n > n_0$. Therefore, it follows from (7) that

$$(1-\lambda)p(v,Fu) < \varepsilon \text{ for all } n > n_0.$$

Therefore we must have $p(v,Fu) = 0 = p(v,v)$, so by Lemma 1.6 we obtain $v = gu \in Fu$. Thus $v$ is a point of coincidence of $F$ and $g$.

For $g = I_X$, the inentity mapping in $X$, we obtain a fixed point result for set-valued generalized contraction in 0-complete partial metric space and a generalization and extension of the results of Altun *et al.* [6] and Aydi *et al.* [8].

**Corollary 2.2.** *Let $(X,p)$ be a 0-complete partial metric space, $F\colon X \to CB^p(X)$ be a mapping. Suppose the following condition holds:*

$$H_p(Fx,Fy) \le \lambda \max\{p(x,y), p(x,Fx), p(y,Fy), \frac{p(x,Fy)+p(y,Fx)}{2}\} \tag{8}$$

*for all $x,y \in X$, where $\lambda \in [0,1)$. Then $F$ has a fixed point $v \in X$.*

Following corollaries are immediate consequence of Theorem 2.1.

**Corollary 2.3.** *Let $(X,p)$ be a 0-complete partial metric space, $F\colon X \to CB^p(X)$ and $g\colon X \to X$ be two mappings such that $Fx \subset g(X)$ for all $x \in X$ and $g(X)$ is a closed subset of $X$. Suppose the following condition holds:*

$$H_p(Fx,Fy) \le \lambda \max\{p(gx,gy), p(gx,Fx), p(gy,Fy)\} \tag{9}$$

*for all $x,y \in X$, where $\lambda \in [0,1)$. Then $F$ and $g$ have a point of coincidence $v \in X$.*

**Corollary 2.4.** *Let $(X,p)$ be a 0-complete partial metric space, $F\colon X \to CB^p(X)$ and $g\colon X \to X$ be two mappings such that $Fx \subset g(X)$ for all $x \in X$ and $g(X)$ is a closed subset of $X$. Suppose the following condition holds:*

$$H_p(Fx,Fy) \le a_1 p(gx,gy) + a_2 p(gx,Fx) + a_3 p(gy,Fy) + a_4[p(gx,Fy)+p(gy,Fx)] \tag{10}$$

*for all $x,y \in X$, where $a_1,a_2,a_3,a_4$ are nonnegative constants such that $a_1+a_2+a_3+2a_4 < 1$. Then $F$ and $g$ have a point of coincidence $v \in X$.*

In the next theorem, we give a sufficient condition for the uniqueness of point of coincidence and common fixed point of mappings $F$ and $g$.

**Theorem 2.5.** *Let $(X,p)$ be a 0-complete partial metric space, $F\colon X \to CB^p(X)$ and $g\colon X \to X$ be two mappings such that all the conditions of Theorem 2.1 are satisfied. Suppose, for any coincidence point $u$ of $F$ and $g$ we have $Fu = \{gu\}$, then $F$ and $g$ have a unique point of coincidence. In addition, if $F$ and $g$ are weakly compatible then they have a unique common fixed point.*

**Proof.** The existence of coincidence point $u$ and point of coincidence $v = gu$ follows from Theorem 2.1. Suppose $F$ and $g$ are weakly compatible in such a way that, for any coincidence point $u$ of $F$ and $g$ we have $Fu = \{gu\} = \{v\}$. We shall show that the point of coincidence $v$ is unique. If $v'$ is another point of coincidence with coincidence point $u'$ of $F$ and $g$, then $Fu' = \{gu'\} = \{v'\}$. Using (1) we obtain

$$\begin{aligned} p(v,v') &= H_p(\{v\},\{v'\}) = H_p(Fu,Fu') \\ &\leq \lambda \max\{p(gu,gu'), p(gu,Fu), p(gu',Fu'), \frac{p(gu,Fu') + p(gu',Fu)}{2}\} \\ &= \lambda \max\{p(v,v'), p(v,Fu), p(v',Fu'), \frac{p(v,Fu') + p(v',Fu)}{2}\}. \end{aligned}$$

As, $Fu = \{gu\} = \{v\}$ and $Fu' = \{gu'\} = \{v'\}$ therefore we have

$$\begin{aligned} p(v,v') &\leq \lambda \max\{p(v,v'), p(v,v), p(v',v'), \frac{p(v,v') + p(v',v)}{2}\} \\ p(v,v') &\leq \lambda p(v,v') < p(v,v'), \end{aligned}$$

a contradiction. Therefore, we must have $p(v,v') = 0$, i.e., $v = v'$. Thus, the point of coincidence of $F$ and $g$ is unique. Again, as $F$ and $g$ are weakly compatible we have $gFu \subseteq Fgu = Fv$, i.e., $\{gv\} \subseteq Fv$. Therefore $gv \in Fv$, which shows that $gv$ is another point of coincidence of $F$ and $g$ and by uniqueness we have $v = gv \in Fv$. Thus, $v$ is the unique common fixed point of $F$ and $g$.

**Example 2.6.** Let $X = [0,1] \bigcap \mathbb{Q}$ and define $p : X \times X \to \mathbb{R}^+$ by

$$p(x,y) = \max\{x,y\} + |x - y| \text{ for all } x,y \in X.$$

Then $(X,p)$ is a 0-complete partial metric space, and every singleton subset of $X$ is closed with respect to $p$. Define $F\colon X\to CB^p(X)$ and $g\colon X\to X$ by

$$Fx=\begin{cases}\{0,\frac{x}{2}\}, & \text{if } x\in[0,\frac{2}{3}];\\[2ex] \{0\}, & \text{if } x\in(\frac{2}{3},1]\end{cases}\quad\text{and}\quad gx=\begin{cases}\frac{3x}{2}, & \text{if } x\in[0,\frac{2}{3}];\\[2ex] \frac{2}{3}, & \text{if } x\in(\frac{2}{3},1].\end{cases}$$

We shall show that the condition (1) is satisfied for all $x,y\in X$ with $\lambda\in[\frac{1}{2},1)$. For this we consider the following cases:

Case I. If $x,y\in[0,\frac{2}{3}]$ and $y\le x$, then

$$\begin{aligned}H_p(Fx,Fy) &= H_p(\{0,\frac{x}{2}\},\{0,\frac{y}{2}\})=\max\{\sup_{a\in\{0,\frac{x}{2}\}}p(a,\{0,\frac{y}{2}\}),\sup_{b\in\{0,\frac{y}{2}\}}p(b,\{0,\frac{x}{2}\})\}\\ &= \max\{\sup\{p(0,\{0,\frac{y}{2}\}),p(\frac{x}{2},\{0,\frac{y}{2}\})\},\sup\{p(0,\{0,\frac{x}{2}\}),p(\frac{y}{2},\{0,\frac{x}{2}\})\}\}\\ &= \max\{\frac{1}{2}(2x-y),\inf\{y,\frac{1}{2}(2x-y)\}\}\\ &= \frac{1}{2}(2x-y)=\frac{1}{3}(3x-\frac{3y}{2})\le\lambda p(gx,gy),\end{aligned}$$

where $\lambda\in[\frac{1}{3},1)$. Similar result is true when $x\le y$.

Case II: If $x,y\in(\frac{2}{3},1]$, then $H_p(Fx,Fy)=H_p(\{0\},\{0\})=0$. Therefore (1) is satisfied trivially.

Case III: If $y\in[0,\frac{2}{3}]$ and $x\in(\frac{2}{3},1]$, then

$$\begin{aligned}H_p(Fx,Fy) &= H_p(\{0\},\{0,\frac{y}{2}\})=\max\{\sup_{a\in\{0\}}p(a,\{0,\frac{y}{2}\}),\sup_{b\in\{0,\frac{y}{2}\}}p(b,\{0\})\}\\ &= \max\{0,\sup\{p(0,\{0\}),p(\frac{y}{2},\{0\})\}\}\\ &= y\le\frac{2}{3}=\frac{1}{2}\cdot\frac{4}{3}\le\lambda p(gx,Fx),\end{aligned}$$

where $\lambda\in[\frac{1}{2},1)$. Similar result is true when $x\in[0,\frac{2}{3}]$ and $y\in(\frac{2}{3},1]$. Note that all the conditions of Theorem 2.4 are satisfied and 0 is the unique common fixed point of $F$ and $g$.

On the other hand, the metric induced by $p$ is given by $d(x,y)=3|x-y|$ for all $x,y\in X$ and the usual metric on $X$ is given by $\rho(x,y)=|x-y|$ for all $x,y\in X$. It is obvious that the induced metric space $(X,d)$ is not complete, therefore the partial metric space $(X,p)$ is not complete and so our theorems are not applicable if $(X,p)$ is taken complete (instead 0-complete) as it was taken by Aydi et al. [8]. Again, since $(X,\rho)$ and $(X,d)$ are not complete therefore the metric versions of our theorems are not applicable.

## 3. Fixed point theorems on ordered partial metric spaces

In this section, we prove some fixed point theorems for set-valued contractions in 0-complete partial metric spaces equipped with a partial order. First we give some definitions about ordered sets.

**Definition 3.1.** Let $(X,\sqsubseteq)$ be any partially ordered set and $F\colon X\to 2^X$ be a mapping. Let $A,B$ be two nonempty subsets of $X$.

(a) If for all $a\in A, b\in B$ we have $a\sqsubseteq b$ then we write $A\sqsubseteq_1 B$.
(b) $F$ is said to be nondecreasing with respect to " $\sqsubseteq$ " if $x,y\in X$, $x\sqsubseteq y\Rightarrow Fx\sqsubseteq_1 Fy$.

It is obvious that the relation " $\sqsubseteq_1$ " is not a partial order relation on $2^X$.

Now we state a fixed point theorem for set-valued generalized contraction in 0-complete partial metric space equipped with a partial order.

**Theorem 3.2.** *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exists a partial metric $p$ on $X$ such that $(X,p)$ is a 0-complete partial metric space and $F\colon X\to CB^p(X)$ be a mapping such that the following conditions hold:*

*(i)*

$$H_p(Fx,Fy)\le\lambda\max\{p(x,y),p(x,Fx),p(y,Fy),\frac{p(x,Fy)+p(y,Fx)}{2}\} \tag{11}$$

*for all $x,y\in X$ with $x\sqsubseteq y$, where $\lambda\in[0,1)$;*
*(ii) $F$ is nondecreasing with respect to " $\sqsubseteq$ ";*
*(iii) there exists $x_0\in X$ such that $\{x_0\}\sqsubseteq_1 Fx_0$;*
*(iv) if $\{x_n\}$ be any nondecreasing sequence converging to $x\in X$ then $x_n\sqsubseteq x$ for all $n$.*

*Then $F$ has a fixed point $v\in X$.*

**Proof.** Let $x_0 \in X$ be such that $\{x_0\} \sqsubseteq_1 Fx_0$. As $Fx_0 \in CB^p(X)$, so let $x_1 \in Fx_0$ and $Fx_1 \in CB^p(X)$, by Lemma 1.11, there exists $x_2 \in Fx_1$ such that

$$p(x_1,x_2) \leq H_p(Fx_0,Fx_1)+\theta_1,$$

where $\theta_1 > 0$ is arbitrary. Similarly, there exists $x_3 \in Fx_2$ such that

$$p(x_2,x_3) \leq H_p(Fx_1,Fx_2)+\theta_2,$$

where $\theta_2 > 0$ is arbitrary. Continuing this process, we obtain a sequence $\{x_n\}$ such that $x_{n+1} \in Fx_n$ and

$$p(x_n,x_{n+1}) \leq H_p(Fx_{n-1},Fx_n)+\theta_n \tag{12}$$

for all $n \in \mathbb{N}$, where $\theta_n > 0$ is arbitrary. Also, as $\{x_0\} \sqsubseteq_1 Fx_0$ and $x_1 \in Fx_0$ we have $x_0 \sqsubseteq x_1$ and $F$ is nondecreasing with respect to " $\sqsubseteq$ " so $Fx_0 \sqsubseteq_1 Fx_1$. As $x_1 \in Fx_0, x_2 \in Fx_1$ we have $x_1 \sqsubseteq x_2$ so $Fx_1 \sqsubseteq_1 Fx_2$. Continuing this process we have $x_n \sqsubseteq x_{n+1}$ for all $n \geq 0$. Thus, $\{x_n\}$ is a nondecreasing sequence with respect to " $\sqsubseteq$ ". Therefore, for any $n \in \mathbb{N}$ it follows from (11) and (12) that

$$\begin{aligned} p(x_n,x_{n+1}) &\leq H_p(Fx_{n-1},Fx_n)+\theta_n \\ &\leq \lambda \max\{p(x_{n-1},x_n), p(x_{n-1},Fx_{n-1}), p(x_n,Fx_n), \\ &\quad \frac{p(x_{n-1},Fx_n)+p(x_n,Fx_{n-1})}{2}\}+\theta_n. \end{aligned}$$

As $x_{n+1} \in Fx_n$ we obtain from the above inequality that

$$\begin{aligned} p(x_n,x_{n+1}) &\leq \lambda \max\{p(x_{n-1},x_n), p(x_{n-1},x_n), p(x_n,x_{n+1}), \\ &\quad \frac{p(x_{n-1},x_{n+1})+p(x_n,x_n)}{2}\}+\theta_n \\ &\leq \lambda \max\{p(x_{n-1},x_n), p(x_n,x_{n+1}), \frac{p(x_{n-1},x_n)+p(x_n,x_{n+1})}{2}\}+\theta_n. \end{aligned}$$

As, $\max\{a,b,\frac{a+b}{2}\} = \max\{a,b\}$ therefore we have

$$p(x_n,x_{n+1}) \leq \lambda \max\{p(x_{n-1},x_n), p(x_n,x_{n+1})\}+\theta_n. \tag{13}$$

Again, as $\theta_n > 0$ is arbitrary and $\lambda \in [0,1)$, we can choose $\delta > 0$ such that

$$\lambda+\delta = \alpha(\text{say}) < 1 \text{ and } \theta_n = \delta \max\{p(x_{n-1},x_n), p(x_n,x_{n+1})\}.$$

Therefore, we obtain from (13) that

$$p(x_n, x_{n+1}) \leq \alpha \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}. \tag{14}$$

Now with a similar process as used in Theorem 2.1 we obtain that

$$\lim_{n,m\to\infty} p(x_n, x_m) = 0.$$

Therefore, $\{x_n\}$ is a 0-Cauchy sequence in $X$. As, $(X, p)$ is 0-complete, there exists $v \in X$ such that

$$\lim_{n\to\infty} p(x_n, v) = \lim_{n,m\to\infty} p(x_n, x_m) = p(v, v) = 0. \tag{15}$$

We shall show that $v$ is a fixed point of $F$. As, $x_n \in Fx_{n-1}$ and by (iv) $x_n \sqsubseteq u$ for all $n \in \mathbb{N}$ we have

$$\begin{aligned}
p(v, Fv) &\leq p(v, x_{n+1}) + p(x_{n+1}, Fv) \\
&\leq p(v, x_{n+1}) + H_p(Fx_n, Fv) \\
&\leq p(v, x_{n+1}) + \lambda \max\{p(x_n, v), p(x_n, Fx_n), p(v, Fv), \\
&\quad \frac{p(x_n, Fv) + p(v, Fx_n)}{2}\} \\
&\leq p(v, x_{n+1}) + \lambda \max\{p(x_n, v), p(x_n, x_{n+1}), p(v, Fv), \\
&\quad \frac{p(x_n, Fv) + p(v, x_{n+1})}{2}\} \\
&\leq p(v, x_{n+1}) + \lambda \max\{p(x_n, v), p(x_n, x_{n+1}), p(v, Fv), \\
&\quad \frac{p(x_n, v) + p(v, Fv) + p(v, x_{n+1})}{2}\}. \qquad (16)
\end{aligned}$$

In view of (15) for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$\max\{p(x_n, v), p(x_n, x_{n+1}), p(v, Fv), \frac{p(x_n, v) + p(v, Fv) + p(v, x_{n+1})}{2}\} = p(v, Fv)$$

and $p(v, x_{n+1}) < \varepsilon$ for all $n > n_0$. Therefore, it follows from (16) that

$$(1-\lambda)p(v, Fv) < \varepsilon \text{ for all } n > n_0.$$

Therefore we must have $p(v, Fv) = 0 = p(v, v)$, so by Lemma 3.1 we obtain $v \in Fu$. Thus $v$ is a fixed point $F$.

**Example 3.3.** Let $X = \{0,1,2,3\}$ be endowed with the partial metric $p\colon X\times X \to \mathbb{R}^+$ defined by

$$p(x,y) = \begin{cases} 0, & \text{if } \ x=y=3; \\ \max\{x,y\}+|x-y|, & \text{otherwise.} \end{cases}$$

Then $(X,p)$ is 0-complete partial metric space and every singleton subset of $X$ is closed with respect to $p$. Define a mapping $F\colon X \to CB^p(X)$ by

$$F0=\{0\}, F1=\{0\}, F2=\{0,1\}, F3=\{0,3\}$$

and a partial order " $\sqsubseteq$ " on $X$ by

$$\sqsubseteq\, = \{(0,0),(1,1),(2,2),(3,3),(0,1),(0,2),(1,2)\}.$$

Then it is easy to see that $F$ is nondecreasing mapping with respect to " $\sqsubseteq$ ". Also, condition (11) is satisfied with $\lambda \in [\frac{2}{3},1)$, and all other conditions of Theorem 3.2 are satisfied and $0,3$ are two fixed points of $F$. Note that the fixed point of $F$ is not unique.

On the other hand, for $x=0, y=3$ we have

$$H_p(Fx,Fy) = H_p(F0,F3) = H_p(\{0\},\{0,3\}) = \max\{p(0,\{0,3\}), p(0,\{0\}), p(3,\{0\})\} = 6,$$

and $p(x,y) = p(0,3) = 6$, $p(0,F0) = 0$, $p(3,F3) = 3$, $p(0,F3) = 0$, $p(3,F0) = 6$. Therefore there is no $\lambda \in [0,1)$ such that

$$H_p(Fx,Fy) \leq \lambda \max\{p(x,y), p(x,Fx), p(y,Fy), \frac{p(x,Fy)+p(y,Fx)}{2}\}.$$

Therefore, the Corollary 2.2 of this paper and the results of Aydi *et al.* [8] are not applicable here.

**Remark 3.4.** Note that in above example there are two fixed points of mapping $F$, namely, 0 and 3 and for both, $H_p(F0,F0) = H_p(F3,F3) = 0$. Indeed, it is true for all fixed points of $F$, i.e., in Theorem 3.2, if $u$ is any fixed point of $F$ then $H_p(Fu,Fu) = 0$.

**Proof.** Let $u \in X$ be any fixed point of $F$, i.e., $u \in Fu \in CB^p(X)$, then by Lemma 1.6 we have $p(u,Fu) = p(u,u)$. If $p(u,u) > 0$ then since $u \sqsubseteq u$ we obtain from (11) that

$$\begin{aligned}
H_p(Fu,Fu) &\leq \lambda \max\{p(u,u), p(u,Fu), p(u,Fu), \frac{p(u,Fu)+p(u,Fu)}{2}\} \\
\delta_p(Fu,Fu) &\leq \lambda \max\{p(u,u), p(u,Fu)\} \\
\sup_{v\in Fu} p(v,v) &\leq \lambda \max\{p(u,u), p(u,u)\} \\
\sup_{v\in Fu} p(v,v) &\leq \lambda p(u,u) \\
\sup_{v\in Fu} p(v,v) &< p(u,u) \quad (\text{as} \quad \lambda \in [0,1)).
\end{aligned}$$

Since $u \in Fu$ therefore the above inequality yields a contradiction and we have $p(u,u) = 0$ and so $H_p(Fu,Fu) = 0$.

Mapping $F\colon X \to CB^p(X)$ is said to be a set-valued weak contraction if

$$H_p(Fx,Fy) \leq \lambda \max\{p(x,y), p(x,Fx), p(y,Fy)\}$$

for all $x,y \in X$, where $\lambda \in [0,1)$. The following corollary is an ordered version of fixed point result for set-valued weak contraction in 0-complete partial metric space and an immediate consequence of Theorem 3.2.

**Corollary 3.5.** *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exists a partial metric $p$ on $X$ such that $(X,p)$ is a 0-complete partial metric space and $F\colon X \to CB^p(X)$ be a mapping such that the following conditions hold:*

*(i)* $H_p(Fx,Fy) \leq \lambda \max\{p(x,y), p(x,Fx), p(y,Fy)\}$ *for all* $x,y \in X$ *with* $x \sqsubseteq y$, *where* $\lambda \in [0,1)$;

*(ii) $F$ is nondecreasing with respect to "$\sqsubseteq$";*

*(iii) there exists $x_0 \in X$ such that $\{x_0\} \sqsubseteq_1 Fx_0$;*

*(iv) if $\{x_n\}$ be any nondecreasing sequence converging to $x \in X$ then $x_n \sqsubseteq x$ for all $n$.*

*Then $F$ has a fixed point $v \in X$.*

**Corollary 3.6.** *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exists a partial metric $p$ on $X$ such that $(X,p)$ is a 0-complete partial metric space and $F\colon X \to CB^p(X)$ be a mapping such that the following conditions hold:*

*(i)* $H_p(Fx,Fy) \leq a_1 p(x,y) + a_2 p(x,Fx) + a_3 p(y,Fy) + a_4[p(x,Fx) + p(y,Fy)]$ *for all* $x,y \in X$ *with* $x \sqsubseteq y$*, where* $a_1, a_2, a_3, a_4$ *are nonnegative constants such that* $a_1 + a_2 + a_3 + 2a_4 < 1$.

*(ii)* $F$ *is nondecreasing with respect to* "$\sqsubseteq$";

*(iii) there exists* $x_0 \in X$ *such that* $\{x_0\} \sqsubseteq_1 Fx_0$;

*(iv) if* $\{x_n\}$ *be any nondecreasing sequence converging to* $x \in X$ *then* $x_n \sqsubseteq x$ *for all* $n$.

*Then* $F$ *has a fixed point* $v \in X$.

Following corollary generalizes the result of Aydi *et al.* in 0-complete partial metric spaces equipped with a partial order.

**Corollary 3.7.** *Let* $(X, \sqsubseteq)$ *be a partially ordered set and suppose that there exists a partial metric* $p$ *on* $X$ *such that* $(X,p)$ *is a 0-complete partial metric space and* $F : X \to CB^p(X)$ *be a mapping such that the following conditions hold:*

*(i)* $H_p(Fx,Fy) \leq \lambda p(x,y)$ *for all* $x,y \in X$ *with* $x \sqsubseteq y$*, where* $\lambda \in [0,1)$;

*(ii)* $F$ *is nondecreasing with respect to* "$\sqsubseteq$";

*(iii) there exists* $x_0 \in X$ *such that* $\{x_0\} \sqsubseteq_1 Fx_0$;

*(iv) if* $\{x_n\}$ *be any nondecreasing sequence converging to* $x \in X$ *then* $x_n \sqsubseteq x$ *for all* $n$.

*Then* $F$ *has a fixed point* $v \in X$.

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