Fractal and EMD based removal of baseline wander and powerline interference from ECG signals

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Abstract

This paper presents novel methods for baseline wander removal and powerline interference removal from electrocardiogram (ECG) signals. Baseline wander and clean ECG have been modeled as 1st and 2nd-order fractional Brownian motion (fBm) processes, respectively. This fractal modeling is utilized to propose projection operator based approach for baseline wander removal. Powerline interference is removed by using a hybrid approach of empirical mode decomposition method (EMD) and wavelet analysis. Simulation results are presented to show the efficacy of both the methods. The proposed methods have been shown to preserve ECG shapes characteristic of heart abnormalities.

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1. Introduction

The heart activity of a person is commonly examined using an Electrocardiogram (or ECG) signal. Different leads (or the electrodes) of ECG recording machine are attached to different positions on the human body and recorded by an external device. An ECG signal can be used to measure the rate and abnormality (if present) of heartbeats, to detect disease or damage to the heart, and also to observe the effect of drugs or devices which alter the heart activity. Generally, the amplitude range of a normal ECG signal is 10 μV to 5 mV and the frequency range is 0.05–100 Hz [1]. In recent times, automated ECG analysis has gained immense popularity in the area of telemedicine [2].

An ECG signal represents heartbeat consisting of P wave, QRS complex, T wave, and U wave. The amplitude, duration, and interval between each of these waves help us distinguish between normal and abnormal waveforms. Certain diseases and their identification from an ECG signal are dependent on the characteristics of these waves.

During recording, an ECG signal generally gets corrupted by different types of noises, namely: (1) baseline wander (BW), (2) powerline interference (PI), and (3) physiological artifact (PA). The presence of these noises in an ECG signal makes the detection of abnormality in the heart beat difficult [3]. In this paper, we focus on (a) the modeling of clean ECG signals, (b) modeling of baseline wander noise, and (3) removal of baseline wander noise and powerline interference from ECG signals.

Powerline (AC) interference is a high frequency additive noise of 50 or 60 Hz. It is typically a sinusoidal wave with random phase but constant frequency [4]. This kind of noise comes from the power line to recording machines and is present even if special care is taken in proper grounding, shielding, and design of amplifiers [5]. It is generally removed by a fixed or adaptive notch filter [4–7]. However, the requirement of reference signal in an adaptive filter makes its hardware and software implementation difficult [7]. The first intrinsic mode function (IMF) of empirical mode decomposition (EMD) is also used for powerline interference removal. However, in this IMF, R wave component is also present besides powerline interference [5]. This poses difficulty in powerline interference removal because any distortion in the amplitude and frequency of R wave can seriously affect disease diagnosis. Thus, in [5], a modified approach using notch filter on the first IMF is suggested for powerline interference removal.

On the other hand, baseline wander noise is a low frequency artifact frequently present in an ECG signal. The frequency range of this noise is roughly between 0.05 and 0.7 Hz [8]. It is mainly caused by movement of patients due to breathing, coughing, anxiety, stress or pain, and motion of electrodes [9]. Large baseline wander can lead to the following problems: (a) original ECG peak clipping by recording instrument amplifiers resulting in loss of signal, (b) change in the duration and shape of ST segment, and (c) loss of lower amplitude peaks such as P- and T-waves. Thus, during elimination of this noise from an ECG signal, extra care has to be taken to preserve the amplitude of R peak, original ST segment, and P and T wave peaks in the ECG waveform. These attributes are characteristic of ECG signals and are critical for disease diagnosis.
Failure to preserve these signal-specific attributes during baseline wander removal may lead to either false alarm or miss of heart ailments such as myocardial infarction, ischemia, etc.

A number of methods have been proposed to remove baseline wander noise from ECG signals [1,7–19]. A classical method of baseline wander noise removal is high pass filtering with cut off frequency of 0.7 Hz [10]. However, the spectrum of baseline wander noise and low frequency component of an ECG signal may overlap [11]. In such a case, baseline wander removal using a highpass filter will alter the original clean ECG signal, particularly, the ST segment. Thus, the use of a highpass filter can lead to wrong diagnosis of a disease. The cubic spline method is one of the other methods that are used to estimate baseline wander noise [12]. This method assumes that the PR segment of an ECG signal is well defined and its position is correctly known for normal beats [11]. However, the detection of exact PR segment is often affected by other artifacts such as electrode motion, muscle artifact, etc. present in the signal. Thus, this method does not lead to efficient denoising in the presence of other noises.

Some other techniques commonly used for removing baseline wander noise from the ECG signal are: (a) adaptive filtering [9], (b) moving average filters [13], (c) wavelet approach [14,15], and (d) empirical mode decomposition (EMD) [7,16–19]. Adaptive filtering requires a reference signal for baseline wander removal [7]. In [7], last few IMFs with sufficient baseline wander were used as input signals, while noisy ECG signal was used as the reference signal. Moving average filters are easy to implement but they tend to distort the original signal wherever there is a sudden change in amplitude especially near R peak onset [13]. The process of baseline wander removal using EMD cannot be automated because baseline wander noise may be distributed over a number of intrinsic mode functions (IMFs) [16,18]. A similar problem arises with the wavelet approach where we cannot determine the amount of baseline wander noise present in different levels of detail coefficients [14,18].

In this paper, new methods are proposed for powerline interference and baseline wander noise removal from ECG signals. First, powerline interference is removed from noisy ECG signals (ECG signals corrupted by baseline wander and powerline interference). Noisy ECG is decomposed using EMD and the first IMF is processed using wavelet approach for powerline interference removal. Thus, we obtain ECG signal corrupted with only baseline wander noise. Next, statistical modeling of baseline wander noise and clean ECG signals is done. This modeling is used to propose projection operator based approach for the removal of baseline wander noise.

The paper is organized into eight sections. Section 2 presents a brief review on empirical mode decomposition (EMD) and the theory of projection operator. Section 3 informs about the database (or source) of ECG signals, powerline interference, and baseline wander noise used in this paper. Clean ECG and the noise recordings are modeled in Section 4. The proposed methods for powerline interference removal and baseline wander removal are presented in Section 5. Section 6 discusses the complete proposed denoising algorithm. Simulation results and comparison with other existing methods are presented in Section 7. In the end, conclusions are presented in Section 8.

1.1. Notations

We use lowercase bold letters and uppercase bold letters to represent vectors and matrices, respectively. The scalar variables are represented by lowercase italicized letters. In addition, $E(\cdot)$ denotes the expectation operator.

2. Preliminaries

2.1. Empirical mode decomposition

Empirical mode decomposition (EMD) is an adaptive data driven approach that decomposes any nonlinear and non-stationary signal (like biomedical signals) into amplitude and frequency modulated (AM–FM) components called intrinsic mode functions (IMFs) [19,20]. Thus, a given signal $x(t)$ can be represented using IMFs as below:

$$x(t) = \sum_{n=1}^{N} c_n(t) + r_N(t)$$

where $N$=total number of IMFs, $c_n(t)$ for $n=1,\ldots, N$ are the IMFs, and $r_N(t)$ is the residual (or the last) IMF. These IMFs can be used for spectral analysis because as the IMF index increases, frequency of IMF decreases. EMD can also be used for subband filtering where the IMFs are adaptive or signal dependent [21]. Currently, this method is being applied to various applications including biomedical, seismic, geological, financial data analysis, etc. [20]. For the complete algorithm, readers may refer to [20,21].

2.2. Theory of projection operator

Consider a two-dimensional ($M=2$) vector space $X_{vs}$, spanned by basis vectors $x_1$ and $x_2$. Given a vector $y$ belonging to the vector space $W_{vs}$ of dimension $N=3$, the aim is to project $y$ onto the space $X_{vs}$ such that

$$y = y + e$$

where $X_{vs}$ is the projection space for $W_{vs}$ with $\dim(X_{vs}) < \dim(W_{vs})$, $\hat{y}$ is the estimate of $y$ that lies in the space $X_{vs}$ and $e$ is the estimation error. The squared length of vector $e$ (or the energy of the signal characterized by $e$) is minimum when $e$ is perpendicular to the space $X_{vs}$ [22]. This is also called as the orthogonal projection of vector $y$ onto the space $X_{vs}$ [22]. From (2), we can write

$$e = y - \hat{y}$$

Since $e$ is orthogonal to space $X_{vs}$, the inner product of $e$ with all the basis vectors of $X_{vs}$ will be zero, i.e.,

$$\langle x_k, e \rangle = x_k^T e = 0 \quad \text{for} \quad 1 \leq k \leq M$$

where $x_k$’s are the basis vectors spanning the space $X_{vs}$.

Or

$$X^T e = 0.$$  \hfill (5)

The estimate $\hat{y}$ (refer to Fig. 1) can be written as

$$\hat{y} = Xc = \sum_{k=1}^{M} c_k x_k$$ \hfill (6)

where $X$ contains the basis vectors of $X_{vs}$ and $c_k$’s are constants. On substituting (3) and (6) in (4), we obtain

$$\langle x_k, e \rangle = x_k^T (y - Xc) = 0 \quad \text{for} \quad 1 \leq k \leq M$$ \hfill (7)

Or,

$$X^T (y - Xc) = 0.$$ \hfill (8a)

$$\Rightarrow X^T y = X^T Xc$$ \hfill (8b)

$$\Rightarrow c = (X^T X)^{-1} X^T y$$ \hfill (9)

On using (6) and (9), we obtain

$$\hat{y} = Xc = X(X^T X)^{-1} X^T y = Py$$ \hfill (10)
is the projection matrix (operator) corresponding to the space $X_{y}$.

Thus, if we would like to have a minimum error length estimate, $\hat{y}$, of a vector $y$ in an $M$ dimensional space $X_{y}$, first we form the projection matrix $P$ as in (11) using the basis vectors of space $X_{y}$ and then use (10) to project vector $y$ onto $X_{y}$.

3. Databases used for ECG signals and noise recordings

Clean ECG records have been taken from the MIT-BIH Arrhythmia Database [23]. This database includes 48 half-hour recordings of two leads of ECG signals. Signals are recorded at a sampling frequency of 360 Hz with 11-bit resolution over a 10 mV range. The database includes records of clean ECG from healthy persons (Records 100, 101, 103, 115, etc.) and from patients suffering with different Arrhythmia diseases. Normal ECG signals have been obtained from 10 records of this database. For each of these records, 15 segments of 5 s (3600 samples) each have been taken from lead ML-II. This provides us 150 segments of clean ECG signals from healthy persons. Results of analysis on these 150 segments have been presented in this paper.

The baseline wander recordings have been taken from the MIT-BIH Noise Stress Test database [24] at 360 Hz sampling frequency. For powerline interference, we have generated sine waves of 50 Hz and 60 Hz, respectively, with variable amplitude and phase.

Noisy ECG records are created (ECG corrupted with baseline wander noise and powerline interference) by adding baseline wander noise and powerline interference to the clean ECG signals (total 150 segments as mentioned above). From hereafter, we will refer to the records mentioned in Table 1 from the MIT database as clean (or original) ECG records. The noisy records will refer to those created as mentioned above.

4. Modeling of clean ECG and noise signals

4.1. Modeling of clean (original) ECG signal

ECG signals have been shown to have self-similar characteristics [25]. Based on this information, we attempt to model clean ECG signals as fBm random processes that are a class of self-similar processes. Fractional Brownian motion (fBm) processes are a class of statistically self similar, Gaussian, zero mean, non-stationary random process [26] whose averaged power spectral density decays by $1/f^\beta$ where $\beta=2H+1$. The characteristics of fBm are dependent on Hurst Exponent $H$, also called as the self similarity index. A higher value of $H$ implies signal spectrum being dominantly lowpass and a lower value of $H$ indicates signal spectrum being dominantly highpass. Fractional Brownian motion with $0<H<1$ are called first order fBm (1-fBm) and with $1<H<2$ are called second order fBm (2-fBm) [27].

We estimated $H$ of clean ECG signals using maximum likelihood (ML) method presented in [28] for the estimation of $H$ of fBm processes. Table 1 shows the estimated values of $H$ corresponding to different segments of clean ECG records of MIT-BIH Arrhythmia Database.

From Table 1, we notice that the value of $H$ of clean ECG signals lies in the range of $1.5<H<2$. This shows that clean ECG signals can be modeled as 2nd order fBm random processes. In addition, the value of $H$ corresponding to clean (original) ECG signals with different types of ST-segments is also observed to lie in the same range of $1.5<H<2$ (Table 2).

Two Records 111 and 108 of MIT-BIH Arrhythmia Database with different types of ST segments were observed to have $H$ less than 1.5. On analysis, these signals are observed to be affected by powerline interference. On removing powerline interference (using the method proposed in Section 5.1), the value of $H$ of denoised ECG signals is again observed to belong to the range $1.5<H<2$ (Table 2).

In the literature, many techniques have been suggested to estimate the value of Hurst exponent for clean (normal) ECG signals [29–32]. For the same MIT-BIH Arrhythmia Database (sampling rate of 360 Hz), the value of $H$ for an ECG of a healthy person is estimated to be approx. 0.34 using rescaled range method [30], 0.42 by Katz algorithm [31], and approx. 0.64 by Higuchi's algorithm [31]. In [29], the value of $H$ for normal ECG with sampling rate of 250 Hz is estimated to be approx. 0.28 using finite variance scaling method. In [32], the value of $H$ estimated by rescaled range analysis for clean (normal) ECG with sampling rate of 1000 Hz is found to be in the range of 0.7–0.9.

![Fig. 1. Vector space interpretation of the projection operator approach.](image-url)
4.1. Discussion

From the above, we observe variation in the estimated value of $H$

(i) over different estimation methods applied on the same database signals (sampled at 360 Hz) and

(ii) over same ECG signals but sampled at different rates.

For (i) above, we note that the methods applied in [29–32] assume the value of Hurst exponent to be strictly between 0 and 1. Most of these methods first estimate fractal dimension $D$ and then compute the Hurst exponent using the relation $D=2−H$. While estimating $D$, some of these methods do not utilize ECG signal statistics, but work on ECG signals similar to the way $D$ would be estimated for a deterministic self-similar signal (or fractal). It is more appropriate to model ECG signals as random processes [33]. Hence, in this work, we have modeled ECG signals as statistically self-similar processes and used ML method for $H$ estimation [28], which is appropriate for estimating non-random parameter in a random process setting.

For (ii) above, it is obvious that with the change in the sampling rate, the estimate of $H$ will vary. This is because at higher sampling rates, we are picking up more number of samples from any fixed duration of continuous time signal. As shown in Table 1, the value of $H$ of clean (original) signal is $1.5 < H < 2$ at 360 Hz. We resampled these signals at 128 Hz and observed the estimated value of $H$ to be $1 < H < 1.5$. We verified our observations by considering clean (normal) ECG signals from [34] recorded at the sampling rate of 128 Hz. The value of $H$ for these signals also lies in the same range of $1 < H < 1.5$. Thus, the estimated value of $H$ changes with the sampling rate of signals. It decreases as the sampling rate is decreased or increases as the sampling rate is increased.

Although ECG signals will not be truly self-similar with one particular value of $H$, the estimated $H$ value is the global (or averaged) value that informs about the spectrum and the characteristic of these signals.

From the above discussion, we conclude that (i) clean ECG signals sampled at 360 Hz can be modeled as 2nd order fBm random processes; and (ii) the change in sampling rate results in different estimated value of $H$. The estimated value of $H$ is $1.5 < H < 2$ for clean ECG signals sampled at 360 Hz, while it is $1 < H < 1.5$ for the same signals sampled at 128 Hz.

4.2. Modeling of baseline wander noise

Next, we estimated the value of $H$ of baseline wander noise. It is observed that the value of $H$ of baseline wander from 360 Hz sampling rate database is $0.3 < H < 1$. Results are tabulated in Table 3.

Similar to clean ECG signals, it is observed that the value of $H$ corresponding to baseline wander noise changes with the sampling rate. On reducing the sampling rate of baseline wander noise record [24] from 360 Hz to 128 Hz, the value of $H$ falls slightly, but still remains in the range of 1st order fBm.

The above results show that baseline wander noise can be modeled as 1st order fBm process. This is to be noted that irrespective of whether the signals are sampled at 360 Hz or 128 Hz, difference in the value of $H$ between clean ECG and baseline wander is nearly one.

Our reference MIT database for ECG signals [23] and noise [24] are sampled at 360 Hz. From hereafter, we will refer the values of $H$ for clean and noisy ECG signals corresponding to the sampling rate of 360 Hz only.

4.3. Modeling of powerline interference

Powerline interference is a sinusoidal signal of 50 Hz or 60 Hz [4]. Its frequency is high compared to the dominant spectrum of clean ECG signals. Because powerline interference is a sinusoidal signal of known frequency, it does not require any further modeling.

5. Noise removal

5.1. Powerline interference removal

As discussed in Section 2, empirical mode decomposition (EMD) is used to decompose a nonstationary signal into IMFs [19]. Powerline interference is a high frequency noise that is observed to be present in the first IMF. Fig. 2 displays the magnitude of frequency spectrum of the first IMF where a distinct peak is observed corresponding to the powerline interference of 60 Hz (Fig. 2, label ‘A’).

Thus, the presence of powerline interference in noisy ECG can be easily detected from the first IMF obtained from EMD algorithm. However, besides powerline interference, some information (or portion) of R wave of clean ECG is also present in this IMF (Label ‘B’ in Fig. 2). This prevents us from removing powerline interference by eliminating first IMF. Instead, a notch filter is used on the first IMF to remove powerline interference [5]. This method has a disadvantage. A slight variation in the center frequency of the notch filter compared to the powerline interference will not
only leave some powerline interference but may also distort the ECG signal by removing a portion of R-wave.

We propose EMD and wavelet based approach to remove powerline interference from the corrupt (noisy) ECG signal.

It is evident from Fig. 2 that the frequency of powerline interference is high compared to the dominant frequency spectrum of clean ECG signal in the frequency spectrum of first IMF. Thus, it should be possible to capture this noise in the detail coefficients of wavelet transform.

Using this argument, we carry out 3-level wavelet decomposition of the first IMF using ‘Coiflet 4’ wavelet (length 24 filters with highest number of vanishing moments possible on scaling and wavelet functions of given support). ‘Coiflet 4’ wavelet is used for noise removal because ECG signal’s QRS complex shape is similar to ‘Coiflet 4’ scaling function [18,35]. We apply universal thresholding [36] on 3rd level detail coefficients of wavelet decomposition to obtain clean IMF and reconstruct the ECG signal that is free of powerline interference.

5.2. Baseline wander removal

In Sections 4.1 and 4.2 on the statistical modeling of clean ECG signals and baseline wander noise, we have modeled baseline wander noise as 1st order fBm random process and clean ECG signals as 2nd order fBm random process [37]. We utilize these results to propose projection operator based approach for the removal of baseline wander noise from noisy ECG signals. The proposed approach consists of two steps: (1) design of noise subspace using the sample functions of a 1st order fBm process and (2) removal of baseline wander noise using the projection operator approach.

5.2.1. Design of the noise subspace

From Sections 4.1 and 4.2, we know that the clean ECG belongs to 2nd order fBm with $1.5 < H < 2$, while baseline wander noise is characterized by $H$ belonging to the range 0.3–1. This shows that there is a clear separation in the range of $H$ corresponding to clean ECG signals and baseline wander noise. We use this modeling of baseline wander noise to design a noise subspace $X_{vs}$.

We generate sample functions of a 1st order fBm process with different values of $H$ in the range of 0.3 < $H$ < 1 and use them as the basis vectors of noise subspace $X_{vs}$. These basis vectors are stacked as columns to form the matrix $X$ (refer to Section 2.2). Thus, the column space of $X$ spans the noise subspace $X_{vs}$.

5.2.2. Removal of baseline wander noise

Now, we consider a noisy ECG signal, $y$, after powerline interference removal:

$$y = s + x$$

where $s = [s(n)]$ is length-$N$ vector of the original (clean) ECG signal, $x = [x(n)]$ is length-$N$ vector of the baseline wander noise, and $y = [y(n)]$ is length-$N$ vector of the noisy ECG signal (free of powerline interference). We use the projection operator basis approach to remove baseline wander noise. To this end, we use the characterization of noise subspace $X_{vs}$ (as discussed in the subsection 1 above). The columns of matrix $X$ depict the basis vectors of noise subspace $X_{vs}$. We use (11) to compute the projection matrix $P$ corresponding to the noise subspace $X_{vs}$. Next, we use the projection matrix $P$ and (10) to project signal $y$ onto the noise subspace and estimate baseline wander noise denoted as $\hat{y}$. This noise component $\hat{y} = [\hat{y}(n)]$ is subtracted from the noisy ECG to estimate denoised signal $\hat{s} = [\hat{s}(n)]$, i.e.

$$\hat{s} = y - \hat{y}$$

Using the method proposed above, we remove baseline wander noise from the noisy ECG signal and obtain the denoised signal [38].

We further note that the proposed approach of baseline wander removal is independent of the number of sample functions of the first order fBm processes taken to generate the $X$ matrix of size $N \times M$ where $N$ is the length of the input signal block and $M$ is the number of sample functions of fBm considered. Table 4 shows the variation of SNRc with the value of $M$. It can be clearly observed that output SNRc is almost constant for $M > 8$. Thus, for our simulation purposes, we have chosen $M = 10$.

In the next section, we present the complete proposed algorithm for ECG denoising.

6. Proposed algorithm

The proposed denoising method for powerline interference and baseline wander noise removal from noisy ECG is depicted pictorially in Fig. 3.

It is observed that if powerline interference is present in an ECG signal, the value of $H$ of the ECG signal falls. Records 108 and 111 of the MIT-BIH Arrhythmia Database are found to have powerline interference. The value of $H$ of Records 108 and 111 (signals corrupted with only with powerline interference) is observed to be in the range of 1st order fBm (refer to Table 5). The baseline wander noise has also been modeled with $H$ in the same range. Thus, if baseline wander noise is removed prior to removing powerline interference, it will lead to loss of signal information.

Further, it is observed that $H$ of these ECG Records (108 and 111) falls in the range of clean ECG signals (Table 2) after the removal of powerline interference. This shows that powerline interference should be removed prior to the removal of baseline wander noise with the proposed approaches.

<table>
<thead>
<tr>
<th>Value of $M$</th>
<th>Output SNR (SNRc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.3</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
</tr>
<tr>
<td>6</td>
<td>3.3</td>
</tr>
<tr>
<td>8</td>
<td>4.1</td>
</tr>
<tr>
<td>10</td>
<td>4.3</td>
</tr>
<tr>
<td>12</td>
<td>4.21</td>
</tr>
<tr>
<td>14</td>
<td>4.23</td>
</tr>
</tbody>
</table>

Table 4: Change in output SNRc with change in the value of $M$ (input SNR = −10 dB).
Algorithm to remove powerline interference and baseline wander noise:

1) Consider a noisy ECG signal, \( r(n) \), corrupted with powerline interference and baseline wander noise.
2) Compute the first IMF of the noisy ECG signal using EMD method.
3) Carry out 3-level wavelet (‘Coiflet-4’) decomposition of the first IMF. Use universal thresholding on the detail coefficients of the 3rd level detail coefficients. This will remove powerline interference. Reconstruct the signal using the modified first IMF and the remaining IMFs. This provides us ECG signal, \( y(n) \), free of powerline interference.
4) For the removal of baseline wander noise, use sample functions of first order fBm corresponding to different values of \( H \) and stack them as columns in a matrix \( X \) of size \( N \times M \) as discussed in Section 5B. Here, we have used \( M = 10 \) and \( N = 1000 \).
5) Use Eq. (11) to generate the projection matrix \( P \) for the noise subspace \( X_{01} \) that is spanned by the columns of \( X \).
6) ECG signal (free of powerline interference but corrupted with baseline wander noise) obtained from step 3 is converted into overlapping blocks of size \( N \) with length of overlap \( N_0 = 50 \). Denote this signal as \( y \).
7) Use (10) to estimate baseline wander noise \( \hat{y} \).
8) Estimate clean ECG signal \( \hat{s} \) using Eq. (13).
9) Unblock estimated clean signal to form \( \hat{s}(n) \).
10) Reject first \( N_0 \) samples of each processed block.
11) Concatenate all the blocks and display the estimated clean ECG signal.

In the next section, we present simulation results on noisy signals of varying input SNR and also compare the performance of the proposed approach with some existing methods.

### 7. Simulation results

In order to evaluate the efficacy of the proposed methods of the removal of powerline interference and baseline wander, we add noise, \( x(n) \), of varying energy to the clean signal, \( s(n) \), and form noisy ECG signal, denoted as \( y(n) \). We compute the input-signal-to-noise ratio (SNR) corresponding to the noisy ECG signal and the output signal-to-noise ratio (SNRo) corresponding to the denoised signal.

The comparison parameters chosen are: SNR, and the cross-correlation coefficient \( (\rho) \) between the clean ECG signal \( s(n) \) and the denoised ECG signal \( \hat{s}(n) \).

#### 7.1 Powerline interference removal method

\[
\text{SNR}_\text{fi} = 10 \log_{10} \left( \frac{\frac{1}{M} \sum_{m=0}^{M-1} |s(m)|^2}{\frac{1}{M} \sum_{m=0}^{M-1} |s(m) - \hat{s}(m)|^2} \right) \tag{14}
\]

\[
\rho = \frac{\sum_{n=0}^{N-1} s(n) \hat{s}(n)}{\left( \sum_{n=0}^{N-1} s(n)^2 \right)^{1/2} \left( \sum_{n=0}^{N-1} \hat{s}(n)^2 \right)^{1/2}} \tag{15}
\]

Where \( N \) is the length of the signals.

We have carried out three experiments to show the efficacy of the proposed noise removal strategies.

**Experiment 1: Comparative performance of Powerline interference removal method**

We compare the performance of the proposed powerline interference removal method with the existing technique of notch filter applied on 1st IMF [5]. The notch filter has a cut off frequency of 50 Hz or 60 Hz depending on the powerline frequency. Table 6 presents results for the removal of powerline interference of 50 Hz. Results for 60 Hz noise removal is presented in Fig. 4 and Table 7. Tables show the averaged output SNR and cross correlation coefficient for 15 segments. From these results, it is observed that the proposed method performs better than the existing method.

**Experiment 2: Comparative performance on Baseline Wander removal**

We compare the performance of our proposed method with three existing techniques: the filterbank method [37], empirical mode decomposition [16], and wavelet analysis [18].

In the filterbank method, a filterbank is designed using the 2nd order fBm model of original ECG signal. The lowpass subband

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### Table 5

<table>
<thead>
<tr>
<th>ECG records</th>
<th>Input SNR (SNRi)</th>
<th>Output SNR (SNRo)</th>
<th>Cross correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Record 111</td>
<td>1.2060</td>
<td>0.9508</td>
<td>0.95</td>
</tr>
<tr>
<td>Record 108</td>
<td>0.9508</td>
<td>0.9312</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Note: The algorithm which performs the best has been marked in bold.

### Table 6

Comparison of proposed powerline interference removal method with EMD + notch filter method [5] (for powerline frequency of 50 Hz) averaged over 15 segments.

<table>
<thead>
<tr>
<th>Input SNR (SNRi)</th>
<th>Output SNR (SNRo)</th>
<th>Cross correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>12.5</td>
<td>7.13</td>
</tr>
<tr>
<td>5</td>
<td>18.07</td>
<td>12.20</td>
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<tr>
<td>0</td>
<td>22.32</td>
<td>17.49</td>
</tr>
<tr>
<td>5</td>
<td>25.86</td>
<td>21.30</td>
</tr>
<tr>
<td>10</td>
<td>26.76</td>
<td>26.23</td>
</tr>
</tbody>
</table>
signal is processed for baseline wander noise removal [37]. In the EMD method, the noisy ECG signal is decomposed into intrinsic mode functions (IMFs) and the last three IMF are discarded [16]. In the wavelet approach, Coiflet-4 (‘4’ is the order of the wavelet) wavelet is used with 10 level decomposition and the detail coefficients of level 9 and level 10 are set to zero as suggested in [18]. Table 8 shows the averaged output SNR (SNRo) and cross correlation coefficient for 15 segments.

It is evident from Table 8 and Fig. 5 that at low input SNR (SNRi) the proposed method performs better than the other three algorithms, both in terms of SNRo and the cross correlation coefficient $\rho$. For clean (original) ECG signals, at high SNRi the filterbank approach [37] performs better than the proposed algorithm.

In the case of trans-abdominal maternal ECG recordings, baseline wander noise is generally very large in the recorded ECG signal. Hence, the SNRi is significantly low. The projection matrix approach as discussed above performs better than the other algorithms in such cases. For example, a noisy ECG signal at low SNRi (−15 dB) is shown along with the signal denoised using different methods in Fig. 6. It is noticed that while the proposed method has been able to remove baseline wander noise, performance of other methods is relatively poor.

In many cases, we need to preserve the R peak of the original (clean) ECG along with the QRS complex and the ST segment for correct heart disease identification. Fig. 7 clearly shows that the denoised signal obtained using the proposed method is better able to follow the original (clean) ECG signal in comparison to the other algorithms.

Experiment 3: Comparative Performance of EMD and Proposed Method of Baseline Wander removal on two heart abnormalities: The performance of the proposed method of baseline wander removal is compared to the EMD method on two ECG signals with heart abnormalities. It should be noted that any distortion introduced by a denoising method in the shape of patient’s ECG signal may lead to either false alarm or miss during disease identification.

Table 7
Comparison of proposed powerline interference removal method with EMD + notch filter method [5] (for powerline frequency of 60 Hz) averaged over 15 segments.

<table>
<thead>
<tr>
<th>Input SNR (SNRi)</th>
<th>Output SNR (SNRo)</th>
<th>Cross correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed method</td>
<td>EMD + notch filtering [5]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Input noisy signal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proposed method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EMD + notch filtering [5]</td>
</tr>
<tr>
<td>−10</td>
<td>15.65</td>
<td>7.067</td>
</tr>
<tr>
<td>−5</td>
<td>16.74</td>
<td>13.05</td>
</tr>
<tr>
<td>0</td>
<td>25.5</td>
<td>18.908</td>
</tr>
<tr>
<td>5</td>
<td>24.67</td>
<td>20.467</td>
</tr>
<tr>
<td>10</td>
<td>25.34</td>
<td>26.87</td>
</tr>
</tbody>
</table>

Note: The algorithm which performs the best has been marked in bold.

Table 8
Comparison of proposed baseline wander removal method with filterbank method [37], EMD [16], and wavelet analysis [18] averaged over 15 segments.

<table>
<thead>
<tr>
<th>Input SNR (SNRi)</th>
<th>Output SNR (SNRo)</th>
<th>Cross correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed method</td>
<td>Filterbank method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Input noisy signal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proposed method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Filterbank method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EMD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wavelet analysis method</td>
</tr>
<tr>
<td>−20</td>
<td>−0.567</td>
<td>−5.78</td>
</tr>
<tr>
<td>−15</td>
<td>1.456</td>
<td>−2.045</td>
</tr>
<tr>
<td>−10</td>
<td>5.56</td>
<td>3.01</td>
</tr>
<tr>
<td>−5</td>
<td>6.789</td>
<td>8.13</td>
</tr>
<tr>
<td>0</td>
<td>7.956</td>
<td>13.567</td>
</tr>
<tr>
<td>5</td>
<td>8.034</td>
<td>19.034</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input SNR (SNRi)</th>
<th>Output SNR (SNRo)</th>
<th>Cross correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Proposed method</td>
<td>Filterbank method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Input noisy signal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proposed method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Filterbank method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EMD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wavelet analysis method</td>
</tr>
<tr>
<td>−20</td>
<td>0.08</td>
<td>0.6114</td>
</tr>
<tr>
<td>−15</td>
<td>0.16</td>
<td>0.7711</td>
</tr>
<tr>
<td>−10</td>
<td>0.29</td>
<td>0.8653</td>
</tr>
<tr>
<td>−5</td>
<td>0.49</td>
<td>0.9047</td>
</tr>
<tr>
<td>0</td>
<td>0.75</td>
<td>0.9189</td>
</tr>
<tr>
<td>5</td>
<td>0.86</td>
<td>0.9223</td>
</tr>
</tbody>
</table>

Note: The algorithm which performs the best has been marked in bold.
From Table 9, it is seen that at low SNR, the QRS duration, a characteristic of denoising signal with EMD method, P wave is either difficult to identify or is completely lost (Fig. 8). This may generate false alarm for heart abnormality known as atrial fibrillation. Thus, denoising with EMD misses a false alarm for atrial fibrillation.

On denoising with proposed method, we notice that the presence of LBBB in patients indicates increased chances of hypertension, enlarged heart, heart failure, or coronary artery disease [39]. It is mainly observed in patients of age greater than 60 years.

Record 214 of MIT-BIH Arrhythmia database [23] shows ECG signal with LBBB. Comparative performance of our proposed algorithm and EMD are shown for the noisy abnormal ECG (clean diseased ECG+baseline wander noise) at low SNR, in Table 9 and Fig. 8. From Table 9, it is seen that at low SNR, the QRS duration, a characteristic of LBBB, gets altered with the EMD method, while it remains preserved with our proposed method. In addition, on denoising signal with EMD method, P wave is either difficult to identify or is completely lost (Fig. 8). This may generate false alarm for heart abnormality known as atrial fibrillation where P-wave is missing. Thus, denoising with EMD misses LBBB and may generate a false alarm for atrial fibrillation. On the other hand, our proposed method is able to retain the characteristics of original (noise free) ECG signal and hence, will correctly help in identifying LBBB.

Thus, a noise removal method should be able to preserve the shape and the characteristics of (abnormal) clean ECG signal.

7.1. **Left bundle branch block (LBBB)**

LBBB is a cardiac abnormality that is mainly caused due to delay in activation of the left ventricle. Hence, the contraction of left ventricle is later than that of right ventricle [38]. ECG recordings of patients suffering with LBBB have the following characteristics: (1) QRS duration is greater than 120 ms; (2) Lead V1 signal shows a slurring of QRS with an initial R wave; (3) ST segment is seen to have displacement; and (4) the direction of T wave is opposite to R wave [40]. The presence of LBBB in patients indicates increased chances of hypertension, enlarged heart, heart failure, or coronary artery disease [39]. It is mainly observed in patients of age greater than 60 years.

Record 214 of MIT-BIH Arrhythmia database [23] shows ECG signal with LBBB. Comparative performance of our proposed algorithm and EMD are shown for the noisy abnormal ECG (clean diseased ECG+baseline wander noise) at low SNR, in Table 9 and Fig. 8. From Table 9, it is seen that at low SNR, the QRS duration, a characteristic of LBBB, gets altered with the EMD method, while it remains preserved with our proposed method. In addition, on denoising signal with EMD method, P wave is either difficult to identify or is completely lost (Fig. 8). This may generate false alarm for heart abnormality known as atrial fibrillation where P-wave is missing. Thus, denoising with EMD misses LBBB and may generate a false alarm for atrial fibrillation. On the other hand, our proposed method is able to retain the characteristics of original (noise free) ECG signal and hence, will correctly help in identifying LBBB.

7.2. **ST elevation myocardial infarction (STEMI)**

ST segment elevation and depression is used for identification of ST elevation myocardial infarction (STEMI) [41]. STEMI is a type of heart attack in which a coronary artery is blocked completely by a blood clot. Some heart muscles which receive oxygen from that coronary artery begin to die. The highly elevated ST segment indicates the amount of heart muscle damage.

As mentioned in Section 1, we know that baseline wander noise affects the ST segment. However, for disease identification such as STEMI, we need to preserve the ST segment in the denoised signal.

To test the efficacy of our proposed method and the EMD method, we use Record 231 with straight elevation (characteristic of STEMI) in the ST segment (Fig. 9a). Noisy ECG signal is created by adding baseline wander noise to the clean ECG signal (Fig. 9b).

From Fig. 9b, we note that the presence of E’ in the noisy signal has got missed, while false alarm for atrial fibrillation [41, 42].

On denoising with proposed method, we notice that the straight elevation of ST segment is restored and STEMI is correctly detected (Fig. 9c). We also notice that false E in the noisy signal has also been removed. On the other hand, because EMD is not able to remove baseline wander to a large extent, ‘SL’ is missing and false E is present. Thus, STEMI has got missed, while false alarm for atrial myocardial infarction may be generated.

8. **Conclusions**

Real-time ECG signals are often corrupted by powerline interference and baseline wander noise that need to be removed before an ECG signal can be used by a doctor for analysis. We have proposed novel methods of removal of baseline wander and powerline interference from noisy ECG signals. First, we removed powerline interference from the noisy ECG signal using a hybrid method of EMD and wavelet transform. We applied universal thresholding on 3rd level wavelet coefficients of the first IMF. Second, we have modeled the original (clean) ECG signal as a 2nd order fBm random process and baseline wander noise as a 1st order fBm random process. Third, we utilized this modeling to propose a projection operator based method for baseline wander...
noise. The proposed methods remove baseline wander noise and powerline interference without altering the characteristics of original (noise free) ECG signals. Performance comparison shows that, compared to other algorithms, the proposed methods provide significant improvement in output signal-to-noise ratio and lead to higher value of cross correlation coefficient between the original (clean) ECG and the denoised ECG signal.

Baseline wander noise affects the ST segment and small amplitude waves i.e. P wave and the T wave of the clean diseased (original) ECG signal. Thus, a baseline wander noise removal method should preserve the P wave, T wave and the ST segment for correct diagnosis. We have also demonstrated that our proposed baseline wander denoising method is able to preserve heart abnormality characteristics when applied on such signals.

Table 9
Comparison of performance of our proposed algorithm and EMD on a noisy, ECG signal with LBBB.

<table>
<thead>
<tr>
<th>ECG signals</th>
<th>Input SNR (SNRi) of noisy ECG signal (dB)</th>
<th>Duration of QRS in clean signal (ms)</th>
<th>Proposed algorithm</th>
<th>Using EMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Duration of QRS from denoised signal (ms)</td>
<td>Other observations from denoised signal</td>
</tr>
</tbody>
</table>
| Record 214 (ECG with LBBB) | − 12                                      | 125                                  | 125                | • Inverted T wave detected  
• P wave detected  
• QRS duration > 120 ms     | 116                                     | • Inverted T wave observed  
• P wave is not clearly observed  
• QRS duration < 120 ms     |
| Record 214 (ECG with LBBB) | − 8                                       | 125                                  | 124                | • Inverted T wave detected  
• P wave detected  
• QRS duration > 120 ms     | 119                                     | • Inverted T wave detected  
• P wave is not clearly observed  
• QRS duration = 120 ms   |

Note: Bold indicates characteristic of LBBB lost in denoising.

Fig. 8. LBBB: (a) The clean (original LBBB) signal. (b) Noisy ECG (SNRi = −8 dB). (c) Denoised signal with proposed algorithm. (d) Denoised signal with EMD approach.

Fig. 9. STEMI: (a) Original STEMI signal; ‘E’ Denotes ‘elevation after P- and before R-wave’; ‘SL’: Denotes ‘straight elevation of ST segment’. (b) Noisy ECG (SNRi = −8 dB). (c) Denoised signal with proposed algorithm. (d) Denoised signal with EMD approach.
Summary

Real-time ECG signals, when recorded, are often corrupted by powerline interference and baseline wander noise that need to be removed before an ECG signal can be used by a doctor for analysis. Powerline interference is a high frequency noise (50 or 60 Hz) which comes from the powerline to the recording machines. During removal of powerline interference, special care has to be taken to preserve the amplitude and frequency of R wave in the ECG signal whose spectrum overlaps with the powerline interference. On the other hand, baseline wander is a low frequency artifact that corrupts ECG signals. It affects the ST segment and small amplitude waves i.e. P wave and the T wave of the clean diseased (original) ECG signal.

We have proposed novel methods for the removal of baseline wander and powerline interference from noisy ECG signals. First, we removed powerline interference from the noisy ECG signal using a hybrid method of EMD and wavelet transform. We applied universal thresholding on 3rd level wavelet coefficients of the first IMF. Second, we have modeled the original (clean) ECG signal as a 2nd order fBm random process and baseline wander noise as a 1st order fBm random process. Third, we utilized this modeling to propose a projection operator based method for baseline wander noise. The proposed methods remove baseline wander noise and powerline interference without altering the characteristics of original (noise free) ECG signals. Performance comparison shows that, compared to other algorithms, the proposed methods provide significant improvement in output signal-to-noise ratio and lead to higher value of cross correlation coefficient between the original (clean) ECG and the denoised ECG signal.

Conflict of interest statement

None declared.

References


