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Shear Behavior of Reinforced Concrete Beams with High-Strength Stirrups

by Jung-Yoon Lee, Im-Jun Choi, and Sang-Woo Kim

ACI 318-08 limits the yield strength of shear reinforcement to control diagonal crack width and prevent possible sudden shear failure due to over-reinforcement. The limitations on the yield strength of shear reinforcement provided by the four current design codes—ACI 318-08, CSA A23.3-04, EC2-02, and JSCE-02—differ substantially from one another. ACI 318-08 limits the yield strength of shear reinforcement to 420 MPa (60,900 psi), whereas JSCE-02 allows the yield strength to reach up to 800 MPa (116,000 psi) when the compressive strength of the concrete is greater than 60 MPa (8700 psi).

This paper presents the effects of the yield strength of shear reinforcement and the compressive strength of concrete on the shear behavior of reinforced concrete (RC) beams. Thirty-two simply supported RC beams with high-strength stirrups were tested. Although the test beams were designed to have a much greater yield strength of shear reinforcement than that required in ACI 318-08, the test beams failed in shear after the yielding of shear reinforcement. In addition, the analytical and experimental results indicated that the diagonal crack width at different load levels of all the tested beams was nearly constant regardless of the yield strength of shear reinforcement.

Keywords: beams; crack width; failure modes; high-strength concrete; reinforced concrete; shear reinforcement; yield strength.

INTRODUCTION

The number of high-rise reinforced concrete (RC) buildings has been steadily increasing since the 1980s. The use of high-strength concrete is indispensable in high-rise RC construction to ensure the sufficient strength of the structure. The effect of high-strength concrete can be significantly improved by the use of high-strength, large-sized reinforcing bars. In particular, footing beams or the beams of an RC frame structure subjected to high lateral force are usually subjected to high shear force and heavier shear reinforcement is needed. The use of high-strength, heavier reinforcement, however, may induce RC members to fail suddenly in a brittle manner without sufficient warning and lead to severe cracking or deflection.

According to ACI 318-08, Section 9.4,[1] for non-prestressed flexural members, the yield strength of longitudinal tension steel used in design calculations shall not exceed 550 MPa (79,750 psi) to reserve adequate deformability and control deflections and cracking. ACI 318-08, Section 11.4.2,[1] also limits the yield strength f_{yt} of shear reinforcement used in shear design to 420 MPa (60,900 psi) for two reasons: 1) to control the diagonal crack width; and 2) to prevent possible sudden shear failure due to concrete crushing before yielding of the stirrups due to over-reinforcement. In ACI 318-08, Section 11.4.2,[1] however, the limitation of 420 MPa (60,900 psi) for shear reinforcement was raised to 550 MPa (79,750 psi) for deformed welded wire reinforcements. Research[2,3] has indicated that the performance of higher-strength steels as shear reinforcements has been satisfactory.

Whereas ACI 318-08[1] limits the yield strength f_{yt} of shear reinforcement to 420 MPa (60,900 psi), EC2-02,[4] which is based on a variable strut inclination method, allows f_{yt} to reach 600 MPa (87,000 psi). JSCE-02[5] also allows the yield strength of shear reinforcement to reach up to 800 MPa (116,000 psi) when the compressive strength of the concrete is greater than 60 MPa (8700 psi). On the other hand, the shear design in CSA A23.3-04[6] limits the yield strength f_{yt} of shear reinforcement to 500 MPa (72,500 psi). The limitations on the yield strength of shear reinforcement provided by the aforementioned four codes differ substantially from one another. For concrete with f'_c = 60 MPa (8700 psi), the maximum f_{yt} required in JSCE-02[5] is nearly double the value of f_{yt} in ACI 318-08.[1]

Although there are many studies regarding the behavior of RC beams subjected to shear, only a limited number of studies on RC beams regarding the yield strength of shear reinforcement are available. Shimono et al.[7] tested four RC beams with high-strength stirrups (f_{yt} = 1048 and 982 MPa [151,960 and 142,390 psi]) subjected to shear. They found that the shear reinforcement in the beams with a smaller number of stirrups reached its yield strain, whereas that of the beams with a greater number of stirrups did not. Hara et al.[8] tested four RC beams with high-strength stirrups (f_{yt} = 785 MPa [113,825 psi]) subjected to shear. They reported that when the compressive strength of the concrete was greater than 50 MPa (7250 psi), the high-strength shear reinforcement in the beams reached its yield strain.

RESEARCH SIGNIFICANCE

Compared with recent research efforts to develop high-strength concrete, studies on the development and practical application of high-strength reinforcing bars seem to be insufficient. Most high-rise RC structures are usually subjected to high axial and shear forces and heavier reinforcement is needed. In addition, reinforcement congestion can be avoided by using high-strength steel bars, resulting in easier construction practices and better quality control of concrete.

To avoid abrupt shear failure due to concrete crushing before the yielding of shear reinforcement and to control the diagonal crack width, design codes specify the limitations on the yield strength of shear reinforcement of RC beams. The

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design limitations of the yield strength of stirrups provided by the four current design codes—ACI 318-08,[1] EC2-02,[4] JSCE-02,[5] and CSA A23.3-04,[6]—differ from one another. This paper presents the test results of 32 simply supported RC beams with high-strength shear reinforcement. Two parameters are considered in this study: the compressive strength of the concrete and the yield strength of shear reinforcement. Shear failure modes, shear strength, and diagonal crack width are studied. In addition, the experimental results of the 49 RC beams reported in the literature are analyzed. The experimental and analytical results indicate that the limitation on the yield strength of shear reinforcement in ACI 318-08[1] is somewhat underestimated and needs to be increased for high-strength concrete beams.

TEST PROGRAM AND MEASUREMENTS

ACI 318-08[1] requires the minimum and maximum amounts of the shear reinforcement ratio to prevent brittle shear failures when inclined cracking occurs shortly afterwards[9] or to prevent the crushing of web concrete before the shear reinforcement yields. This code also limits the yield strength of shear reinforcement. Thirty-two simply supported RC beams were cast, instrumented, and tested to investigate the influences of the compressive strength of the concrete and the yield strength of shear reinforcement. The 32 RC specimens were divided into six groups: Groups-S20, -S30, -S35, -S40, -S50, and -S80, as shown in Table 1.

Test program

Thirty-two simply supported RC beams with different yield strengths of shear reinforcement and compressive strengths of concrete were prepared. The specimens were divided into six groups depending on the compressive strength of concrete f'_c = 25.0, 33.3, 35.0, 38.2, 50.3, and 81.4 MPa ([3625, 4829, 5075, 5539, 7294, and 11,803 psi] for Groups-S20, -S30, -S35, -S40, -S50, and -S80, respectively). Groups-S30, -S40, -S50, and -S80 each consisted of six RC beams, whereas Groups-S20 and -S35 each consisted of four RC beams. Ready mixed concrete was produced with Type 1 portland cement. A 25 mm (0.98 in.) maximum-size coarse aggregate was used for Groups-S20, -S30, -S35, and -S40, whereas a 20 mm (0.79 in.) maximum-size coarse aggregate was used for Groups-S50 and -S80. The cross-sectional dimensions of the specimens were 300 x 600, 300 x 450, and 250 x 350 mm (11.81 x 23.62, 11.81 x 17.72, and 9.84 x 13.78 in.) for Group-S30; Groups-S40, -S50, and -S80; and Groups-S20 and -S35, respectively. Figure 1 shows the details of Beams S20-3, S30-3, and S40-3.

Five types of deformed steel bars (D10) (A_s = 71.3 mm^2 [0.11 in.^{2}]) with different yield strengths were used for the shear steel bars, as shown in Table 1. Figure 2 shows the tensile stress versus tensile strain curves of the steel bars for shear reinforcement. As shown in Fig. 2, all the curves, with the exception of the high-strength steel bars (f_{yt} = 750.1 MPa [108,765 psi]), displayed a well-defined yield plateau. Although this plateau presents for the high-strength steel bars (f_{yt} = 634.1 MPa [91,945 psi]), it is shorter than that of normal-strength steel bars. Moreover, this plateau depends not only on the type of steel (manufacturing process), but also on the yield strength.

Two types of deformed steel reinforcements (D25 and D29) (A_s = 506.7 and 642.4 mm^2 [0.79 and 1.0 in.^{2}]) were used for the longitudinal tensile steel bars. To more clearly investigate the effect of high-strength materials and prevent the flexural yielding of the tensile steel bars before shear failure, a high reinforcement ratio was used for the longitudinal steel bars.

All the specimens, with the exception of the beams without stirrups, were shear reinforced with closed stirrups having 135-degree standard hooks according to the minimum and maximum shear requirements specified in ACI 318-08, Sections 11.4.6 and 11.4.7.9.[1] Twenty-eight beams in Groups-S30, -S40, -S50, and -S80, except Groups-S20 and -S35, were designed according to the maximum spacing requirement of the vertical stirrups specified in ACI 318-08, Section 11.4.5.[1] The shear span-depth ratio (a/d) of all the beams varied from 2.5 to 4.0. All the beams in each group were designed to share the same a/d, percentage of longitudinal reinforcement, spacing of stirrups, and percentage of shear reinforcement but had different yield strengths of shear reinforcement.

Considering the inherent scattered nature of concrete shear strength, a pair of specimens with the same experimental variables was designed in Group-S30. Concrete cylinders were tested on the first and last days of each beam test group. The beam tests were performed within 2 months after the placement of the concrete. The average compressive strengths of the concrete obtained from the cylinder tests are shown in Table 1.

Loading system and measurements

The schematic diagram of the experimental setup and the locations of the linear variable differential transducers (LVDTs) are shown in Fig. 3. The specimens were simply supported and subjected to two-point concentrated loads. In the test, a strain-controlled test procedure was adopted. Electrical strain gauges were attached to the surfaces of the steel bars to record the strains of the vertical and longitudinal steel bars in the test beams, as shown in Fig. 1. Six LVDTs were attached to each face of the beam near the shear-critical region to measure the longitudinal, transverse, and shear strains of each region. One LVDT was attached to the bottom surface at the midspan of the test beam to measure the midspan deflection of the beam.

The load was applied monotonically with occasional pauses to take the crack width measurements. A handheld microscope with a resolution of 0.02 mm (0.00079 in.) was used to measure the crack width.

TEST RESULTS

All the beams, with the exceptions of Beams S40-2 and S40-6, failed in shear without the flexural yielding of the longitudinal reinforcements. Beams S30-5 and S30-6 failed in shear nearly simultaneously when the longitudinal tensile steel bars reached their yield strain. Beam S40-6, however, failed after the longitudinal tensile steel bar reached its yield

Table 1—Specification of specimens and material properties

Beams		f'_c, MPa	Longitudinal tensile bars ρ_l, %	Longitudinal tensile bars f_{yl}, MPa	Shear steel bars s, mm	Shear steel bars ρ_t, %	Shear steel bars f_{yt}, MPa	b, mm	h, mm	a/d	Results, Test P_{max}, kN	Results, Test Δ_{max}, mm	Results, Test FM	Results, FE analysis P_{max}, kN	Results, FE analysis Δ_{max}, mm	Results, FE analysis FM
Group-S20	S20-1	25.0	2.70	707.1	200	0.30	484.4	250	350	2.5	374.1	8.57	SYCF	430.2	8.21	SYCF
	S20-2	25.0	2.70	707.1	200	0.30	555.3	250	350	2.5	401.7	14.79	SYCF	451.4	10.76	SYCF
	S20-3	25.0	2.70	707.1	200	0.30	634.1	250	350	2.5	441.7	15.18	SYCF	474.0	12.04	SYCF
	S20-4	25.0	2.70	707.1	200	0.30	750.1	250	350	2.5	502.5	14.48	SCF	492.2	11.37	SYCF
Group-S30	S30-1	33.3	2.92	530.0	—	—	—	300	600	4.0	380.0	10.0	SF	—	—	—
	S30-2	33.3	2.92	530.0	—	—	—	300	600	4.0	409.6	10.4	SF	—	—	—
	S30-3	33.3	2.92	530.0	320	0.15	580.0	300	600	4.0	600.2	24.1	SYCF	718.6	23.88	SYCF
	S30-4	33.3	2.92	530.0	320	0.15	580.0	300	600	4.0	620.8	22.3	SYCF	718.6	23.88	SYCF
	S30-5	33.3	2.92	530.0	160	0.30	580.0	300	600	4.0	893.1	29.5	SCF and FF	879.0	21.97	SYCF
	S30-6	33.3	2.92	530.0	160	0.30	580.0	300	600	4.0	893.7	30.4	SYCF and FF	879.0	21.97	SYCF
Group-S35	S35-1	35.0	2.70	707.1	200	0.30	484.4	250	350	2.5	451.6	8.69	SYCF	534.6	8.81	SYCF
	S35-2	35.0	2.70	707.1	200	0.30	555.3	250	350	2.5	489.3	9.99	SYCF	545.2	9.33	SYCF
	S35-3	35.0	2.70	707.1	200	0.30	634.1	250	350	2.5	516.0	10.73	SYCF	546.7	9.42	SYCF
	S35-4	35.0	2.70	707.1	200	0.30	750.1	250	350	2.5	507.0	10.60	SYCF	575.8	11.46	SYCF
Group-S40[10]	S40-1	38.2	4.65	554.0	—	—	—	300	450	2.76	351.6	3.72	SF	—	—	—
	S40-2	38.2	4.65	554.0	95	0.50	378.8	300	450	2.76	795.8	13.98	—	1184.2	12.68	SYCF
	S40-3	38.2	4.65	554.0	95	0.50	484.4	300	450	2.76	1073.9	16.36	SYCF	1258.2	12.69	SYCF
	S40-4	38.2	4.65	554.0	95	0.50	555.3	300	450	2.76	1133.4	17.99	SYCF	1283.4	12.71	SYCF
	S40-5	38.2	4.65	554.0	95	0.50	634.1	300	450	2.76	1183.4	18.58	SYCF	1379.6	14.68	SYCF
	S40-6	38.2	4.65	554.0	95	0.50	750.1	300	450	2.76	981.1	15.78	FF	1421.4	14.65	SYCF
Group-S50[10]	S50-1	50.3	4.65	554.0	—	—	—	300	450	2.76	373.6	4.16	SF	—	—	—
	S50-2	50.3	4.65	554.0	95	0.50	378.8	300	450	2.76	1174.4	16.53	SYCF	1198.0	12.63	SYCF
	S50-3	50.3	4.65	554.0	95	0.50	484.4	300	450	2.76	1281.7	14.62	SYCF	1307.6	12.69	SYCF
	S50-4	50.3	4.65	554.0	95	0.50	555.3	300	450	2.76	1313.5	16.15	SYCF	1364.2	13.68	SYCF
	S50-5	50.3	4.65	554.0	95	0.50	634.1	300	450	2.76	1420.2	16.41	SYCF	1429.2	14.18	SYCF
	S50-6	50.3	4.65	554.0	95	0.50	750.1	300	450	2.76	1517.3	17.74	SYCF	1503.6	15.27	SYCF
Group-S80[10]	S80-1	81.4	4.65	554.0	—	—	—	300	450	2.76	523.2	5.10	SF	—	—	—
	S80-2	81.4	4.65	554.0	95	0.50	378.8	300	450	2.76	1336.3	16.14	SYCF	1403.0	13.77	SYCF
	S80-3	81.4	4.65	554.0	95	0.50	484.4	300	450	2.76	1444.9	16.85	SYCF	1522.4	14.27	SYCF
	S80-4	81.4	4.65	554.0	95	0.50	555.3	300	450	2.76	1566.5	17.87	SYCF	1617.4	15.37	SYCF
	S80-5	81.4	4.65	554.0	95	0.50	634.1	300	450	2.76	1674.8	18.13	SYCF	1648.8	15.36	SYCF
	S80-6	81.4	4.65	554.0	95	0.50	750.1	300	450	2.76	1736.8	20.48	SYCF and FF	1700.6	15.33	FF

Notes: FM is failure mode; SF is shear failure; SYCF is shear failure after yielding of shear reinforcement; SCF is shear failure before yielding of shear reinforcement; FF is flexural failure; 1 MPa = 145 psi; 1 mm = 0.0394 in.; 1 kN = 0.225 kips.

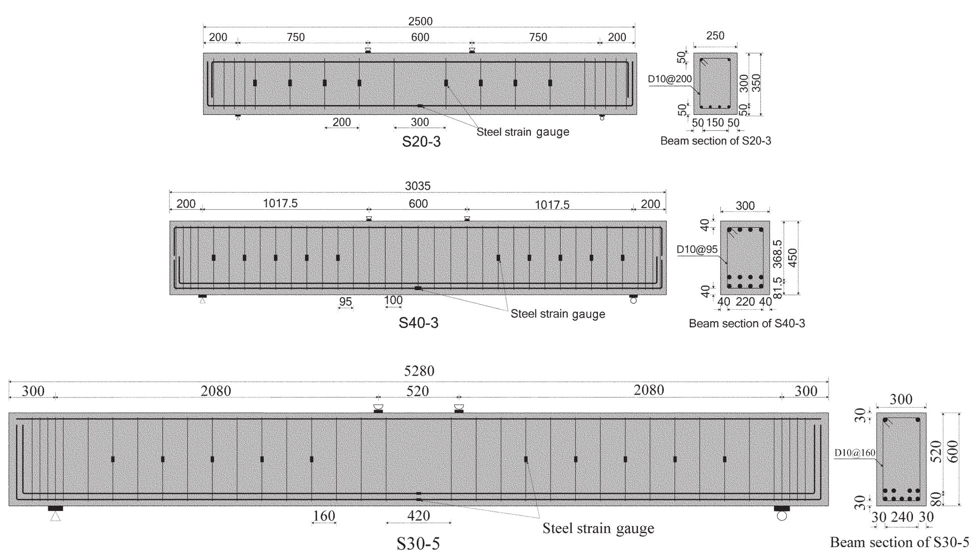


Fig. 1—Dimensions and reinforcement of test beams. (Note: Dimensions in mm; 1 mm = 0.0394 in.)

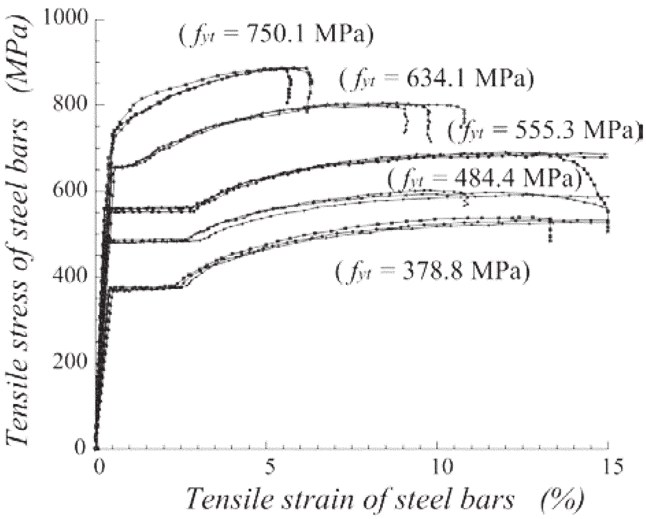


Fig. 2—Tensile stress-tensile strain curves of steel bars for shear reinforcement. (Note: 1 MPa = 145 psi.)

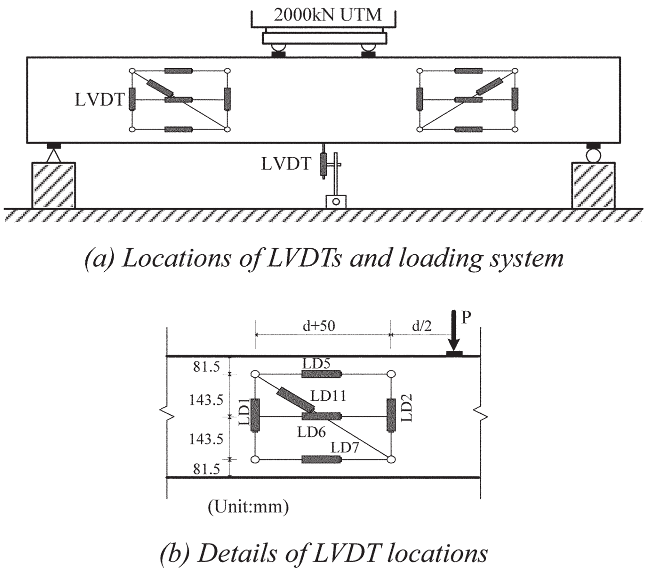


Fig. 3—Test setup and instrumentation of test Beam S40-3. (Note: 1 mm = 0.0394 in.; 1 kN = 0.225 kips.)

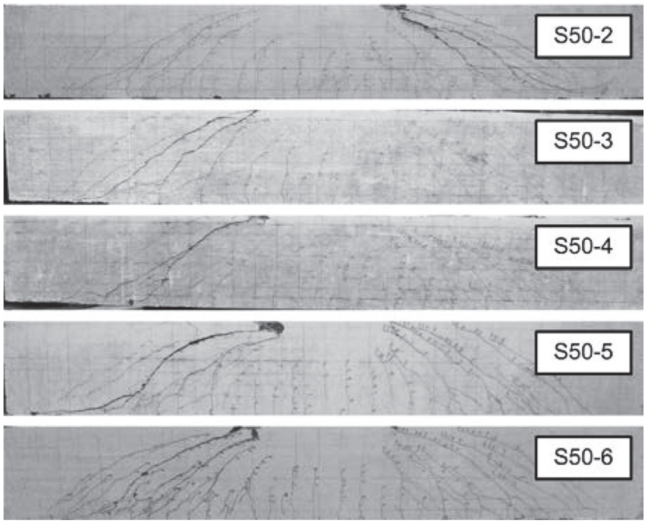


Fig. 4—Photos of tested beams after test.

strain. Beam S40-2 showed premature failure due to the error of the specimen's setting position. The test results for Beams S40-2 and S40-6 were not included in the analysis of the test results. The values of the strain gauges attached to the longitudinal reinforcements of all the beams, with the exception of Beams S40-2 and S40-6, did not reach their yield strains.

Flexural cracks first appeared in the maximum moment region. As the load increased, some of these cracks were gradually inclined towards the loading point, as shown in photographs of the beams after testing (Fig. 4), which were considered to represent the typical results of the 30 successful beams in the tests. After the first diagonal crack developed, the shear forces of all the specimens in the groups gradually increased up to the maximum load. Failure was observed after the formation of two or more significant diagonal cracks near the midspan of the test beam. Furthermore, it can be seen from Fig. 4, which shows the main shear cracks developing across more than one stirrup, that the shear truss mechanism is formed in the test specimens.

In the beams with high-strength shear reinforcements, as the midspan load increased, more diagonal cracks—mostly of the web-shear type originating at middepth—appeared in sequence from the midspan towards the supports. The number of flexural-diagonal cracks increased with the increase of the yield strength of shear reinforcement, which will be discussed in detail in a following section of this paper.

The load-deflection curves at the midspans of the 30 simply supported RC beams with different compressive strengths of concrete are shown in Fig. 5. The ultimate load and the midspan deflection corresponding to the ultimate midspan load of the beams increased as the yield strength of shear reinforcement $\rho_t f_{yt}$ increased. For example, the ultimate midspan load of Beam S50-6 (f_{yt} = 750.1 MPa [108,765 psi]) was nearly 230% greater than that of Beam S50-2 (f_{yt} = 378.8 MPa [54,926 psi]). In addition, the ultimate load of the high-strength concrete beams (Group-S80) was greater than that of the normal-strength concrete beams (Group-S40). The maximum load P_{max}, midspan deflection Δ_{max} corresponding to the ultimate midspan load, and the failure modes of the beams are shown in detail in Table 1.

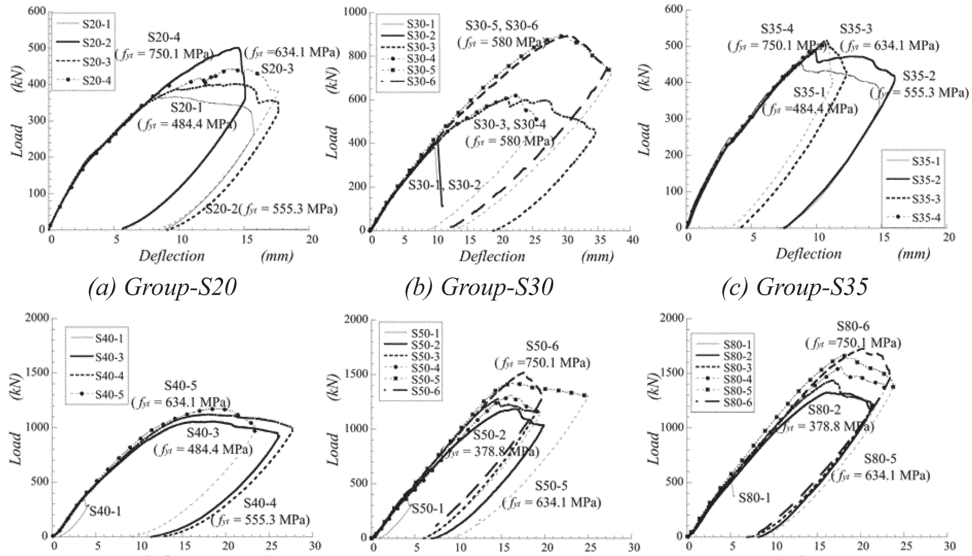
DISCUSSION OF TEST RESULTS

Strain distribution of stirrups

One of the reasons to limit the yield strength of shear reinforcement in the codes[1,4-6] is to prevent possible sudden shear failure due to concrete crushing before the yielding of stirrups because of the following two reasons:

1. Greater yield strain of high-strength shear reinforcement—The yield strain ε_y of steel bars is proportional to the yield strength of the steel bars. Because the high-strength shear reinforcement has a greater yield strain, the web concrete may crush before the shear reinforcement reaches its yield strain. In this case, the shear resistance of stirrups V_s of the beam could not be calculated by substituting the yield strength of stirrup f_{yt} into Eq. (11-15) in ACI 318-08[1] ($V_s = A_v f_{yt} d/s$).

2. Over-reinforcement shear failure—If a beam is over-reinforced, the web concrete crushes before the yielding of shear reinforcement, which leads to brittle shear failure. This failure mode violates the requirement in ACI 318-08[1] to calculate V_s because the stress of the stirrup does not reach f_{yt} ($f_t < f_{yt}$ in Eq. (11-15) in ACI 318-08[1]). Therefore, ACI 318-08, Section 11.4.7.9,[1] requires a maximum amount ρ_{max} of shear



(a) Group-S20 (b) Group-S30 (c) Group-S35

(d) Group-S40 (e) Group-S50 (f) Group-S80

Fig. 5—Load-versus-deflection curves of tested beams. (Note: 1 kN = 0.225 kips; 1 mm = 0.0394 in.; 1 MPa = 145 psi.)

reinforcement in RC beams to ensure adequate reserve shear strength and prevent possible sudden shear failure due to the concrete crushing before the yielding of the stirrups due to over-reinforcement for shear. ACI 318-08[1] limits the maximum amount of shear reinforcement as

$$\rho_{max} = \frac{2}{3}\frac{\sqrt{f'_c}}{f_{yt}} \text{ (MPa)}$$
$$\rho_{max} = 8\frac{\sqrt{f'_c}}{f_{yt}} \text{ (psi)} \tag{1}$$

In this study, to observe the actual mode of shear failure, the strain of the stirrups was measured by the electrical strain gauges attached to the stirrups. Figure 6 indicates the strain distributions of the stirrups in the tested beams at different load levels. In Fig. 6, the x-axis represents the location of the stirrups (the distance to the section from the left support), whereas the y-axis represents the measured strain values. The strain distributions are nonuniform and show a rapid increase as the applied load nears the maximum load level. The measurements indicated that the stirrups of all the beams, except Beams S20-4 and S30-5, reached the yield strain, and these stirrups were located where the significant diagonal cracks developed.

The rate of the strain of the stirrup at the peak load to the yield strain of the stirrup $\varepsilon_{peak}/\varepsilon_y$ with different compressive strengths of concrete is shown in Fig. 7. As shown in Fig. 6 and 7, the $\varepsilon_{peak}/\varepsilon_y$ increases as the compressive strength of the concrete increases. The $\varepsilon_{peak}/\varepsilon_y$ value of a beam in Group-S80 is 9.32, whereas that of a beam in Group-S20 is 0.52. The high-strength stirrups in all the beams that had high-strength concrete greater than f'_c = 35 MPa (5075 psi) reached the yield strain, whereas the stirrups of Beam S20-4 that had high-strength shear reinforcement (f_{yt} = 750.1 MPa [108,765 psi]) but normal-strength concrete (f'_c = 25.3 MPa [3669 psi]) did not reach the yield strain. The stirrups in Beam S20-3 (f_{yt} = 634.1 MPa [91,945 psi] and f'_c = 25.3 MPa [3669 psi]) reached the yield strain. It should be noted that even if the yield strength of shear reinforcement in Beams S20-2 and -3; S30-3, -4, and -6; S35-3, -4, and -5; S40-4 and -5; S50-4, -5, and -6; and S80-4, -5, and -6 was much greater than the maximum yield strength of shear reinforcement required by ACI 318-08,[1] the shear reinforcement of these beams reached their yield strains.

Similar results were also observed in the experimental results of the 49 RC beams[9,11-20] reported in the literature. The amount of shear reinforcement $\rho_t f_{yt}$ of the 49 beams was smaller than $\rho_{max} f_{yt}$ required by Eq. (1) in ACI 318-08.[1] All the beams failed in shear prior to the yielding of longitudinal reinforcement without bond-splitting failure. The test results of only the beams with high-strength shear reinforcement greater than 411.8 MPa (59,711 psi), which is approximately the maximum yield strength of shear reinforcement required by ACI 318-08,[1] were used in this analysis. The yield strength of the stirrups varied from 411.8 to 1421.6 MPa (59,711 to 206,132 psi), whereas the concrete compressive strength varied from 18.0 to 139.2 MPa (2610 to 20,184 psi). All the beams were tested under a concentrated load. The 49 beams had various end-supporting conditions (restrained beams, overhanging beams, and simply supported beams); loading types (anti-symmetric moment on the restrained beams and three- and four-point loads on the simply supported beams); and cross-sectional shapes (T-shape and rectangular shape). The observed failure modes of the 49 beams are listed in Table 2 and shown in Fig. 8.

Figure 8 shows the observed shear failure modes of the 49 test beams. Figure 8(a) shows a histogram that illustrates the observed shear failure modes of the 49 RC beams reported in the literature with different yield strengths of the stirrups. In these figures, the solid bars represent the number of beams in which the shear reinforcement yields before crushing of the web concrete, whereas the hatched bars represent the number of beams in which the shear reinforcement does not yield before crushing of the web concrete. As shown in Fig. 8(a), all the beams with shear reinforcement of less than 800 MPa (116,000 psi) showed

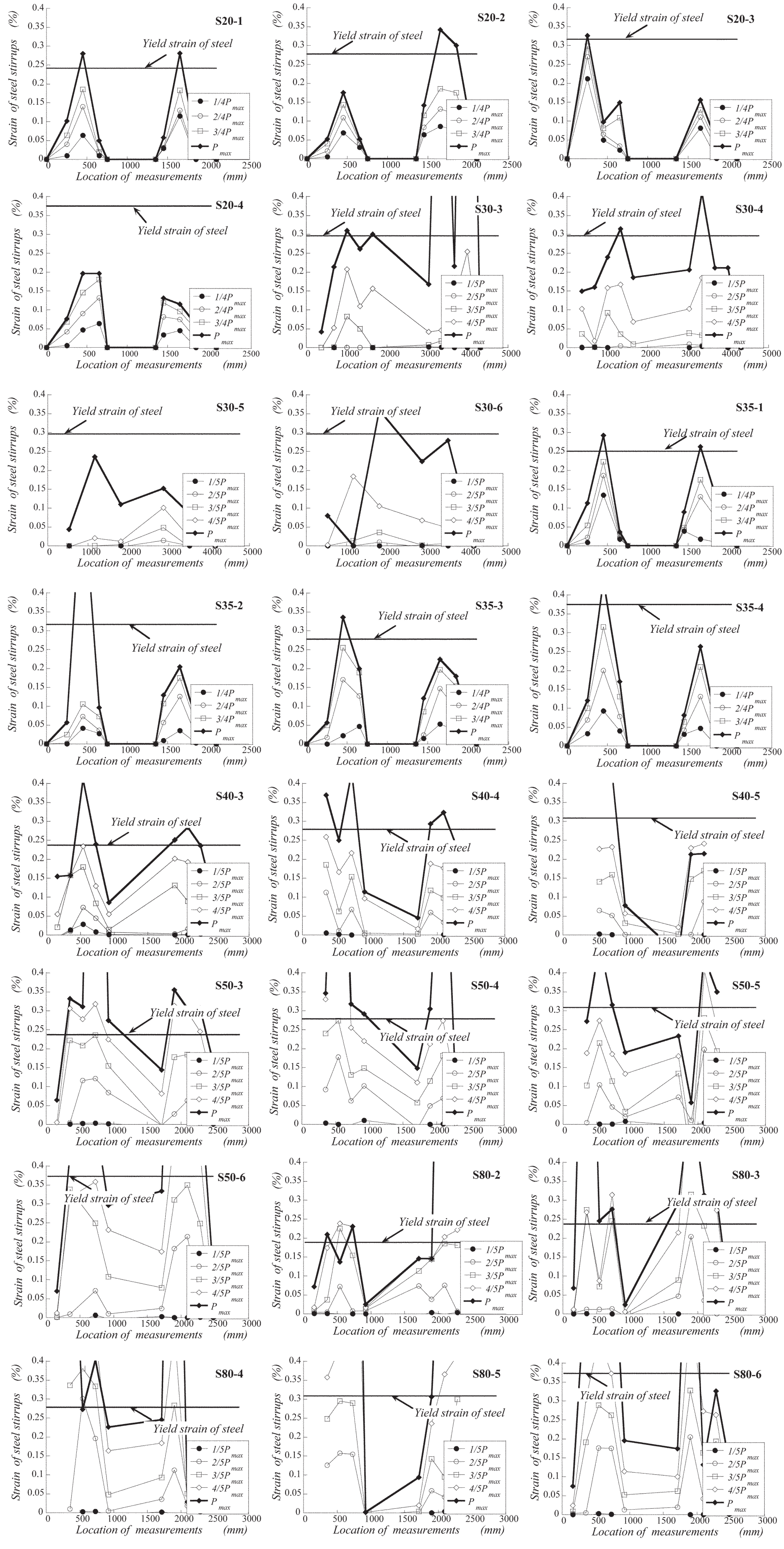


Fig. 6—Strain distributions of transverse steel bars. (Note: 1 mm = 0.0394 in.)

the yielding of the stirrups, whereas seven of the 27 beams with shear reinforcement greater than 800 MPa (116,000 psi) did not reach the yielding of the stirrups.

Figure 8(b) shows a histogram that indicates the observed shear failure modes of the 49 RC beams with different compressive strengths of concrete. It can be clearly seen from the figure that the shear reinforcement yields before crushing of the web concrete for high-strength concrete greater than 40 MPa (5800 psi). The stirrups of the beams that had high-strength shear reinforcement but normal-strength concrete, however, did not reach the yield strain.

Based on Fig. 7 and 8, it can be concluded that all the beams with stirrups with a yield strength $f_{yt} \leq 700$ MPa (101,500 psi) failed after reaching their yield strains, regardless of the compressive strength of the concrete, whereas the shear failure mode of the beams with a yield strength $f_{yt} > 700$ MPa (101,500 psi) is influenced by the compressive strength of the concrete.

Shear strength

Figure 9 compares the experimental shear strengths of the test beams in the six groups. The test results showed a

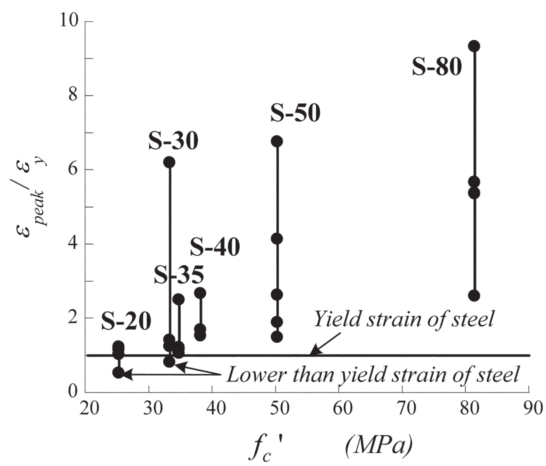


Fig. 7—Strain rates of stirrups with different compressive strengths of concrete. (Note: 1 MPa = 145 psi.)

constant trend of the shear strengths of all the beams except Beam S35-4. The shear strengths of 30 simply supported beams (except Beam S35-4) increased almost linearly with the increase of $\rho_t f_{yt}$. The shear strength of the tested beams with a constant $\rho_t f_{yt}$ increased as the compressive strength of the concrete f_c' increased. For example, the shear strengths of Beams S40-3, S50-3, and S80-3 (f_c'=38.2, 50.3, and 81.4 MPa [5539, 7294, and 11,803 psi]), which all had the same shear reinforcement but different f_c', were 4.55, 5.43, and 6.13 MPa (660, 787, and 889 psi), respectively.

The figure also shows the maximum yield strength of shear reinforcement required by ACI 318-08.[1] Some of the beams in each group with much greater yield strengths than those required by ACI 318-08[1] showed under-reinforced shear failure.

Diagonal crack width

One of the reasons to limit the yield strength of shear reinforcement in the codes[1,4-6] is to reduce the excessive width of cracks. The diagonal crack widths of the tested beams were measured by a handheld microscope with a resolution of 0.02 mm (0.00079 in.). Figure 10 shows the maximum diagonal crack widths on the right and left sides of the tested beams at different load levels. In Fig. 10, the load level in the x-axis represents the ratio of the load to the maximum load. As shown in this figure, the maximum crack width of the beam with relatively greater f_{yt} is approximately the same as (or a little narrower than) that of the beam with lower f_{yt}. The crack width at different load levels of all the tested beams was nearly constant, regardless of f_{yt}. All the beams reached the maximum load when the crack width was approximately 1.5 mm (0.059 in.).

Figure 11 shows the number of cracks versus the $\rho_t f_{yt}$ relationships of the test beams that have the same shear reinforcement ratio ρ_t. In the test, the number of diagonal cracks in the web between the midspan and support of a beam at the peak load was counted. As shown in Fig. 11, the number of cracks increases with the increase of f_{yt}. In the case where the spacing of the steel stirrups is the same

Table 2—Specification of specimens and material properties

Beams	f_c', MPa	f_{yt}, MPa	FM	Beams	f_c', MPa	f_{yt}, MPa	FM	Beams	f_c', MPa	f_{yt}, MPa	FM
IA-2R[11]	18.0	526.4	UR	B-150-019[13]	34.9	1235.3	UR	SH-1[19]	139.5	1453.9	UR
IC-2R[11]	33.8	526.4	UR	B-1.5-022[13]	35.3	823.5	UR	SH-2[19]	139.5	1453.9	OR
ID-2R[11]	33.8	526.4	UR	S36-40[14]	29.3	869.6	OR	(2)-5[20]	31.7	1323.5	UR
IA-2[11]	18.0	526.4	UR	B-210-6.0[15]	20.4	1333.3	OR	(2)-6[20]	31.7	1323.5	UR
IC-2[11]	33.8	526.4	UR	B-360-4.1[15]	37.5	1392.2	UR	(2)-15[20]	31.7	673.5	UR
IIA-2[11]	18.0	526.4	UR	B-360-5.1[15]	37.5	1421.7	UR	(1)-3[20]	27.5	1360.8	UR
IIB-2[11]	16.7	526.4	UR	B-360-6.0[15]	37.5	1333.3	UR	(1)-4[20]	27.5	1360.8	UR
IIC-2[11]	37.9	526.4	UR	B-360-7.4[15]	37.5	1421.7	OR	(1)-7[20]	27.5	1360.8	UR
IID-2[11]	37.9	526.4	UR	B-570-4.1[15]	53.8	1392.2	UR	(1)-9[20]	27.5	1360.8	OR
G3[12]	26.1	454.3	UR	B-570-6.0[15]	53.8	1333.3	UR	(1)-10[20]	27.5	1360.8	OR
G4[12]	26.6	454.3	UR	B-570-7.4[15]	53.8	1421.6	UR	(1)-11[20]	27.5	1413.7	OR
G5[12]	26.0	454.3	UR	210-0.19[16]	22.9	683.3	UR	(1)-12[20]	27.5	1413.7	OR
B-60-030[13]	32.6	492.2	UR	360-0.19[16]	37.0	679.4	UR	F20-1[9]	26.8	508.0	UR
B-80-019[13]	33.3	865.7	UR	570-0.89[16]	65.9	683.3	UR	F20-2[9]	26.8	508.0	UR
B-80-022S[13]	33.6	823.5	UR	B-90-041[17]	36.9	886.3	UR	F20-3[9]	26.8	508.0	UR
B-120-019[13]	34.5	1061.8	UR	B-6[18]	73.5	411.8	UR	F60-1[9]	63.0	508.0	UR
B-120-030[13]	34.8	1061.8	UR	—	—	—	—	—	—	—	—

Notes: FM is failure mode; UR is under-reinforced shear failure; OR is over-reinforced shear failure; 1 MPa = 145 psi.

but the yield strength of shear reinforcement differs, a larger number of diagonal cracks developed in the web of the beams with greater f_{yt} than the beams with lower f_{yt}.

Based on Fig. 10 and 11, it can be concluded that when the spacing of the steel stirrups is the same but the yield strength of shear reinforcement differs, the maximum crack width of the beam with relatively greater f_{yt} was approximately the same as the crack width of the beam with lower f_{yt} because a larger number of diagonal cracks developed in the web of the beams with greater f_{yt} than the beams with lower f_{yt}. This tendency was also observed from the analytical results from a finite element (FE) method, which will be explained in detail in the following sections.

DISCUSSION OF ANALYTICAL RESULTS BY FE ANALYSIS

Modeling of RC beams

This study used the two-dimensional (2-D) nonlinear finite element (FE) analysis VecTor2 program based on the disturbed stress field model (DSFM)[21] to perform the numerical analysis for the shear-critical RC beams. The DSFM, which has a hybrid formulation between a rotating crack model and a fixed crack model, extended the equilibrium, compatibility, and constitutive formulations of the modified compression field theory (MCFT)[22] by considering shear-slip deformations on crack surfaces. The VecTor2 program has been successfully used to analyze shear-critical RC structures using the DSFM.[23]

An FE representation of the typical Beam S50-3 is presented in Fig. 12. The quadrilateral elements were used to model both RC and plain concrete. Considering the position of the reinforcements, the width and depth of the elements were designed from 20 to 35 mm (0.79 and 1.38 in.), respectively. Furthermore, the thickness of the concrete elements was the same as that of the beam specimen. Whereas the shear reinforcements within the test region and all the longitudinal steel bars were modeled discretely using truss bar elements connected directly to the concrete elements, the transverse reinforcements located outside of the test region were included in the concrete elements as smeared reinforcement. The default constitutive models designated in VecTor2 were used, except the Popovics model, for the compression prepeak curve of high-strength concrete. The details of the constitutive models and their implementation into the FE program have been reported elsewhere.[24]

Analytical results

All beams tested in this study were analyzed using the VecTor2 FE program. The calculated maximum load ρ_{max}, midspan deflection Δ_{max} corresponding to the ultimate midspan load, and failure modes of the beams are shown in detail in Table 1. Figure 13 shows the observed and analyzed load-versus-deflection curves of Group-S50, which were considered to represent the typical results of the 32 simply supported beams. As shown in Table 1 and Fig. 13, the VecTor2 FE program can predict the shear behavior of the beams with reasonable agreement.

The numerical analysis results obtained from the VecTor2 program indicated that even if the yield strength of shear reinforcement in the beams was much greater than the maximum yield strength of shear reinforcement required by ACI 318-08,[1] the shear reinforcement of these beams reached their yield strains, as shown in Table 1. All the beams with shear reinforcement, with the exception of Beam S80-6, showed the shear failure after the yielding of shear reinforcement mode

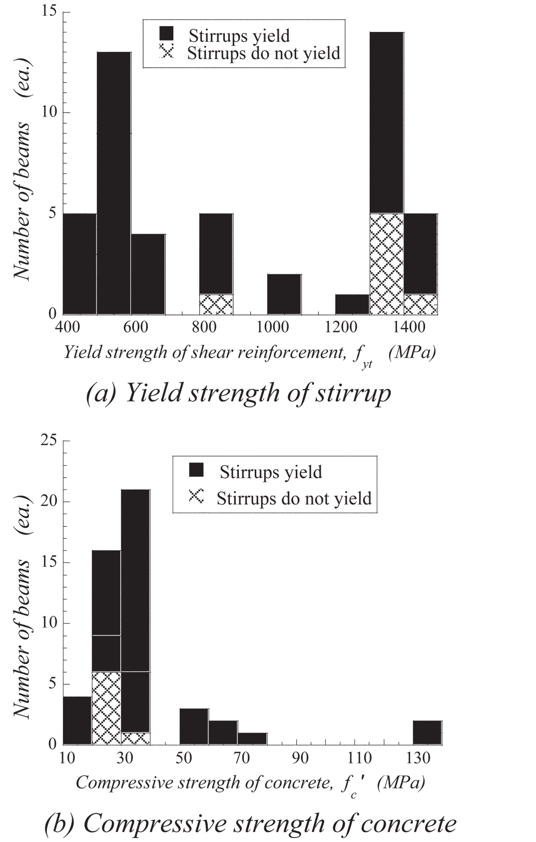


Fig. 8—Shear failure modes of beams in the literature.[9,11-20] (Note: 1 MPa = 145 psi.)

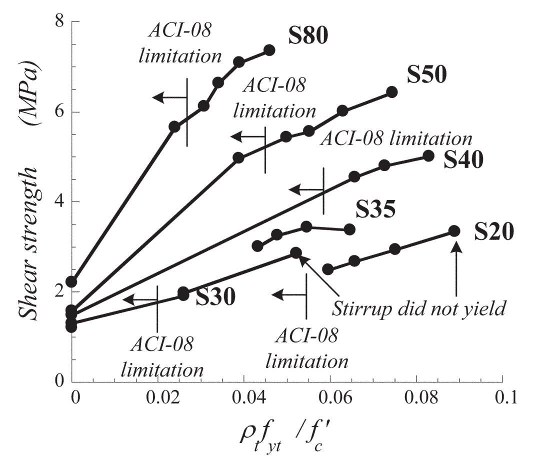


Fig. 9—Shear strength versus $\rho_t f_{yt}/f_c'$ of tested beams. (Note: 1 MPa = 145 psi.)

(SYCF), whereas Beam S80-6 failed after the longitudinal tensile steel bar reached its yield strain.

Figure 14 shows the maximum diagonal crack widths of the typical beams in Group-S50 at different load levels predicted by the VecTor2 FE program. As shown in this figure, the maximum crack width of the beam with relatively greater f_{yt} is approximately the same as (or a little narrower than) that of the beam with lower f_{yt}. The crack width at different load levels of all the tested beams was nearly constant, regardless of f_{yt}. This tendency was also observed from the experimental test results, as shown in Fig. 10.

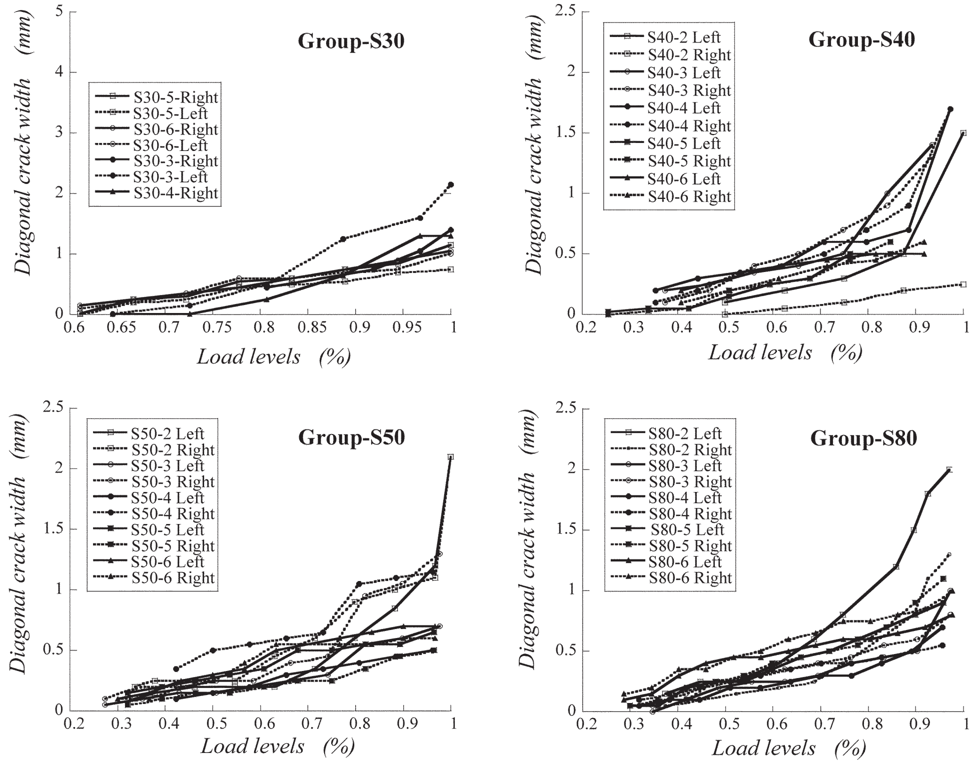


Fig. 10—Diagonal crack width of tested beams. (Note: 1 mm = 0.0394 in.)

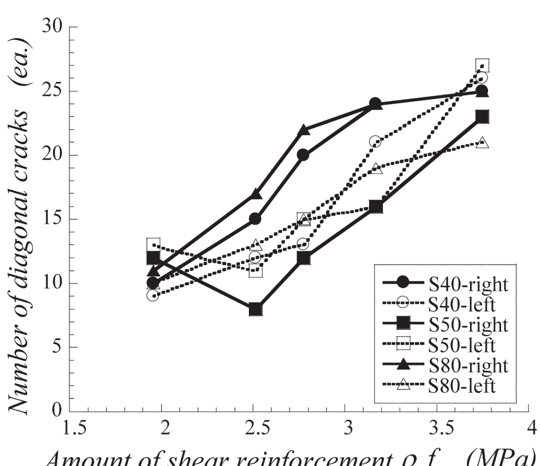


Fig. 11—Number of diagonal cracks versus $\rho_t f_{yt}$ of tested beams. (Note: 1 MPa = 145 psi.)

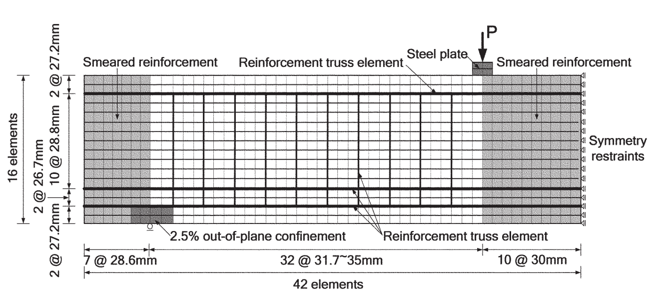


Fig. 12—FE representation of typical Beam S50-3. (Note: 1 mm = 0.0394 in.)

CONCLUSIONS

The existing design codes[1,4-6] provide an expression for the maximum yield strengths of shear reinforcement f_{yt} that were derived from experimental investigations. This may cause the f_{yt} calculated by the design codes[1,4-6] differ substantially from one another. The maximum yield strengths of shear reinforcement of ACI 318-08,[1] EC2-02,[4] and CSA A23.3-04[6] are a constant value, regardless of the compressive strength of the concrete, whereas those in JSCE-02[5] increase up to 800 MPa (116,000 psi) when the compressive strength of the concrete is greater than 60 MPa (8700 psi).

In this paper, 32 simply supported RC beams were tested to verify the influences of the yield strength of shear reinforcement and the compressive strength of the concrete on the shear behavior of RC beams. The 32 RC beams were also analyzed using a 2-D nonlinear FE analysis program (VecTor2). In addition, the test results of the 49 RC beams reported in the literature were analyzed. Based on the analytical and experimental results, the following conclusions are drawn:

1. The experimental and FE analytical results of the 32 simply supported RC beams indicated that even if the yield strength of shear reinforcement in the beams was much greater than the maximum yield strength of shear reinforcement required by ACI 318-08[1] and CSA A23.3-04,[6] the shear reinforcement of these beams reached their yield strains.

2. The test results of the high-strength concrete specimens indicated that JSCE-02[5] more reasonably captures the effect of concrete strength on the shear behavior of RC beams with high-strength stirrups than ACI 318-08,[1] EC2-02,[4] and CSA A23.3-04[6] using a constant f_{yt}.

3. The VecTor2 FE program and experimental results showed that the maximum crack width of the simply supported beam with relatively greater f_{yt} is approximately the same as (or a little narrower than) that of the beam with

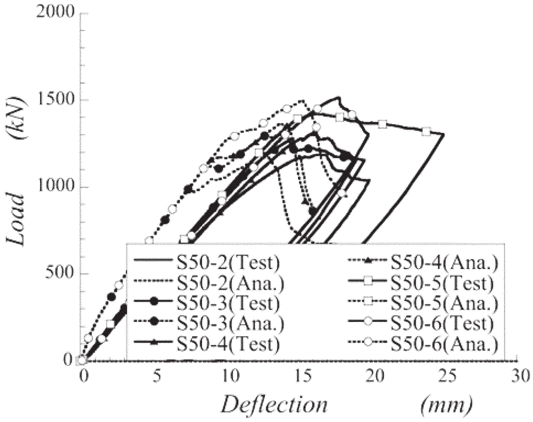


Fig. 13—Comparisons between predicted and observed load versus deflection curves. (Note: 1 kN = 0.225 kip; 1 mm = 0.0394 in.)

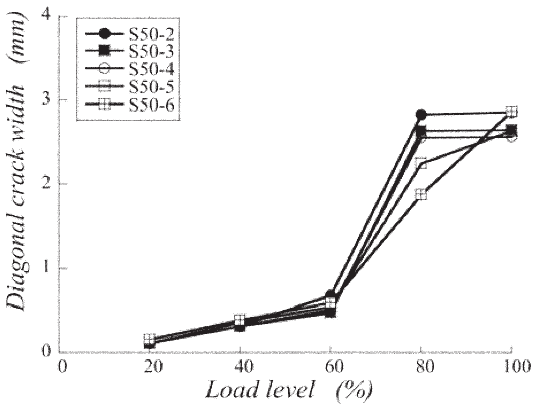


Fig. 14—Predicted diagonal crack width of beams. (Note: 1 mm = 0.0394 in.)

lower f_{yt}. The crack width at different load levels of all the tested beams was nearly constant, regardless of f_{yt}.

4. The experimental results of the 81 RC beams indicated that all the beams with stirrups with a yield strength $f_{yt} \leq 700$ MPa (101,500 psi) failed after reaching their yield strains, regardless of the compressive strength of the concrete, whereas the shear failure mode of the beams with a yield strength $f_{yt} > 700$ MPa (101,500 psi) is influenced by the compressive strength of the concrete. Even if the test results show the boundary between two failure modes, however, additional analytical and experimental works on the shear behavior of RC members with high-strength stirrups for various compressive strengths of concrete (especially high-strength concrete), *a*/*d*, shear reinforcement ratios, and the maximum spacing of stirrups are necessary to find a more rational evaluation equation for the maximum strength of shear reinforcement.

Furthermore, further research should be conducted on the effects of the shear reinforcement ratio and the stirrups' maximum spacing for the shear behavior of RC beams with high-strength stirrups.

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