Abstract

The Extended Voronoi Transform and the Fast Marching Method combination provide potential maps for robot navigation in previously unexplored dynamic environments. The Extended Voronoi Transform of a binary image of the environment gives a grey scale that is darker near obstacles and walls and lighter when far from them. The Logarithm of the Extended Voronoi Transform imitates the repulsive electric potential from walls and obstacles. The method proposed, called the Voronoi Fast Marching method, uses a Fast Marching technique on the Extended Voronoi Transform of the environment’s image provided by sensors to determine a motion plan. The computational efficiency of the method lets the planner operate at high rate sensor frequencies. This avoids the need for collision avoidance algorithms. The robot is directed towards the most unexplored and free zones of the environment so as to be able to explore all the workspace. This method is very fast and reliable and the trajectories are similar to the human trajectories: smooth and not very close to obstacles and walls. In this article we propose its application in the exploration task of unknown environments.

Keywords:

Navigation map, mobile robots, global localization, evolutive algorithm, robot mapping.
I. INTRODUCTION

Sensor based exploration is a fundamental function for mobile robot intelligence. There is a variety of potential applications for autonomous mobile robots in such diverse areas as forestry, space, nuclear reactors, environmental disasters, industry, and offices. Tasks in these environments are often hazardous to humans, in a remote located area, or tedious to perform. Potential tasks for autonomous mobile robots include maintenance, delivery, and security surveillance, which all require some form of intelligent navigational capabilities. A mobile robot is a useful addition to these domains only when it is capable of functioning robustly under a wide variety of environmental conditions, operating without human intervention for long periods of time, and providing some guarantee of task performance. The environments in which mobile robots must function are dynamic, unpredictable and not completely specifiable by a map beforehand. In order for the robot to successfully complete a set of tasks, it must dynamically adapt to changing environmental circumstances. Sensor-based discovery path planning is the guidance of an agent - a robot - without a complete a priori map, by discovering and negotiating the environment so as to reach a goal location while avoiding all encountered obstacles. Sensor-based discovery (i.e., dynamic) path planning is problematic because the path needs to be continually recomputed as new information is discovered.

In order to explore an unknown environment, this paper presents an exploration and path planning method based on the Logarithm of the Extended Voronoi Transform and the Fast Marching Method. In each step of the exploration process the sensors provide a binary image of the visible environment having distinguished the detected obstacles of the free space. The Extended Voronoi Transform of an image gives a grey scale that is darker near the obstacles and walls and lighter when far from them. The Logarithm of the Extended Voronoi Transform imitates the repulsive electric potential in 2D from walls and obstacles. This potential impels the robot to follow a trajectory far from obstacles.

The last step is to calculate the trajectory in the image generated by the Logarithm of the Extended Voronoi Transform using the Fast Marching Method. Then, the path obtained verifies the smoothness and safety considerations required for mobile robot path planning.

The Fast Marching Method has been applied to Path Planning [30], and their trajectories are of minimal distance, but they are not very safe because the path is too close to obstacles and what is more important, the path is not
In order to improve the safety of the trajectories calculated by the Fast Marching Method, two solutions are possible:

The first possibility, in order to avoid unrealistic trajectories produced when the areas are narrower than the robot, the segments with distances to the obstacles and walls smaller than the size of the robot need to be removed from the Voronoi diagram previous to the Extended Voronoi Transform.

The second possibility, used in this work, is to dilate the objects and walls in a security distance that assure that the robot not collide and does not accept passages narrower than the robot’s size.

The advantages of this method are its easy implementation, its speed and the quality of the trajectories. The method works in 2D and 3D, and can be used on a local scale operating with sensor information.

II. PREVIOUS AND RELATED WORKS

A. Representations of the world

Roughly speaking there are two main forms for representing the spatial relations in an environment: metric maps and topological maps. Metric maps are characterized by a representation where the position of the obstacles are indicated by coordinates in a global frame of reference. Some of them represent the environment with grids of points, defining regions that can be occupied or not by obstacles or goals [18], [20]. Topological maps represent the environment with graphs that connect landmarks or places with special features [24] [3]. In our approach we choose the grid-based map to represent the environment. The clear advantage is that with grids we already have a discrete environment representation and ready to be used in conjunction with the Extended Voronoi Transform function and Fast Marching Method for path planning. The pioneer method for environment representation in a grid-based model was the certainty grid method developed at Carnegie Mellon University [18] by Moravec. He represents the environment as a 3D or 2D array of cells. Each cell stores the probability of the related region being occupied. The uncertainty related to the position of objects is described in the grid as a spatial distribution of these probabilities within the occupancy grid. The larger the spatial uncertainty, the greater the number of cells occupied by the observed object. The update of these cells is performed during the navigation of the robot or
through the exploration process by using an update rule function. Many researchers have proposed their own grid-based methods. The main difference among them is the function used to update the cell. Some of them are, for example: Fuzzy [13], Bayesian [19], Heuristic Probability [10], Gaussian [11], etc. In the Histogramic In-Motion Mapping (HIMM), each cell, has a certainty value, which is updated whenever it is being observed by the robots sensors. The update is performed by increasing the certainty value by 3 (in the case of detection of an object) or by decreasing it by 1 (when no object is detected), where the certainty value is an integer between 0 and 15.

B. Approaches to exploration

This section relates some interesting techniques used for exploratory mapping. They mix different localization methods, data structures, search strategies and map representations. Kuipers and Byun [4] proposed an approach to explore an environment and to represent it in a structure based on layers called Spatial Semantic Hierarchy (SSH) [3]. The algorithm defines distinctive places and paths, which are linked to form an environmental topological description. After this, a geometrical description is extracted. The traditional approaches focus on geometric description before the topological one. The distinctive places are defined by their properties and the distinctive paths are defined by the twofold robot control strategy: follow-the-mid-line or follow-the-left-wall. The algorithm uses a lookup table to keep information about the place visited and the direction taken. This allows a search in the environment for unvisited places. Lee [23] developed an approach based on Kuipers work [4] on a real robot. This approach is successfully tested in indoor office-like spaces. This environment is relatively static during the mapping process. Lee’s approach assumes that walls are parallel or perpendicular to each other. Furthermore, the system operates in a very simple environment comprised of cardboard barriers. Mataric [24] proposed a map learning method based on a subsumption architecture. Her approach models the world as a graph, where the nodes correspond to landmarks and the edges indicate topological adjacencies. The landmarks are detected from the robot movement. The basic exploration process is wall-following combined with obstacle avoidance. Oriolo et al. [14] developed a grid-based environment mapping process that uses fuzzy logic to update the grid cells. The mapping process runs on-line [13], and the local maps are built from the data obtained by the sensors and integrated into the environment map as the robot travels along the path defined by the $A^*$ algorithm to the goal. The algorithm has two phases. The first one is the perception phase. The robot acquires data from the sensors and
updates its environment map. The second phase is the planning phase. The planning module re-plans a new safe path to the goal from the new explored area. Thrun and Bucken [28][29] developed an exploration system which integrates both evidence grids and topological maps. The integration of the two approaches has the advantage of disambiguating different positions through the grid-based representation and performing fast planning through the topological representation. The exploration process is performed through the identification and generation of the shortest paths between unoccupied regions and the robot. This approach works well in dynamic environments, although, the walls have to be flat and cannot form angles that differ more than 15° from the perpendicular. Feder et al. [17] proposed a probabilistic approach to treat the concurrent mapping and localization using a sonar. This approach is an example of a feature-based approach. It uses the extended Kalman filter to estimate the localization of the robot. The essence of this approach is to take actions that maximize the total knowledge about the system in the presence of measurement and navigational uncertainties. This approach was tested successfully in wheeled land robot and autonomous underwater vehicles (AUVs). Yamauchi [35][5] developed the Frontier-Based Exploration to build maps based on grids. This method uses a concept of frontier, which consists of boundaries that separate the explored free space from the unexplored space. When a frontier is explored, the algorithm detects the nearest unexplored frontier and attempts to navigate towards it by planning an obstacle free path. The planner uses a depth-first search on the grid to reach that frontier. This process continues until all the frontiers are explored. Zelek [37] proposed a hybrid method that combines a local planner based on a harmonic function calculation in a restricted window with a global planning module that performs a search in a graph representation of the environment created from a CAD map. The harmonic function module is employed to generate the best path given the local conditions of the environment. The goal is projected by the global planner in the local windows to direct the robot. Recently, Prestes el al. [33] have investigated the performance of an algorithm for exploration based on partial updates of a harmonic potential in an occupancy grid. They consider that while the robot moves, it carries along an activation window whose size is of the order of the sensors range.

Prestes and coworkers [34] propose an architecture for an autonomous mobile agent that explores while mapping a two-dimensional environment. The map is a discretized model for the localization of obstacles, on top of which a harmonic potential field is computed. The potential field serves as a fundamental link between the modeled
(discrete) space and the real (continuous) space where the agent operates.

The proposed method in this paper can be included in the sensor-based global planner paradigm. It is a potential method but it does not have the typical problems of these methods enumerated by Koren-Borenstein [3]: 1) Trap situations due to local minima (cyclic behavior). 2) No passage between closely spaced obstacles. 3) Oscillations in the presence of obstacles. 4) Oscillations in narrow passages. The proposed method is conceptually close to the navigation functions of Rimon-Koditscheck [6], because the potential field has only one local minimum located at the single goal point. This potential and the paths are smooth (the same as the repulsive potential function) and there are no degenerate critical points in the field. These properties are similar to the characteristics of the electromagnetic waves propagation in Geometrical Optics (for monochromatic waves with the approximation that length wave is much smaller than obstacles and without considering reflections nor diffractions).

The Fast Marching Method has been used previously in Path Planning by Sethian [32], [31], but using only an attractive potential. This method has some problems. The most important one that typically arises in mobile robotics is that optimal motion plans may bring robots too close to obstacles (including people), which is not safe. This problem has been dealt with by Latombe [22], and the resulting navigation function is called NF2. The Voronoi Method also tries to follow a maximum clearance map [16]. Melchior, Poty and Oustaloup [26], [1], present a fractional potential to diminish the obstacle danger level and improve the smoothness of the trajectories, Philippsen [27] introduces an interpolated Navigation Function, but with trajectories too close to obstacles and without smooth properties and Petres [7], introduces efficient path-planning algorithms for Underwater Vehicles taking advantage of the underwaters currents.

To achieve a smooth and safe path, it is necessary to have smooth attractive and repulsive potentials, connected in such a way that the resulting potential and the trajectories have no local minima and curvature continuity to facilitate path tracking design. The main improvement of the proposed method are these good properties of smoothness and safety of the trajectory. Moreover, the associated vector field allows the introduction of nonholonomic constraints.

It is important to note that in the proposed method the important ingredients are the attractive and the repulsive potentials, the way of connecting them describing the attractive potential using the wave equation (or in a simplified way, the eikonal equation). This equation can be solved in other ways: Mauch[25] uses a Marching with Correctness
Criterion with a computational complexity that can reduced to $O(N)$. Covello[8] presents a method that can be used on nodes that are located on highly distorted grids or on nodes that are randomly located.

III. INTRODUCTION TO THE EXTENDED VORONOI TRANSFORM

The Distance Transform[2] is a useful tool in digital picture processing. It has found a wide range of uses in image analysis, pattern recognition, and robotics. In Computer Vision, it is known as Distance Transform, but this term is also used in Robotics to designate a different concept. For this reason, in Robotics, this concept is called Extended Voronoi Transform.

The Extended Voronoi Transform computes the Euclidean Distance of the binary image. For each pixel in the image, the Extended Voronoi Transform assigns a number which is the distance to the nearest nonzero pixel of the image. In a binary image, a pixel is referred to as background if its value is zero. For a given distance metric, the Extended Voronoi Transform of an image produces a distance map of the same size. For each pixel inside the objects in the binary image, the corresponding pixel in the distance map has a value equal to the minimum distance to the background.

Clearly, the Extended Voronoi Transform is closely related to the Voronoi diagram. The Voronoi diagram concept is involved in many Extended Voronoi Transform approaches either explicitly or implicitly.

For any topologically discrete set $S$ of points in Euclidean space and for almost any point $x$, there is one point of $S$ to which $x$ is closer than $x$ is to any other point of $S$. The word “almost” is occasioned by the fact that a point $x$ may be equally close to two or more points of $S$. If $S$ contains only two points, $a$, and $b$, then the set of all points equidistant from $a$ and $b$ is a hyperplane, i.e. an affine subspace of codimension 1. That hyperplane is the boundary between the set of all points closer to $a$ than to $b$, and the set of all points closer to $b$ than to $a$.

In general, the set of all points closer to a point $c$ of $S$ than to any other point of $S$ is the interior of a convex polytope (in some cases unbounded) called the Dirichlet domain or Voronoi cell for $c$. The set of such polytopes tesselates the whole space, and is the Voronoi tessellation corresponding to the set $S$. If the dimension of the space is only 2, then it is easy to draw pictures of Voronoi tessellations, and in that case they are sometimes called Voronoi diagrams.
The close relation between EVT and the Voronoi Diagram implies that it is possible to compute one of them from the other. The majority of the EVT methods use the Voronoi Diagram as an intermediate step.

For more than two dimensions, the Extended Voronoi Transform uses a nearest-neighbor search on an optimized kd-tree, as described by Friedman[12].

The algorithm used in this work for two dimensions, is based on the second algorithm work carried out by Breu et al. [6]. In this work, the special properties of the Euclidean metric are exploited. They designed two linear-time algorithms based on Voronoi transforms where the second algorithm could have been improved if they had used the result of the previous row to reduce the set of possible candidates. It is an $O(m \times n)$ algorithm, where the image size is $m \times n$.

The Voronoi approach to path planning has the advantage of providing the safest trajectories in terms of distance to obstacles but because its nature is purely geometric and it does not achieve enough smoothness.

IV. INTUITIVE INTRODUCTION OF THE EIKONAL EQUATION AND THE FAST MARCHING PLANNING METHOD

Intuitively, Fast Marching Method gives the propagation of a front wave in an inhomogeneous media as shown in fig 1a and b.

Let us imagine that the curve or surface moves in its normal direction with a known speed $F$(see fig. 1c). The objective would be to follow the movement of the interface while this one evolves. A large part of the challenge, in the problems modeled as fronts in evolution, consists in defining a suitable speed, which faithfully represents
A way to characterize the position of a front in expansion is to compute the time of arrival $T$, in which the front reaches each point of the underlying mathematical space of the interface. It is evident that for one dimension (see fig. 1d) the equation for the arrival function $T$ can be obtained in an easy way, simply by considering the fact that the distance $x$ is the product of the speed $F$ by the time $T$: $x = F \cdot T$. The spatial derivative of the solution function becomes the gradient: $\frac{1}{F} = \frac{dT}{dx}$ and therefore we have that the magnitude of the gradient of the arrival function $T(x)$ is inversely proportional to the speed, $\frac{1}{F} = |\nabla T|$. For multiple dimensions, the same concept is valid because the gradient is orthogonal to the level sets of the arrival function $T(x)$. In this way, the movement of the front can be characterized as the solution of a boundary conditions problem. The speed $F$ depends only on the position, then the equation $\frac{1}{F} = |\nabla T|$ or the Eikonal equation:

$$|\nabla T| \cdot F = 1. \quad (1)$$

As a simple example we define a circular front $\gamma_t = \{(x, y)/T(x, y) = t\}$ for two dimensions that advance with unitary speed. The evolution of the value of the arrival function $T(\theta)$ can be seen as the time increases (i.e. $T = 0, T = 1, T = 2, ...$) and the arrival function comes to points of the plane in more external regions of the surface as can be seen in fig. 2. The boundary condition is that the value of the wave front is zero in the initial curve.

The direct use of the Fast Marching method does not guarantee a smooth and safe trajectory. Due to the way the front wave is propagated the shortest geometrical path is determined. This makes the trajectory unsafe because it touches corners, walls and obstacles, as is shown in figure 5. This problem can be easily solved by enlarging the

![Fig. 2. Movement of a circular wavefront, as a problem of boundary conditions](image-url)
obstacles, but even in that case the trajectory tends to get close to the walls and it is not smooth and safe enough.

The use of the Fast Marching method over a slowness (refraction or inverse of velocity) potential improves the quality of the calculated trajectory considerably. On one hand, the trajectories tend to go close to the Voronoi skeleton because of the optimal conditions of this area for robot motion[15]. On the other hand, the trajectories are also considerably smooth. For a small and easy H-shaped environment, the slowness (velocity inverse) potential in 3D is shown in fig. 3b and the funnel shaped potential given by the wave propagation of and the trajectory calculated by the gradient method is shown in fig. 3a.

For further details and summaries of level set and fast marching techniques for numerical purposes, see (Sethian[31]). The Fast Marching Method is an $O(n)$ algorithm as has been demonstrated by Yatziv [36].

V. INTUITIVE INTRODUCTION TO THE VORONOI FAST MARCHING METHOD (VFM)

Which properties and characteristics are desirable for a Motion Planner of a mobile robot? The first one is that the planner always drives the robot in a smooth and safe way to the goal point. In Nature there are phenomena with the same way of working: electromagnetic waves. If in the goal point, there is an antenna that emits an electromagnetic wave, then the robot could drive himself to the destination following the waves to the source. The concept of the electromagnetic wave is especially interesting because the potential and its associated vector field,
see figure 3, have all the good properties desired for the trajectory, such as smoothness (it is $C^\infty$) and the absence of local minima.

This attractive potential still has some problems. The most important one that typically arises in mobile robotics is that optimal motion plans may bring robots too close to obstacles, which is not safe. This problem has been dealt with by Latombe [22], and the resulting navigation function is called NF2. The Voronoi Method also tries to follow a maximum clearance map [16]. To get a safe path, it is necessary to add a component that repels the robot away from obstacles. In addition, this repulsive potential and its associated vector field should have good properties such those of electrical field. If we consider that the robot has an electrical charge of the same sign as obstacles, then the robot would be pushed away from obstacles. The properties of this electric field are very good because it is smooth and there are no singular points in the interest space ($C_{\text{free}}$).

The third part of the problem consists in how to mix the two fields together. This union between an attractive and a repulsive fields has been the biggest problem for the potential fields in path planning since the works of Khatib [21]. In our exposition, this problem has been solved in the same way that Nature does so: the electromagnetic waves, as light, have a propagation velocity that depends on the media. For example flint glass has a refraction index of 1.6, while in the air it is approximately one. This refraction index of a medium is the quotient between the velocity of light in the vacuum and the velocity in the medium. That is the slowness index of the front wave propagation of a medium.

For this reason, in the VFM proposed method, the repulsive potential is used as refraction index of the wave emitted from the goal point. This way a unique field is obtained and its associated vector field is attractive to the goal point and repulsive from the obstacles. This method inherits the properties of the electromagnetic field. Intuitively, the VFM Method gives the propagation of a front wave in an inhomogeneous media as shown in fig 1a and b.

**VI. ALMOST FLAT WAVE APPROACH OF THE WAVE EQUATION: THE EIKONAL EQUATION**

Maxwell’s laws govern the propagation of the electromagnetic waves and can be modeled with the wave equation:

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \nabla^2 \phi$$
In electrodynamics, each component of the fields satisfies the wave equation where \( c^2 = \frac{c_0^2}{\mu \epsilon} \). (\( c_0 \) is the speed of light in a vacuum, \( \mu \) is the permeability and \( \epsilon \) is the dielectric constant). A solution of the wave equation is called a wave. The moving boundary of a disturbance is called a wave front. In this section we will show how the wave front can be described by an eikonal equation. We follow the reasoning presented in [9]. If \( c \) is constant, then there are plane traveling wave solutions of the form

\[
\phi = \phi_0 e^{i(kx - \omega t)}
\]

(We can take the real or imaginary part to obtain a real-valued solution). Here the constant \( \phi_0 \) is the amplitude and \( \omega \) is the frequency. The wave number vector \( \mathbf{k} \) is in the direction of the wave. This is perpendicular to the wave fronts which satisfy \( kx - \omega t = \text{constant} \). The wave number \( k \) is the length of the wave vector, \( k = \sqrt{\mathbf{k} \cdot \mathbf{k}} \) and satisfies \( k = \frac{\omega}{c} \). The index of refraction \( \eta \) is defined by \( c = \frac{c_0}{\eta} \). Let \( k_0 \) be the wave number in a vacuum where the index of refraction is unity. For simplicity, consider a wave propagating in the first coordinate direction.

\[
\phi = \phi_0 e^{ik_0(\eta x - c_0 t)}
\]

(2)

Here we have factorized out \( k_0 \) because we will be considering the case where the wave number is large. Now we consider the case that the index of refraction \( \eta \) is spatially dependent. We seek a solution that is similar to the plane wave in eq. 2.

\[
\phi = \exp(A(x) + ik_0(\psi(x) - c_0 t))
\]

(3)

Here the amplitude \( e^A \) and the phase \( k_0\psi \) are determined by the slowly varying functions \( A(x) \) and \( \psi(x) \).

To study the transmission of rays, a useful approach is the "almost flat waves" (i.e. the optical wave propagates at wavelengths much smaller than image objects so that ray optics can approximate wave optics) in isotropic and possibly non homogeneous media for a monochromatic wave. These equations along with the mentioned approach, allow us to develop the theories based on rays such as: geometrical optics, the theory of sound waves, etc. We compute the derivatives of this approximate almost flat wave.

\[
\nabla \phi = \phi \nabla (A + ik_0\psi)
\]
$$\nabla^2 \phi = \phi (\nabla^2 A + i k_0 \nabla^2 \psi + (\nabla A)^2 - k_0^2 (\nabla \psi)^2 + i 2 k_0 \nabla A \cdot \nabla \psi)$$

We substitute eq. 3 into the wave equation.

$$\frac{\eta^2}{c_0^2} = \nabla^2 \phi$$

$$-k_0^2 \eta^2 = \nabla^2 A + i k_0 \nabla^2 \psi + (\nabla A)^2 - k_0^2 (\nabla \psi)^2 + i 2 k_0 \nabla A \cdot \nabla \psi$$

Since $A$ and $\psi$ are real-valued, we equate the real and imaginary parts.

$$\nabla^2 A + (\nabla A)^2 + k_0 (\eta^2 - (\nabla \psi)^2) = 0$$

$$2 \nabla A \cdot \nabla \psi + \nabla^2 \psi = 0$$

We assume that $\eta$ varies slowly on the length scale of a wavelength, $\lambda = 2\pi/k$. Alternatively, for a fixed function $\eta$, we assume that the frequency is high (the wavelength is short). This is the geometrical optics approximation.

For large $k_0$, the first equation is approximately solved by an eikonal equation:

$$|\nabla \psi|^2 = \eta^2$$

We rewrite this eikonal equation in terms of the phase $u$ of the wave.

$$\phi = \exp (A(x) + i (u(x) - \omega t))$$

$$|\nabla u|^2 = \frac{\omega^2}{c^2}$$

Surfaces of constant $u$ describe the wave fronts. In the Sethian [30] notation

$$|\nabla T| \cdot F = 1$$

where $T(x)$ represents the wavefront (time), $F(x)$ is the slowness index of the medium.

In Geometrical Optics, Fermat’s least time principle for light propagation in a medium with space varying refractive index $\eta(x)$ is equivalent to the eikonal equation and can be written as $|\nabla \Phi(x)| = \eta(x)$ where the eikonal $\Phi(x)$ is a scalar function whose isolevel contours are normal to the light rays. This equation is also known as Fundamental Equation of the Geometrical Optics.

The eikonal (from the Greek "eikon", which means "image") is the phase function in a situation for which the phase and amplitude are slowly varying functions of position. Constant values of the eikonal represent surfaces of
constant phase, or wavefronts. The normals to these surfaces are rays (the paths of energy flux). Thus the eikonal equation provides a method for "ray tracing" in a medium of slowly varying index of refractive (or the equivalent for other kinds of waves).

VII. DETAILS OF THE VFM ALGORITHM

This method starts with the calculation of the Logarithm of the Extended Voronoi Transform of the 2D a priori map of the environment (or the Extended Voronoi Transform in case of 3D maps). Each white point of the initial image (which represents free cells in the map) is associated with a level of grey that is the logarithm of the 2D distance to nearest obstacles (or the Extended Voronoi Transform in 3D). As a result of this process, a kind of potential proportional to the distance to the nearest obstacles to each cell is obtained, see figure 4. Zero potential indicates that a given cell is part of an obstacle and maxima potential cells corresponds to cells located in the Voronoi diagrams (which are the cells located equidistant to the obstacles).

This function introduces a potential similar to a repulsive electric potential (in 2D) as shown in figure 4, that can be expressed by

$$ \phi = c_1 \log(r) + c_2. $$

(4)

where $c_1$ is a negative constant.

If $n > 2$, ($n$ is the space dimension), the potential is

$$ \phi = \frac{c_3}{r^{n-1}} + c_4. $$

(5)

where $r$ is the distance from the origin.

In a second step, the technique proposed here uses Fast Marching to calculate the shortest trajectory in the potential surface defined by logarithm of the Extended Voronoi Transform. The calculated trajectory is the geodesic one in the potential surface, i.e. with a viscous distance. This viscosity is done by the grey level. If the Fast Marching Method were used directly on the environment map, we would obtain the shortest geometrical trajectory, as shown in fig. 5, but the trajectory is not safe nor smooth.

The potential created has local minima as shown in fig. 6 and 7, but the trajectories are not stuck in these points because the Fast Marching Method gives the trajectories that correspond to the propagation of a wave front which
Fig. 4. Potential of the Logarithm of the inverse of Extended Voronoi Transform.

Fig. 5. Trajectory calculated with Fast Marching without the Logarithm Extended Voronoi Transform.

is faster in lighter regions and slower in the darker ones.

The trajectories obtained by using the logarithm of the EVT tend to go by the Voronoi diagram but properly smoothed as shown figure 6.

VIII. PROPERTIES

The proposed VFM algorithm has the following key properties:

- **Fast response.** The planner needs to be fast enough to be used reactively in case unexpected obstacles make it necessary to plan a new trajectory. To obtain this fast response, a fast planning algorithm and fast and simple treatment of the sensor information is necessary. This requires a low complexity order algorithm for a real time response to unexpected situations. As shown in table I, the proposed algorithm has a fast response time
to allow its implementation on real time, even in environments with moving obstacles using a normal PC computer.

**Smooth trajectories.** The planner must be able to provide a smooth motion plan which can be executed by the robot motion controller. In other words, the plan does not need to be refined, avoiding the need for a local refinement of the trajectory. The solution of the eikonal equation used in the proposed method is given by equation 2:

$$\phi = \phi_0 e^{ik_0 (\eta x - c_0 t)}$$

As this solution is an exponential, if the potential $\eta(x)$ is $C^\infty$ then the potential $\phi$ is also $C^\infty$ and therefore the trajectories calculated by the gradient method over this potential would be of the same class.

This smoothness property can be observed in figure 6, where trajectory is clearly good, safe and smooth. One advantage of the method is that it not only generates the optimum path, but also the velocity of the robot at each point of the path. The velocity reaches its highest values in the light areas and minimum values in the greyer zones. The $VFM$ Method simultaneously provides the path and maximum allowable velocity for a mobile robot between the current location and the goal.

- **Reliable trajectories.** The proposed planner provides a safe (reasonably far from a priori and detected obstacles) and reliable trajectory (free from local traps). This avoids the coordination problem between the local collision avoidance controllers and the global planners, when local traps or blocked trajectories exist in the environment.
This is due to the refraction index, which causes higher velocities far from obstacles.

- **Completeness.** As the method consists of the propagation of a wave, if there is a path from the initial position to the objective, the method is capable of finding it.

- **Stability.** The VFM 1st-step builds the slowness potential of the robot: \( \dot{x} = f(x) \) and the second step builds the corresponding Lyapunov potential \( V(x) \) that is radially unbounded. By construction this potential has zero value in the destination point and as it is the expansion of a wave with time as its last axis, it has positive values and it increases with time (see figure 2a).

Using the Lyapunov Stability theorem:

If \( V: D \to \mathbb{R} \) is a continuously differentiable scalar function in a neighborhood \( D \) of \( x = 0 \) such that:

\[
\begin{cases}
V(0) = 0 & \text{and } V(x) > 0, \quad \forall x \in D - \{0\} \\
\dot{V}(x) \leq 0, & \forall x \in D
\end{cases}
\]  

then the origin is globally asymptotically stable.

Therefore, we can establish that the destination point \((x = 0)\) is globally asymptotically stable.

**IX. IMPLEMENTATION OF THE EXPLORER**

In order to solve the problem of the exploration of an unknown environment, our algorithm can work in two different ways. In the first one, the initial information is the localization of the final goal. In this way, the robot has a general direction of movement towards the goal. In each movement of the robot, information about the environment is used to build a binary image distinguishing occupied space represented by value 0 (obstacles and walls) from free space, with value 1. The Extended Voronoi Transform of the known map at that moment, gives a grey scale that is darker near the obstacles and walls and lighter far from them. The Voronoi Fast Marching Method gives the trajectory from the pose of the robot to the goal point using the known information.

In this first case, the robot has a final goal: the exploration process the robot performs in the algorithm described in the flowchart of figure 7.

In the second way of working of the algorithm, the goal location is unknown and robot behavior is truly exploratory. We propose an approach based on the incremental calculation of a map for path planning. We define
an neighborhood window, that travels with the robot, with roughly the size of its laser sensor range. This window indicates the new grid cells that are recruited for update, i.e., if a cell was at a given time in the neighborhood window, it becomes part of the explored space by participating in the EVT and Fast Marching Method calculation for all times. The set of activated cells that compose the explored space is called the neighborhood region. Cells that were never inside the neighborhood window indicate unexplored regions. Their potential values are set to zero and define the knowledge frontier of the state space, the real space in our case. The detection of the nearest unexplored frontier comes naturally from the Extended Voronoi Transform calculation. It can also be understood from the physical analogy with electrical potentials that obstacles repel while frontiers attract.

Consider that the robot starts from a given position in an initially unknown environment. In this second method, there is no direction of the place where the robot must go. The first step consists of calculating a first matrix $W$ that gives us the EVT of the obstacles found up until the moment. A matrix with zeros in the obstacles and value 1 in the free zones is considered. The EVT is applied and a matrix $W$ of grays with values between 0 (obstacles) and 1 is obtained. The second matrix $VT$ is built darkening the zones that the robot already has visited for which it assigns value 1 to the points of the trajectory. Then, it calculates the EVT of the obtained image. Finally, matrix $WV$ is the sum of the matrices $VT$ and $W$, with weights 0.5 and 1 respectively. In this way, it is possible to darken
the zones already visited for the robot and impel it to go to the unexplored zones. The whitest point of matrix \( WV \) is calculated as \( \max(WV) \), that is, the most unexplored region that is in a free space. This is the point chosen as the new goal point. Applying Fast Marching method on \( WV \), the trajectory towards that goal is calculated. The robot moves following this trajectory. In the following steps, the trajectory to follow is computed, calculating at every moment first \( W \) and \( VT \), and therefore \( WV \), but without changing the objective point. Once the robot has been arrived at the objective, (that is to say, that path calculated is very small), a new objective is selected as \( \max(WV) \).

Therefore, the robot moves maximizing knowledge gain. In this case or in any other situation where there is no gradient to guide the robot, it simply follows the forward direction. The exploration process the robot performs in the second method described is summarized in the flowchart of figure 8.

The algorithms laid out in Figure 7 (Flowchart of Case 1) can be inefficient in very large environments. To increase speed it is possible to pick a goal point, put a neighborhood window the size of the sensor range, then run into the goal point, then look at the maximal initial boundary, recast and terminate when one reaches the boundary of the computed region. Similar improvements can be made to Algorithm 2.

X. Results

The method proposed, can be used for sensor based planning, working directly on a raw sensor image of the environment, as shown in figures 9 and 10.

To illustrate the method possibilities, a trajectory in a typical office’s indoor environment has been used for planning as shown in figure 11. The dimensions of the environment are 116x14 meters (the cell resolution is 12 cm), that is the image has 966x120 pixels. For this environment the first step (Log of inverse Extended Voronoi Transform) takes 0.06 seconds in a Pentium 4 at 2.2 Ghz, and the second step (Fast Marching) takes 0.20 seconds for a long trajectory.

The method provides smooth trajectories that can be used at low control levels without any additional smooth interpolation process. The results are shown in figure 12 (Log of the inverse Extended Voronoi Transform of the environment map of the Robotics Lab known a priori), some of the steps of the process in the figure 13 (the red line represents the crossed path and the blue one represents the calculated trajectory from the present position to the destination point). In each moment the illuminated area represents the front wave propagation from the present
Fig. 8. Flowchart of algorithm 2.

position of the robot to the destination point. The EVT computation is made over the sensory map that the robot has in its memory. Finally in figure 14 the path obtained after applying the Fast Marching method to the previous potential image is shown.

For the case of exploration that this paper contemplates, the results of two different tests are presented to illustrate both cases described for the application of the proposed method.

Figure 15 represents the first case for implementing the exploration method. A final goal is provided for the robot, which is located with respect to a global reference system; the starting point of the robot movement is also known with respect to that reference system. The algorithm allows calculating the trajectory towards that final goal with the updated information of the surroundings that the sensors obtain in each step of the movement. When the robot reaches the defined goal, a new destination in an unexplored zone is defined, as it can be seen in the seventh image of the figure. In this case the size of image is 628x412.

The results of one of the tests done for the second case of exploration described are shown in figure 16. Any
Fig. 9. Laser data read by the robot.

Fig. 10. Trajectory calculated with Fast Marching using the laser data (Local map).

Fig. 11. Environment map of the Robotics Lab.

Fig. 12. Log of the Extended Voronoi Transform applied of the environment map of the Robotics Lab.
Fig. 13. Consecutive steps of the process (the red line represents the crossed path and the blue one represents the calculated trajectory from the present position to the destination point).
Fig. 14. Trajectory calculated to avoid obstacles in a cluttered environment with Fast Marching and the Logarithm Extended Voronoi Transform (Global map).

final goal is defined. The algorithm leads the robot towards the zones that are free of obstacles and unexplored simultaneously.

The proposed method is highly efficient from a computational point of view because the method operates directly over a 2D image map (without extracting adjacency maps), and due to the fact that Fast Marching complexity is $O(m \times n)$ and the Extended Voronoi Transform is also of complexity $O(m \times n)$, where $m \times n$ is the number of cells in the environment map. In table I, orientative results of the cost average in time appear (measured in ms), and each step of the algorithm for different trajectory lengths to calculate (the computational cost is depending on the number of points of the image).

<table>
<thead>
<tr>
<th>Alg. Step/Trajectory length</th>
<th>Long (ms)</th>
<th>Medium (ms)</th>
<th>Short (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obst. Enlarging</td>
<td>0.008</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>Ext. Voronoi Transf.</td>
<td>0.039</td>
<td>0.039</td>
<td>0.039</td>
</tr>
<tr>
<td>FM Exploration</td>
<td>0.172</td>
<td>0.078</td>
<td>0.031</td>
</tr>
<tr>
<td>Path Extraction</td>
<td>0.125</td>
<td>0.065</td>
<td>0.035</td>
</tr>
<tr>
<td>Total time</td>
<td>0.344</td>
<td>0.190</td>
<td>0.113</td>
</tr>
</tbody>
</table>

XI. CONCLUSION

The results obtained show that the Logarithm of Extended Voronoi Transform can be used to improve the results obtained with Fast Marching method applied to environment exploration, providing smooth and safe trajectories
Fig. 15. Simulation results with method 1, with final objective.
Fig. 16. Simulation results with method 2, without final objective
along the exploratory process.

The algorithm complexity is $O(m \times n)$, where $m \times n$ is the number of cells in the environment map, which let us use the algorithm on line. Furthermore, the algorithm can be used directly with raw sensor data to implement a sensor based local path planning exploratory module.

REFERENCES


