Abstract: Recent wireless networks offer more bandwidth than ever. High quality services are developed and provided by the operators, the number of users is constantly increasing. As the transferred data and the number of terminals are growing, the network providers have to face the increasing complexity of the network management and operation tasks.

In this paper we observe the estimation and prediction of users’ distribution in a cellular network. We compare the accuracy of mobility models serving for location prediction with our enhanced user motion prediction algorithm.

1. Introduction

In a wireless mobile network, two major problems arise: the poor performance of the wireless channel and the effect of user mobility. Enhanced radio techniques (e.g. CDMA and OFDMA) applied mobile systems provide enough bandwidth for present mobile multimedia applications. However such applications are sensitive to the degradation of QoS parameters. Graceful degradation could happen when too many mobiles arrive to the same radio cell. To avoid such situations the Call Admission Control (CAC) has to limit the number of newly accepted connections, and proper dimensioning is also an important issue. To address these issues, accurate, yet simple mobility models are required [1].

We introduce a generalized Markovian mobility model, which is able to capture the typical movement and cell dwell time patterns in arbitrary shaped mobile cell clusters. As a simplification to this model, we also present an equivalent extension to the standard Random Walk model, to make it capable to predict the future number of terminals in each cell. Based on these models, more effective CAC algorithms can be applied in order to ensure user’s satisfaction and optimal resource usage.

2. User Motion Parameters

Our work is based on the utilization of time-series of mobile users’ movement patterns in cellular mobile networks. In our work we assume that there is a given trace of a mobile service provider’s network signaling history. This dataset consists of all signals that were transferred in the network in the examined time interval. Beside many other network parameters and properties, two main information sets can be recovered:

- the cell-path that each user visited before
- the time intervals users have spent in each cell

The series of visited cells is crucial to analyze the similarities in the users’ motion. Based on the motion patterns of the terminals amongst the cells, we can describe some drifts of the users’ motion in a given cell or point. A drift may be caused by geographical or infrastructural objects (like highways, etc.) or some time-dependent circumstances (like mass events, concert, football matches, etc.).

In a more generalized way, the drift of the motion can be interpreted as different transition probabilities from one cell to another. A probability-vector can be defined for each cell that describes the probabilities of moving from the source cell to an adjacent one. This vector is called Handover Vector (HOV) [2]. The HOV is a discrete distribution, contains transition probabilities to neighbouring cells on the condition that the user moves out from the given cell [3]. The HOV might have time dependent components (e.g. the morning or the afternoon rush hours). We describe the usage of the HOV in Section 5.

The cell dwell time is an important parameter that helps to estimate the velocity of the motion. The velocity can vary in a wide range since the users frequently change their speed or motion direction. From the network operator’s point of view the accurate prediction of the cell dwell time is useful to estimate the users’ position in the near future, especially in case of handoffs. In this paper we show two different methods for modeling the motion velocity through cell dwell time prediction.

3. Mobility Models

In the literature several mobility models can be found which describe aggregate or individual movement behavior [1][4].

The most general mobility model is the traditional Random Walk (RW) model. The traditional discrete time random walk model is a series of steps, each taken into a uniformly distributed random direction. In one-dimensional case, the random walk is the movement of an individual on a straight line. A decision is made in each point of time, the model steps forward with probability p and backwards with (1-p). The two-dimensional RW model the movement space is an infinite square grid where four possible directions are available in each state. The model can be extended in the similar way to higher dimensions.
Although it is easy to be used, the traditional random walk model has disadvantages in user motion modeling:

- Without any improvements, the RW model is not capable of simulating the mobility in an environment where geographical or infrastructural objects determine the motion behavior. Beside this, the RW model simulates the user distribution in the network in a uniform way which is clearly not applicable in real-life situations.
- The model steps in each time slot. The previously visited states or the origin state are recurrent in one and two dimensions (Pólya, 1921), but the model does not allow the same state in two following time slots. Thus the time spent in each state is not taken into account, each time tick means a transition to another state that can be interpreted in a cellular network as a constant velocity motion.
- The model does not include the user’s motion history, the states that were visited in the past. The uniformly distributed successor directions in a given state are not precise enough in a real-life application. It can be seen that the possibility of moving forward in a user drift is higher than stepping backward. The sophisticated weighted possibilities of successor directions can be constructed by knowing the previously visited cells [5].

4. MARKOVIAN APPROACH

The random walk model, as it was analyzed in the previous section, is the most commonly used mobility model to describe individual movement behavior. To avoid the mentioned disadvantages of the random walk model, we introduce our Discrete Time Markov model. With our method an optional Markov model could be created depending on the network parameters, precision and complexity [6].

In our method the direction of a user is identically distributed between 0 and 2\(\pi\). The user’s speed is between 0 and \(V_{\text{max}}\). After moving in a direction with a randomly chosen speed for a given \(\Delta t\) time, the user changes its direction and speed. \(N_{\text{cell}}\) denotes the number of cells in the network.

In a simple Markovian model a user can be located in three different states during each time slot: the stationary state (S), the left-move state (L) and the right-move state (R). This classification of cells can be seen in Figure 1.

![Figure 1 - Neighbour cells separated into two groups](image)

Let us define \(X(t)\) random variable, which represents the movement state of a given terminal during time slot \(t\). We assume that \(X(t), t=0,1,2,...\) is a Markov chain with transition probabilities \(p, q, v\).

We assume that a user is in cell \(i\) at the beginning of a timeslot. If the user is in state \(S\), similarly to the previous model, it remains in the given cell in the next slot. If the user is in state \(R\), it moves to one of the cells on the right-hand side, if in state \(L\), it moves to the left-hand side of the dividing line.

Since the transition propensities are not symmetric, the left-move state and the right-move state have different probabilities. Figure 2 depicts the Markov chain and transition (\(\Pi\)) matrix.

![Figure 2 - State diagram and \(\Pi\) matrix of 3-state M-model](image)

Transition probabilities \(p, q, v\) and \(v\) can be determined based on the network parameters.

The balance equations for this Markov chain are given in Eq. (1).

\[
P_S \cdot (p_1 + p_2) = P_L \cdot (1-q_1-v_1) + P_R \cdot (1-q_2-v_2) \\
P_L \cdot (1-q_1) = P_S \cdot p_1 + P_R \cdot v_1 \\
P_R \cdot (1-q_2) = P_S \cdot p_2 + P_L \cdot v_2
\]

We also know that \(P_S + P_L + P_R = 1\) thus the steady state probabilities can be calculated.

With knowledge of the result we can predict the number of mobile terminals for time slot \(t+1\) for each cell, using Eq. (2), where \(S'_{\text{adj}(r)}\) is the set of right neighbours of cell \(i\).

\[
N_i(t+1) = N_i(t) \cdot P_S(i) + \frac{1}{3} \sum_{l, C_j \in S'_{\text{adj}(r)}} N_j(t) \cdot P_R(l) + \frac{1}{3} \sum_{r, C_j \in S'_{\text{adj}(r)}} N_j(t) \cdot P_M(r)
\]

As we mentioned earlier this model performs well when the user’s movement has only one typical direction, because in this case the handover intensities of the right-move cells does not differ significantly. It is the same in the aspect of the left-move cells.

If we try to predict the user’s distribution in a city having irregular, dense road system, or in a big park where people are able to move anywhere then the handover intensities could differ thus the calculations above could produce errors. From this point of view the best way is if we represent all of the neighbour cells as a separated Markov state, so we generalized for a common case when a cell has \(n\) neighbour cells. We expanded our previous mentioned model to \(n\)-neighbour case (Figure 3), when all the \(n\) neighbours are represented with a Markov state:

- stationary state (S)
- move to neighbour 1..n state (\(M_{1\ldots n}\))
The steady state probabilities can be calculated as in the previous cases (Eq. 7).

\[
P_{S} = \sum_{i=1}^{n} p_{i} = \sum_{j=1}^{n} (1-q_{i} - v_{i,j+1} \text{mod} n - v_{i,j-1} \text{mod} n) p_{Ni}
\]

\[
P_{Nk}(1-q_{k}) = P_{S} \cdot p_{n} + \sum_{i \neq k} P_{Ni} \cdot v_{i,k} \quad 1 \leq k \leq n
\]

Using the result the predicted number of users in the next time slot is given in Eq. (4), where \( P_{Mi}(j) \) denotes the steady state probabilities of moving to the cell \( i \) from cell \( j \).

\[
N_{i}(t+1) = N_{i}(t) \cdot P_{S}(i) + \sum_{j \in \text{adj}S} N_{j}(t) \cdot P_{Mi}(j)
\]  (4)

\[
\Pi = \begin{bmatrix}
1 - \sum_{s=1}^{w} \frac{v_{i,s}}{m_{K}} & p_{1} & \cdots & \cdots & \cdots & p_{e} \\
1-q_{i} - v_{i,s} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
1-q_{i} - v_{i,s} & \cdots & \cdots & \cdots & \cdots & \cdots \\
1-q_{i} - v_{i,s} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

Figure 3 - State diagram and \( \Pi \) matrix of \( n+1 \)-state Markov model

As a result of the algorithm the simplified equation is Eq. (5), where \( K \) denotes a group of cells, \( m_{K} \) the number of elements in group \( K \), and \( P_{M_{K}}(j) \) is the steady state probability of moving to group \( K \) from cell \( j \).

\[
N_{i}(t+1) = N_{i}(t) \cdot P_{S}(i) + \sum_{j \in \text{adj}S} N_{j}(t) \cdot \frac{1}{m_{K}} P_{M_{K}}(j)
\]  (5)

\( C_{i} \in K \)

In Figure 4 the trend line of the theoretical error rate is plotted in the function of the number of Markov states.

We introduced a Markov model generator method, which can generate an optional model depending on the complexity and the precision. A method was proposed to improve the model with an optimizing algorithm.

5. Random Walk Model Extension

We decide to use the RW model since it can be used in a wide range of motion patterns (from pedestrian walk to highways) and can be specialized for uncommon scenarios also.

An important extension of the RW model is substituting the uniformly distributed random successor direction to a special distribution that is valid only in the given cell. This distribution is represented by the HOV. The HOV can be determined from the trace of handoffs made in an arbitrary network. The given data of the trace provides measures of the relative frequency of handoffs between each adjacent cell-pairs. Based on the relative frequency, the probabilities of the HOV can be easily calculated. The HOV is cell-specific, each cell has its own geographical properties that affects the users’ motion drift and the transition probabilities into the adjacent cells.

The Random Walk model weighted with the handover vector can simulate the constant or time-dependent changing density of the users distributed in the different cells of the network.

An additional drawback of the random walk model is the lack of the handling of different cell dwell times. Since the RW model assumes a motion with constant absolute value of velocity it cannot simulate different movement speeds. The best method to implement this feature is ensuring the possibility of staying in the same RW state for arbitrary amount of time. This can be achieved with two different approaches that are giving mostly similar results.

In the basic RW model the user stays in the actual cell until the end of the actual time slot. The lengths of slots are equal, since it is a discrete time system. We propose two methods to replace the deterministic distribution of the
discrete time slot-lengths with a distribution that converges better to real-life motion patterns.

The handoff trace can be used to calculate several dwell times for each cell. Based on this derived data-series, we modeled each cell by a phase-type (PH) system, which produces a distribution of the cell dwell time with the appropriate absorption time. The phase-type system is defined in Eq. (6) with a transient matrix \( A \), a vector that contains the transition intensities to the absorbing state \( a \) and the initializing vector \( \alpha \).

\[
Q = \begin{bmatrix} A & a \\ 0 & 0 \end{bmatrix} \quad \text{pdf}_{PH} = f(A, \alpha)
\]  

(6)

The Cell Phase Type System is defined based on the trace, a unique distribution can be created for each cell. It is started when a user registers in the given cell, and the model unregisters the user when the PH system absorbs. Immediately after the absorption the user registers into one of the adjacent cells. The movement direction is derived from the previously defined HOV of the cell.

In Figure 5 the simplest PH system can be seen, combined with the most common HOV structure. The PH contains one transient state which means that the absorption time is exponentially distributed.

Based on the handoff trace another dwell time simulation method can be derived. We provided the possibility of staying in the same cell for the next fixed length time slot. The RW model does not allow the user to stay in the same state, so we propose an extension of the HOV with a new parameter which symbolizes the dwell time. Formally it means the probability of stepping into the same cell at the end of the slot, such as a looping step shown in Figure 6.

\[
\sum_{i} h[i] = 1
\]

Figure 5 - Exponentially distributed (exp(\( \lambda \))) dwell time simulator with HOV

With the correct tuning of this extra HOV value, arbitrary distributions of cell dwell times can be simulated (if the time slot length is appropriately chosen).

\[
\begin{align*}
h' &= [h'[0], h'[1], h'[2], h'[3], h'[4], h'[5], h'[6]] \\
h'[0] &= p_{stay} \\
h'[i] &= (1 - p_{stay}) \cdot h[i]
\end{align*}
\]  

(7)

Eq. (7) shows that the outgoing transition probabilities have to be weighted in order to keep HOV as a distribution.

The model can be made more accurate with the feature of history-dependent HOV. The actual transition probabilities might depend on the series of previous cells that the user has visited in the past. The simple HOV implements the Markov approach, as the future states are depending only on the actual state. The real-life user movement patterns cannot be simulated well with this approach, since in practice all users travel with a destination in mind. The user steps forward with a significantly higher probability than backwards.

6. ACCURACY MEASUREMENTS WITH SIMULATION

The inaccuracy of the Random Walk mobility model based calculations and simulations depend on the properties of the handover vector. The RW model is only capable of accurate prediction of user movements in case of uniform movement distributions (e.g. all elements in the vector are equal to 1/6).

According to the calculations, the Markovian approach of user movement modeling and the Extended Random Walk produces better estimation of the users’ distribution in a cell cluster.

The estimation procedure was validated by a simulation environment of a cell cluster shown on Figure 7. The cluster consisted of 61 named cells, the simulation environment included geographical data that is interpreted as streets on the cluster area. The drift of the movement is heading to the streets from neutral areas.

The simulation used 610 mobile terminal (10 for each cell), in the initial state uniformly distributed in the cluster. The average motion velocity of the users is parameterized with the PH cell dwell time simulator (reciprocal of exponentially distributed values).

The simulation consists of two parts. The reference simulation is the series of the steps that the simulated mobiles have taken. It produces a time-trace that contains the actual location data for each mobile terminal in the network. We have used this reference simulation as if it was a provider’s real network trace. The distribution of users in the cells during the reference simulation is shown on Figure 8, where higher peaks mean more users in the same cell.

The estimation procedure uses the reference simulation to estimate the current and future number of users in each cell.
The estimation error is interpreted as the measure of accuracy of each mobility model in this paper. The results on Figure 9 show that the simple Random Walk model works with a significant error rate at all time. Since the user movement patterns in the simulation are not completely random due to the streets and geographical circumstances, the uniformly distributed Random Walk pattern cannot model it.

The Markov mobility model is more accurate in the estimation process. The figures show that the correlation between the error rate and the number of user in a cell is smaller than in the case of RW. In crowded cells the Markovian model gives better significantly better estimation than RW.

The Extended Random Walk model produces the best results. By the extension the model is able to accurately simulate cell dwell time, and the six elements of the outgoing HOV are correctly weighted based on the warm-up data trace. The accurate dwell time and movement direction estimation makes the Extended Random Walk model the best of the three simulated approaches.

7. Conclusion

With our proposed extensions the RW model becomes capable of simulating any real-life user movement. The network operator may use this model to make predictions on the future distribution and location of users among radio cells to justify CAC or other QoS decisions.

8. Acknowledgements

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9. References