Reliability Modeling of Tree Topology
IP Micro Mobility Networks

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Abstract
Telecommunications, computer sciences and media of today seem to converge to an IP network. Not only IP backbone will be used but also IP access networks. At the same time there is an increasing need for mobility. MobileIP can not provide fast handovers in an always-on scenario. Therefore IP micro mobility solutions are needed. IP micro mobility networks have several special requirements. They have to provide fast handovers and special routing is needed. Most of the IP micro mobility solutions are based upon a tree topology network. The most important weakness of the tree topology networks is vulnerability. In this paper we introduce a reliability model for micro mobility networks and an algorithm to compute the reliability of tree topology micro mobility networks. With the help of this algorithm the reliability of different topologies can be compared and it can be a helpful tool when designing the topology of micro mobility access networks.

1 Introduction
Nowadays three separate areas: telecommunications, computer sciences and media are converging towards a common so-called ‘infocom’ network. The common aspect of the trends lies in the network layer, where IP will be the common base of the systems. Therefore there is an increasing need for IP mobility. The mobility provided by IETF MobileIP [1] cannot properly fulfill the requirements in an always-on scenario; hence micro mobility is needed to extend macro mobility. There are several micro mobility protocol recommendations introduced in the literature, see [2,3]. Most of them are based on a physical or logical tree topology network. The most important and severe weakness of tree topology is its poor reliability. In this paper we present an approach for modeling and comparing the reliability of some network topologies. Our measure is especially designed for micro mobility networks.

This paper is organized as follows:
After a short introduction we define a reliability measure and a reliability function for tree topology micro mobility networks. In Section 3 an appropriate reliability measure and a recursive algorithm is introduced that can be used to exactly compute the reliability function of tree topology networks. In Section 4 we examine how the changing of various parameters of the topology affects reliability. In Section 5 we explain how this method can be extended to study different topologies and to compare their reliability. Using our algorithm a tool can be developed for analyzing and designing reliable micro mobility network topologies.

2 Related Work

2.1 IP Micro Mobility
The Internet and most telecommunication networks have or will have an IP backbone. In the future IP will be taken to the terminals. At the same time user requirements are changing, there is a growing need for security and mobility for example. The standard IP mobility solution, Mobile IP (Mobile IPv4 or Mobile IPv6) is not suitable for an always-on scenario with frequent handovers [4]. When handovers are frequent micro mobility has to be used. Micro mobility provides local mobility within a well-defined area, for example in an access network. While the mobile equipment stays in the same micro mobility domain handovers are handled locally, the Mobile IP home agent or the corresponding nodes are not notified, as the IP address of the mobile node is not changed.

There are several published IP micro mobility protocol recommendations, see [2,3]. Most of them are
designed for IPv4 but with modifications they can be applied in IPv6 networks too.

In a micro mobility network the positions of the mobile nodes have to be stored in a database to route packets correctly. Because of routing considerations almost all micro mobility solutions are based on a tree topology network. The root of the tree functions as the gateway to the IP backbone, and the leaves of the tree are the base stations, see Figure 1.

The tree topology suits most of the requirements of a micro mobility network (e.g. efficient routing, scalability), the chief disadvantage is, however, the poor reliability [9]. If a link or node breaks down a whole subtree is separated from the network. If the gateway breaks down, the whole micro mobility network is separated from the IP backbone.

2.2 Reliability
Reliability modeling and analysis can be briefly summarized as follows [5]:
- definition of adequate reliability measures,
- determination of the possible states of the network,
- determination of the impact of failures on reliability measures.

A graph model of the network will be used. The edges of the graph represent the links connecting the nodes. In our model only links break down, the nodes are totally reliable. This makes the model much simpler and in a tree topology micro mobility network the breakdown of a node has exactly the same effect as the breakdown of the link that connects that node to its parent. A binary model will be used for unreliable links. The links have two states: up (working) state, and down (broken) state. In our simple model all the links have an independent (and very low) probability of being in the down state.

Reliability means that faults in the system do not degrade the performance of the system too much. To formalize this statement a performance function is introduced as a reliability measure. This performance function tells the performance of the system in a state, [6].

Maximum performance is the value of the performance function in the faultless state of the system. If the performance of the system in a given state is divided by the maximum performance we get the relative performance.

Given the probability of the states of the network and the reliability measure there are several methods to get the expected value or the distribution of the relative performance. Some of these methods are exact, some are stochastic and some give only upper and lower bounds for the mean value [7].

A reliability measure and an algorithm for getting the exact distribution of the performance for tree topology networks will be introduced in Section 3.

3 Tree Topology

3.1 The Reliability Measure
The general graph model of the network was given in Section 2.2. In a given state some of the links are up and some of them are down. Our reliability measure for tree topology micro mobility networks will be the following: The performance of the network in a given state is the number of base stations that can reach the backbone.

Let \( B \) denote the number of base stations in the network. \( B^* \) is the number of base stations that can reach the backbone. The performance is:

\[ Perf = B^* \]

The maximum performance is obviously the number of base stations in the network.

\[ Perf_{\text{max}} = B \]

The relative performance is the proportion of the base stations that can reach the backbone among all of the base stations.

\[ Perf_r = \frac{B^*}{B} \]

Figure 2 shows a 4-level binary tree topology network. It has 8 base stations and 7 routers including the gateway.
If the link at the first level (the link connecting the gateway to the core network) breaks down, the relative performance is 0, because none of the base stations can reach the backbone. If there is a one-link failure at the fourth level (lowest level link, connecting a base station to a router), the relative performance is 7/8 as only 7 out of 8 base stations can reach the backbone.

### 3.2 The Infinite Binary Tree

Assume that all the links have an error probability of $p$. What happens when the depth of the binary tree is growing? It is obvious that as the base stations get further from the gateway degradation will occur in reliability. With our formal method it can be proved:

If our binary tree is $n$ levels deep, there are $B = 2^{n-1}$ base stations. Consider one particular base station. It is $n$ links away from the backbone. What is the probability of that this base station can reach the backbone? It is the probability of all the links leading to the backbone being up. We have $n$ independent links, all of them up with a probability of $(1-p)^n$. The probability is $(1-p)^n$.

The expected number of the base stations that can reach the backbone can be calculated as:

$$E[B] = 2^{n-1} \cdot (1-p)^n,$$

because the expected value of the sum of random variables is the sum of the expected values, even if the variables are not independent.

Then the expected relative performance is

$$Q = 2^{n-1} \cdot (1-p)^n / 2^{n-1} = (1-p)^n / 2,$$

and from here:

$$\lim_{n \to \infty}(1-p)^n / 2 = 0 \quad \text{if} \quad 0 < p < 1.$$

So the expected relative performance approaches 0 as the tree grows.

### 3.3 The Distribution Function

For general tree topology micro mobility networks with the reliability measure described earlier we constructed an algorithm in order to calculate the exact distribution function of the relative performance.

First we define a vector $m(b_0,...,b_n)$ for each tree topology network, where $b$ refers to the number of base stations in the network. The length of the vector is $(b+1)$ and the elements are indexed from 0 to $b$. The element $m_i$ equals to the probability of exactly $i$ base stations being able to reach the backbone. This is called the performance vector. For example the performance vector of the network of Figure 2 with a link error probability of 0.1 is the following:

$$m=(0.1110, 0.0063, 0.0341, 0.0519, 0.1325, 0.0911, 0.1841, 0.1830, 0.2059)$$

Note that the sum of the elements of $m$ has to equal to 1 and that the distribution function of the performance can be computed easily from this vector, as this vector is actually the density function of performance.

Now we define a recursive algorithm for computing the performance vector of any tree topology network. Complex tree topologies can be built up recursively from simple building blocks.

We apply two basic operations of the algorithm for tree topology networks and for their vectors.

The first operation is joining the roots of two trees to get one tree. Figure 3 shows an example for this operation. The performance vector of the resulting tree is the convolution of the two performance vectors.

For example if the link error probability is 0.01, then the performance vectors of the networks in Figure 3 are:

- $m_1=(0.0001, 0.0004, 0.0200, 0.0380, 0.9415)$
- $m_2=(0.0001, 0.0198, 0.9801)$
- $m_3=(0.0000, 0.0000, 0.0001, 0.0008, 0.0204, 0.0559, 0.9227)$

![Figure 3. The first operation type: joining the roots](image-url)
performance is zero. To get the new performance vector $m'$ of the new network $m$ has to be multiplied by $(1-p)$ then $m_0$ has to be incremented by $p$.

Figure 4. The second operation type: adding an edge to the root

By means of these two operations all tree topology networks can obviously be built from basic building blocks. The basic building block is one base station that is connected to the backbone by a link. If the error probability of this link is $p$, the performance vector is $(p, 1-p)$.

Figure 5 shows how a general network can be built from these basic building blocks.

Figure 5. Building a network using the two operations

4 Tree Parameter Considerations

4.1 The Program

We have written a function in Mathworks MATLAB[8] to calculate the distribution function of performance of tree topology networks and to validate our analysis. The function is called "trel", the name stands for "tree reliability". It works only for networks where all of the base stations are at the same depth and the branching of the tree is the same at every node and the link error probability is the same for every link. This limited function is not for computing the performance of a specific network but rather to examine the effect of parameter changes to the reliability of the tree topology network. The definition of the function is the following:

```matlab
function [s,t]=trel(p,depth,branch)
```

The meaning of the parameters is the following: $p$ is the error probability of the links, $depth$ is the depth of the tree, $branch$ is the branching of the tree, for example 2 for binary.

A network of this topology has "branch to the power of $depth$" base stations. The function returns two vectors; each of them is the number of base stations plus one of length. Return value $s$ refers to the distribution function of performance, $t$ is for scaling, if $(s,t)$ pairs are plotted the result is scaled down to the $[0,1]$ interval and instead of the relative performance we get the relative unavailability which is one minus the relative performance. In normal situations the relative performance should be close to 1 with a high probability, so the relative unavailability should be close to 0. Usually logarithmic scale is used on the probability axis.

In this Section it is elaborated how the parameter changes affect the reliability of a tree topology micro mobility network.

4.2 Link Error Probability

Of course the smaller the link error probability is the more reliable network we have. The probability of the network being totally down (relative performance equals to 0) is approximately the link error probability. This is because if the link connecting the gateway to the backbone breaks down, the network is completely separated, so the relative performance becomes 0. The probability of 100% unavailable capacity is actually slightly higher than this.

Our example network is a binary three with a depth of 9. The network has 256 base stations. Four reliability functions were computed with different link error probabilities. The link error probabilities were: 0.1, 0.01, 0.001 and 0.0001. Our function written in MATLAB generated the plot shown in Figure 6.

Figure 6. The effect of link error probability
The plot shows what we have expected, and some more qualities of the reliability can be read from it. It is surprising that even with a link error probability of 0.0001 the probability of the relative performance being over 99% is 0.0031. This result shows that the tree topology is even more unreliable than one would expect.

### 4.3 Depth

After analytical considerations we expect that the plot of the function of a tree with given depth always runs below the plot of a deeper tree if the link error probability is the same. We have already seen in Section 3.2 that it is not a good idea to use deep trees for network topology. Figure 7 shows the unavailability functions of binary trees with different depths. The link error probability was set to 0.01 in all three cases. The depth of the trees are: 3, 6, 9, 12.

![Figure 7. The effect of different tree depths](image)

We get exactly what we have expected. The functions of deeper trees always run below the functions of shorter trees.

As the depth is increased the unavailability probabilities should approach 1, as we have seen in Section 3.2. The topology with the depth of 12 has 2048 base stations, and the 1% as error probability is rather high. The network is still far from unusable, which means that the convergence is slow.

### 4.4 Branching Factor

The effect of the branching factor should also be examined. It is hard to predict the effect of the branching factor on reliability. There are two separate effects of increasing the branching factor of a tree. The number of base stations is increasing while the number of links at each level is also increasing. However, the distance of the base stations from the root (depth) remains the same.

In our example the depth of the tree is always 4. The branching factor runs from 2 to 7. The distance of the base stations from the gateway is always 4 links, the number of base stations is increasing from 8 to 343. There are 6 plots in Figure 8, the unavailability function for the 7-brach tree was drawn on all of the plots for reference.
As we have expected the functions intersect each other several times running above and below each other.

4.5 Weights

In a micro mobility network some of the base stations may be more important than others. To model this effect we extended our model so that a positive (integer) weight can be assigned to all of the base stations. We extended the MATLAB code that not all the base stations have identical weights. In our extension one of the base stations can have a special weight while 1 is assigned to all the other base stations. This is similar to the case when there are more than one base stations at one of the leaves of the tree.

**Figure 8. The effect of the branching factor of the tree**
Figure 9 shows the effect of different weights of one of the base stations in a network. The topology of the network remains the same: it is a four-deep binary tree (16 base stations). The two curves show the performance for weights 1 and 100. Weight 1 means that all the base stations are weighted 1, weight 100 means that there are 15 base stations weighted 1 and one base station weighted 100.

The weighted model should be refined and further examinations are needed to study the effect of different weights in the network.

5 Conclusions and Future Work
A method was proposed that enables to study the reliability of tree topology micro mobility networks. A simple reliability measure was defined then an algorithm was introduced to compute the exact distribution function of the relative performance of the network. MATLAB source code for this algorithm was also given.

We are planning to extend and generalize the MATLAB code to handle any tree topology networks with base stations not only at the leaves. At the same time algorithms for other network topologies would be developed.

We have introduced a reliable micro mobility topology earlier in [10]. This new topology is called the hierarchy of rings. The reliability of the hierarchy topology, the tree topology and other micro mobility topologies will be compared using the algorithm presented here. This way the good reliability qualities of the hierarchy could be proven analytically and not only by simulations, see [11].

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