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Stability analysis of embankments and slopes

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A simple but accurate method of stability analysis of embankments and slopes is developed to determine the critical earthquake acceleration that is required to bring a mass of soil, bounded by a slip line of any shape and the free surface, to a state of limiting equilibrium. At the same time, the usual factor of safety can be determined. It is based on the principle of limiting equilibrium and the method of slices. Effective stress strength parameters are used. A distribution of internal body forces is found based on a simple assumption. This depends on the geometry of the dam and the sliding surface as well as on the strength of the material. Though a computer is used for the calculations presented in the Paper, it is not essential. As in any solution, the physical acceptability of the complete solution must be checked before accepting the result. It is suggested to use the critical acceleration as a measure of the factors of safety.

On a mis au point une méthode simple mais précise pour l'analyse de la stabilité des terrassements et talus, pour déterminer l'accélération sismique critique requise pour amener une masse de sol limitée par une ligne de glissement de forme quelconque et la surface libre, à un état d'équilibre limite. On peut calculer en même temps les coefficients de sécurité habituels. Cette méthode est basée sur le principe de l'équilibre limite et sur la méthode des tranches. On se sert des paramètres de résistance de contrainte effective. On trouve une distribution des forces volumiques internes qui est basée sur une hypothèse simple, dépendant de la géométrie du barrage et de la surface de glissement ainsi que de la résistance du matériau. Bien qu'on se soit servi d'un ordinateur pour résoudre les équations presentées dans cette Communication, on n'en a pas absolument besoin. Comme dans toute solution, le degré de validité physique de la solution complète doit être vérifié avant d'en accepter les résultats. On suggère de se servir de l'accélération critique comme mesure des coefficients de sécurité.

INTRODUCTION

The slip surfaces that may develop during failure in natural or man-made slopes are generally non-circular. Regardless of whether failure is caused by static or earthquake forces, slip surfaces will be of a 'general' shape rather than strictly circular. With increasing non-uniformity and degree of zoning of the slope material, departure from circularity of the failing surface increases. Ambraseys (1959) has shown that even for homogeneous elastic cases, the potential failure surface during earthquake shaking is non-circular. As earthquake inertia forces become larger, or zoning of the slope material becomes stronger, so results from circular slip analysis become more unrealistic. The failure surface of the Lower San Fernando dam, produced by the San Fernando Earthquake of 9 February, 1971, is indeed non-circular (Jennings, 1971). Case histories studied by Morgenstern and Price (1965) show that even without earthquake loading, there are cases of non-circular slip surfaces both in natural slopes and in embankments.

So far, stability analysis methods based on general slip surfaces aim at the determination of the minimum factor of safety of the slope. These methods are based on the principle of limiting equilibrium and the method of slices. Kenney (1956), Janbu (1957) and Nonveiller (1965) developed techniques of analyses based on these methods which, however, do not satisfy all conditions of equilibrium. It is obvious, therefore, that these methods are in error

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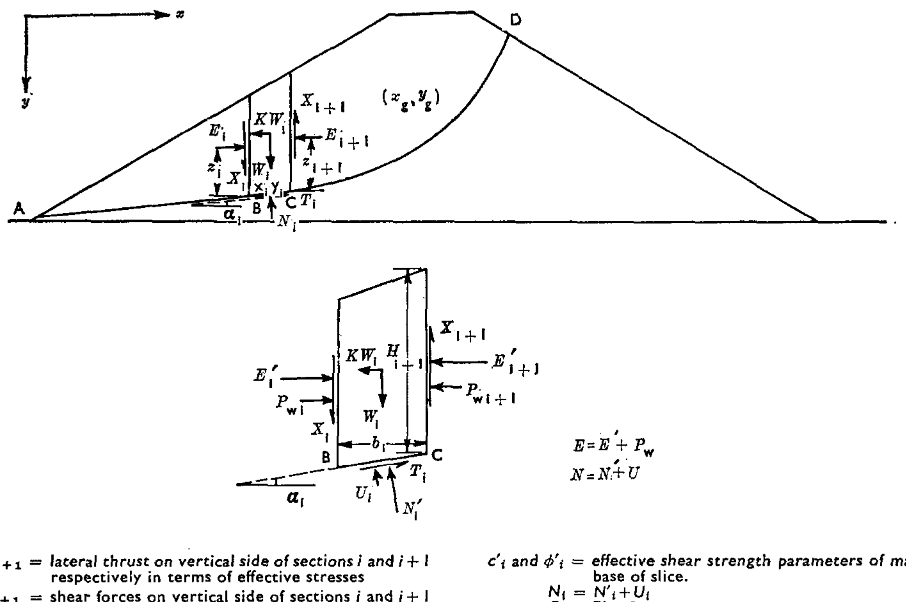
by a certain unknown amount. The degree of inaccuracy involved in these methods depends on the extent of inaccuracy associated with the assumptions made. The first known methods which satisfy all conditions of equilibrium are those of Bishop (1955) for circular arc slip surfaces and of Morgenstern and Price (1965) for general slip surfaces. The Morgenstern–Price method involves many iterations and cannot be used without the aid of a computer. Bell (1968) produced another statically accurate solution which also cannot be applied without a computer. Madej (1971) modified Janbu's method to satisfy equilibrium but needs 20–25 iterations to determine a factor of safety. Therefore, there is still need of a simple method of analysis of a general slip surface which is accurate in the sense that it satisfies all the conditions of equilibrium.

The method proposed in this Paper is developed as a means of computing the critical horizontal acceleration that is required to bring the mass of soil bounded by the slip surface and the free surface to a state of limiting equilibrium. This critical acceleration is therefore a measure of the static factor of safety. To determine the critical acceleration, no iteration is necessary. The usual factor of safety is obtained on a simple iterative basis which requires only about three iterations and does not contain any problem of convergence. If, however, the critical acceleration is to be determined by the other methods mentioned already, the amount of computational work involved becomes too large. The simplicity of the proposed stability analysis method will become apparent from the formulation of the problem.



E'_i and E'_{i+1} = lateral thrust on vertical side of sections i and $i+1$ respectively in terms of effective stresses
X_i and X_{i+1} = shear forces on vertical side of sections i and $i+1$ respectively
P_{wi} and P_{wi+1} = resultant water pressure acting on vertical side of sections i and $i+1$ respectively
W_i = weight of ith slice
KW_i = horizontal force on ith slice acting at centre of gravity of slice
N'_i = normal force at base of slice in terms of effective stresses
U_i = force due to pore pressure at base of slice
T_i = shear force at base of slice
z_i = height of point of application of E_i above slip line
c'_i and ϕ'_i = effective shear strength parameters of material at base of slice.
$N_i = N'_i + U_i$
$E_i = E'_i + P_{wi}$
H_i = height of sliding mass at section i
K_c = critical horizontal accelerations, as fraction of gravity, required to bring mass of soil above slip line to state of limiting equilibrium
b_i = breadth of slice
α_i = angle made by slip line BC with the horizontal
x_i and y_i = co-ordinate of mid point of base of slice with respect to the axes
x_g and y_g = co-ordinate of centre of gravity of sliding mass with respect to the axes
$DE_i = E_{i+1} - E_i$
$DX_i = X_{i+1} - X_i$

Fig. 1

The advantage of the method is that the aid of a computer is not essential and the problem of non-convergence does not exist. The accuracy associated with the Morgenstern–Price method is not lost in the simplicity of the present method. However, in the present solution, as in any stability analysis solution, the physical acceptability of the solution must be checked before the result itself is accepted, e.g. internal forces obtained from the solution must not violate failure criteria and tension must not be implied within the soil mass; see, for instance, Whitman and Bailey (1967).

FORMULATION

Consider the equilibrium of the potential sliding mass shown in Fig. 1. The sliding mass is bounded by the slip line ABCD and the free surface. Just before failure the body of the mass must be in equilibrium. The sliding mass is divided into n vertical slices. The ith slice and the forces acting on it are shown in the figure. It is assumed that the slices are of thickness sufficiently small to assume that the normal force N_i acts at the mid point of the slice.

Since there are no other external forces on the free surface

$$\sum DE_i = 0 \qquad (1a)$$

$$\sum DX_i = 0 \qquad (1b)$$

From the vertical and horizontal equilibrium of the slice, the following may be obtained

$$N_i \cos \alpha_i + T_i \sin \alpha_i = W_i - DX_i \qquad (2)$$

$$T_i \cos \alpha_i - N_i \sin \alpha_i = KW_i + DE_i \qquad (3)$$

It is assumed that, under the action of the force KW, the complete shear strength of the surface is mobilized. This means that the factor of safety is equal to one and therefore K represents the critical acceleration K_c as a fraction of gravity.

Then

$$T_i = (N_i - U_i) \tan \phi'_i + c'_i b_i \sec \alpha_i \qquad (4)$$

where

$$U_i = R_{ui} W_i \sec \alpha_i \qquad (5)$$

and where R_{ui} is the pore pressure ratio as defined by Bishop and Morgenstern (1960).

From equations (2), (3), (4), (5)

$$N_i = [-c'_i b_i \tan \alpha_i + W_i(1+R_{ui} \tan \alpha_i \tan \phi_i') \cos \phi'_i/\cos (\phi'_i-\alpha_i)] - DX_i \cos \phi'_i/\cos (\phi'_i-\alpha_i) \qquad (6)$$

$$T_i = [c'_i b_i + W_i(1-R_{ui}) \tan \phi'_i] \cos \phi'_i/\cos (\phi'_i-\alpha_i) - DX_i \sin \phi'_i/\cos (\phi'_i-\alpha_i) \qquad (7)$$

$$DX_i \tan (\phi'_i-\alpha_i) + DE_i = D_i - KW_i \qquad (8)$$

where

$$D_i = W_i \tan (\phi'_i-\alpha_i) + (c'_i b_i \cos \phi'_i - R_{ui} W_i \sin \phi'_i) \sec \alpha_i/\cos (\phi'_i-\alpha_i) \qquad (9)$$

Considering the equilibrium of the whole mass

$$\sum DX_i \tan (\phi'_i-\alpha_i) + \sum KW_i = \sum D_i \qquad (10)$$

For the complete equilibrium of the whole mass the moment equilibrium condition must also be satisfied. For this purpose moments may be taken about any arbitrary point. Taking the moment of all the forces about the centre of gravity of the sliding mass and noting that the sum of the moments of W_i and KW_i about the centre of gravity vanishes and also that the interslice body forces do not produce any net moment one obtains

$$\sum (T_i \cos \alpha_i - N_i \sin \alpha_i)(y_i - y_g) + \sum (N_i \cos \alpha_i + T_i \sin \alpha_i)(x_i - x_g) = 0 \qquad (11)$$

Using equations (2), (3) and (8), this becomes

$$\sum DX_i[(y_i-y_g)\tan(\phi'_i-\alpha_i)+(x_i-x_g)] = \sum W_i(x_i-x_g)+\sum D_i(y_i-y_g) \quad . \quad (12)$$

The choice of the centre of gravity as the moment centre is to show the effect of DX on the moment equation. If the slices are of sufficiently small thickness, then $\sum W_i(x_i-x_g)$ would vanish. However, the second term of the right hand side of equation (12) remains.

The right-hand sides of equations (10) and (12) are known. Should it be possible to find a set of X forces which will satisfy equations (1b) and (12), then the critical acceleration $K=(K_c)$ can be determined, using equation (10) with this set of X forces; then equations (10) and (12) are both satisfied and equilibrium is achieved.

If it is written that

$$DX_i = \lambda F_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where F_i is assumed to be known so that

$$\sum F_i = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

then equation (10) and (12) become

$$\lambda \sum F_i \tan(\phi'_i-\alpha_i)+K\sum W_i = \sum D_i \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\lambda \sum F_i\,[(y_i-y_g)\tan(\phi'_i-\alpha_i)+(x_i-x_g)] = \sum W_i(x_i-x_g)+\sum D_i(y_i-y_g) \quad . \quad (16)$$

Equation (13) suggests that the shape of distribution of the X forces, but not their magnitude, is known. The magnitudes will be known from the solution of equation (16). Since F_i is assumed to be known, equations (15) and (16) can be solved simultaneously to obtain λ and K.

$$\lambda = S_2/S_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$K = (S_1-\lambda S_4)/\sum W_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where

$$S_1 = \sum D_i \quad . \quad . \quad . \quad . \quad (19)$$

$$S_2 = \sum W_i(x_i-x_g)+\sum D_i(y_i-y_g) \quad . \quad . \quad . \quad . \quad (20)$$

$$S_3 = \sum F_i[(y_i-y_g)\tan(\phi'_i-\alpha_i)+(x_i-x_g)] \quad . \quad . \quad . \quad . \quad (21)$$

$$S_4 = \sum F_i \tan(\phi'_i-\alpha_i) \quad . \quad . \quad . \quad . \quad (22)$$

The value of K gives the critical acceleration K_c for the surface as a fraction of gravity, λF_i gives the change of X force across the ith slice. From equation (8) the change of E force across the slice is obtained.

$$DX_i = \lambda F_i$$

$$DE_i = D_i-KW_i-DX_i\tan(\phi'_i-\alpha_i) \quad . \quad . \quad . \quad . \quad . \quad (23)$$

From the known initial condition at point A where $X_1=E_1=0$ all X and E forces can now be determined. These E forces will satisfy equation (1a). The factor of safety on the vertical sections of the slices (i.e. the local factor of safety) can now be calculated.

$$F_{Li} = [(E_i-P_{wi})\tan\bar{\phi}_i+\bar{c}_i H_i]/X_i \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Where $\tan\bar{\phi}_i$ and $\bar{c}_i$ are the weighted average shear strength parameters of the section i. Taking moment equilibrium of the ith slice alone, the point of application of the E forces can be determined. Taking moment about the centre of the base of the slice one obtains

$$z_{i+1} = [E_i z_i - 0{\cdot}5b_i\tan\alpha_i(E_i+E_{i+1})-0{\cdot}5b_i(X_i+X_{i+1})]/E_{i+1} \quad . \quad . \quad (25)$$

Again starting from known initial condition $z_1=0$ all points of application can be determined.

The accuracy of the proposed method was checked against that of the Morgenstern–Price method by the following procedure. For a particular surface, the earthquake acceleration used in the Morgenstern–Price solution was increased progressively until the factor of safety was reduced to one. The X values that were obtained from this solution were used to determine the F_i values which were subsequently used in equations (21) and (22), to determine the values of λ and K. The results obtained differed only in the fourth significant figure.

It is now apparent that the problem has an infinite number of solutions which depend on the infinite variation of F_i. This is a situation similar to that arising in the Morgenstern–Price solution. Of this infinite number, only a few solutions are acceptable; the rest must be rejected on the grounds of physical reasoning. As argued by Morgenstern and Price (1965) and Whitman and Bailey (1967), only those solutions are acceptable which do not violate the failure criterion of the soil mass above the slip surface and which at the same time do not cause tension in the material. The solution is acceptable if F_{Li} is greater than unity and $0 \leqslant z_i/H_i \leqslant 1$ in all sections. It has been found that the following definition of X_i gives satisfactory results. The derivation of the formula is given in the Appendix.

$$X_i = \lambda f_i[K'_i - R_{ui})\gamma H_i^2 \tan \phi'_i/2 + c'_i H_i] \qquad (26)$$

where

$$K'_i = \frac{1-\sin \beta_i(1-2R_{ui}) \sin \phi'_i + 4c'_i \cos \phi'_i/\gamma H_i)}{1+\sin \phi'_i \sin \beta_i} \qquad (27)$$

$$\beta_i = 2\alpha_i - \phi'_i \qquad (28)$$

f_i is a number to be selected, usually 1. This represents the reciprocal of the expected local factor of safety. However, the true local factor of safety is obtained as part of the solution which is not necessarily the same as the expected one.

$$\lambda F_i = X_{i+1} - X_i \qquad (29)$$

Formulæ (26), (27), (28) are derived for the case in which the material and the pore pressures are homogeneous along the vertical boundaries of the slices. Other formulæ which take into consideration the non-homogeneity along these boundaries are given in the Appendix.

In order to obtain the *static factor of safety*, the strength parameters of the material along the slip surface must be reduced by a known factor of safety, and the critical acceleration computed. The value of the factor which gives zero critical acceleration is the factor of safety, which obtains without the earthquake forces. However, the critical acceleration itself may be used as a measure of the static factor of safety and this iteration is then unnecessary.

APPLICATION

In almost all cases that are tried, $f_i=1$ gives satisfactory results. In those cases, where X and E forces violated the failure criterion, the value of f_i was changed so as to eliminate this violation. It is observed that if the slip surface is a reasonable one, an acceptable solution is obtained with $f_i=1$. If the failure criterion is violated at one or two of the vertical sections only, this may be due to the discontinuity of the slip surface. In this case, a satisfactory solution can be obtained by smoothing the surface or by reducing the value of f for those sections to less than unity. If the failure criterion is violated in a larger proportion of the mass, no reasonable X forces can be easily found for that surface. However, this point needs further investigation.

Tables 1(a) and 1(b) compare values obtained by the Morgenstern–Price method and the proposed method using equation (26). The K_c values were obtained in the Morgenstern–Price method by gradually increasing the earthquake acceleration coefficient until the factor of safety was reduced to one. The corresponding static factor of safety obtained in this process is shown.

In Table 2 results obtained for non-homogeneous cases are shown. The results of the proposed method were obtained using equation (48). The factors of safety in the proposed method were obtained by reducing the strength of the material on the slip surface until K_c became zero. The corresponding K_c value for the full strength is also shown.

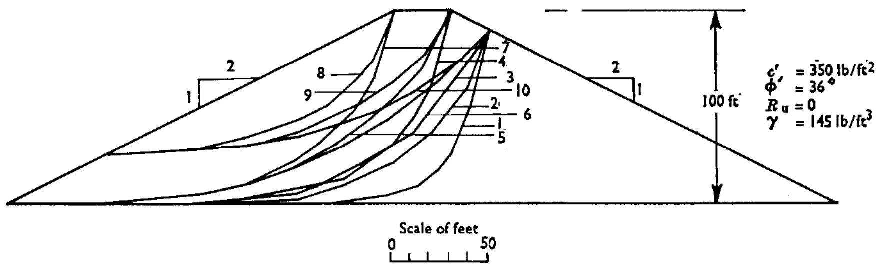


Fig. 2

Table 1(a). Homogeneous dam $\phi'=40°$, $c'=350$ lb/ft^2, $R_u=0$ (Fig. 2)

No.	Morgenstern–Price[1]		Author[2]
	F.S.	K_c	K_c
1	4·213	0·806[4]	0·807[3]
2	3·126	0·676	0·666
3	2·594	0·529	0·527
4	3·018	0·678	0·666
5	2·485	0·539	0·535
6	2·913	0·635	0·629
7	2·562	0·578	0·570
8	2·460	0·533	0·532
9	2·569	0·542	0·541
10	2·908	0·561	0·563

Table 1(b). $R_u=0{\cdot}40$ (Fig. 2)

No.	Morgenstern–Price[1]		Author[2]
	F.S.	K_c	K_c
4	1·742[4]	0·271[4]	0·245
5	1·449[4]	0·170[4]	0·164
6	1·711[4]	0·248[4]	0·235
7	1·480	0·190[4]	0·176
8	1·461	0·176	0·170

[1] $f(x) = 1$ for all cases.
[2] $f_i = 1$.
[3] No satisfactory solution obtained with varying f_i.
[4] Solution not acceptable.

Table 2. Non-homogeneous case

	Morgenstern–Price F.S.	Author F.S.	Author K_c	Note
1	1·014 (see Fig. 3)	0·98 1·00	−0·005 0·000	$f_i=1$ f_i=varied $f(x)=1$, Morgenstern–Price method
2	1·557 (see Fig. 4)	1·542	0·224	$f_i=1$; $f(x)=1$ in the Morgenstern–Price method. Whitman and Bailey give F.S.=1·24–1·31 for Morgenstern and Price solution. It is suspected that the large differences are due to the surfaces not being exactly the same. See also Bell (1968) F.S.=1·49

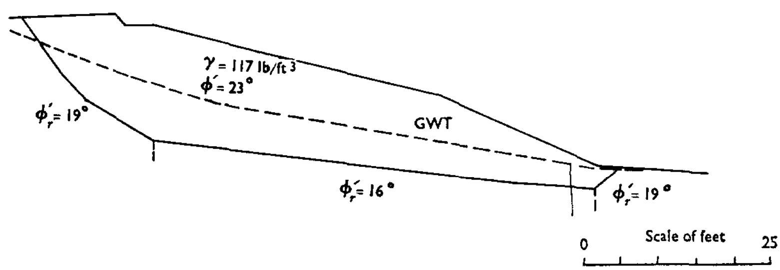


Fig. 3

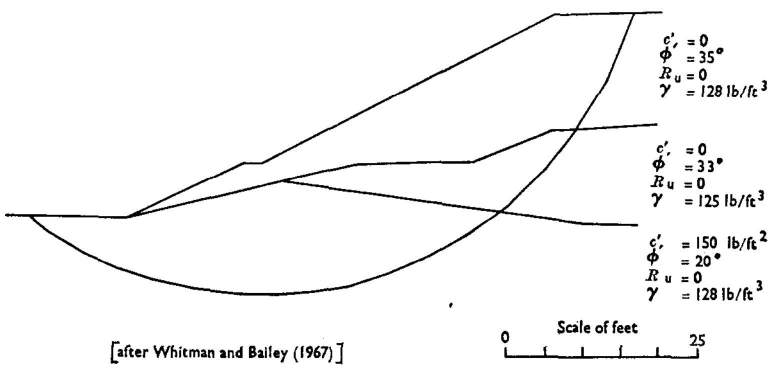


Fig. 4

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The calculations were carried out on the CDC 6400–6600 computer at Imperial College.

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APPENDIX

Derivation of the formula for X

Figure 5(b) shows that the plane inclined at an angle α to the horizontal (α is positive as shown) represents a failure plane. It can be shown from the Mohr's circle of stresses (Fig. 5(a)) that for such a plane

$$\sigma'_y = \frac{\sigma'_1+\sigma'_3}{2}+\frac{\sigma'_1-\sigma'_3}{2}\sin(2\alpha-\phi') \quad . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

$$\sigma'_x = \frac{\sigma'_1+\sigma'_3}{2}-\frac{\sigma'_1-\sigma'_3}{2}\sin(2\alpha-\phi') \quad . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

If on the failure plane the amount of pore pressure developed is Δu then adding Δu to both sides of equations (30) and (31)

$$\sigma_y = \frac{\sigma_1+\sigma_3}{2}+\frac{\sigma_1-\sigma_3}{2}\sin(2\alpha-\phi')$$

$$\sigma_x = \frac{\sigma_1+\sigma_3}{2}-\frac{\sigma_1-\sigma_3}{2}\sin(2\alpha-\phi')$$

therefore

$$K = \frac{\sigma_x}{\sigma_y} = \left[1-\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3}\sin(2\alpha-\phi')\right]\Big/\left[1+\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3}\sin(2\alpha-\phi')\right] \quad . \; . \; . \quad (32)$$

From the Mohr's circle of stresses (Fig. 5(a)) it can also be shown that

$$\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3} = \sin\phi'+\frac{2c'\cos\phi'}{\sigma_1+\sigma_3}-2\frac{\Delta u\sin\phi'}{\sigma_1+\sigma_3} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

Using equation (33) and the invariant $\sigma_1+\sigma_3=\sigma_x+\sigma_y$ and rearranging terms

$$K = \frac{\sigma_x}{\sigma_y} = \frac{1-\sin\beta\,(\sin\phi'+2c'\cos\phi'/\sigma_y-2\Delta u\sin\phi'/\sigma_y)}{1+\sin\beta\sin\phi'} \quad . \; . \; . \; . \quad (34)$$

where

$$\beta = 2\alpha-\phi'$$

therefore

$$\sigma_x = \sigma_y\left[\frac{1-\sin\beta\sin\phi'}{1+\sin\beta\sin\phi'}\right]-\frac{2c'\cos\phi'\sin\beta}{1+\sin\phi'\sin\beta}+\frac{2\Delta u\sin\phi'\sin\beta}{1+\sin\phi'\sin\beta} \quad . \; . \; . \; . \; . \; . \quad (35)$$

Homogeneous case

Let it be assumed that within the sliding mass along the vertical axis, all planes inclined at an angle α to the horizontal are in a state of limiting equilibrium.

In earth dams and slopes, we may approximate $\sigma_y=\gamma h$ and Δu is defined by $\Delta u=R_u\gamma h$ where h is the depth of a point from the free surface.

Substituting in equation (35)

$$\sigma_x = \gamma h\frac{1-\sin\beta\sin\phi'(1-2R_u)}{1+\sin\beta\sin\phi'}-\frac{2c'\cos\phi'\sin\beta}{1+\sin\beta\sin\phi'} \quad . \; . \; . \; . \; . \; . \quad (36)$$

then

$$E = \int_O^H \sigma_x\,\mathrm{d}h = \frac{\gamma H^2}{2}\left\{\frac{1-\sin\beta[(1-2R_u)\sin\phi'+(4c'/\gamma H)\cos\phi']}{1+\sin\beta\sin\phi'}\right\}$$

or

$$E = K'\gamma H^2/2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (37)$$

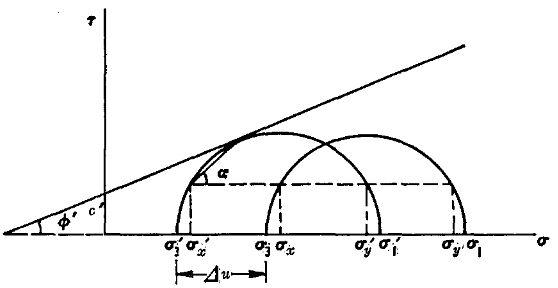

Fig. 5(a)

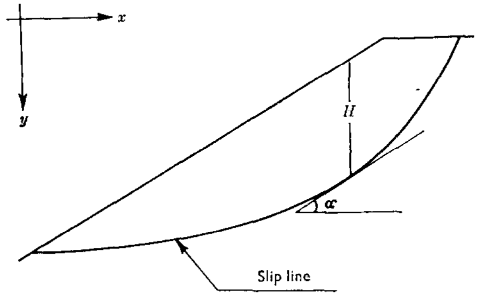

Fig. 5(b)

$$K' = \frac{1-\sin\beta[(1-2R_u)\sin\phi'+(4c'/\gamma H)\cos\phi']}{1+\sin\beta\sin\phi'} \quad . \quad . \quad . \quad . \quad . \quad (38)$$

It is assumed that the pore pressure is also homogeneous. This means that R_u is constant with depth. Total pore pressure on the vertical section is then

$$P_w = R_u\gamma H^2/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

Then

$$X = \left[(K'-R_u)\frac{\gamma H^2}{2}\tan\phi'+c'H\right]\frac{1}{F_L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where F_L is the average factor of safety along the vertical boundary of the slice.

Equation (40) may be written as

$$X = \left[(K'-R_u)\frac{\gamma H^2}{2}\tan\phi'+c'H\right]\lambda f(x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

where $\lambda f(x)$ is the reciprocal of F_L. The pattern of the variation of the local factor of safety at the vertical section along the x-axis is represented by $f(x)$. A solution to the problem reduces to the determination of the value of λ which will satisfy equation (16).

From the cases studied, it appears that $f(x)=1$ gives physically acceptable solutions but the resulting value of E computed from equation (23) may not exactly equal $K'H^2/2$. The local factor of safety is then determined from

$$F_L = [(E-P_w)\tan\phi'+c'H]/X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

which will not equal $1/\lambda f(x)$.

The formulæ given above applies to homogeneous cases only (homogeneous along the vertical sections not necessarily homogeneous along the sliding surface).

Non-homogeneous case

To extend the method to non-homogeneous cases, the following procedure is applied (Fig. 6).

Equation (35) may be written as

$$\sigma_x = a\sigma_y+b+d.\Delta u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42a)$$

where

$$a = \frac{1-\sin\beta\sin\phi'}{1+\sin\beta\sin\phi'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

$$b = -\frac{2c'\cos\phi'\sin\beta}{1+\sin\beta\sin\phi'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

$$d = \frac{2\sin\phi'\sin\beta}{1+\sin\beta\sin\phi'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

It is assumed that, if the shear strength along the vertical section were the same as that in the failure plane, all planes inclined at an angle α to the horizontal would be in a state of limiting equilibrium. Then a, b and d are constants along the vertical section and c' and ϕ' are those that apply at the slip surface. Then

$$E_j = \int_{h_j}^{h_{j+1}} \sigma_x \, dh = aw_j(h_{j+1}-h_j)+a_j(h_{j+1}-h_j)^2/2+b(h_{j+1}-h_j)+dP_{wj} \quad . \quad . \quad (46)$$

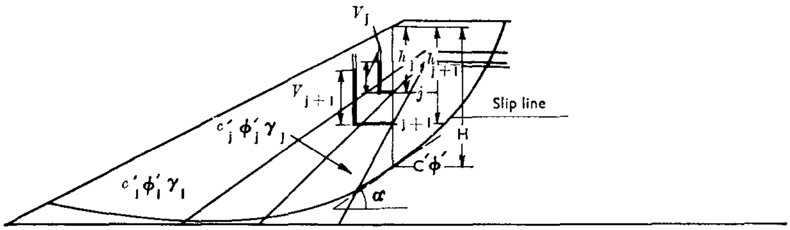

Fig. 6

where

$$w_j = \text{weight per unit area above the level } j = \sum_{k=1}^{j-1} \gamma_k DH_k$$

$$DH_k = h_{k+1} - h_k$$

$$P_{wj} = \tfrac{1}{2}\gamma_w(V_j + V_{j+1})(h_{j+1} - h_j) \quad (47)$$

and V_j is the piezometric height at level j, therefore

$$X = \sum X_j = \{\sum [(E_j - P_{wj}) \tan \phi'_j + c'_j(h_{j+1} - h_j)]\}\lambda.f(x) \quad (48)$$

where $\sum_j$ represents summation along the height.

An average density $\bar{\gamma}$ and an average pore pressure ratio $\bar{R}_u$ for the section can be computed by

$$\bar{\gamma} = \sum \gamma_j DH_j / H \quad (49)$$

where $\quad DH_j = h_{j+1} - h_j \quad (50)$

H=total height of the section.

$$\bar{R}_u = \frac{\sum P_{wj}}{\gamma H^2/2} \quad (51)$$

Similarly an average $\bar{K}'$, $\bar{\phi}$ and $\bar{c}$ for the section may be computed by

$$\bar{K}' = \frac{\sum E_j}{\bar{\gamma} H^2/2} \quad (52)$$

$$\tan \bar{\phi} = \frac{\sum (E_j - P_{wj}) \tan \phi'_j}{\sum (E_j - P_{wj})} \quad (53)$$

$$\bar{c} = \frac{\sum c'_j . DH_j}{H} \quad (54)$$

Then equation (48) becomes

$$X = \lambda.f(x)[(\bar{K}' - \bar{R}_u)(\bar{\gamma} H^2/2) \tan \bar{\phi} + \bar{c} H] \quad (55)$$

The local factor of safety F_L is computed from

$$F_L = [(E - \sum P_{wj}) \tan \bar{\phi} + \bar{c} H]/X \quad (56)$$

where E and X are obtained from the solution.

Partially submerged slopes

In the cases of partially submerged slopes, the method suggested by Bishop (1955) can be adopted. In this case the vertical weight is computed on the basis of the submerged weight below the line ab and saturated weight above the line. In computing the horizontal load KW, the saturated weight for the whole column must be used (Fig. 7). The pore pressures are expressed as excess over the hydrostatic water level. In this case, the abscissa of the centre of gravity x_g is computed on the basis of the vertical weight and the ordinate y_g is computed on the basis of full saturated weight. In case the slip surface appears below the line ab in the other side, the line ab should be assumed to follow the free surface bD.

W_1 = saturated weight of the column below the line ab (dotted)
W_2 = saturated weight above the line
W_{sub} = submerged weight of the column below the line
$W_{vertical} = W_{sub} + W_2$
$KW = K(W_1 + W_2)$

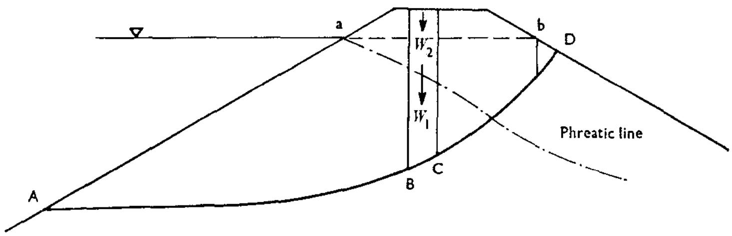

Fig. 7

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