DESIGN OF OPTIMAL LINEAR QUADRATIC REGULATOR (LQR) CONTROL FOR FULL CAR ACTIVE SUSPENSION SYSTEM USING REDUCED ORDER

By

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Certificate

This is to certify that the thesis prepared by **Mr. SAIROEL AMERTET** entitled **"Design of optimal linear quadratic regulator (LQR) Control for full car Active Suspension System using Reduced order"** and submitted as a partial fulfillment for the Degree of Master of Science complies with the regulations of the University and meets the accepted standards with respect to originality, content and quality

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Abstract

Vehicle dynamics and control has become a key research area by many researchers around the globe. Most of the researchers proven the effectiveness of different controllers for full order model without considering the actuator dynamics. However, this paper investigates the design and analysis of Linear Quadratic Regulator for reduced order full car model incorporating the dynamics of the actuator to improve system performance. The vehicle model is reduced to a minimal order using minimal realization technique. The entire system response were simulated in MATLAB/ Simulink environment. The effectiveness of LQR controller was compared for the system model with and without actuator dynamics for different road profiles. The simulation results indicates that the percentage reduction in the peak value of vertical and horizontal velocity for the LQR with actuator dynamics relative to LQR without actuator dynamics is 66.67%. Overall simulation results demonstrates that the proposed control scheme has able to improve the effectiveness of the vehicle active suspension system for both ride comfort and vehicle stability compared to PID and Fuzzy controllers.
Keywords: LQR,suspension system,minimal realization

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Abbreviations and Acronyms

ASS	Active Suspension System
CG	Center of Gravity
DOF	Degree of Freedom
GA	Genetic Algorithm
LQR	Linear quadratic regulator
LQG	Linear quadratic Gaussian
PID	Proportional integral derivative
SMC	slide mode control
MIMO	Multiple input multiple output
PSO	Particle Swarm Optimization
PSS	Passive Suspension System

LIST OF SYMBOLS

Ms	Sprung mass (mass of the car body) (kg)
Mur1	Front left unsprung mass (mass of the wheel) (kg)
Mur2	Front right unsprung mass (mass of the wheel) (kg)
Mur3	Rear right unsprung mass (mass of the wheel) (kg)
Mur4	Rear left unsprung mass (mass of the wheel) (kg)
Ir	Pitch moment of inertia (kgm^2)
Ip	Roll moment of inertia (kgm^2)
k1	Front left suspension stiffness (N/m)
k2	Front right suspension stiffness (N/m)
k3	Rear right suspension stiffness (N/m)
k4	Rear left suspension stiffness (N/m)
C1	Front left suspension damping coefficient (Ns/m)
C2	Front right suspension damping coefficient (Ns/m)
C3	Rear right suspension damping coefficient (Ns/m)
C4	Rear left suspension damping coefficient (Ns/m)
kur1	Front left tire stiffness (N/m)
kur2	Front right tire stiffness (N/m)
kur3	Rear right tire stiffness (N/m)
kur4	Rear left tire stiffness (N/m)
a1	Distance of the front wheel from the CG (m)
a2	Distance of the rear wheel from the CG (m)
b1	Distance of the right wheel from the CG (m)
b2	Distance of the left wheel from the CG (m)

Z0	Vertical displacement of CG of Vehicle body (m)
θ	Pitch of vehicle along x axis from the C.G in degree
θ	Roll of vehicle along z axis from the C.G in degree
z01	Front left vertical displacement of the corner of the vehicle body (m)
z02	Front right vertical displacement of the corner of the vehicle body (m)
z03	Rear right vertical displacement of the corner of the vehicle body (m)
z04	Rear left vertical displacement of the corner of the vehicle body (m)
m_{b1}	mass of hydraulic cylinder(kg)
m_{b2}, m_{b3}	mass of pads(kg)
J_{b1}, J_{b2}	inertia mass moments of disk and tire(kgm^2)
k_{b1}, k_{b2}	stiffness of inner pads(N/m)
C_{b1}, C_{b2}	damping coefficient of inner pads(Ns/m)
k_{b3}, C_{b3}	stiffness and damping coefficient of housing(N/m,Ns/m)
$\triangle_1, \triangle_2$	initial gaps b/n pad and disk (mm)
F_{N1}, F_{N2}	normal forces b/n pads and disk(N)
F_{fl4}	friction force between cylinder and pads (N)
F_{fr12}	Friction force between cylinder and piston(N)
f	frictional coefficient between pad and disk
M_b	brake torque(Nm)
R_a	An average radius(mm)
$Q_{(t)}$	Amount of discharge(m^3/s)
V_0	initial volume of the cylinder

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Chapter 1

Introduction

Traditionally automotive suspension designs have been compromise between the three conflicting criteria's namely road handling, load carrying, and passenger comfort. The suspension system must support the vehicle, provide directional control using handling maneuvers and provide effective isolation of passengers,load disturbance and easily to control the position and velocity of the vehicle. Good ride comfort requires a soft suspension, where as insensitivity to apply loads require stiff suspension. Good handling requires a suspension setting somewhere between it and better to control the vehicle at desire position and condition. Due to these conflicting demands, suspension design has to be something that can compromise of these problems. A passive suspension has the ability to store energy via a spring and to dissipate it via a damper. Its parameters are generally fixed, being chosen to achieve a certain level of compromise between road handling, load carrying and ride comfort. An active suspension system has the ability to store, dissipate and to introduce energy to the system. It may vary its parameters depending upon operating conditions.
Suspension consists of the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. In other meaning, suspension system is a mechanism that physically separates the car body from the car wheel. The main function of vehicle suspension system is to minimize the vertical and horizontal acceleration transmitted to the passenger which directly provides road comfort and for safe travel as well as well controllable if the impact is suddenly occurs.There are three types of suspension system; passive, semi-active and active suspension system. Traditional suspension consists springs and dampers are referred to as passive suspension, then if the suspension is externally controlled it is known as a semi active or active suspension.

1.1 Motivation

Due to the ever-increasing demand for automobile vehicles and the impact of irregularity road surface on vehicle active suspension system has attracted a growing research interest. It is multi-physics area which involves multidisciplinary concepts which the control technique is depend on. Nowadays, the world tries to investigate a smart way of comfort ride. Design of a powerful control technique for active suspension system so that comfort ride is a current hot research area. Thus, it needs depth research in the area of vibration control and depth understanding of the dynamics of the system. In addition to this when I was attended my Bachelor degree in Hawassa university the course vibration is not involving in the curriculum of electromechanical program,and when observing the modern technology its basement is vibration ,so I eager to know the physics of the vibration,since science of vibration is mostly affects the mechatronics system throughout all direction. This motivates me behind this research thesis to conduct.

1.2 Statement of the problem

There were a passive suspension element in which huge energy was dissipating and can't controllable as well as observable due its fixed parameters. Semi active and active suspension elements were dissipated a reasonable energy and had a controllable variable on it. Moreover the researcher's were tested using a different controller algorithm in order to achieve the best control performance and extract the vehicle information. However, they were not considering the state controllable and state observable in which the state feedback was possible.

1.3 Objectives

1.3.1 General Objective

The general objective of this research paper was to design an LQR Controller for active Suspension System of Automobile vehicles using Full car model by integrating the dynamics of the tires on the system model to improve the performance of the system; thereby the ride comfort ensured.

1.3.2 Specific Objectives

- To model mathematically the vehicle dynamic suspension system including tire system, and actuator dynamics.
- To reduced the states order for suitable control design
- To design the controller (Linear Quadratic Regulator) based on reduced order model
- To test the performance of the controller through computer simulation using matlab/Simulink

1.4 Methodology

To achieve the general and specific objectives mentioned above the methodology followed clearly stated. The first task : to meet the objectives was understanding about the suspension systems by conducting a literature review on passive and active suspension systems. Mathematical model of the dynamics of active suspension systems for full car model was formulated using physical laws and some basic assumptions. The tire dynamics was included in the system model. Then, the state space representation for the system created. A literature review on optimal linear quadratic regulator (LQR) was carried out. An LQR controller was designed based on the system model including tire dynamics.The design of the proposed LQR controller was also based on the tuning the values of the weighting matrices i.e. Q and R matrices. These values were found using manual approach method. By choosing diagonal Q matrix each state was penalized by its respective diagonal element. So, Selecting Q and R and simulating the system and observing the status of the states. This was the basic systematic way of choosing the Q and R parameters. Computer simulation was carried out using MATLABR/SIMULINK environment for the designed controller based on the state space representation to investigate its performance. Finally, the result analyzed and interpreted by obtaining the proper feedback gain matrix to compare and observe the car body motion with active suspension systems.

1.5 Scope of the research

The nature of this research was multidisciplinary because, in general, it dealt with Mechanical Engineering and Electrical Engineering (Control engineering).The scope of the study focused on Mechatronics Engineering perspective that includes actuator model and integrated with the controller design,however the sensors were not consider here because already the sensors were existing for the vehicle active suspension system and to reduce the complexity of the model. In this research, modeling and simulation of various parts and the whole system was done.The source of the ride comfort problem arise from the vibration of vehicle body, which may be produced by internal vehicle subsystems, such as engine, powertrain, non-uniformities of the tire/wheel assembly or the suspension mechanisms, or they may be produced from external factors, such as road roughness or aerodynamics forces. In this research paper, the main caused of the vehicle active suspension problem which was road surface irregularities was only considered. The other mentioned factors(aerodynamic resistance,friction forces,air drag force,aerodynamic pitching moment,Rolling resistance of the front and rear tyres,Drawbar loads,Aerodynamic forces acting on the body of the vehicle,and Vertical load of the vehicle at the front and rear tyres) are ignored. This research paper also didn't explained about the robustness of the controller. The analysis was also limited for the linear region only.

1.6 Organization of the research paper

Chapter two dealt with overview of vehicle active suspension system and the Literature Reviews of the active suspension system. In this chapter, overviews of the active vehicle suspension system and the Literature Reviews was done based on different journal papers clearly understood the multi physics of the active vehicle dynamics . Chapter three was concerned with the methodology of suspension system includes(mathematical modeling of the active suspension system, actuators.The mathematical modeling was done based on the assumption of linear passive elements and the design of LQR controller). The design of LQR controller was described starting from the theoretical backgrounds and mathematical description. Chapter four explained the results and discussions with simulation in detail. In this chapter, the performance of the LQR controller with actuator dynamics included in the system model, LQR controller without including the actuator dynamics compared. Chapter five presented about conclusions drawn which done in this thesis and recommendations on further works.

Chapter 2

An overview of vehicle active suspension system and literature reviews

2.1 An overview of vehicle active suspension system

Vehicle active Suspension system is a mechanism that physically separates the car body from the car wheels. It is the most important part of the vehicle which heavily affects the ride quality and used to isolate the vehicle structure from shocks and vibration due to irregularities of the road surface[1]. The main purpose of vehicle active suspension system is to minimize the horizontal displacement, velocity or acceleration transmitted to the passenger which directly provides ride comfort[2],[3]. Design and development of automotive active suspension system has been of great interest for many years. Vehicle active suspension systems using different types of springs, dampers, and linkages with tailored flexibility in various directions have been developed over the last century since the beginning of the automobile age[4]. There are three types of suspension systems, namely passive suspension system, semi- active suspension system and active suspension system [5]. Traditionally, there is passive suspension system that contains springs and dampers. It stores energy through spring and dissipates it via a damper. Since its parameters have a fixed rates, it has a limitation to prevent the vibration of the vehicles body for different types of road disturbance. Depending on the road excitation, however, it is desirable to adjust this property to increase performance. Therefore, another way of improvement which is called semi – active suspension system is developed in which a variable damping element that can be changed to some extent by an external control according to actual demands. Even the semi-active suspension system has a limited range of control but it is better than passive suspension system[6]. A vehicle active suspension system contains separate actuator that exert extra force on the suspension system to improve the ride comfort further. It provides high performance compared to the passive and semi-active suspension systems[7]. In semi-active suspension system the damping coefficient can be controlled by a switch so that the amount of energy dissipation of the damper can be changed. However, it does not supply energy to the system. Active suspension systems have an additional elements such as electric motor, hydraulic or pneumatic cylinders of which their damping coefficients can be changed and have an ability to provide energy to the system continuously by using different types of controllers[8].

2.1.1 Vehicle active Suspension Systems

vehicle active suspension control has been one of the favorite research in the automotive area[5]. The active suspension systems contain sensors, actuators, controllers in addition to the passive elements such as springs and dampers.Gradually, the purpose of the active suspension system is to replace the classical passive elements by a controlled system, which can supply a regulated force to the system. The active suspension system dynamically responds to the changing road surface due to its ability to supply energy, which is used to achieve the relative motion between the body and wheel[9].

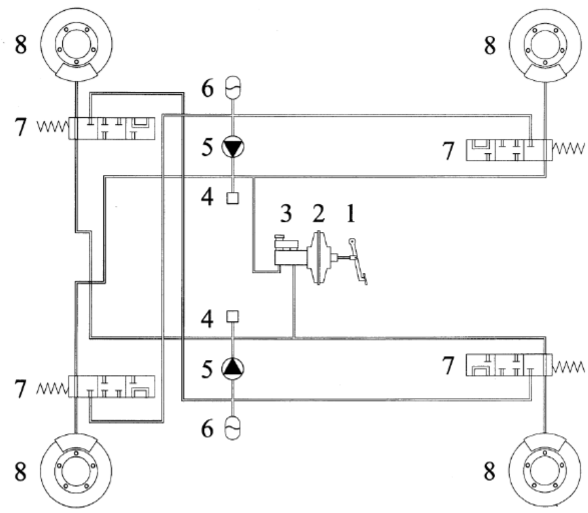


Fig. 2.1: The main scheme of Automobile model system with electro hydraulic actuator[11]

2.1.2 Actuators

There are different types of actuators are available for vehicle active suspension system to apply a required force to the vehicle wheels. many actuator types have been studied in the area of vehicle active suspension system evolution during the years. Hydraulic , and pneumatic are among the widely used actuators. Hydraulic and pneumatic actuators have been investigated and used for many years. In this thesis, an electro hydraulic that can provide best incompressible performance, greater control flexibility,to improve efficiency is selected as an actuator.Actuator can be used to generate the control forces necessary for rolling and pitch motion in active suspension system by converting electric energy in to a motion or force. It actuates a load to move decelerate and accelerate[10],[11].

working mechanism of actuator

As shown in Fig 2.1 the selonoid valve be on and of many times in microsecond up to the brake pedal made free by driver or limiter meanwhile hydraulic inside the reservoir is forcing fluid through calipers which acts upon one or more calipers piston,sealed by O-rings in order to prevent leakage of fluid. The following components are pressed corresponding their sequence numbers[11]. 1)braking pedal,2)booster,3)main cylinder,4)damper,5)pump,6)hydraulic accumulator,7)electromagnetic solenoid valve,8)disk brake mechanism.

2.1.3 Sensors

A sensor measures a physical vehicle dynamics measurement (Horizontal displacement, velocity and acceleration) of the chassis as well as the wheel and changes it in to an electrical signal. The sensor measurements are used to instantaneously counteract the road undulations. In this system, four types of sensors are used. The car body side sensors gives a direct measure of slip of the vehicle. The wheel sensor is installed to estimate the state of the tire since it is not possible to measure the tire compression directly[12].Hence, the operation of the system is carried out based on the information provided by these two sensors.Front sensor and back sensors are used to sense accelerating and decelerating respectively.

2.1.4 Controller

Many researchers investigate different types of control techniques and strategies to increase the performance of the vehicle active suspension system.
PID controller, LQR controller, Fuzzy PID,LQG controller, sliding mode controller and Fuzzy Logic controller are some of the frequently used. Quarter car and half car models are proposed to check and improve the performance of the controllers for active suspension system of automobile vehicles. But, most of the studies considered a quarter-car model to test the performance of the controller due to its simplicity[13]. In this thesis, the problem is a regulator problem. The full car model is multiple input multiple output (MIMO) system. The active suspension system also consumes energy which needs an optimal controller to minimize the energy. Therefore, an Optimal Linear Quadratic Regulator (LQR) is proposed as controller which supports full vehicle system.

2.1.5 Operating Principle of Vehicle Active Suspension Systems

Active suspension systems are Mechatronics systems that control the vertical movement of the vehicle body or chassis relative to the wheels. The active suspension systems considered in this thesis are applied for enhancing the Possibility ride comfort of automobile vehicles. Therefore, they are placed in between the chassis and the wheels through attachment of their two ends to the car body and the wheels. In active suspension system the vehicle's decelerate and accelerate motion is controlled by LQR feedback controller according to the road conditions; the controller output controls the actuator to compensate the road oscillations and increases the vehicle stability as well.As a result of the road disturbance, the vehicle body has been oscillated during brake for some time. The sensors measure the amplitude of the vibration from the equilibrium position. The LQR controller processed the electrical signal information obtained from the sensors and it provides a control signal which controls the action of the actuator for fine response in real time.

2.2 Literature Reviews

2.2.1 Linear Passive Suspension System

As shown in the Fig2.2 the Passive suspension doesn't delivery the energy to the system but can limit the relative heave between vehicle body towards wheels,and its has significant limitation in structure application,fix characteristics.If the suspension system is heavily damped then only this better interns of control ,however it dissipates huge energy into vehicle body for a reasonable road disturbance.Testing the vehicle travels a limited speed on a rough road surface,or unlimited speed on straight line dramatically it is perceived as damage the road.If the damped suspension system is lightly design will give more comfortable ride but loss stability of the vehicle during turn and road line changing[14]-[17].

2.2.2 Linear Semi-active Suspension System

Around 1970's the semi-active suspension system was developed . As shown in the Fig 2.3 the semi-active suspension system has dual functions as passive and active suspension system. As passive suspension system it doesn't provide any energy to the system;the damper coefficient is changed by controllable damper due to this reason it provides fast response where as acting as active suspension system it determines the control action in order to regulate the desired levels with the help of sensors and actuators which detect the road profile[14]-[17].

2.2.3 Linear Active Suspension System

Active suspension system consists of damper and spring system which interceded by the force actuator. The force actuators are weather add the energy or dissipate the energy from the system. Thus the force actuators are determined by different types of controls.To make comfort and vehicle stability ,the better control performance is required[14]-[17]. Fig 2.4 shows simple block diagram to explain how the active suspension can achieve better performance. Figure 2.5 describe basic component of active suspension. In this type of suspension the controller can modify the system dynamics by activating the actuators. All these three types of suspension systems have merit and demerit. But almost all researchers are works on the active car suspension system ,since obtaining it's performance is better than the other two types of suspension systems as mentioned before. It's also contains closed loop feedback to correct the error and gave the output to the desired level and it has ability to give ride comfort.Having force actuators can control by the controller[14]-[17].

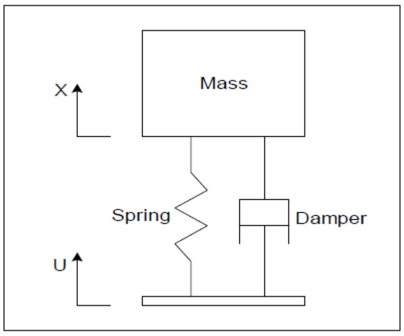


Fig. 2.2: Passive Suspension Component[14]

Table 2.1: comparison table[14]

Vehicle suspension types	Energy consumptions	Applying control action	Vehicle performance
Passive	No dissipated	No control applied	not good
Semi active	partially dissipated	partially applied	better
Active	fully dissipated	fully applied	best

2.2.4 Vehicle Models

The reason of developing different types of the vehicle dynamics model is to obtained the basic information; to expend full car model and to test the performance of various control types in order to select the best one. Quarter-car model as shown in Fig 2.6 is frequently used to extract the vehicle dynamics information;since it's simple in structure and easily capture important natures of full vehicle system.This model consists sprung mass,unsprung mass which provide bounce roll and pitch angle can be represented in the X, Y and Z axis,and variable force generating elements are located among sprung mass and unsprung mass which constitutes suspension system, and based on this component the governing equations are developed[14]-[17]. A paper titled "Passive Suspension Modeling and Analysis of a Full -Car Model" by M. Rababah and A. Bhuyan has presented the design of the Quarter-car model and simulated using MATLAB/SIMULINK, which allows analyzing the behavior of the suspension system. The paper is extended in to half car model, and eventually the passive suspension system is extrapolated into modeling of a full car model by connecting the link between the sprung mass to the four unsprung masses, with a similar basic modeling technique and performing similar exercises. After the evaluation of the behavior of these passive models, a semi-active suspension system is introduced for controllable suspension system. However, it has not explained anything about full active suspension system which can give better performance[18].

A. Mitra, N. Benerjee, H. A. Khalane, M. A. Sonawane, D. R. Joshi and G. Bagul[19] on the paper named, "Simulation and Analysis of Full Car Model for various Road profile on analytically validated MATLAB/SIMULINK model", have validated the solution of analytical methods with the Simulink model. The validated simulation model is used as a platform to analyze the performance of vehicle dynamics for different road profile. For the analysis purpose, they concerned the passive suspension system of the full car model with seven degrees of freedom. But, the paper does not report about active suspension systems and actuator dynamics.

S. K. Sharma, V. Pare, M. Chouksey and B. Rawal in their paper called "Numerical studies using full car model for combined primary and cabin suspension", have designed and developed primary and secondary passive suspension system of full car model. The Primary suspension system isolates vehicle's chassis from road irregularities, while cabin suspension system isolates cabin from chassis vibrations. The paper reports the comparison between the response of the primary and cabin suspension with the response of primary suspension under step excitation. The results show that the incorporation of secondary suspension system results in improvement of the performance of the vehicle ride comfort. Although in this paper the model is considered with ten degree of freedom to analyze the system, it is more complex and the step input is not a real time excitation. Therefore, designing of a controller with dynamic analysis of an actuator may be better than the proposed model to improve the performance[20].

Table 2.2: Comparison of vehicle model[21]

Vehicle model types	DOF	model structure	Vehicle feature extraction
Quarter car	2	simple	good
Half car	3	medium	better
Full car	7	difficult	best

2.2.5 Controls for vehicle suspension models

As the modeling of the car growth from quarter to half and to full, the number of degrees of freedom (DOF) (parameter) increase as well as the accuracy of the model increases, however, it needs more time and data to analyze and solve it. Quarter and half car modeling are good for initial studies of the systems but one has to choose a full car modeling for better modeling and representation of the vehicle body and suspension system. Quarter and half car models are approximations of suspension system and vehicle body. These models cannot capture the behavior of vehicle body in all situations. Because road input conditions are not considered at remaining 3 and 2 tires respectively. But, these models can be used for analyzing control algorithms[21] .

M. Nagarkar, G. Vikhe, K. Borole and V. Nandedkar[22], have proposed "Active control of quarter car suspension system using Linear Quadratic Regulator". This paper analyzed passive suspension system and active suspension system using a linear quadratic regulator (LQR) controller. The simulation results showed that the designed active suspension system using LQR improved the ride comfort quality. They have dealt with mathematical modeling using a two degree of freedom model of a quarter car. More over they used a quarter car model to analyzed for ride comfort because of its simplicity. The paper considered each quarter car independently to drive the mathematical model and design a Linear Quadratic Regulator (LQR) controller to achieve the better performance of active suspension system. In this case, the four quarter parts considered as a four independent systems. However, all the four quarter car models were interdependent to each other's. They should be combined to form a single full car system and the four wheels considered as states in full car system. Besides, in practical implementation four controllers required to each quarter part in case of quarter car model whereas a single controller needed for full car model. So, cost of the controller has considered. Based on the perspective of the modeling system full car model was better than quarter and half car models. R. Darus and Y. Md. Sam[23],[24], in their paper titled "Modeling and Control Active Suspension System for a Full Car Model", have designed and analyzed the LQR controller for full car model and compared the performance of the controller with the passive suspension system. The result of their studied shown that the active suspension system with LQR controller was able to provide better performance than passive suspension system. The proposed strategy utilized mathematical model of the suspension system to design the controller. The studied of actuators and sensors included in the system model to improve the real time performance and focused on the body displacement to measure the ride quality but pitch angle which has a significant role to measure passenger comfort not considered. R. Binti Darus[23][24], has proposed "Modeling and Control of Active Suspension for a Full Car Model", in this paper, an LQR controller designed based on the full car model. Active and passive suspension systems were compared to evaluate the performance of the controller. It showed, that the controller gave better performance than passive suspension system in terms of vertical displacement and acceleration to measure passenger comfort. Fourteen number of states used to describe the dynamic behavior of the system which was complex to handle the calculations as well as it required fourteen sensors to measure each states which was bulky and asked high cost. For implementation purpose, further reduction of the order of the system required. Practical implementation was not feasible. Gang Wang,and Zhijin Zhou[25] have proposed "Design and Implementation of $H\infty$ Miscellaneous Information Feedback Control for Vehicle Suspension System",in this paper,$H\infty$ Miscellaneous Information Feedback Control designed based on the quarter car model.The $H\infty$ Miscellaneous Information

Feedback Control and $H\infty$ controls been compared to evaluate the performance of the quarter car model.The result showed that the performance of $H\infty$ Miscellaneous Information Feedback Control was better than $H\infty$. on the paper named, "LQR Tuning by Particle Swarm Optimization of Full Car Suspension System"[26] , have validated the LQR tuning via Particle Swarm Optimization control algorithm using full Car dynamics model. The validated simulation model been used as a platform to analyze the performance of vehicle dynamics for different tuning optimization until it achieved the desired results. For the analyzed purpose, they concerned the full car suspension model with seven degrees of freedom. S.F. Youness, E.C. Lobusov[27]-[30], have proposed "Networked Control for Active Suspension System" . This paper analyzed active suspension system using a linear quadratic regulator (LQR) and proportional integral derivative (PID) controller. The simulation results showed that the designed active suspension system using LQR improved the ride comfort quality better than PID. They have dealt with mathematical modeling using a seven degree of freedom model of a full car. They used a full car model to analysis for ride comfort because of extracting more vehicle nature and to test the control algorithms. The paper considered each full car independently to drive the mathematical model and design a Linear Quadratic Regulator (LQR) and PID controller in order to achieve better performance of active suspension system. In this case, the seven degree full car parts has been considered as a seven independent systems. A paper titled "Enhancing vehicle suspension system control performance based on the improved extension control" by Hongbo Wang has presented the design of the full-car model and simulated using MATLAB/SIMULINK, which allowed analyzing the behavior of the suspension system. After the evaluation of the behavior of these vehicle suspension system introduced for controllable suspension system. The paper entitled "enhancing vehicle dynamic control performance based on the improved extension control(characteristic variable extraction (CVE), correlation degree calculation (CDC), measure mode division (MMD), control reasoning mechanism (CRM), and control state(CS)"), and eventually the extension control system extrapolated into the "Takagi–Sugeno– Kang (TSK)" fuzzy control to smooth over all control action for vehicle dynamics. The result showed that extension control with an integration TSK gave better improvement in ride comfortable[31]. P. SENTHILKUMAR, K. SIVAKUMAR, R. KANAGARAJAN, S. KUBERAN, in their paper titled "Fuzzy Control of Active Suspension System using Full Car Model", have designed and analyzed the Fuzzy controller for full car model and compared the performance of the controller with the PID control. The result of their studied showed that the active suspension system with Fuzzy controller was able to provide better performance than PID control. The proposed strategy utilized mathematical model of the suspension system to design the controller[32]. Kaldas, M.M.S., Soliman, A.M.A., Abdallah, S.A., and Amien, F.F., proposed entitled "Model Reference Control for Active Suspension System,",this paper discussed about the Model Reference Control for Active Suspension System.In this paper the full vehicle suspension developed and MRC and PID performances was tested ,the finding showed that MRC was better than the PID control[33]. The Fuzzy logic proposed by Lotfi, on this seminar paper the main focused based on the values between true(1) and false(0). The activity of fuzzy to change according to type and behavioral of linguistic variable in which the values of variable crossed using membership function. Mamdani proposed the fuzzy logic in control system for real time . The control rules in fuzzy logic wrote depend on the knowledge of system. Fuzzy logic has capacity to derive controller with no mathematical representation equation of a system. The difficulty of linear nature of actuator and its dynamics has been controlled successfully using fuzzy logic controller. In order to design the fuzzy logic sys-

Table 2.3: Comparison of vehicle model in different control types are applied on different vehicle model[36]

Control types	research conducted authors	Type of Vehicle model to test control algorithm	Comparison with	Results/Finding Showed that
LQR	Aref Soliman (2011)	nonlinear system of the vehicle	adaptive LQR control system	adaptive LQR gave better ride performance
LQG	D. Hrovat(2018)	One DOF and two DOF	two DOF and One DOF	two DOF was better than One DOF
Fuzzy	2003, Abdelhady	two DOF	LQR	improved both the ride comfort and road holding parameters
Impedance control	Fateh and Alavi	passive suspension system	IR	IR showed important advantages

tem the expert's knowledge and experience needed. Quarter vehicle model based on active suspension stability of the system didn't significantly improved[34]. The paper entitled "Improvement of Vehicle Ride Performance Using a Hydro-pneumatic Active Suspension System" this paper considering full vehicle model.Based on the model the linear quadratic regulator (LQR) performance tested.The result showed that the peak values of the parameters(position,velocity,and acceleration) were reduced.But this paper didn't consider the effect of unsprung mass system[35]. A paper titled "Analysis of Active Suspension System: A Review" by Amit, Nausad Khan[36], and ABarkat Ali has presented the reviewed on different control algorithm(Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian (LQG) control, Adaptive sliding control, H∞ control, sliding mode control, fuzzy logic, preview control, optimal control and neural network) I terms of ride comfort.According to this paper thought neural network control algorithm became the most effective as compared to the rest of control algorithm as mention before. The paper is extended in to different control algorithm, and eventually the neural network control appreciated control algorithm for vehicle models.

From the above papers, we can infer that the presented LQR controller considers the full car model into independent four quarter car models. The LQR controller showed a good performance for each quarter car model independently. However, the coupling effect of each quarter model was not taken into account. Moreover, most of the papers were focusing on controller design using fourteen states in case of full car model which leads to unbalanced ranks of the controllable and observable matrices. Practically, in such conditions the state feedback controller design is not possible directly, hence minimal realization technique is required to design the LQR controller.

Table 2.4: Comparison of vehicle model in different control types are applied on different vehicle model[36]

neural network	Eski and Yildirim	full vehicle model.	PID	neural network was better than vehicle performance
stochastic parameters optimization	Demic et al	active suspension systems using spatial vehicle model	No	minimization of sprung mass vibration and standard deviation of forces in tire-to-ground contact area and vehicle handling
H∞control	Nguyen et al	Two DOF	No	reduce considerably the gains from road disturbance to car body
fuzzy sliding mode controller (FSMC)	Yagiz et al.	Active suspension system	traditional fuzzy controller (TFC)	fuzzy sliding mode controller (FSMC) appreciated

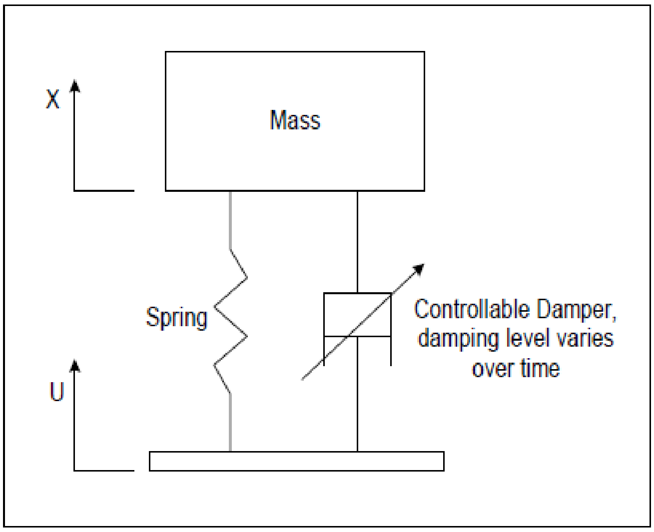


Fig. 2.3: Semi-Active Suspension Component[14]

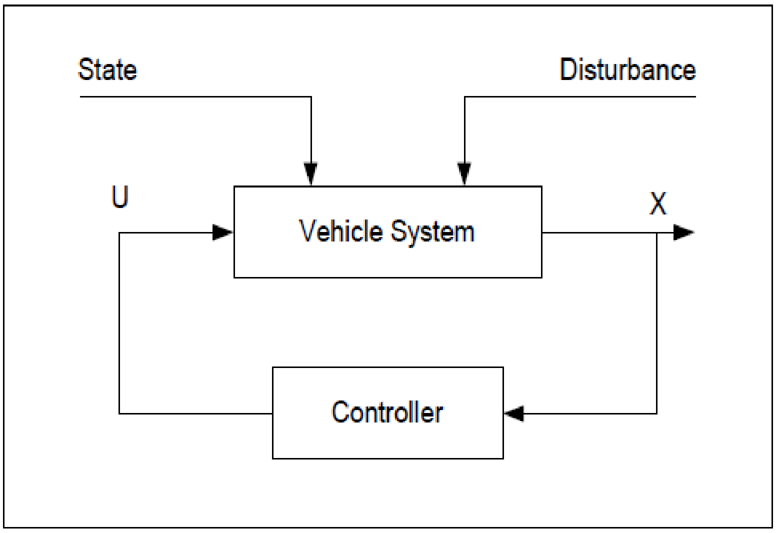


Fig. 2.4: Active Suspension Control System[14]

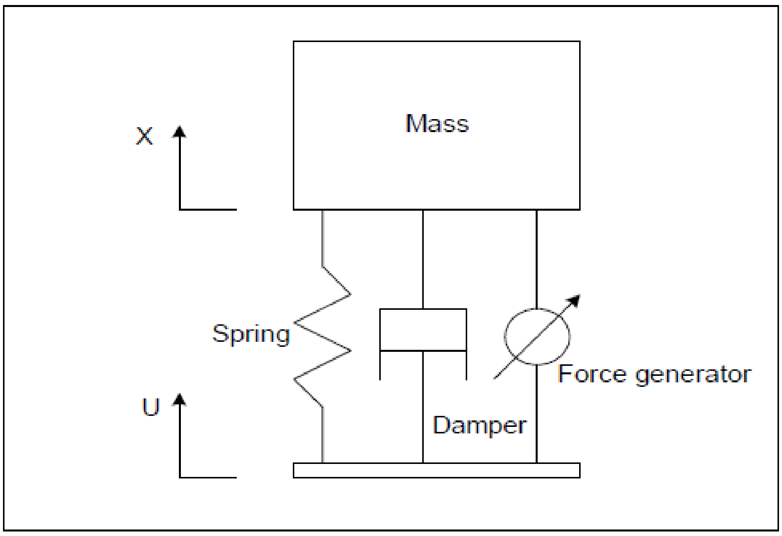


Fig. 2.5: Active Suspension Component[14]

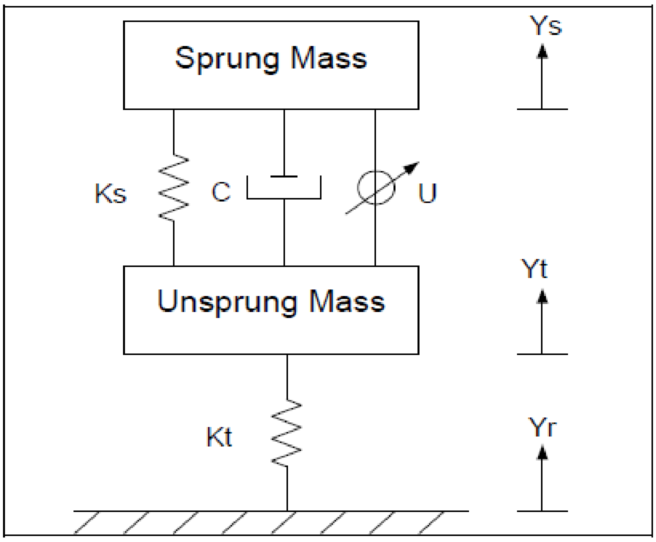


Fig. 2.6: Quarter Car Model[14].

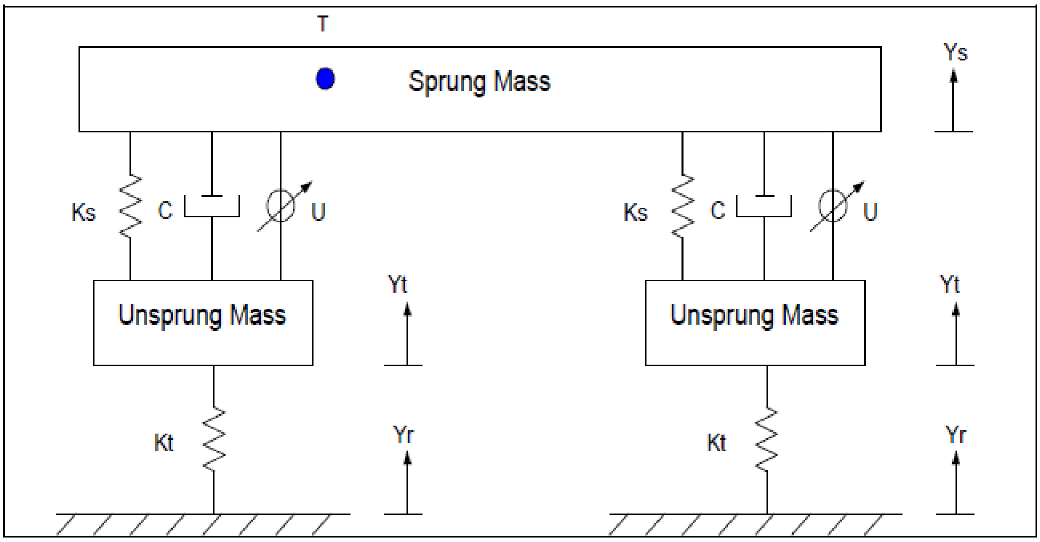


Fig. 2.7: Half Car Model [14]

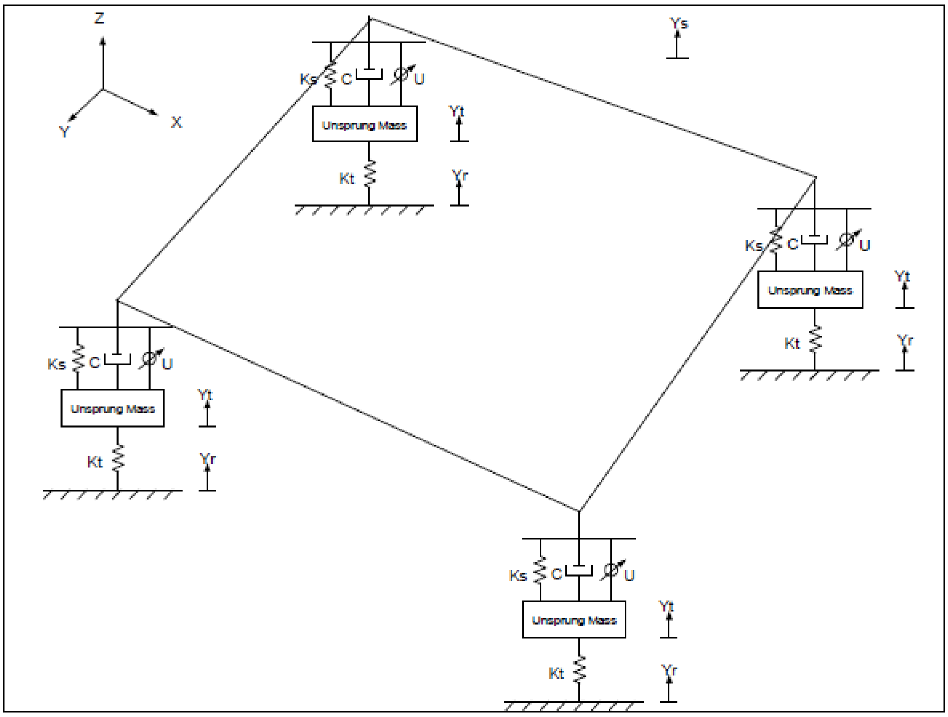


Fig. 2.8: Full Car Model [14]

Table 2.5: Comparison of vehicle model in different control types are applied on different vehicle model[36]

Control types	research conducted authors	Type of Vehicle model to test control algorithm	Comparison with	Results/Finding Showed that
EFSMC	Lin et al	active suspension system	Both FSMC, TFC	The EFSMC exhibits better control performance than either the TFC or the FSMC
genetic algorithm (GA),tunne PID	2007, Hany et al	active suspension system	No	obtain the better coefficients of a virtual damper and a skyhook damper for its effective searching ability
T–S (Takagi–Sugeno) fuzzy	H. Du and N. Zhang	nonlinear system of the vehicle	parallel-distributed compensation (PDC)	T–S fuzzy becomes powerful engineering tools for the modeling and control of complex dynamic systems

Chapter 3

Mathematical models of full vehicle system and its control design

In this chapter, the mathematical model of the full vehicle systems using full car model derived based on the Fig3.4. Besides, the actuator mechanism modeled based on the Fig 2.1 and the Road profile presented. More over vehicle system identified by mathematical model of each component. Finally, the overall model of the vehicle System obtained by combining all the models of the sub-systems. Various types of car models such as quarter car model, half car model and full car model have been used to simulate the performance of vehicle active suspension systems. In the different research studied the quarter car model was frequently used because of its simplicity, however, the half car model showed more appropriate vertical motion, including either the pitch or the roll effects. The full car model is the best accurate one, but it requires more computation than the others as a result very few studied have been carried out based on it[36]. Lot of common vehicles today used active suspension system to control the dynamics of a vehicle's vertical motion as well as spinning (pitch) and tilting (roll)[37].
The Fig3.4 is selected as the vehicle model based on the following assumption to reduce the complexity of the system[38].

- The vehicle was like plain(its must be like a table)
- The vehicle was at stationary
- Damping properties of the vehicle tires were not considered
- The physical shapes of the vehicle considered as rectangular shape
- Non linearity of tires properties not considered
- The vehicle non linearity not considered here
- External influences factories (aerodynamics resistance)not considered

3.1 Mathematical modeling of passive suspension system for a full Car model

The slight shaking movement (vibration), the easiest way of model be entitled a system is a linear system model, which to need something of noticeable(considerable) analytical and computational task(effort) on systems with seven degrees of freedom. In such case, the advantage of software instruction, such as MATLAB is necessary to obtaining numerical outcomes in order to verify and feasible system's physical behavior.The natural frequencies and mode shapes of a seven degree of freedom for full car suspension system could be, a pairs of difficult to understand conjugates for which hand finding solution and extractions is a difficult task. Such studies can be easily done in MATLAB environment. The main objective of this part is, to help us verify and understanding the basics of vibrations through an effective combing of software instruction, MATLAB and SIMULINK,

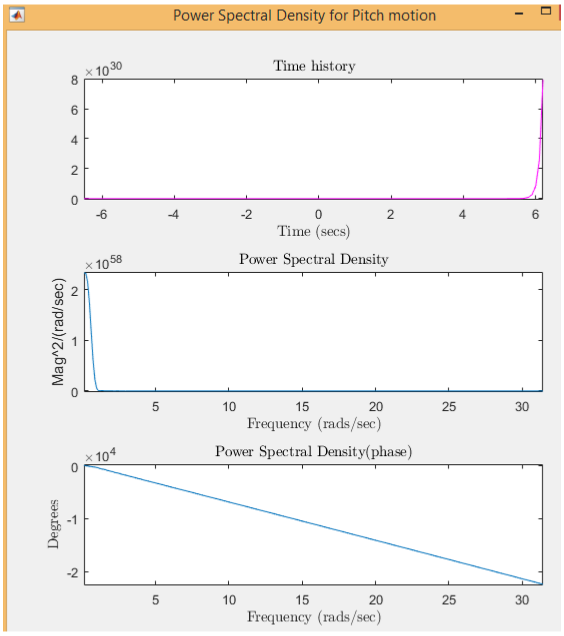


Fig. 3.1: power spectral density for vehicle pitch infinite resonance frequency

with theory to develop well mathematical models of full car suspension system. This further simplified the way of integration between mathematical analysis and engineering system design.A car on a rough when considering its natural features(terrain), such as the one shown in the Figure 3.4, shows bounce, pitch, and roll on top of its rigid body motion.Thus have different natural frequency as depicted in the figure 3.1,3.2 and 3.3.The natural frequencies and damped frequencies are equal and the damping ratio is zero due this reason the resonance frequency becomes infinity[39]-[42].The natural frequency of pitch,roll ,bounce,front wheels,and rear wheels motions are as follow.
$\omega_1 = \{$49.7704i,19.2477i ,17.4477i,8.7195,18.0831,21.3583,and 24.5957 $\}$
$\omega_2 = \{$42.9085i,17.4786i,12.8505i,6.4665,11.7310,15.8397,and 16.4932$\}$
To know the damping ratio and natural frequency of the system we will procedure as follows[39]-[42].

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = complexroot \tag{3.1}$$

$$-\omega_n \zeta = realroot \tag{3.2}$$

Therefore the damping ratio for pitch, roll and bouncing motion is zero($\zeta = 0$) and for the four wheels the damping ratio is unity($\zeta = 1$).The damping frequency in the case of four wheels are zero and the natural frequency is real roots and due to this reason the magnitude of the resonance frequency of all four wheels are the same as bouncing motion.Because the magnitude of resonance frequency in the case of bounce motion is relatively finite and real as depicted in the figure 3.3.And they have similar mode shapes.The mode shapes indicate that there is no phasing in the modes as expected in the proportional damping case[39]-[42].The following are the mode shape of the vehicle suspension system depicted in the figure 3.1-3.3. $U_{modeshape} = \{$0.1260,0.0297 ,0.1260,0.0204,0.1291,0.050,and 0.1291$\}$.From the mode shape we understand that the speeds of the wheels and bounce motions are similar because their modes are unity.

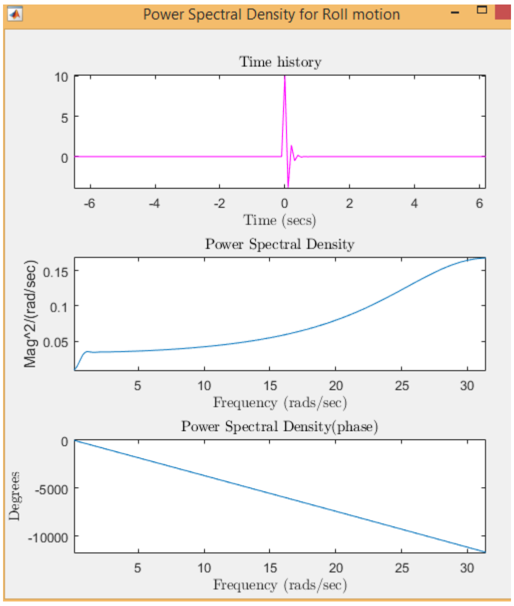


Fig. 3.2: power spectral density for vehicle roll infinite resonance frequency

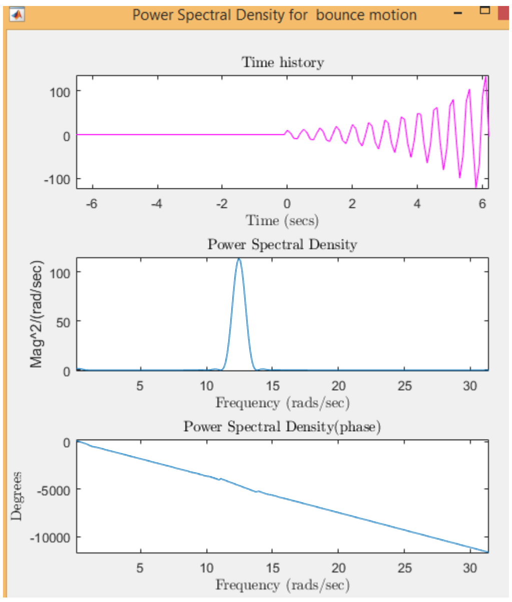


Fig. 3.3: power spectral density for vehicle bounce finite resonance frequency

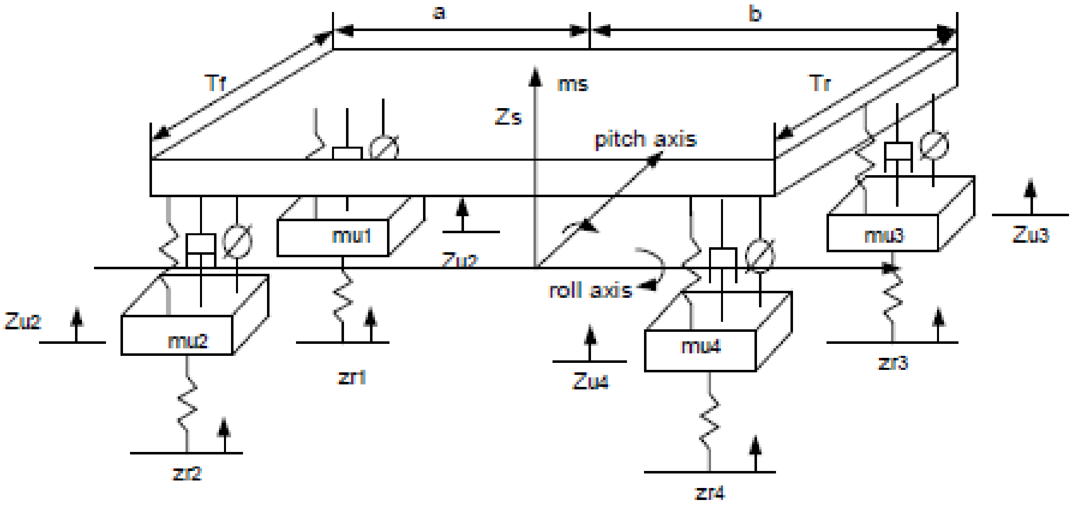


Fig. 3.4: Full Car Model[14]

3.1.1 For rolling motion of the sprung mass

Based on the figure 3.1 to 3.3 and 3.4 we going to create the relation between the C.g and the roll center of the vehicle and how the distance between them creates more torque or force around the roll center.And this is an important concept to grasp because this will help us to understand the forces that are affecting the vehicle as its in a corner,which will help to set up our suspension system.The force and the torque and how it acts on our roll center is if we just take a look at a combination wrench and how we use it to loosen a bolt that will give us a good idea.Because what we redoing is applying force to the end of the wrench and it creates twist or torque , around the pivot point,which is the center of the bolt.Now if we are having a hard time loosening that bolt ,what we do is get a longer wrench, or handle on the wrench,or we end up putting a piece of pipe over the end of the wrench to create a longer handle what this does is the length of the moment arm ,and the moment arm is the distance between the pivot point and where it intersects the line of forces at right angle the perpendicular distance between the pivot point and the line of forces.Now lets tie that in to how the force affect the vehicle and instead of the pivot point being on the bolt ,then our pivot point on the vehicle would be the roll center.And instead of the end of the wrench we have our center of gravity.So the forces acting on the car are going through the center of gravity instead of like our hand on the wrench and then our pivot point,once again,is our roll center for the vehicle.So ,as you can see we want that center of gravity height,the distance between the center of gravity height and our roll center to be as short as possible to limit the forces that are coming in the corner and affecting our body roll by creating that torque on the roll center of our vehicle[14],[39]-[42].The design goals are:

- minimize vehicle body roll angle and rate
- minimize camber/ geometry changes with suspension moment
- minimize roll center moment for more predictable driving feel
- balance front/rear roll center heights for desired slip angle characteristics

Now lets stating the mathematical derivations of rolling motion based on the above notation.

$$I_r\ddot{\Phi}_s = -b_f T_f(\dot{Z}_{s1}-\dot{Z}_{u1}) + b_f T_f(\dot{Z}_{s2}-\dot{Z}_{u2}) - b_r T_r(\dot{Z}_{s3}-\dot{Z}_{u3}) + b_r T_r(\dot{Z}_{s4}-\dot{Z}_{u4}) - k_f T_f(Z_{s1}-Z_{u1}) + k_f T_f(Z_{s2}-Z_{u2}) - k_r T_r(Z_{s3}-Z_{u3}) + k_r T_r(Z_{s4}-Z_{u4}) \quad (3.3)$$

$$\begin{aligned} I_r\ddot{\Phi}_s = & -b_f T_f(\dot{Z}_{s1}-\dot{Z}_{u1}) + b_f T_f(\dot{Z}_{s2}-\dot{Z}_{u2}) \\ & - b_r T_r(\dot{Z}_{s3}-\dot{Z}_{u3}) + b_r T_r(\dot{Z}_{s4}-\dot{Z}_{u4}) \\ & - k_f T_f(Z_{s1}-Z_{u1}) + k_f T_f(Z_{s2}-Z_{u2}) \\ & - k_r T_r(Z_{s3}-Z_{u3}) + k_r T_r(Z_{s4}-Z_{u4}) \end{aligned}$$

3.1.2 For pitching motion of the sprung mass

Car's nose goes up or down a certain angle when it facing on the irregular road profiles.When vehicle is passing a speed bump,pitch angle depends on the speed of the vehicle[14],[39]-[42].The design goal of pitch angle is

- minimize vehicle body pitch
- minimize up or down geometry changes with suspension moment
- minimize pitching angle
- balance front/rear pitch angle

Based on the above goals we going derive the mathematical models.

$$I_p\ddot{\theta}_s = -b_f a(\dot{Z}_{s1}-\dot{Z}_{u1}) - b_f a(\dot{Z}_{s2}-\dot{Z}_{u2}) + b_r b(\dot{Z}_{s3}-\dot{Z}_{u3}) + b_r b(\dot{Z}_{s4}-\dot{Z}_{u4}) - k_f a(Z_{s1}-Z_{u1}) - k_f a(Z_{s2}-Z_{u2}) + k_r b(Z_{s3}-Z_{u3}) + k_r b(Z_{s4}-Z_{u4}) \quad (3.4)$$

$$\begin{aligned} I_p\ddot{\theta}_s = & -b_f a(\dot{Z}_{s1}-\dot{Z}_{u1}) - b_f a(\dot{Z}_{s2}-\dot{Z}_{u2}) \\ & + b_r b(\dot{Z}_{s3}-\dot{Z}_{u3}) + b_r b(\dot{Z}_{s4}-\dot{Z}_{u4}) \\ & - k_f a(Z_{s1}-Z_{u1}) - k_f a(Z_{s2}-Z_{u2}) \\ & + k_r b(Z_{s3}-Z_{u3}) + k_r b(Z_{s4}-Z_{u4}) \end{aligned}$$

3.1.3 For bouncing of the sprung mass

The total mass that the vehicle's suspension supports.It moves up and down like a ball bouncing which is not good comfort for the passenger.So to control such bouncing motion we going to develop the mathematical models of vehicle bouncing[14],[39]-[42].

$$m_s\ddot{Z}_s = -b_f(\dot{Z}_{s1} - \dot{Z}_{u1}) - b_f(\dot{Z}_{s2} - \dot{Z}_{u2}) \\ - b_r(\dot{Z}_{s3} - \dot{Z}_{u3}) - b_r(\dot{Z}_{s4} - \dot{Z}_{u4}) \\ - k_f(Z_{s1} - Z_{u1}) - k_f(Z_{s2} - Z_{u2}) \\ - k_r(Z_{s3} - Z_{u3}) - k_r(Z_{s4} - Z_{u4}) \quad (3.5)$$

$$\begin{aligned} m_s\ddot{Z}_s =& - b_f(\dot{Z}_{s1} - \dot{Z}_{u1}) - b_f(\dot{Z}_{s2} - \dot{Z}_{u2}) \\ & - b_r(\dot{Z}_{s3} - \dot{Z}_{u3}) - b_r(\dot{Z}_{s4} - \dot{Z}_{u4}) \\ & - k_f(Z_{s1} - Z_{u1}) - k_f(Z_{s2} - Z_{u2}) \\ & - k_r(Z_{s3} - Z_{u3}) - k_r(Z_{s4} - Z_{u4}). \end{aligned}$$

3.1.4 For each side of wheel motion (vertical direction)

The mass of a vehicle's suspensions and other connects components,not supported by the damping of the suspensions that is hub,tire wheel.Thus all unsprung mass have independently move up or down which make not comfort for the passenger as result we going to develop the mathematical models for the each side of wheel motion as vertical direction[14],[39]-[42].

$$\begin{aligned} m_{uf}\ddot{Z}_{u1} =& b_f(\dot{Z}_{s1} - \dot{Z}_{u1}) + k_f(Z_{s1-Z_{u1}})) - k_{tf}Z_{u1} + k_{tfZ_{r1}} \\ m_{uf}\ddot{Z}_{u2} =& b_f(\dot{Z}_{s2} - \dot{Z}_{u2}) + k_f(Z_{s2-Z_{u2}}) - k_{tf}Z_{u2} + k_{tfZ_{r2}} \\ m_{uf}\ddot{Z}_{u3} =& b_r(\dot{Z}_{s3} - \dot{Z}_{u3}) + k_f(Z_{s3-Z_{u3}}) - k_{tf}Z_{u3} + k_{tfZ_{r3}} \\ m_{uf}\ddot{Z}_{u4} =& b_r(\dot{Z}_{s4} - \dot{Z}_{u4}) + k_f(Z_{s4-Z_{u4}}) - k_{tf}Z_{u4} + k_{tfZ_{r4}} \end{aligned}$$

where [14],[39]-[42]

$$\begin{aligned} Z_{s1} =& T_f\Phi_s + a\theta_s + z_s \\ \dot{Z_{s1}} =& T_f\dot{\Phi}_s + a\dot{\theta}_s + \dot{z}_s \\ Z_{s3} =& - T_f\Phi_s + a\theta_s + z_s \\ \dot{Z_{s1}} =& T_f\dot{\Phi}_s + a\dot{\theta}_s + \dot{z}_s \\ \dot{Z_{s3}} =& T_f\dot{\Phi}_s - b\dot{\theta}_s + \dot{z}_s \\ Z_{s4} =& - T_f\Phi_s - b\theta_s + z_s \\ \dot{Z_{s4}} =& - T_f\dot{\Phi}_s - b\dot{\theta}_s + \dot{z}_s \end{aligned}$$

where
ms= mass of the car body or sprung mass (kg)
m_{uf} =front and rear mass of the wheel or unsprung mass (kg)
$I_r\&I_P$pitch and roll of moment of inertia $(kgm)^2$
Z_s =car body displacement (m)
$Z_{s1}, Z_{s2}, Z_{s3}, andZ_{s4}$ = are car body displacement for each corner (m)
$Z_{u1}, Z_{u2}, Z_{u3}, andZ_{u4}$ = are wheel displacement
$T_f andT_r$ = front and rear treat (m)

$a =$ distance from center of sprung mass to front wheel (m)
$b =$ distance from center of sprung mass to rear wheel (m)
$b_r and b_f =$ front and rear damping (Nm/s)
$k_r and k_f =$ stiffness of car body spring for front and rear(N/m)
$k_{tr} and k_{tf} =$ tire stiffness (N/m)
$u_1 and u_2 =$ front right and left force actuators
$u_3 and u_4 =$ rear right and left force actuators

$$
\begin{aligned}
\dot{x}1 &= \dot{\Phi}_s \approx x_8 \\
\dot{x}2 &= \dot{\theta}_s \approx x_9 \\
\dot{x}3 &= \dot{Z}_s \approx x_{10} \\
\dot{x}4 &= \dot{Z}_{u1} \approx x_{11} \\
\dot{x}5 &= \dot{Z}_{u2} \approx x_{12} \\
\dot{x}6 &= \dot{Z}_{u3} \approx x_{13} \\
\dot{x}7 &= \dot{Z}_{u4} \approx x_{14}
\end{aligned}
$$

Back substitution into the equation[3.3-3.5] described under mathematical models then we got the following state equation[14],[39]-[42].

$$
\begin{aligned}
\dot{x}_8 = \ddot{\Phi}_s \approx [&-b_f T_f((T_f x_8 + a x_9 \\
&+ x_{10}) - x_{11}) + b_f T_f((-T_f x_8 + a x_9 + x_{10}) \\
&- x_{12}) - b_r T_r((T_r x_8 - b x_9 \\
&+ x_{10}) - x_{13} + b_r T_r((-T_r x_8 - b x_9 + x_{10}) - x_{14} \\
&- k_f T_f((T_f x_1 + a x_2 + x_3) - x_4) - k_f T_f((-T_f x_1 \\
&+ a x_2 + x_3) - x_5) + k_r b((T_f x_1 + a x_2 + x_3) - x_6) \\
&+ k_r b((-T_f x_1 + a x_2 + x_3) - x_7) + T_f u_1 - T_f u_2 + T_r u_3 - T_r u_4]/I_r
\end{aligned} \tag{3.6}
$$

$$
\begin{aligned}
\dot{x}_8 = \ddot{\Phi}_s \approx [&-b_f T_f((T_f x_8 + a x_9 \\
&+ x_{10}) - x_{11}) + b_f T_f((-T_f x_8 + a x_9 + x_{10}) \\
&- x_{12}) - b_r T_r((T_r x_8 - b x_9 \\
&+ x_{10}) - x_{13} + b_r T_r((-T_r x_8 - b x_9 + x_{10}) - x_{14} \\
&- k_f T_f((T_f x_1 + a x_2 + x_3) - x_4) - k_f T_f((-T_f x_1 \\
&+ a x_2 + x_3) - x_5) + k_r b((T_f x_1 + a x_2 + x_3) - x_6) \\
&+ k_r b((-T_f x_1 + a x_2 + x_3) - x_7) + T_f u_1 - T_f u_2 + T_r u_3 - T_r u_4]/I_r
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_9 = \ddot{\theta}_s \approx [&-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{11}) + b_f T_f((-T_f x_8 \\
&+ a x_9 + x_{10}) - x_{11}) + b_f T_f((-T_f x_8 + a x_9 + x_{10}) - x_{12}) \\
&- b_r T_r((T_r x_8 - b x_9 + x_{10}) - x_{13} + b_r T_r((-T_r x_8 - b x_9 \\
&+ x_{10}) - x_{13} + b_r T_r((-T_r x_8 - b x_9 + x_{10}) \\
&- x_{14} - k_f T_f((T_f x_1 + a x_2 + x_3) - x_4) - k_f T_f((-T_f x_1 \\
&+ a x_2 + x_3) - x_5) + k_r b((T_f x_1 + a x_2 + x_3) - x_6) + k_r b((-T_f x_1 \\
&+ a x_2 + x_3) - x_7) + a u_1 + a u_2 - b u_3 - b u_4]/I_p
\end{aligned} \tag{3.7}
$$

$$
\begin{aligned}
\dot{x}_9 = &\ddot{\theta}_s \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{11}) + b_f T_f((-T_f x_8 \\
&+ a x_9 + x_{10}) - x_{11}) + b_f T_f((-T_f x_8 + a x_9 + x_{10}) - x_{12}) \\
&- b_r T_r((T_r x_8 - b x_9 + x_{10}) - x_{13} + b_r T_r((-T_r x_8 - b x_9 \\
&+ x_{10}) - x_{13} + b_r T_r((-T_r x_8 - b x_9 + x_{10}) \\
&- x_{14} - k_f T_f((T_f x_1 + a x_2 + x_3) - x_4) - k_f T_f((-T_f x_1 \\
&+ a x_2 + x_3) - x_5) + k_r b((T_f x_1 + a x_2 + x_3) - x_6) + k_r b((-T_f x_1 \\
&+ a x_2 + x_3) - x_7) + a u_1 + a u_2 - b u_3 - b u_4]/I_p
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_{10} = \ddot{Z}_s \approx [&-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{11}) \\
&+ b_f T_f((-T_f x_8 + a x_9 + x_{10}) - x_{12}) \\
&- b_r T_r((T_r x_8 - b x_9 + x_{10}) - x_{13} \\
&+ b_r T_r((-T_r x_8 - b x_9 + x_{10}) - x_{14} - k_f T_f((T_f x_1 + a x_2 \\
&+ x_3) - x_4) - k_f T_f((-T_f x_1 + a x_2 + x_3) - x_5) \\
&+ k_r b((T_f x_1 + a x_2 + x_3) - x_6) \\
&+ k_r b((-T_f x_1 + a x_2 + x_3) - x_7) + u_1 \\
&+ u_2 + u_3 + u_4]/m_s
\end{aligned} \tag{3.8}
$$

$$
\begin{aligned}
\dot{x}_{10} = &\ddot{Z}_s \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{11}) \\
&+ b_f T_f((-T_f x_8 + a x_9 + x_{10}) - x_{12}) \\
&- b_r T_r((T_r x_8 - b x_9 + x_{10}) - x_{13} \\
&+ b_r T_r((-T_r x_8 - b x_9 + x_{10}) - x_{14} - k_f T_f((T_f x_1 + a x_2 \\
&+ x_3) - x_4) - k_f T_f((-T_f x_1 + a x_2 + x_3) - x_5) \\
&+ k_r b((T_f x_1 + a x_2 + x_3) - x_6) \\
&+ k_r b((-T_f x_1 + a x_2 + x_3) - x_7) + u_1 \\
&+ u_2 + u_3 + u_4]/m_s
\end{aligned}
$$

$$
\begin{aligned}
\dot{x}_{11} = \ddot{Z}_{u1} \approx [&-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{11}) \\
&+ k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) \\
&- x_4 - k_{tf} x_4 - u_1 + k_{tf} \dot{Z}_{r1}]/m_{uf}
\end{aligned} \tag{3.9}
$$

$$\dot{x}_{11} = \ddot{Z}_{u1} \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{11}) + k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) - x_4 - k_{tf} x_4 - u_1 + k_{tf} \dot{Z}_{r1}]/m_{uf}$$

$$\dot{x}_{12} = \ddot{Z}_{u2} \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{12}) + k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) - x_5 - k_{tf} x_5 - u_1 + k_{tf} \dot{Z}_{r2}]/m_{uf} \quad (3.10)$$

$$\dot{x}_{12} = \ddot{Z}_{u2} \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{12}) + k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) - x_5 - k_{tf} x_5 - u_1 + k_{tf} \dot{Z}_{r2}]/m_{uf}$$

$$\dot{x}_{13} = \ddot{Z}_{u3} \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{13}) + k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) - x_6 - k_{tf} x_6 - u_3 + k_{tf} \dot{Z}_{r3}]/m_{uf} \quad (3.11)$$

$$\dot{x}_{13} = \ddot{Z}_{u3} \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{13}) + k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) - x_6 - k_{tf} x_6 - u_3 + k_{tf} \dot{Z}_{r3}]/m_{uf}$$

$$\dot{x}_{14} = \ddot{Z}_{u4} \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{14}) + k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) - x_7 - k_{tf} x_7 - u_4 + k_{tf} \dot{Z}_{r4}]/m_{uf} \quad (3.12)$$

$$\dot{x}_{14} = \ddot{Z}_{u4} \approx [-b_f T_f((T_f x_8 + a x_9 + x_{10}) - x_{14}) + k_f T_f((T_f x_1 + a x_9 + a x_2) + x_3) - x_7 - k_{tf} x_7 - u_4 + k_{tf} \dot{Z}_{r4}]/m_{uf}$$

From the state equation[3.6-3.12] we got the following matrix

$$A11 = 0_{7X7}$$

$$A12 = eye(7X7)$$

$$A21 = \begin{bmatrix} \frac{2(-k_f T_f T_f - k_r T_r T_r)}{I_r} & 0 & 0 & \frac{-1}{I_r} & \frac{-1}{I_r} & \frac{-1}{I_r} & \frac{-1}{I_r} \\ 0 & \frac{2(-k_f aa - k_r bb)}{I_p} & \frac{2(-k_f a + k_r)}{I_p} & \frac{-1}{I_p} & \frac{-1}{I_p} & \frac{-1}{I_p} & \frac{-1}{I_p} \\ 0 & \frac{2(-k_f a + k_r)}{m_s} & \frac{2(-k_f - k_r)}{m_s} & \frac{-1}{m_s} & \frac{-1}{m_s} & \frac{-1}{m_s} & \frac{-1}{m_s} \\ \frac{k_f T_f}{m_{uf}} & \frac{k_f a}{m_{uf}} & \frac{k_f}{m_{uf}} & \frac{-1-k_f}{m_{uf}} & \frac{2(-k_f a + k_r)}{m_s} & 0 & 0 \\ \frac{-k_f T_f}{m_{uf}} & \frac{k_f a}{m_{uf}} & \frac{k_f}{m_{uf}} & 0 & \frac{-1}{m_{uf}} & 0 & 0 \\ \frac{k_r T_r}{m_{ur}} & \frac{-k_r b}{m_{ur}} & \frac{k_r}{m_{ur}} & 0 & 0 & \frac{-k_r b}{m_{ur}} & \frac{-1-k_r}{m_{ur}} \\ \frac{-k_f T_f}{m_{uf}} & \frac{-k_r b}{m_{ur}} & \frac{k_r}{m_{ur}} & 0 & 0 & 0 & \frac{-1-k_r}{m_{ur}} \end{bmatrix}$$

$$A22 = \begin{bmatrix} \frac{2(-b_f T_f T_f - b_r T_r T_r)}{I_r} & 0 & 0 & \frac{-1}{I_r} & \frac{-1}{I_r} & \frac{-1}{I_r} & \frac{-1}{I_r} \\ 0 & \frac{2(-b_f aa - b_r bb)}{I_p} & \frac{2(-b_f a + b_r b)}{I_p} & \frac{-1}{I_p} & \frac{-1}{I_p} & \frac{-1}{I_p} & \frac{-1}{I_p} \\ 0 & \frac{2(-b_f a + b_r b)}{m_s} & \frac{2(-b_f - b_r)}{m_s} & \frac{-1}{m_s} & \frac{-1}{m_s} & \frac{-1}{m_s} & \frac{-1}{m_s} \\ \frac{b_f T_f}{m_{uf}} & \frac{b_f a}{m_{uf}} & \frac{b_f}{m_{uf}} & \frac{-1}{m_{uf}} & 0 & 0 & 0 \\ \frac{-b_f T_f}{m_{uf}} & \frac{b_f a}{m_{uf}} & \frac{b_f}{m_{uf}} & 0 & \frac{-1}{m_{uf}} & 0 & 0 \\ \frac{b_r T_r}{m_{ur}} & \frac{-b_r b}{m_{ur}} & \frac{b_r}{m_{ur}} & 0 & 0 & \frac{-1}{m_{ur}} & 0 \\ \frac{-b_r T_r}{m_{ur}} & \frac{-b_r b}{m_{ur}} & \frac{b_r}{m_{ur}} & 0 & 0 & 0 & \frac{-1}{m_{ur}} \end{bmatrix}$$

Then the State Space Equation for a Suspension system matrix

$$\dot{x}(t) = \begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} \begin{bmatrix} x(t) \end{bmatrix} + fu(t)$$

$$y(t) = \begin{bmatrix} Cx(t) \end{bmatrix} + Du(t)$$

For the passive suspension system matrix

$$F1u(t) = 0_{7X4}$$

$$F2u(t) = \left(\begin{pmatrix} 2896.34 & 0 & 0 & 0 \\ 0 & 2896.34 & 0 & 0 \\ 0 & 0 & 3041.16 & 0 \\ 0 & 0 & 0 & 3041.16 \end{pmatrix} \right)$$

Then the passive suspension system matrix becomes[**?**]

$$Fu(t) = \begin{bmatrix} F1u(t) \\ F2u(t) \end{bmatrix}$$

where

$$\dot{x}(t) = \begin{bmatrix} \dot{x}1 & \dot{x}2 & ... & \dot{x}13 & \dot{x}14 \end{bmatrix}^T$$

$$\dot{x}(t) \approx \begin{bmatrix} \dot{\Phi}s & \dot{\theta}s & \dot{Z}s & \dot{Z}1 & ...\dot{Z}4 & \ddot{\Phi}1 & ... & \ddot{Z}4 \end{bmatrix}^T$$

$$x(t) = \begin{bmatrix} x1 & x2 & ... & x13 & x14 \end{bmatrix}^T, u(t) = \begin{bmatrix} u1 & u2 & u3 & u4 \end{bmatrix}^T$$

$$x(t) \approx \begin{bmatrix} \Phi s & \theta s & Zs & \dot{Z}1 & ...Z4 & \dot{\Phi}1 & ... & \dot{Z}4 \end{bmatrix}^T$$

$$\dot{x}(t) = \begin{bmatrix} 14 \times 1 \end{bmatrix}, x(t) = \begin{bmatrix} 14 \times 1 \end{bmatrix}, u(t) = \begin{bmatrix} 4 \times 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 14 \times 14 \end{bmatrix}, F = \begin{bmatrix} 14 \times 4 \end{bmatrix}, C = \begin{bmatrix} eye_{2 \times 14} \end{bmatrix}, D = \begin{bmatrix} 0_{2 \times 4} \end{bmatrix}$$

The substitute values of vehichle suspension system parameters are available at appendix5.2C and its matrix is at appendix5.2A

3.2 Mathematical modeling of active suspension system for a full Car Model including actuator

3.2.1 Actuator model

In this part the actuator is considering in modeling part.Even if the actuators are affecting the vehicle system its not part of the states.In this theses paper we couldn't consider the actuator as part of the states but can affect the inputs of the vehicle system.

Electro hydraulic anti lock brake system proposed in this theses paper provides the relative motion between the sprung mass and unsprung mass of the car is converted in to a linear motion of the hydraulic pressure, which acts as a control for the wheels The relation between relative motion of the sprung and unsprung masses of the car to the angular velocity of the hydraulic is given by equation[14].

$$Pressure = \frac{Forceexerted}{Area} \tag{3.13}$$

$F = (pressue)(Area) = (P)(A)$

$$Power = Force(F) * length(L) = (F)(L) \tag{3.14}$$

$Power = (Area) * (pressure) * (length)$
The relation between force from the sprung and unsprung masses and torque exerted by the wheel is $Power = (Torque) * (omega) = (T) * (\omega)$
$\omega = \frac{pressure*Area*Lenght}{Torque} = \frac{(P)(A)(L)}{T} = \frac{(Pressure)(Volume)}{T}$
The selonoid valve be on or off many times in microsecond,then the fluid through inside reservoir pipe is forced to move in one or more calipers acts upon one or more calipers piston.This effect doesn't affect the state of the vehicle but play a great role for the input. Therefore the fluid continuity equation becomes[**?**]

$$\frac{\partial}{\partial t}(S(x)\rho) + \frac{\partial}{\partial t}(S(x)\rho V) = F_1(x) \tag{3.15}$$

$$\frac{\partial}{\partial t}(S(x)\rho V) + \frac{\partial}{\partial t}(S(x)(p + \rho V^2)) + \pi(x)\tau + S(x\rho a_x) = F_2(x)p\frac{\partial S}{\partial x} \tag{3.16}$$

where[**?**] $\rho and V$ are density and velocity of fluid, $S(x)$ is cross section area of the pipeline, $F_1(x)$ is discharge of fluid mass to the unit of the length, in the pipeline,P is fluid pressure,π is perimeter of cross section of the pipeline, τ is tangential fluid stress in the inner surface of the pipeline a_x is acceleration along x-axis,$F_2(x)$ is kinetic energy of the fluid flow in the pipeline to the unit of area.The system of equations above can be written as the system second-order quasi-linear differential equations.

$$\begin{bmatrix} A \end{bmatrix} (\frac{\partial u}{\partial t}) + (\frac{\partial u}{\partial x}) = f$$

where [14]A and B are matrix and f is vector which depend on t,x and elements u_i of vector

$$\begin{Bmatrix} U \end{Bmatrix}^T = \begin{bmatrix} P & V \end{bmatrix}$$

The electro-hydraulic model provides the quality of volumetric losses of pressure in every cavity and used to interceded the vehicle during brake. The governing equation becomes[14].

$$\begin{bmatrix} M \end{bmatrix} \begin{Bmatrix} \ddot{q} \end{Bmatrix} + \begin{bmatrix} C \end{bmatrix} \begin{Bmatrix} \dot{q} \end{Bmatrix} + \begin{bmatrix} K \end{bmatrix} \begin{Bmatrix} q \end{Bmatrix} = \begin{Bmatrix} F(t,q,\dot{q}) \end{Bmatrix}$$

where[14] M,C,K are mass, damping and stiffness matrices, respectively;$F(t,q,\dot{q})$nonlinear load vector;are displacement, velocity and acceleration vectors, respectively.
$mb_1\ddot{q}b_1 = -P_1 s_1 - kb_3(qb_1 + qb_4) - Cb_3(\dot{q}b_1 + \dot{q}b_4) - F_{frl}sign(\dot{q}b_1 - \dot{q}b_2) - F_{fr4}sign(\dot{q}b_1 + \dot{q}b_4)$

$mb_2\ddot{q}b_2 = P_1 S_1 - F_{frl}sign(\dot{q}b_2 - \dot{q}b_1) - FN_1$

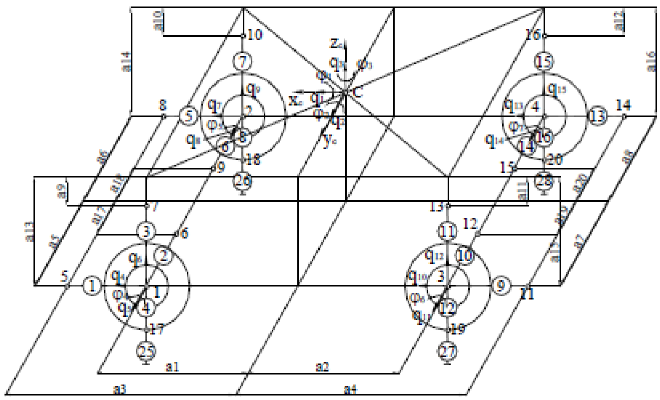


Fig. 3.5: Principal scheme of hydraulic brake system [14]

$$FN_1 = \begin{Bmatrix} 0 & qb_3 < \Delta_1 \\ kb_1(qb_2-(qb_3-\Delta_1))+ & \\ Cb_1(\dot{qb_2}-\dot{q}b_3), qb_3 > \Delta_1 & \end{Bmatrix}$$

$$mb_3\ddot{q}b_3 = -kb_3(qb_1+qb_4) - Cb_3(\dot{q}b_1+\dot{q}b_1) - F_{fr4}sign(\dot{q}b_1+\dot{q}b_4) - FN_2$$

$$FN_2 = \begin{Bmatrix} 0, qb_5 \leq \triangle_2 \\ kb_2(qb_4-(qb_5-\triangle_2)) + Cb_2(\dot{q}b_4-\dot{q}b_5), qb_5 >_2 \end{Bmatrix}$$

$$\dot{p} = \frac{k(p)}{V0+S(qb_2-qb_1)}(Q_{in} - S(\dot{q}b_2-\dot{q}b_1))$$
$$jb_1\ddot{q}b_6 = -Mbsign(\dot{q}b_6) - kb_4(qb_6-qb_7) - cb_4(\dot{q}b_6-\dot{q}b_7)$$
$$Mb = R_a(FN_1+FN_2)f$$

where mb_1 is the mass of a hydraulic cylinder;mb_2, and mb_3 are masses of pads;Jb_1 and Jb_2 are inertia mass moments of disk and tire;kb_1, kb_2 and cb_1, cb_2 are stiffness and damping coefficients of inner pads, respectively;kb_3, cb_3 are stiffness and damping coefficients of housing;$\triangle_1, \triangle_2$ are initial gaps between a pad and a disk;$Q(t)$ is discharge of fluid;F_{frl2} is friction force between a cylinder and a piston;F_{frl4} is friction force between a cylinder and a pad;FN_1, FN_2 are normal forces between pads and the disk S is a cross-section area of the piston;$V0$ is initial volume of the cylinder;M_b is brake torque;R_a is an average radius;f is friction coefficient between pad and disk;and $qb_1, qb_2, ...qb_7$ are generalized coordinates based on the figure3.5.

3.2.2 For rolling motion of the sprung mass with actuator dynamics

The relation between the C.g and the roll center of the vehicle and how the distance between them creates more torque or force around the roll center.And this is an important concept to grasp because this will help us to understand the forces that are affecting the

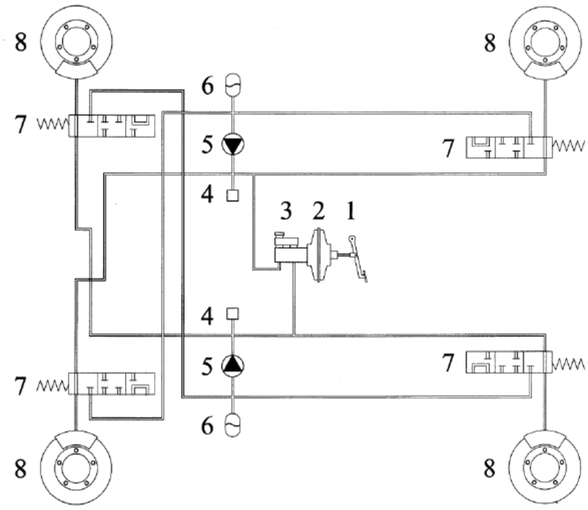


Fig. 3.6: The main scheme of Automobile model system with hydraulic actuator[14]

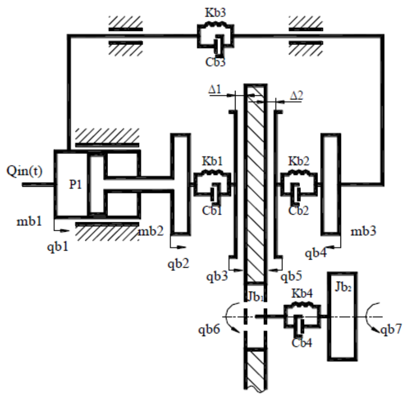


Fig. 3.7: The main scheme of the braking system with hydraulic actuator[14]

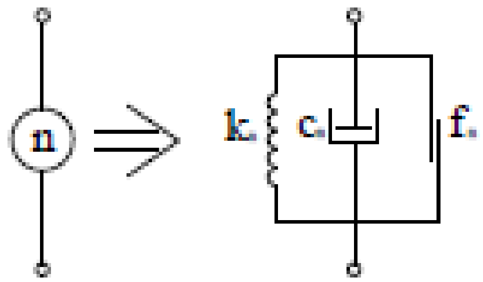


Fig. 3.8: The with a tire model in hydraulic actuator[14]

vehicle as its in a corner,which will help to set up our suspension system.The force and the torque and how it acts on our roll center is if we just take a look at a combination wrench and how we use it to loosen a bolt that will give us a good idea.Because what we redoing is applying force to the end of the wrench and it creates twist or torque , around the pivot point,which is the center of the bolt.Now if we are having a hard time loosening that bolt ,what we do is get a longer wrench, or handle on the wrench,or we end up putting a piece of pipe over the end of the wrench to create a longer handle what this does is the length of the moment arm ,and the moment arm is the distance between the pivot point and where it intersects the line of forces at right angle the perpendicular distance between the pivot point and the line of forces.Now lets tie that in to how the force affect the vehicle and instead of the pivot point being on the bolt ,then our pivot point on the vehicle would be the roll center.And instead of the end of the wrench we have our center of gravity.So the forces acting on the car are going through the center of gravity instead of like our hand on the wrench and then our pivot point,once again,is our roll center for the vehicle.So ,as you can see we want that center of gravity height,the distance between the center of gravity height and our roll center to be as short as possible to limit the forces that are coming in the corner and affecting our body roll by creating that torque on the roll center of our vehicle[39]-[42].The design goals are:

- minimize vehicle body roll angle and rate
- minimize camber/ geometry changes with suspension moment
- minimize roll center moment for more predictable driving feel
- balance front/rear roll center heights for desired slip angle characteristics

Now lets stating the mathematical derivations of rolling motion based on the above notation[39]-[42].

$$\begin{aligned} I_r\ddot{\Phi}_s = & -b_fT_f(\dot{Z}_{s1}-\dot{Z}_{u1})+b_fT_f(\dot{Z}_{s2}-\dot{Z}_{u2})-b_rT_r(\dot{Z}_{s3}-\dot{Z}_{u3})+b_rT_r(\dot{Z}_{s4}-\dot{Z}_{u4}) \\ & -k_fT_f(Z_{s1}-Z_{u1})+k_fT_f(Z_{s2}-Z_{u2})-k_rT_r(Z_{s3}-Z_{u3})+k_rT_r(Z_{s4}-Z_{u4}) \\ & +T_{fu1}-T_{fu2}+T_{ru3}-T_{ru4}+f1-f2+f3-f4 \end{aligned} \tag{3.17}$$

$$\begin{aligned} I_r\ddot{\Phi}_s = & -b_fT_f(\dot{Z}_{s1}-\dot{Z}_{u1})+b_fT_f(\dot{Z}_{s2}-\dot{Z}_{u2}) \\ & -b_rT_r(\dot{Z}_{s3}-\dot{Z}_{u3})+b_rT_r(\dot{Z}_{s4}-\dot{Z}_{u4})-k_fT_f(Z_{s1}-Z_{u1})+k_fT_f(Z_{s2} \\ & -Z_{u2})-k_rT_r(Z_{s3}-Z_{u3})+k_rT_r(Z_{s4}-Z_{u4}) \\ & +T_{fu1}-T_{fu2}+T_{ru3}-T_{ru4}+f1-f2+f3-f4 \end{aligned}$$

3.2.3 For pitching motion of the sprung mass with actuator dynamics

Car's nose goes up or down a certain angle when it facing on the irregular road profiles.When vehicle is passing a speed bump,pitch angle depends on the speed of the vehicle[39]-[42].The design goal of pitch angle is

- minimize vehicle body pitch
- minimize up or down geometry changes with suspension moment

- minimize pitching angle
- balance front/rear pitch angle

Based on the above goals we going derive the mathematical models[39]-[42].

$$I_p\ddot{\theta}_s = -b_f a(\dot{Z}_{s1}-\dot{Z}_{u1}) - b_f a(\dot{Z}_{s2}-\dot{Z}_{u2}) + b_r b(\dot{Z}_{s3}-\dot{Z}_{u3}) + b_r b(\dot{Z}_{s4}-\dot{Z}_{u4}) - k_f a(Z_{s1}-Z_{u1}) - k_f a(Z_{s2}-Z_{u2}) + k_r b(Z_{s3}-Z_{u3}) + k_r b(Z_{s4}-Z_{u4}) + a_{u1} + a_{u2} - b_{u3} - b_{u4} + f1 + f2 - f3 - f4 \quad (3.18)$$

$$\begin{aligned} I_p\ddot{\theta}_s = &- b_f a(\dot{Z}_{s1}-\dot{Z}_{u1}) - b_f a(\dot{Z}_{s2}-\dot{Z}_{u2}) \\ &+ b_r b(\dot{Z}_{s3}-\dot{Z}_{u3}) + b_r b(\dot{Z}_{s4}-\dot{Z}_{u4}) \\ &- k_f a(Z_{s1}-Z_{u1}) - k_f a(Z_{s2}-Z_{u2}) \\ &+ k_r b(Z_{s3}-Z_{u3}) + k_r b(Z_{s4}-Z_{u4}) + a_{u1} \\ &+ a_{u2} - b_{u3} - b_{u4} + f1 + f2 - f3 - f4. \end{aligned}$$

3.2.4 For bouncing of the sprung mass with actuator dynamics

The total mass that the vehicle's suspension supports.It moves up and down like a ball bouncing which is not good comfort for the passenger[39]-[42].So to control such bouncing motion we going to develop the mathematical models of vehicle bouncing.

$$m_s\ddot{Z}_s = -b_f(\dot{Z}_{s1}-\dot{Z}_{u1}) - b_f(\dot{Z}_{s2} - \dot{Z}_{u2}) - b_r(\dot{Z}_{s3}-\dot{Z}_{u3}) - b_r(\dot{Z}_{s4}-\dot{Z}_{u4}) - k_f(Z_{s1}-Z_{u1}) - k_f(Z_{s2}-Z_{u2}) - k_r(Z_{s3}-Z_{u3}) - k_r(Z_{s4}-Z_{u4}) + u_1 + u_2 + u_3 + u_4 + f1 + f2 + f3 + f4 \quad (3.19)$$

$$\begin{aligned} m_s\ddot{Z}_s = &- b_f(\dot{Z}_{s1}-\dot{Z}_{u1}) - b_f(\dot{Z}_{s2}-\dot{Z}_{u2}) \\ &- b_r(\dot{Z}_{s3}-\dot{Z}_{u3}) - b_r(\dot{Z}_{s4}-\dot{Z}_{u4}) \\ &- k_f(Z_{s1}-Z_{u1}) - k_f(Z_{s2}-Z_{u2}) \\ &- k_r(Z_{s3}-Z_{u3}) - k_r(Z_{s4}-Z_{u4}) \\ &+ u_1 + u_2 + u_3 + u_4 + f1 + f2 + f3 + f4 \end{aligned}$$

3.2.5 For each side of wheel motion (vertical direction with actuator dynamics)

The mass of a vehicle's suspensions and other connects components,not supported by the damping of the suspensions.Thus all unsprung mass have independently move up or down which make not comfort for the passenger as result we going to develop the mathematical models for the each side of wheel motion as vertical direction [39]-[42].

$$m_{uf}\ddot{Z}_{u1} = b_f(\dot{Z}_{s1}-\dot{Z}_{u1}) + k_f(Z_{s1-Z_{u1}})) - k_{tf}Z_{u1} + k_{tfZ_{r1}} - k_{tr}Z_{u1} - u_1 + k_{tr}Z_{r1} - f1 \quad (3.20)$$

$$m_{uf}\ddot{Z}_{u1} = b_f(\dot{Z}_{s1} - \dot{Z}_{u1}) + k_f(Z_{s1-Z_{u1}})) - k_{tf}Z_{u1} + k_{tfZ_{r1}} \\ - k_{tr}Z_{u1} - u_1 + k_{tr}Z_{r1} - f1$$

$$m_{uf}\ddot{Z}_{u2} = b_f(\dot{Z}_{s2} - \dot{Z}_{u2}) + k_f(Z_{s2-Z_{u2}}) - k_{tf}Z_{u2} + k_{tfZ_{r2}} \\ k_{tfZ_{r2}} - k_{tr}Z_{u2} - u_2 + k_{tr}Z_{r2} - f2 \quad (3.21)$$

$$m_{uf}\ddot{Z}_{u2} = b_f(\dot{Z}_{s2} - \dot{Z}_{u2}) + k_f(Z_{s2-Z_{u2}}) - k_{tf}Z_{u2} + k_{tfZ_{r2}} + k_{tfZ_{r2}} \\ - k_{tr}Z_{u2} - u_2 + k_{tr}Z_{r2} - f2$$

$$m_{uf}\ddot{Z}_{u3} = b_r(\dot{Z}_{s3} - \dot{Z}_{u3}) + k_f(Z_{s3-Z_{u3}}) - k_{tf}Z_{u3} + k_{tfZ_{r3}} \\ k_{tfZ_{r1}} - k_{tr}Z_{u3} - u_3 + k_{tr}Z_{r3} - f3 \quad (3.22)$$

$$m_{uf}\ddot{Z}_{u3} = b_r(\dot{Z}_{s3} - \dot{Z}_{u3}) + k_f(Z_{s3-Z_{u3}}) - k_{tf}Z_{u3} + k_{tfZ_{r3}} \\ k_{tfZ_{r1}} - k_{tr}Z_{u3} - u_3 + k_{tr}Z_{r3} - f3$$

$$m_{uf}\ddot{Z}_{u4} = b_r(\dot{Z}_{s4} - \dot{Z}_{u4}) + k_f(Z_{s4-Z_{u4}}) - k_{tf}Z_{u4} + k_{tfZ_{r4}} \\ k_{tfZ_{r1}} - k_{tr}Z_{u4} - u_4 + k_{tr}Z_{r4} - f4 \quad (3.23)$$

$$m_{uf}\ddot{Z}_{u4} = b_r(\dot{Z}_{s4} - \dot{Z}_{u4}) + k_f(Z_{s4-Z_{u4}}) - k_{tf}Z_{u4} + k_{tfZ_{r4}} \\ k_{tfZ_{r1}} - k_{tr}Z_{u4} - u_4 + k_{tr}Z_{r4} - f4$$

Where
f1,f2,f3,f4 are ,each actuator force or each wheels actuating force
developed from the actuator models. Now we putting all the state equation from equation [3.17-3.22] in the form of matrix ,we got the following matrix
Then the Active suspension system matrix becomes

$$Bu1(t) = 0_{7X4}$$

$$Bu2(t) = \begin{bmatrix} \frac{T_f}{I_r} & \frac{-T_f}{I_r} & \frac{T_f}{I_r} & \frac{-T_f}{I_r} \\ \frac{a}{I_p} + \frac{\triangle_1}{J_{b1}} & \frac{a}{I_p} + \frac{\triangle_1}{J_{b1}} & \frac{-b}{I_p} - \frac{\triangle_1}{J_{b2}} & \frac{-b}{I_p} \\ \frac{kb3+Cb3}{m_s} & \frac{kb3+Cb3}{m_s} & \frac{kb3+Cb3}{m_s} & \frac{kb3+Cb3}{m_s} \\ \frac{-1}{m_{uf}} + \frac{-kb1-Cb1}{m_{b1}} & 0 & 0 & 0 \\ 0 & \frac{-1}{m_{uf}} + \frac{-kb1-Cb1}{m_{b2}} & 0 & 0 \\ 0 & 0 & \frac{-1}{m_{uf}} + \frac{-kb2-Cb2}{m_{b3}} & 0 \\ 0 & 0 & 0 & \frac{-1}{m_{uf}} + \frac{-kb2-Cb2}{m_{b4}} \end{bmatrix} \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \end{bmatrix}$$

$$Bu(t) = \begin{bmatrix} Bu1(t) \\ \\ Bu2(t) \end{bmatrix}$$

The state matrix is the same for the vehicle suspension system without considering the coupling effects and the vehicle suspension system with coupling effects, whereas the input matrixes are different that is why the coupling effects is only affecting the input matrix, but isn't affecting the state. Therefore the input matrix with a cooling effect is

$$B_a = \begin{bmatrix} 14 \times 4 \end{bmatrix}$$

The substitute values of vehichle suspension system including actuator dynamic parameters are available at appendix5.2C and its matrix is at appendix5.2A

3.3 Road profile

In order to study the dynamic behavior of the vehicle and to analysis the performance of the rolling suspension system an external excitation input for the model is required. In this study, different types of sinusoidal function road profiles is used as excitation for simulation purpose[43],[44]. In this simulation the road disturbance is sinusoidal bumps[2]. The time delay for the rear wheels can be computed using equation3.24.where, c1 and c2 are the distances of the front and rear axles from the center of gravity of body mass and V is the real-time estimated forward velocity of vehicle[43],[44].

$$Timedelay(\tau_d) = \frac{c1 + c2}{v} \tag{3.24}$$

It is assumed that both front wheels (right and left) reach the road bump at the same time and after some delay both rear wheels (right and left) also reach the bump at the same time. Besides, the amplitude of the front right and left road disturbances are the same. The same assumption is also considered for the rear road input disturbances.

3.3.1 System analysis and control design

In this chapter, system analysis and design of linear quadratic regulator (LQR) controller for the full vehicle system that stabilizes the car body in the equilibrium required position will be developed. Before the controller design, the pre-requirement system analyses such as system controllability system stability ,and system observable are must be achieved.

Fig. 3.9: Road disturbance which can affect vehicle motion[2].

3.3.2 Vehicle Active Suspension System Analysis

The active suspension system model has been developed in this chapter . As derived and stated in this chapter, the model has fourteen (14) state variables. The state matrix (A) of the state space representation of the system is fourteen by fourteen (14x14). Therefore, it needs 14 sensors to measure each states which is not feasible practically. This is costly, so the system should be described with a minimal number of states and this is achieved using a minimal realization principle. Why becuase, it is difficult to manipulate the system. Therefore, to handle the system, the order of the system state space representation is reduced to tenth (10th) by using minimal realization technique(data deriven) without affecting the characteristics of the original model system.This is proved to be true by checking the response of the 14th order system and the 10th order system for the same input disturbance of single bump as shown in the Fig3.10.

3.3.3 System Minimal Realization

Due to the more state variables are used to describe the system (redundancy of state variables), too much symmetry the system and the system has physically uncontrollable components, It is difficult (not feasible) to implement the system physically in real time application. Therefore, these problems are reduced using minimal realization technique. A minimal realization principle is a means of describing the system with a small number of states. Therefore, the system is implemented with a minimal number of components.Minimal realization technique eliminates uncontrollable or unobservable state in state-space models, or cancels pole-zero pairs in transfer functions. It describes the system with the minimum number of states. Thus, the obtained minimal realization model has minimal order and the same response characteristics as the original model system. A minimal realized system is both controllable and observable.The computational of the Minimality is realization of (A, B, C, D) of a transfer function/matrix H(s) is said to be minimal if no other realization of H(s) has smaller dimension[45]-[51].In order to implement the thoery we should have consider the following points

- (A, B, C, D) is a minimal realization of H(s)
- (A, B) is controllable and (C, A) is observable
  Let (A, B, C, D) be a realization of H(s). The following statements are equivalent:

- (A, B, C, D) is minimal.
- The poles of H(s) are the eigenvalues of A.

A realization is minimal if and only if it is reachable and observable according to the above statement.As stated in the mathematical models, the state is not observable as well as not reachable,we can use the Kalman decomposition to extract its realization order part, and thereby obtain a realization of smaller order.For the converse, suppose (A,B,C,D) is a reachable, observable realization of order n, but is not minimal. Then there is a minimal realization $(A*,B,*C*,D*)$of order $n* < n$(and necessarily reachable and observable[45]-[51].

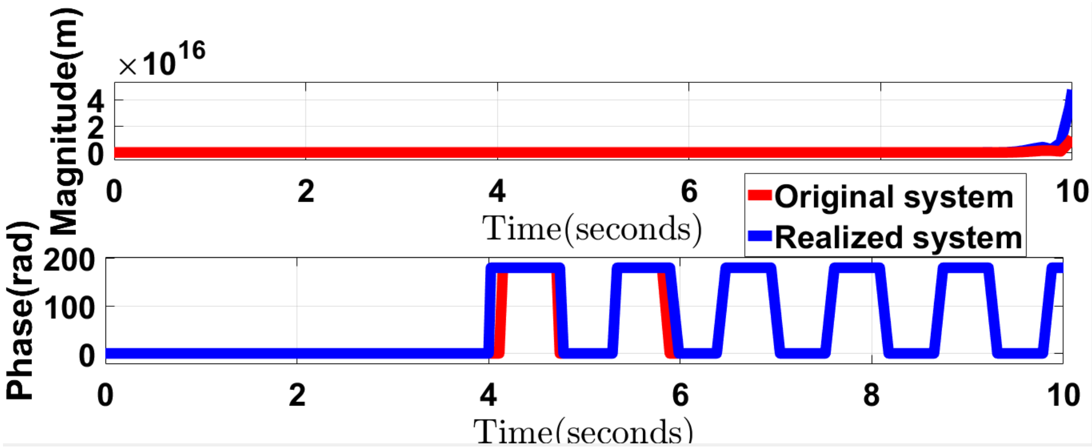


Fig. 3.10: Comparison the response of original and minimal realized systems,the matlab simulink is available at appendix5.1a

$$o_nR_n = \begin{bmatrix} C \\ CA \\ CA^2 \\ . \\ . \\ . \\ CA^{N-1} \end{bmatrix} \begin{bmatrix} B & AB & A^2BA^3B & . & . & . & A^{N-1}B \end{bmatrix} = \begin{bmatrix} H1 & H2 & . & . & . & Hn \\ H2 & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ Hn & . & . & . & . & H(2n-1) \end{bmatrix} = o_n^*R_n^*$$

Where N is the number of states that is 10.
The reachability and observability of (A,B,C,D) ensures that rank($O_nR_n = n$) as can be verified using Sylvester's inequality while rank ($O_n^*R_n^* = n^*$).then,
$D = D*$, $CA^KB = C^*A^{*k}B^*, k \geq 0$,so
$o_nR_n = o_n^*R_n^*$
Also,
$o_nAR_n = o_n^*A^*R_n^*$
Let us introduce the notation M+ to denote the ("Moore-Penrose") pseudo-inverse of a matrix M. If Mhas full column rank, then $M^+ = (M'M)^{-1}M'$ and in the general case the pseudo-inverse can be explicitly written in terms of the SVD of M.
$R_nR_n^{*+} = O_nO_n^{*+} = T$
$T^{-1} = R_n^{*+}R_n = O_n^{*+}O_n$
Invoking the reachability and observability of the minimal realization
$AT = TA^{-1}, B = TB^{*-1}, C^* = CT$
That is the realizations are similar.
Hand caculation is tidous ,So the computations conducted on matlab build system. As it can be observed in Fig 3.10 the simulation results of both the original model system

(14X14 matrix) and minimal realized system (10X10 matrix) are the same in both angle and magintde as shown Fig3.10 and 3.11 . This shows that minimal realization technique doesn't alter the behavior of the original model system.Based on a minimal realized technique some of the state variables are removed from some of the state variables are not affect the state. Thus, states variables are not affected status of the system. These state variables are

$$\begin{aligned} \dot{x}1 &= \dot{\phi}_s \dot{x}4 = \dot{z}_{u1} \dot{x}7 = \dot{z}_{u4} \\ \dot{x}2 &= \dot{\theta}_s \dot{x}5 = \dot{z}_{u2} \dot{x}8 = \ddot{x}_1 \, \dot{x}10 = \ddot{x}_s = \ddot{z}_{u1} = ...\ddot{z}_{u4} \\ \dot{x}3 &= \dot{z}_s \dot{x}6 = \dot{z}_{u3} \dot{x}9 = \ddot{x}_2 \end{aligned}$$

The removed state variables are relative acceleration of the sprung mass system of the unsprung mass system. The magnitude and direction of the relative motions are equal. Hence the sum of the relative motion becomes zero. So, the removed state variables are not affecting the state. However the input matrix becomes added, this is because the data driven technique leads to push variables in the optimizer direction in which the system become more observable or controllable. Therefore, the new state space model becomes as follow.

$$A = 10 \times 10 = \begin{bmatrix} A1 & A2 \end{bmatrix}, B = 10 \times 8 = \begin{bmatrix} B1 & B2 \end{bmatrix}, C = 2 \times 10 = \begin{bmatrix} C1 & C2 \end{bmatrix}, D = 0_{2\times 8}$$

$$\dot{x}(t) = \begin{bmatrix} 10 \times 1 \end{bmatrix}, x(t) = \begin{bmatrix} 10 \times 1 \end{bmatrix}, u(t) = \begin{bmatrix} 8 \times 1 \end{bmatrix}$$

using matlab the minimal realized state values are shown in appendix 5.2B

3.4 System Controllability and observable

In order to design an LQR controller the system must be fully controllable. This is verified by determining the rank of the controllability matrix (A, B). Therefore, the controllability matrix (A, B) of the full vehicle system has full rank (10), which makes it fully controllable. then it is fully accessible.

3.4.1 System Stability

Stability is an important property that a system is required to have. It is usually not desirable that a small change in the input, initial condition, or parameters of the system produces a very large change in the response of the system. If the response increases indefinitely with time, the system is said to be unstable.The open-loop response of the system to a road profile1 can be used to verify stability of the system.

From the Fig 3.10 and 3.11, it can be observed that the response of the suspension system without feedback controller is unbounded. This shows that the system is unstable because, for the unbounded input the system is producing unbounded output. The system stability is also verified by determining the poles of the system. If the poles of the system are located in the left half of s-plane, then the system is stable.When its at the right half of s plane then

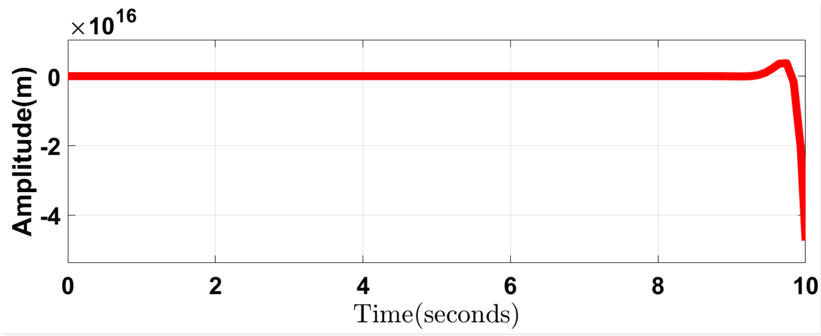


Fig. 3.11: Open-loop pitch angle response of the suspension system model for road profile,the matlab simulink is available at appendix5.1a

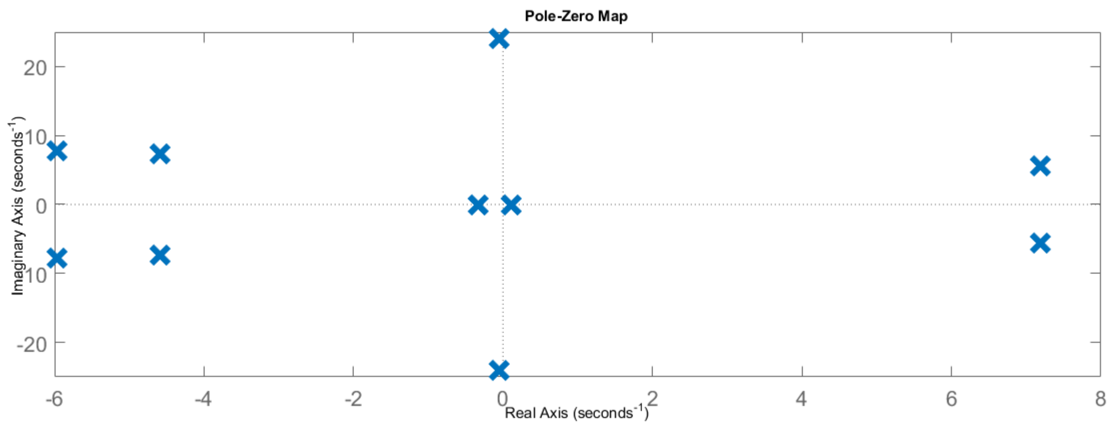


Fig. 3.12: Pole location of open loop suspension model on s-plane

its unstable.Its possible to make unstable part to stable parts of the system if and only if when the system is controllable and observable,that is we provide the input and can see the out put.As earilier told the system is fully accessible and measureable, due this information we will bring the poles which are located at the right half s plane to the left half s plane.At least we will bring the right half splane pole location towards the origin.To bring the poles located at the right half s plane we going to design gain the right feedback gain. From the Figure 3.12 the system has poles at $-0.0498+24.0446i$,$-0.0498-24.0446i$,$7.1944+5.5706i$,$7.1944-5.5706i$,0.1079,-0.3316,$-4.5918+7.3487i$,$-4.5918-7.3487i$,$-5.9700+7.7766i$,and $-5.9700-7.7766i$.
Therefore, it is clearly observed that the poles are located on the left hand side of the s-plane and right hand side of s plane.

3.5 Controller Design

As it is observed in Fig 3.10 and 3.11 the suspension system without active controller is unstable, so, it needs to be stable. Therefore, in order to enhance the performance a suitable controller must be designed.The controller generates an appropriate control signal to maintain the car body at the desired equilibrium position in response to road disturbances. The controller designed will help the system to be insensitive to the road disturbances. Therefore, the controller will try to make the system stable and perform well regardless of the disturbance.

3.5.1 Linear Quadratic Regulator (LQR) Design

The main objective of this section is to design LQR controller for the full vehicle model system modeled in this chapter. One of the state space based optimal control method is Linear Quadratic Regulator (LQR).The LQR is the extension of pole placement technique that tends to find the control input $u(t)$.so as to place the poles of the system at a desired optimal position. The main idea in LQR control design is to minimize the quadratic cost function of J[14],[51].

$$\dot{x} = Ax(t) + Bu(t) \tag{3.25}$$

$$y(t) = Cx(t) + Du(t) \tag{3.26}$$

$$y(t) = Cx(t) + Du(t)$$

$$\dot{x} = \frac{d}{dt}x = Ax(t) + Bu(t)$$

If something is controllable means we are able to control or influence it or push the initial value to a desire final destination as time goes to infinity. it means An output controllable system is not necessarily state controllable. For example, if the dimension of the state space is greater than the dimension of the output, then there will be a set of possible state configurations for each individual output.
In order to be able to do whatever we want with the given dynamic system under control input, the system must be controllable

First we suppose to check the controllable and observable of the system.When the system is controllable observable its rank must be full rank.Then we should proceeded the task,if not we should have make to controllable and observable form.As thought the main function of control is to control the system safely with best optimal control laws.control law describes us a system must be optimize the cost function of the system.In order to determine control law we have dig to find out the constant gain feed back control with different techniques such as place,compensator PID,by tunny until we get the desired values.These consumes the time and our efforts.so that we need the effective time and effort,as result we finding the another controller which will minimize our time and effort not only that it optimize both vehicle model state and fuel by weighting constant scale matrix,that is LQR controller.

The LQR algorithm is essentially an automated way of finding an appropriate state-feedback controller. Difficulty in finding the right weighting factors limits the application of the LQR based controller synthesis.
So we are putting the entire vehicle state problem that we get from the vehicle model into optimization frame work.
Constraints of the problem is describe as fellow

time final $=(tf) is free$
final state $=x(tf) is free$

Boundary conditions are

$$x^*(t0) = t0$$

$$\frac{\partial}{\partial x}h(x^*(tf), u^*(tf), p^*(tf), tf) + \frac{\partial}{\partial x}h(x^*(tf), tf) = 0$$

$$J = \int_0^\infty \left(x^T Qx + u^T Ru + 2x^T Nu\right) dt \tag{3.27}$$

tf is an element of feasible sets. Based on the feasible set we measure the goodness of the each item feasible sets,this concept is cost function.In order to proceeded the task; this paper claims the four case study.

Case-1 : **minimizing the time**
$J_1(tf)$ = measure the time required for the action of feasible sets(Z)
ρ_1 = minimize the $J_1(tf)$
tf is an element of feasible sets
Case-2 : **minimizing the fuel**
$J_2(tf)$ = measure the cost required for the action of feasible sets(tf)
ρ_2 = minimize the $J_2(tf)$
tf is an element of feasible sets
Case-3 : **minimizing the time and fuel trade off**
$J_3(tf) = Q * J_1(tf) + R * J_2(tf)$
where Q and R are scaler weighting
ρ_3 = minimize the $J_3(tf)$

- Large Q relative to R
- Small Q relative to R
- Trade off between Q and R

tf is an element of feasible sets
Case-4: **Looking case 3 add the constraints**
ρ_4 = minimize the $J_3(tf)$
tfis an element of feasible sets
Function is a time associate with feasible sets. x(tf) is free

$$x(t) = 10X1$$
$$u(t) = 8X1$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$J = \int_0^\infty \left(x^T Qx + u^T Ru + 2x^T Nu\right) dt$$
$$x(t) = nX1, statevector$$
$$u(t) = mX1, controlvector$$

If Q is bigger than R ⇒Fast regulation of $x(t) \Rightarrow 0$
u is large ⇒ Aggressive action
If R is bigger than Q ⇒Slow regulation of $x(t) \Rightarrow 0$

u is small $\Rightarrow$ Conservative action
The feedback control law that minimizes the value of the cost functional defined as
is subject to the vehicle state of
$\dot{x} = Ax + Bu$
$y = Cx + Du$
and the the control law that minimize the cost function is

$$u = -Kx \tag{3.28}$$

where k is given by

$$K = R^{-1}(B^T P(t) + N^T) \tag{3.29}$$

J is a cost function that measure the solution of feasible sets of the vehicle states.Optimal problem involves finding the feasible solution to minimize the cost function. and P is found by solving the continuous time Riccati differential equation.
$A^T P(t) + P(t)A - (P(t)B + N)R^{-1}(B^T P(t) + N^T) + Q = -\dot{P}(t) A^T P(t) + P(t)A - (P(t)B + N)R^{-1}(B^T P(t) + N^T) + Q = -\dot{P}(t)$
The first order conditions for Jmin are

- State equation
  $\dot{x} = Ax + Bu$

- Co-state equation
  $-\dot{\lambda} = Qx + Nu + A^T \lambda$

- Boundary conditions
  $0 = Ru + N^T x + B^T \lambda 0 = Ru + N^T x + B^T \lambda$
  $K = R^{-1}(B^T P + N^T) K = R^{-1}(B^T P + N^T)$
  $x(t_0) = x_0 x(t_0) = x_0$
  and
  $\lambda(t_1) = F(t_1)x(t_1)\lambda\ (t_1) = F(t_1)x(t_1)$

and Q is found by solving the continuous time algebraic Riccati equation

$A^T P + PA - (PB + N)R^{-1}(B^T P + N^T) + Q = 0$
$A^T P + PA - (PB + N)R^{-1}(B^T P + N^T) + Q = 0$
This can be also written as
$\mathscr{A}^T P + P\mathscr{A} - PBR^{-1}B^T P + \mathscr{Q} = 0$
with
$\mathscr{A} = A - BR^{-1}N^T \qquad ,Q = Q - NR^{-1}N^T$
The solution of ARE is obtained by setting the elements of the P matrix and weighting matrix Q .
The Q matrix is the state weighting matrix and must be the same the size of state matrix and the R matrix which weighting the input parameters and therefore must be the same

size of input matrix.Q and R are weighting matrices and should be positive semi define and positive definite respectively. They are also symmetric matrices $Q = Q^T, R = R^T$

$$Q = x_{\{1X10\}} * [QiXQj] * x_{\{1X10\}}$$

Where $i, j = 1, 2, ...10$
and the p matrix (solution of ARE) it would be positive definite,positive definite,negative definite,and negative definite.When its said to be positive definite all the diagonal and the manor matrix are positive ,when semi-positive definite all the diagonal matrix is positive and the manor matrix are positive and zero where as negative definite all the diagonal and the manor matrix are negative ,when semi-positive definite all the diagonal matrix is negative and the manor matrix are negative and zero.if the p is semi definite the system is infinite where as negative or positive definite its finite system.

$$x^T Qx \geq 0 ... positivesemidefinite$$

$$x^T Qx \leq 0 ... negativesemidefinite$$

$$u^T Ru > 0 ... positivedefinite$$

$$P = [PiXPj]$$

Where $i, j = 1, 2, ...10$

$$\begin{aligned} Q =& Q1x_1^2 + Q2x_2^2 + Q1x_3^2 + Q2x_4^2 \\ &+ 2Q5x_1x_2 + 2Q6x_1x_3 + 2Q7x_1x_4 \\ &+ 2Q8x_2x_3 + 2Q9x_2x_4 + 2Q10x_3x_4 \end{aligned}$$

Let assumed that the product of x1 and x2,x1 and x3,x1 and x4,x2 and x3,x2 and x4,x3 and x4 ...(mutual products)are not much affect on the state or its effect is very small and we ignore its value.So ,the Q is becoming diagonal matrix

$$Q = 10000 * eye[iXj]$$

Where $i, j = 1, 2, ...10$
Therefore, since the outputs of the system are combination of these individual states, by penalize each individual state independently using its respective Q value and observing the combination effect on the outputs the required performance can be achieved. In fact, penalize one state has an effect on another but it is small.

Chapter 4

Simulation results and discussions

This chapter discussed about linear active suspension system's performance as described in chapter 2.2.5. Simulation based on the mathematical model for full car model by using MATLAB/SIMULINK software addressed. Performances of the suspension system in term of stability observed,based on lyapunov energy principle. Parameters that observed were the suspension travel, wheel deflection and the car body acceleration for full car model.The aim was to achieve small amplitude value for suspension travel, wheel deflection and the car body acceleration with in fast settling time response. Using LQR to address practical full state feedback control issue for vehicle active suspension dynamics discussed. The comparison of LQR performance with (PID,Fuzzy) in-terms of error and generalization drawn.

4.1 checking the stability of closed loop vehicle suspension system based on Lyapunov stability(Internal stability)

The following lyapunov energy stability proved was to validate the internal stability of the closed loop vehicle active suspension system's performance as describe in chapter 2.2.5 and simulated using matlab software ,LQR controller including actuator dynamics as shown in the Fig4.1.The proved was based on linear system in order to achieve the stability analysis and to dedicate the vehicle suspension sysytem with the controller was converge or not.It was based on ball radius in which the lyapunov energy stability principle(internal) was originated and for justification and to arrive the principle of the stability of the vehicle suspension system as shown in Fig 4.1[52],[53].

Let's considering the Fig4.1 the Vehicle dynamic systems with respect to its controllers.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$u = -kx$ susbstituting this equation into the above we got the following equations

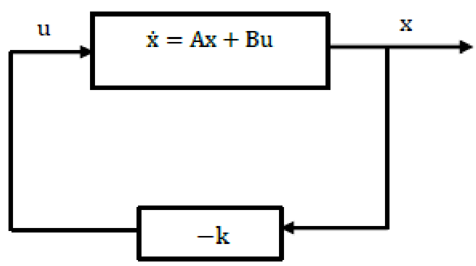


Fig. 4.1: Block diagram of LQR control scheme[54]

$$\dot{x} = (A - Bk)x$$

$$\dot{x} = (A - Bk)x = f(x), f : R \longrightarrow R^n, C'$$

- We can obtain the linearization as f(x)=Ax+h(x) where

$$||x|| \underset{\longrightarrow}{\lim} 0 \frac{||h(x)||}{||x||}$$

if fuction f is differentiable,then

$$A = \frac{\partial f}{\partial x} |_{x=0} = a, \dot{x} = ax$$

Where a is constant coefficient matrix of (A-Bk)'s. Since A is hurwitz, then there exist $p > 0$, $Q > 0$,and the integration is existing such that

$$\begin{aligned} A^T P + PA + Q = 0 \\ A^T P + PA = -Q \\ P = \int_0^\infty e^{A^T t} Q e^{At} dt P^T \\ P = \int_0^\infty \left(e^{A^T t} Q e^{At} dt \Rightarrow A^T P + PA = -Q \right) \\ = \int_0^\infty \left(A^T e^{A^T t} Q e^{At} + e^{A^T t} Q e^{At} A \right) dt \\ = \int_0^\infty \frac{d}{dt} \left(e^{A^T t} Q e^{At} \right) dt \\ = \left[e^{A^T t} Q e^{At} \right]_0^\infty \\ = 0 - Q = -Q \end{aligned}$$

,by the definition of h,we know that for any $\varepsilon > 0$,there exists $r > 0$ such that $||h(x)|| < \varepsilon ||x||, \forall ||x|| < r$ that is that for ε small enough,
$\left(\varepsilon < \frac{\lambda_{min}(Q)}{2\lambda_{min}} \right) \lambda_{min}(Q)||x||^2 > 2||x||||h(x)||\lambda_{max}(p)$
in this case ,let $m = \lambda_{min}(Q)||x||^2 - 2||x||||h(x)||\lambda_{max}(p)$
Candite lyapunov function $V(x) = x^T Px$,derivative along the system trajectory

$$\begin{aligned} \dot{V}(x(t)) = \dot{x}^T Px + x^T P\dot{x} \\ V(x) > 0, x(t) \neq 0 \\ = (Ax + h(x))^T Px + x^T Ph(x) \leq -\lambda_{min}(Q)||x||^2 + 2||x||||h(x)||\lambda_{max}(p) \\ = -m(negativedefinite) \end{aligned}$$

- $\lambda = a$ then the eigen values of the closed loop system are
$\{-400390, -22450, -22300, -5930, -580, -230, -90, -10 \pm 10j, -10\}$

- $\exists\forall R(\lambda) > 0 \Rightarrow x = 0$ is locally asymptotically stable.

According Lyapunov's [52],[53]indirect method the closed loop system is locally asymptotically stable. This prove that the lyapunov's condions based on closed loop system,so the mathematical models of vehicle suspension system develoved under the chapter three is stable in lyapunov stability sense.The trajectories move toward from infinite-distant out and eventually converge to the critical point (when r are both negative).

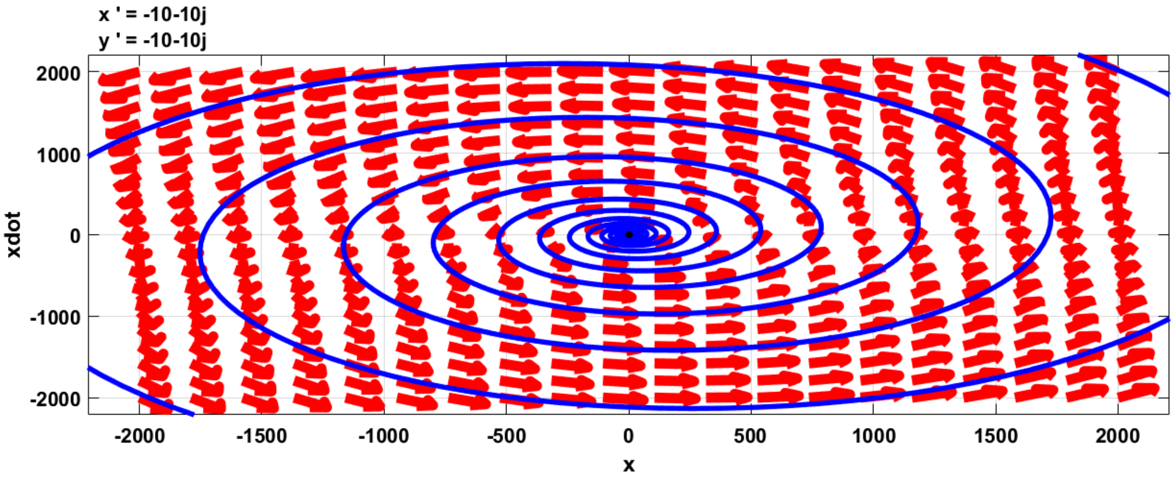
Based on the phase plane dipected in the Fig4.2a,4.2b ,and 4.2c the equlibrium point $x = 0$ is stable,further if $\dot{\mathbf{V}}(\mathbf{x}) < \mathbf{0}$ in $\mathbf{D} - \{0\}$ then the equilibrium point $x = 0$ is asymtotically stable.
From Khalil's non linear system book.
Given $\varepsilon > 0$ we need to construct $\sigma > 0$ such that any trajectory starting in $\mathsf{B}(0,\sigma)$ doesn't leave$\mathsf{B}(0,\varepsilon)$.We will construct one set $\Omega \subset \mathsf{B}(0,\varepsilon)$[52],[53]. We will show that Ω is (positively)invariant that is trajectories startingwith in Ω doesnot leave Ω. Finally ,we deduce existence of a $\mathsf{B}(0,\sigma)$ inside Ω.
Ω construction will crucially use properties of V(x).$\mathsf{B}(0,\sigma)$ existence will use continuty of V at 0 and $V(0) = 0$.Verify(later)that all assumptions in theorem indeed got utilized:(or else,some statement under"less assumption")[52],[53].
(simplfing) notation $\mathbf{B}_\sigma$ open ball of radius σ around the origin
Let $\varepsilon > 0$ be given check that $\mathbf{B}_\varepsilon$ is contained in $\mathbf{D}$ (if not choose a smaller$\varepsilon > 0$).
$\partial\mathbf{B}_\varepsilon = \{x \in R^n\}||x|| = \varepsilon)$ then the value of lyapunov function $\mathbf{V}$ on the boundary $\partial\mathbf{B}_\varepsilon$ is strictly positive.Why? Because we assumed that $\mathbf{V}(\mathbf{x}) > 0$ for all x except $x = 0$.All points in $\partial\mathbf{B}_\varepsilon$ are distance ε away from origin.Hence ,$\mathbf{V}(\mathbf{x})$ for all $x \in \partial\mathbf{B}_\varepsilon$. Let$\alpha := minV(x)\underline{\lim}||x|| = r$
Then $\alpha > 0$
Take any $\mathbf{B}_\varepsilon(0,\alpha)$ and define $\Omega_c := \{x \in\}\mathbf{B}_\varepsilon|V(x) \leq \beta$ then
Claim 1:Ω_c is in interior of $\mathbf{B}_\varepsilon$
Claim2:The set Ω_c satisfies any trajectory in Ω_c at $t = 0$ stays in Ω_c for all $t \geq 0$,that is Ω_c is positively invariant (with respect to) the dynamics of (A-B*k)x(t).

Claim 1's proof

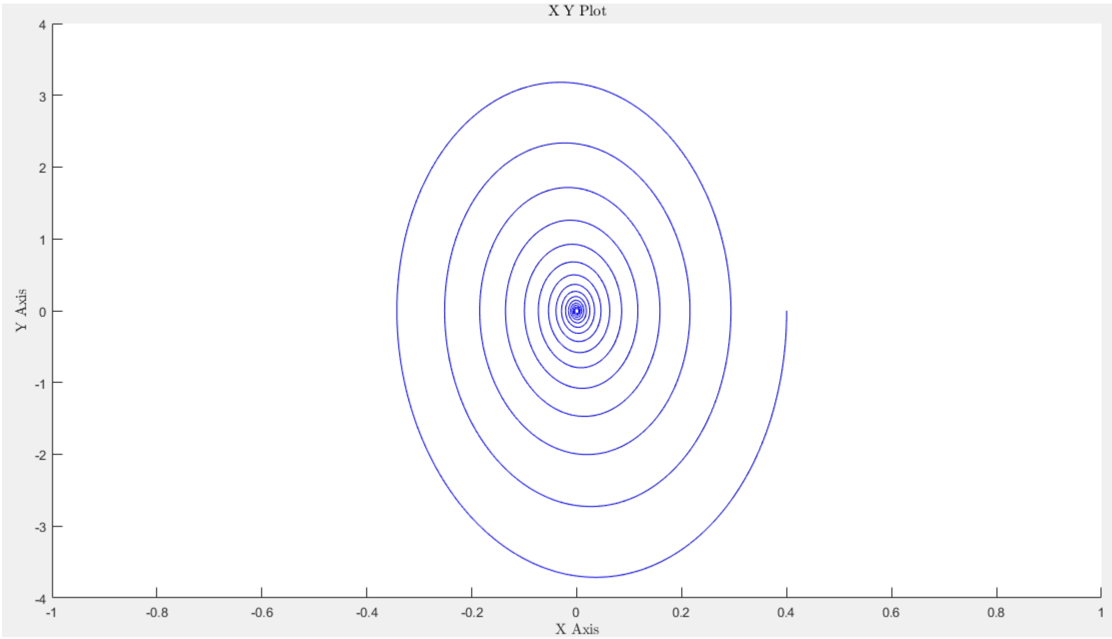
Suppose Ω_c was not in the interior of $\mathbf{B}_\varepsilon$,then there would be a point $p \in \partial\mathbf{B}_\varepsilon \bigcap \Omega_c$ $p \in \partial\mathbf{B}_\varepsilon$ implies $V(p) \geq \alpha(:= minV(x)_{||x||=r})$
$P \in \Omega_c$ implies that $V(p) \leq \beta \leq \alpha!$.
This contradiction thus proves that there cannot be a point p in $\partial\mathbf{B}_\varepsilon \bigcap \Omega_c$ is bounded:$\Omega_c \subset \mathbf{B}_\varepsilon$.Hence Ω_c is compact(useful soon).

Claim 2's proof

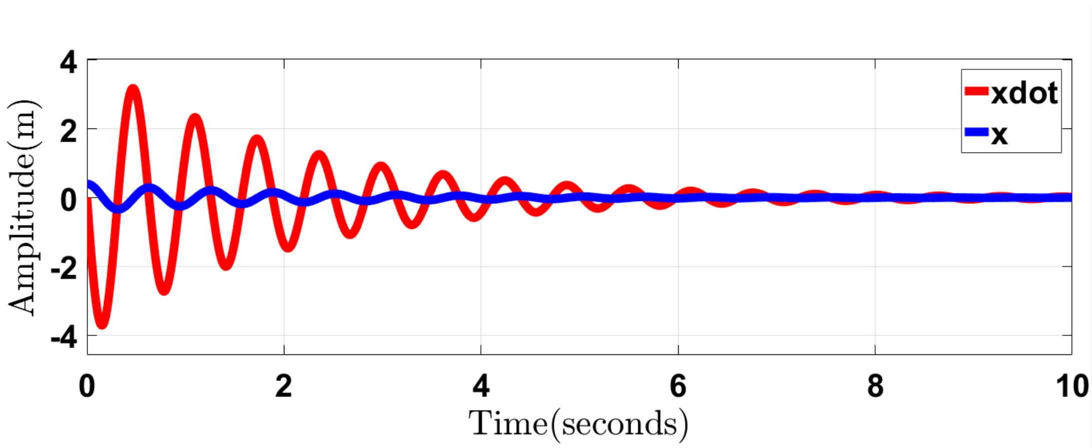
$\dot{V}(x(t)) \leq 0 \Longrightarrow V(x(t)) \leq V(x(0)) < \beta$ for all $t \geq 0$.
It means that $\dot{V}(x) \leq 0$
$\int_0^t \dot{V}(x)dt \leq 0$
$V(x(t)) - V(x(0)) \leq 0$
$V(x(t)) \leq V(x(0))$,for all $t \geq 0$
This proves Ω_c is positively invariant (with respect to dynamics of $(A - B * k)x(t)$.
Since Ω_c is also a compact set ,$\dot{x} = (A - B * k)x(t)$ has a unique solution defined for all $t \geq 0$,for each $x(0) \in \Omega_c$.(We saw this theorem before). Any solution starting in Ω_c stays in Ω_c and hence in B_ε.
Does this prove 0 is stable?No not yet ,now to find a $\sigma \geq$ such that $B_\sigma \bigcap .\Omega_c$.As V(x)is continuous and $V(0) = 0$,V(x) is close to zero for all x in some B_ε also.More precisely,V is continous at $x = 0$ if and only if for every $\beta > 0$ there exists a $\sigma > 0$ such that x$\in B_\sigma \Longrightarrow |V(x) - V(0)| < \beta$(The so-called $\varepsilon - \sigma$ definition of continuity$\varepsilon \to \beta$)



(a) Complex eigenvalues, with negative real part



(b) all phase portraits starts from initial to original



(c) time histories of phase portraits

Using $V(0) = 0$ and$V(x) > 0$ for other $x \in D$ this means V(x) is continuous at $x = 0$ implies that for any $\beta > 0$,there exists $\sigma > 0$ such that $x \in B_\sigma \Rightarrow V(x) < \beta$.
Thus there exists a ball B_σ contained inside Ω_c for some $\sigma > 0$.We have shown: for every $\varepsilon > 0$,there exists a $\sigma > 0$ such that $B_\sigma \subset \Omega_C \subset B_\varepsilon$ and $x(0) \in B_\sigma \Rightarrow x(0) \in \Omega_C$
Which means for all $t \in [0,\infty)$ we have $x(t) \in \Omega_C$ and hence $x(t) \in B_\sigma$.This completes proof of the stability.

Asymptotically stability

Based on the Fig4.2a,4.2b,and 4.2c and strong concuilision for the following case $\forall Re(\lambda) < 0 \Rightarrow x = 0$ locally exponentially stable, if $\dot{V}(x) < 0$ in $D - \{0\}$ also holds to now show asymtotically stable we need to show that $x(t) \to 0$ as $t \to 0$ .Since $V(x) = 0 \Leftrightarrow x = 0$,we can instead show $V(x) \to 0$
$\dot{V}x(t) < 0$ means V(x(t)) is monotonically decreasing with time.Hence a limit does exist.As $t \to \infty, V(x(t)) \to C$
To show that $C = 0$, a contradiction argument is used.Suppose ,$C > 0$,by continuity of $V(x(t))$ there is $d > 0$such that $B_d \subset \Omega_C$
The limit $V(x(t)) \geq C$ for all $t \geq 0$ this implies that trajectory x(t) lies out side the ballB_d for all $t \geq 0$
Let $-\gamma = max\dot{V}x((t)), d \leq ||x|| \leq \varepsilon$ Does the maximium exists?
(Over a set maximium/supremum may/ may not exist).Over a compact set a continuous function acheives its maximum and minimium.The compact set:$d \leq ||x|| \leq r$ the continuous function on this set is $\dot{V}x(t)$
$V(x(t)) = V(x(0)) + \int_0^t V(x(\tau))d\tau \leq V(0) - \gamma t$
$V(x(0)) - \gamma t$ for $x(0) \in B_\varepsilon$,Recall $-\gamma = max\dot{V}x(t)$ for all x satisfying $d \leq ||x|| \leq \varepsilon the \varepsilon > 0$.Hence RHS eventually becomes negative.
Hence V(x(t)) is also becomes (further) negative.Thus the set $d \leq ||x|| \leq \varepsilon the \varepsilon > 0$ can not be invariant ,and our assumption about $C > 0$ causes this contradiction.Thus we showed $V(x(t)) \to 0$ as $t \to \infty$, and hence $x(t) \to 0$ also this proves asymptotically stable.Based on this prove we arrived the function V(x(t)) satisfying

- $V(0) = 0$
- $V(0) \geq 0 for all x \neq 0$ is said to be positive definite

Rephrasing lyapunov's theorem:
The origin is stable if there is a continuously defferentiable positive definite function V(x) such that $\dot{V}x(t)$ is negative semidefinite ,and is asymptotically stable if $\dot{V}(x)$ is negative definite.
There fore, lyapunov theorem's conditions are sufficient and necessary condition for the particular work of this thesis paper.Global asymptotically stability ,if an equilibrium point is asymptoticly stable then inside a neighborhood, it is the only equilibrium point (locally)asymptotically stable.Globally can lyapunov's theorem tell some thing?yes The region of attraction (for an asymptotically stable equilibrium point).Level sets of V(x) give (possible conservative) region.Some property of V(x) ensures global asymptotical stability is V(x) 'radial unbounded . V(x) called 'radial unbounded if along any radial,direction,V(x) becomes unbounded that is[52],[53] $||x|| \to \infty \implies V(x) \to \infty$ not closed contours for any level set is ruled out.
Theorem: let $x = 0$ be an equlibrium point for $\dot{x} = Ax(t)$.Let $\mathbf{V(x)} : \mathbf{R^n} \to \mathbf{R}$ be continuously differentiable function such that
$\mathbf{V(0) = 0} and \mathbf{V(x) > 0}, \forall \mathbf{x} \neq \mathbf{0}$

$\mathbf{V}(\dot{\mathbf{x}}) < \mathbf{0}, \forall \mathbf{x} \neq \mathbf{0}$
$||x|| \to \infty \Rightarrow V(x) \to \infty$ then $V(x) \to 0$ is globally asymptotically stable.(Not just asymptotically stable , but infact ,globally asymptotically stable)[**?** **?**].For more pricise this thesis paper dynamics A is said to be Hurwitz if all its eigen values have real part negative .$\dot{x} = Ax(t)$ has origin as an equilibrium point origin is asymptotically stable if and only if A is Hurwitz.Based ob this principle we conculded that lyapunov's theorem is necessary and sufficient condition for contex of this thesis paper.Consider $\dot{x} = Ax(t)$ for $A \in R^{nxn}$ then A is Hurwitz if and only if there exists $\mathbf{V}(\mathbf{x}) : \mathbf{R^n} \to \mathbf{R}$ a continuously differentiable fuction [**?**]such that $\mathbf{V}(\mathbf{0}) = \mathbf{0}$and $\mathbf{V}(\mathbf{x}) > \mathbf{0}, \forall \mathbf{x} \neq \mathbf{0}$
$\mathbf{V}(\mathbf{x}) < \mathbf{0}, \forall \neq \mathbf{0}$.
To sum up the stability

Table 4.1: stability testing based on lyapunov energy principle table

stability characteristics	vehicle models developed in chapter three with LQR	sufficient condition	Necessary condition
Converge	YES	YES	YES
Asymptotically stable	YES	YES	YES
locally stable	YES	YES	YES
exponetially stable	YES	YES	YES
exponetially locally stable	YES	YES	YES

4.2 Testing the vehicle active suspension model stability based on different road profile

Under this section we going to test the mathematically developed vehicle suspension dynamics systems models in terms of stability will be observed based on the different road profiles(single bump up,single bump down,double bump up,and double bump down).The parameters that will be observed are the suspension travel(bouncing),pitch travel,roll travel, and the force required by the rear wheels and front wheels will more detail discuss.

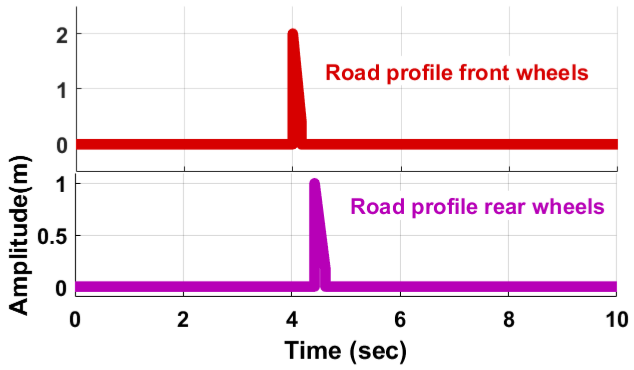
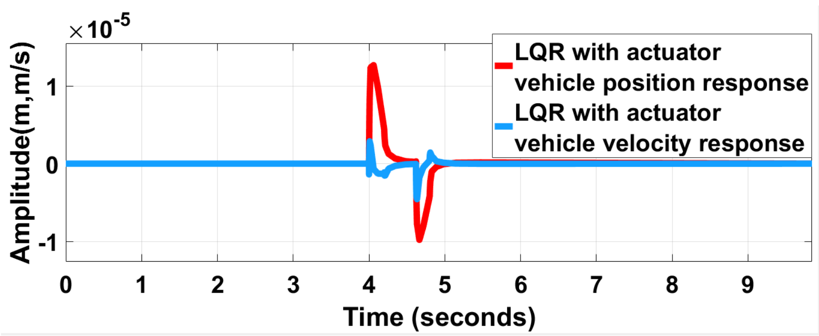
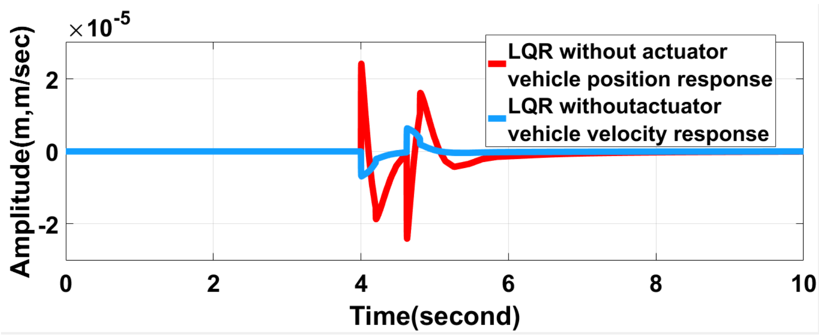


Fig. 4.3: Road input disturbance of a single bump



(a) Simulation result of vehicle bouncing dynamics with actuator body displacement using single bump for vehicle dynamics with actuator



(b) Simulation result of vehicle bouncing dynamics body displacement using single bump for vehicle dynamics without actuator

For the single bump upward road profile inputs

The road bump profile in Fig 4.3 is appear for $4second \leq t \leq 4.25second$ for front right and left wheels and $4.3second \leq t \leq 4.55second$ for rear right and left wheels. The width of each bump '(t)' in this case 0.25 sec indicates the duration of the road bump at each wheel .From Fig 4.4a and 4.4b, it can be seen that the peak value response of chassis displacement for active suspension system for including actuator and excluding actuator are $1.5*10^{-5}m$,$2.5*10^{-5}m$ respectively for the same road input and the same controller gains.

It can be also observed that the peak value response of the passive suspension system is ∞.Because the suspension system is unstable and its out put doesn't store for the specification memory location.The reduction (improvement) in percentage for the displacement of the chassis can be calculated as follows:

$$Re = \frac{AV - EV}{Ev} \tag{4.1}$$

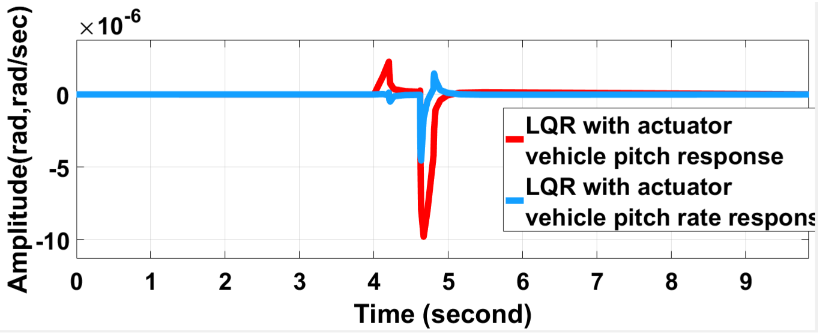
Where RE is Reduction in peak value from active excluding actuator dynamics
Ev is Active (LQR) excluding actuator dynamics value
Av is active (LQR) with actuator dynamics value.
$Re = \frac{2.5*10^{-}5 - 1.5*10^{-}5}{1.5*10^{-}5} = 0.6667 = \mathbf{66.67}\%$
Thus, the active suspension system with out actuator displacement (peak value) is reduced by **33.33**% and **66.67**% in case of an active suspension system with actuator dynamics which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over passive suspension system and active suspension system without actuator dynamics.
The settling time, as we can observe from Fig 4.4a and 4.4b,is 5.8second and 4.9seconds,for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 0.6667 = **66.67**%,and as compared to LQR excluding actuator dynamics. As observing from the simulation result

(a) Comparison of LQR without and with actuator dynamics for bouncing position

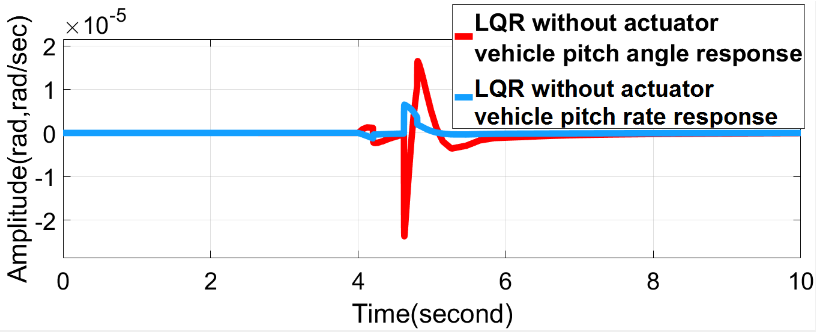
Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m)	$2.5*10^{-5}$	$1.5*10^{-5}$	66.67%
Settling time (sec)	5.8*second*	4.9*second*	18.36%
rise time (sec)	4	4	0%

(b) Comparison of LQR without and with actuator dynamics for bouncing velocity

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m/s)	$1.5*10^{-5}$	$0.5*10^{-5}$	66.67%
Settling time (sec)	5.8*second*	4.9*second*	18.36%
rise time (sec)	5.05	4.04	25%



(a) Simulation result of vehicle pitch dynamics with actuator with actuator response using a single bump



(b) Simulation result of vehicle pitch dynamics with actuator without actuator response using a single bump

of Fig 4.4a and 4.4b, the peak overshoot of sprung mass velocity for the LQR without actuator dynamics and LQR with actuator dynamics are $1.5*10^{-5}m/s$ and $0.5*10^{-5}m/s$ respectively. From these values, it is found that for active suspension system (LQR) with actuator dynamics the peak value of the velocity of the sprung mass is reduced by **66.67**% as compared to active suspension system (LQR) without actuator dynamics the reduction is **33.33**%.The passive suspension system and active suspension without LQR dynamics have no settling time for the displacement whereas the settling time for the active suspension system with LQR with actuator dynamics and without actuator dynamics are 4.9*second* and 5.8*second* respectively. Therefore, the reduction settling time in active controller (LQR) with actuator dynamics which is included in the system model is 81.64% as compared with LQR without actuator dynamics is 18.36%.The rise time for LQR including actuator and with out actuator dynamics is 4.04seconds,and 5.05 seconds respectively.The reduction rise time in active controller (LQR) with actuator dynamics is75% as compared with LQR with out actuator dynamics is 25%.

From the Fig4.5b thus, the chassis displacement (peak value) is reduced by 92.35%,,80% and 7.65% ,20% in case of an active suspension system with actuator dynamics and excluding actuator dynamics pitch angle and pitch rate respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.5b is 5.7sec , and 5sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 86% and 14% LQR excluding actuator dynamics respectively.The reduction(improvement) in rise time for the active suspension including actuator dynamics are 75%,88.89%,and 25%,11.11% in case of an active suspension system with actuator dynamics and excluding actuator dynamics pitch angle and pitch rate respectively.

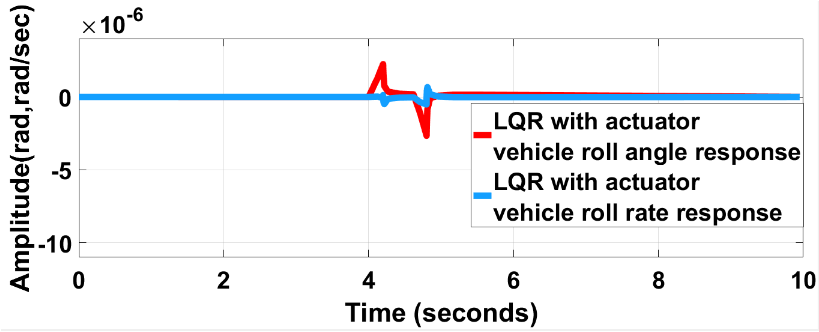
(a) Comparison of LQR without and with actuator dynamics for Pitch angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$1.96*10^{-5}$	$1.5*10^{-6}$	92.35%
Settling time (sec)	$5.7second$	$5second$	14%
rise time (sec)	5	4	25%

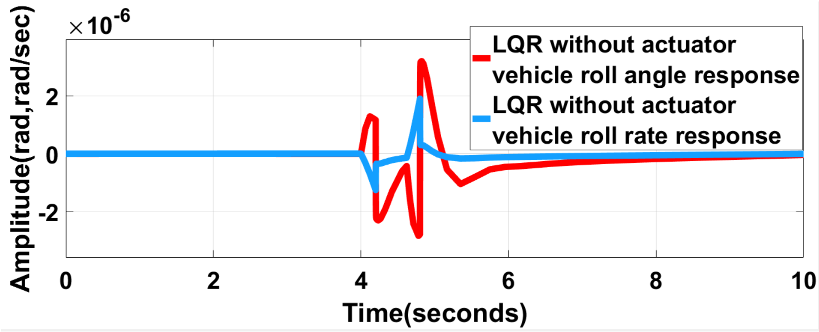
(b) Comparison of LQR without and with actuator dynamics for Pitch velocity

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/sec)	$0.9*10^{-5}$	$0.5*10^{-5}$	80%
Settling time (sec)	$5second$	$5second$	0%
rise time (sec)	4.5	4.05	11.11%

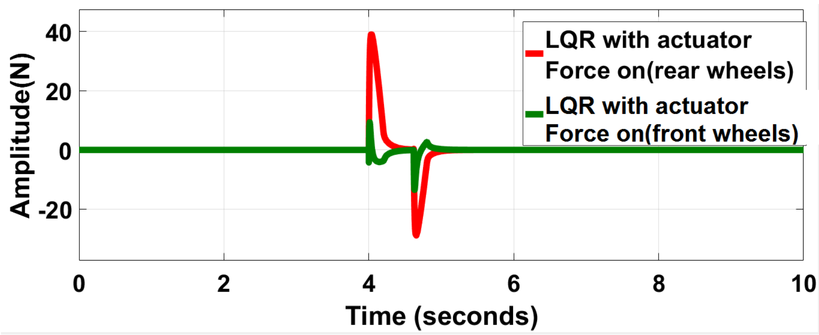
From the Fig4.6a,and 4.6b its clear that, the chassis displacement (peak value) is reduced by 60%,,66.67% and 40% ,33.33% in case of an active suspension system with actuator dynamics and excluding actuator dynamics roll angle and roll rate respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.6a ,and 4.6b are 6sec ,5.05sec, and 5sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 80%,99% and 20% ,1% LQR excluding actuator dynamics respectively.The rise time are 4sec,4.8sec,and 4sec, 4.6sec for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 100%,95.65%,and 0%,4.35% in case of an active suspension system with actuator dynamics and excluding actuator dynamics roll angle and roll rate respectively. Clearly vehicle suspension system with actuator is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.



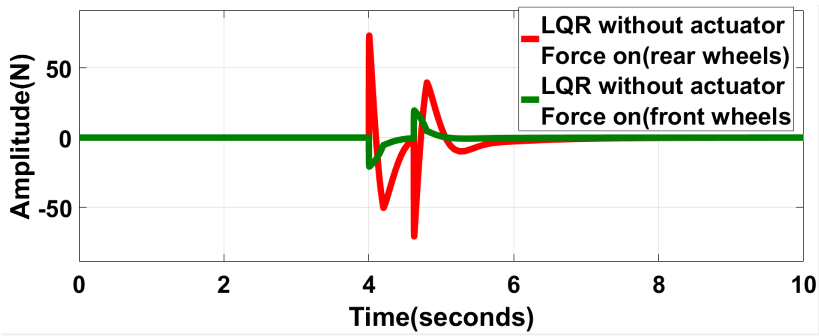
(a) Simulation result of vehicle roll dynamics with actuator using upward single bump



(b) Simulation result of vehicle roll dynamics without actuator using upward single bump



(a) Simulation result of force generated from front and rear wheels with actuator using upward single bump



(b) Simulation result of force generated from front and rear wheels without actuator using upward single bump

From the Fig4.7a,and 4.7b its clear that, the chassis displacement (peak value) is reduced by 62.5%,50% and 37.5% ,50% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels and front wheels respectively, which is included in the system model. This is a direct indication of the su-

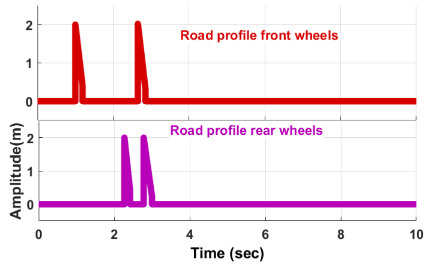


Fig. 4.8: Road input disturbance of a double up ward bump

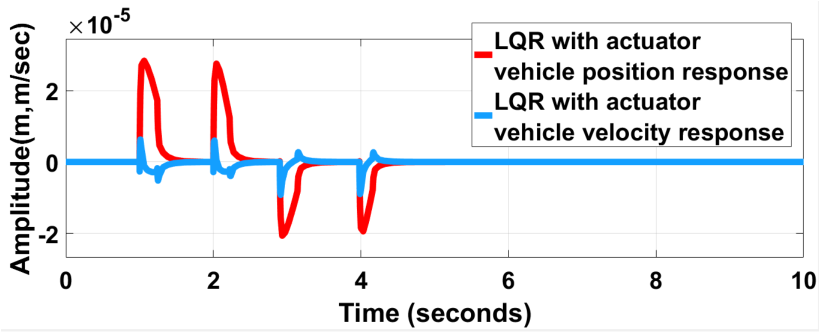
periority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.7a ,and 4.7b are 5.3sec ,5.8sec, and 5sec for rear wheels and front wheels for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 96%,84% and 6% ,16% LQR excluding actuator dynamics respectively.The rise time are 5sec,4sec,and 4sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 75%,100%,and 25%,0% in case of an active suspension system with actuator dynamics and excluding actuator dynamics roll angle and roll rate respectively.

4.2.1 For the double bump upward road profile inputs

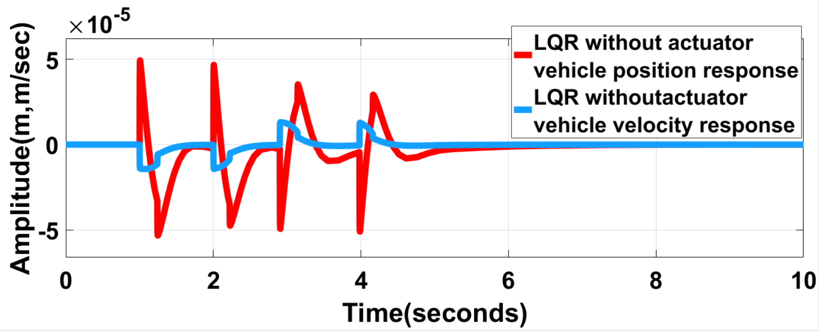
The road bump profile in Fig 4.8 is appear for $1sec \leq t \leq 1.35sec$, and $2.5 \leq t \leq 2.75sec$ for front right and left wheels and $1.5sec \leq t \leq 1.75sec$ and $2.8sec \leq t \leq 3sec$ for rear right and left wheels. The width of each bump '(t)' in this case 0.35 sec ,0.25sec ,0.2sec,these indicates the duration of the road bump at each wheel.

From the Fig4.9a,and 4.9b its clear that, the chassis displacement (peak value) is reduced by 78.6%,66.67% and 21.4% ,33.33% in case of an active suspension system with actuator dynamics and excluding actuator dynamics respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.9a ,and 4.9b are 5.4sec ,4.5sec, and 4.5sec and 4.4sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 80%,97.73% and 20% ,2.27% LQR excluding actuator dynamics respectively.The rise time are 1.5sec,3sec,and 1.35sec,1.6sec for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 88.89%,87.5%,and 11.11%,12.5% in case of an active suspension system with actuator dynamics and excluding actuator dynamics bouncing position and bouncing velocity respectively.

From the Fig4.10a,and 4.10b its clear that, the chassis displacement (peak value) is reduced by 55%,96% and 45% ,4% in case of an active suspension system with actuator dynamics and excluding actuator dynamics respectively, which is included in the

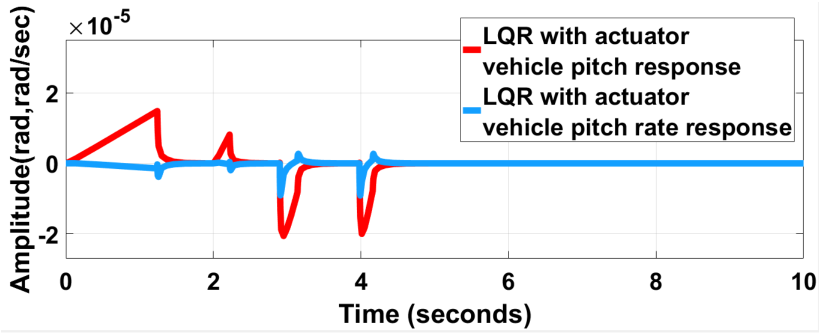


(a) Simulation result of vehicle bouncing dynamics with actuator body displacement using double bump for vehicle dynamics with actuator

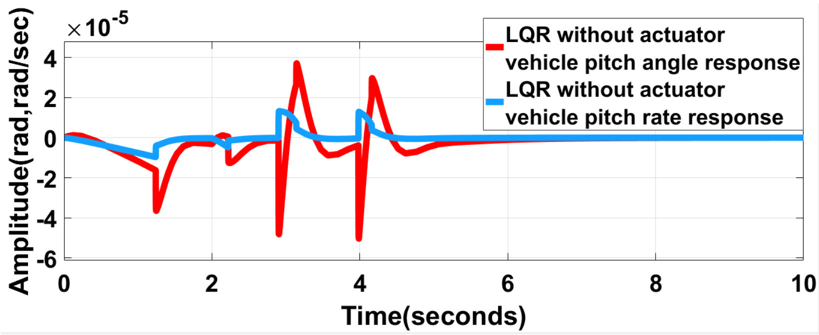


(b) Simulation result of vehicle bouncing dynamics body velocity using double bump for vehicle dynamics without actuator

system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.10a ,and 4.10b are 5.5sec ,4.5sec, and 4.98sec,4.4sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 89.56%,97.73% and 10.44% ,2.27% LQR excluding actuator dynamics respectively.The rise time are 3.5sec,3.6sec,and 1.4sec,3.5sec for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 60%,97.15%,and 40%,2.85% in case of an active suspension system with actuator dynamics and excluding actuator dynamics pitch angle and picth rate respectively. From the Fig4.11a,and 4.11b its clear that, the chassis displacement (peak value) is reduced by 98.99%,57.2% and 1.01% ,42.8% in case of an active suspension system with actuator dynamics and excluding actuator dynamics respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.11a ,and 4.11b are 5.6sec ,4.3sec, and 4.2sec, 4.1sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 66.67%,95.1% and 33.33% ,4.9% LQR excluding actuator dynamics respectively.The rise time are 3.5secand 1.8sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 94.44%,and 5.56% in case of an active suspension system with actuator dynamics and



(a) Simulation result of vehicle pitch angle dynamics with actuator body displacement using double bump for vehicle dynamics with actuator



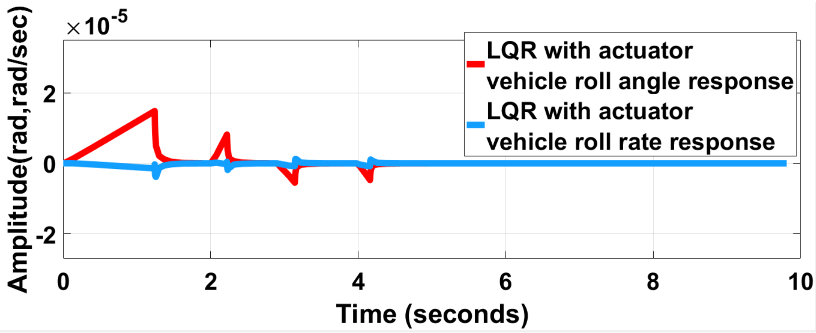
(b) Simulation result of vehicle pitch rate dynamics body displacement using double bump for vehicle dynamics without actuator

excluding actuator dynamics roll angle and roll rate respectively. From the Fig4.12a,and 4.12b its clear that, the chassis displacement (peak value) is reduced by 55.55%,50% and 44.45% ,50% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels and front wheels respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.12a ,and 4.12b are 4.4sec ,4.6sec, and 4.1sec,4.5sec for rear wheels and front wheels for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 92.68%,97.78% and 7.32% ,2.22% LQR excluding actuator dynamics respectively.The rise time are 3.5sec,2.1sec,and 2sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 75%,95%,and 25%,5% in case of an active suspension system with actuator dynamics and excluding actuator dynamics rear wheels and front wheels respectively.

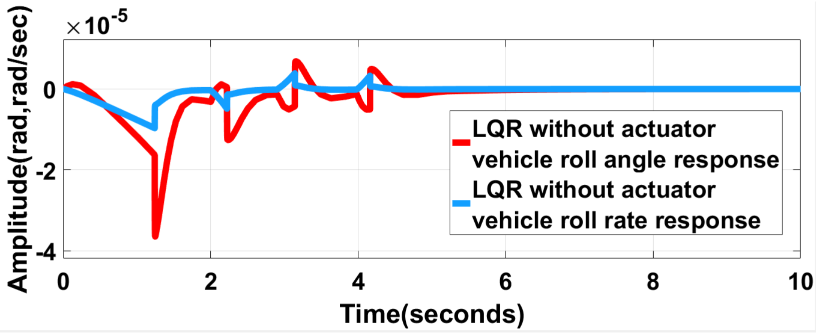
4.2.2 For the single bump downward road profile inputs

The road bump profile in Fig 4.8 is appear for $1sec \leq t \leq 1.25sec$, for front right and left wheels and $1.5sec \leq t \leq 1.75sec$ and for rear right and left wheels. The width of each bump '(t)' in this case ,0.25sec these indicates the duration of the road bump at each wheel.

From the Fig4.14a,and 4.14b its clear that, the chassis displacement (peak value) is reduced by 66.67%,89.58% and 33.33% ,10.42% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels

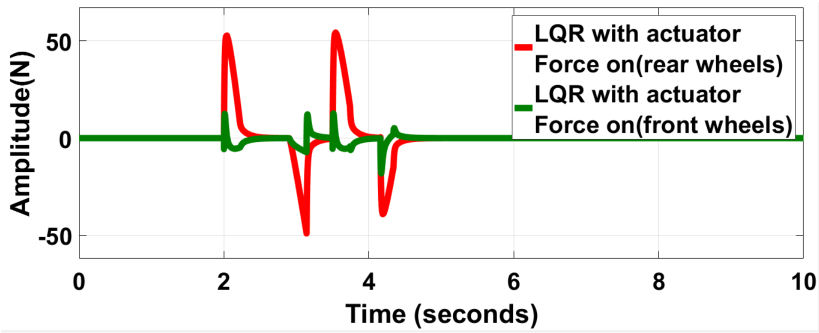


(a) Simulation result of vehicle roll angle dynamics with actuator body displacement using double bump for vehicle dynamics with actuator

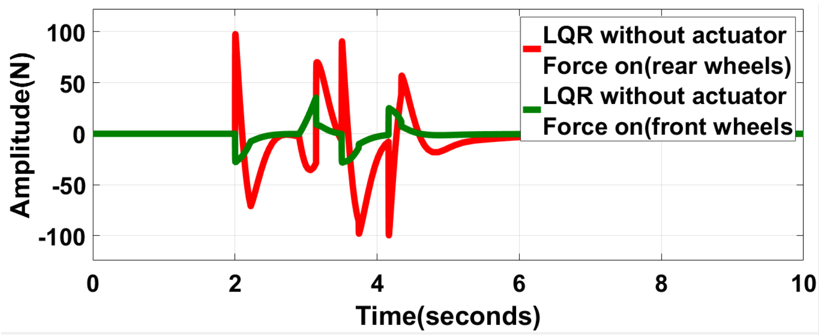


(b) Simulation result of vehicle roll rate dynamics body displacement using double bump for vehicle dynamics without actuator

and front wheels respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.14a ,and 4.14b are 6sec ,5.2sec, and 5sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 80%,96% and 20% ,4% LQR excluding actuator dynamics respectively.The rise time are 4.7sec,4.5sec,and 4.5sec,4.05sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 95.56%,88.89%,and 4.44%,11.11% in case of an active suspension system with actuator dynamics and excluding actuator dynamics respectively. From the Fig4.15a,and 4.15b its clear that, the chassis displacement (peak value) is reduced by 78.57%,75% and 21.5% ,25% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels and front wheels respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.15a ,and 4.15b are 5.89sec ,5sec, and 2.8sec,2.5sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 52.46%,50% and 47.54% ,50% LQR excluding actuator dynamics respectively.The rise time are 4.5sec,4.5sec,and 2sec,1.6sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 55.55%,64.44%,and



(a) Simulation result of force generated from front and rear wheels with actuator using upward double bump



(b) Simulation result of force generated from front and rear wheels without actuator using upward double bump

44.45%,35.56% in case of an active suspension system with actuator dynamics and excluding actuator dynamics pitch angle and pitch rate respectively.

From the Fig4.16a,and 4.16b its clear that, the chassis displacement (peak value) is reduced by 96.67%,95% and 3.33% ,5% in case of an active suspension system with actuator dynamics and excluding actuator dynamics respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.16a ,and 4.16b are 6sec ,5.2sec, and 5.2sec,5sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 84.7%,96% and 15.3% ,4% LQR excluding actuator dynamics respectively.The rise time are 4.5sec,4.5sec,and 4.44sec,4.4sec for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 98.65%,97.73%,and 1.35%,2.27% in case of an active suspension system with actuator dynamics and excluding actuator dynamics roll angle and roll rate respectively.

From the Fig4.17a,and 4.17b its clear that, the chassis displacement (peak value) is reduced by 63.33%,60% and 36.67% ,40% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels and front wheels respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.17a ,and 4.17b are 2.5sec ,3.5sec, and 2.3sec,3.2sec for rear wheels and front wheels for active suspension excluding actuator dynamics and active suspension including

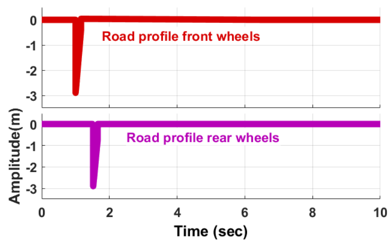
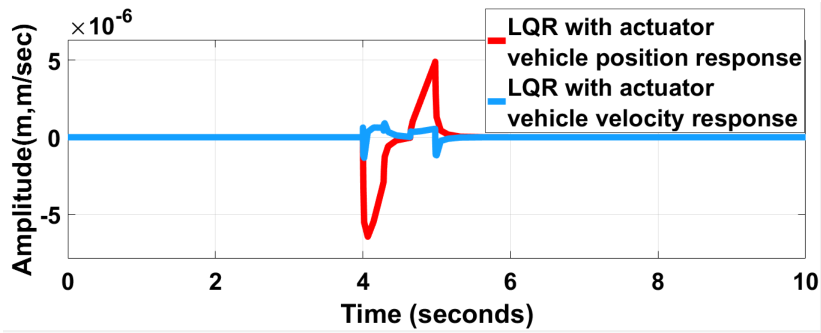
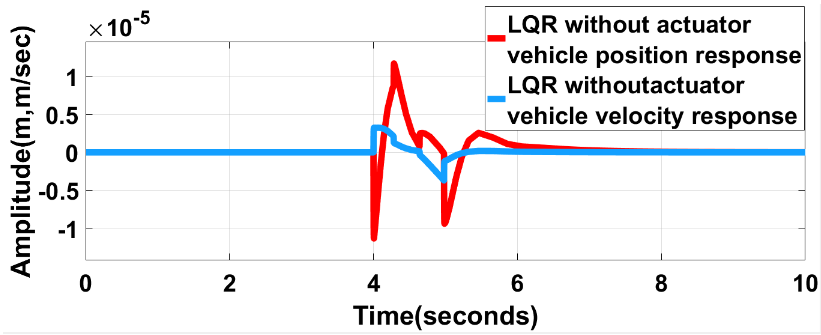


Fig. 4.13: Road input disturbance of a single down ward bump

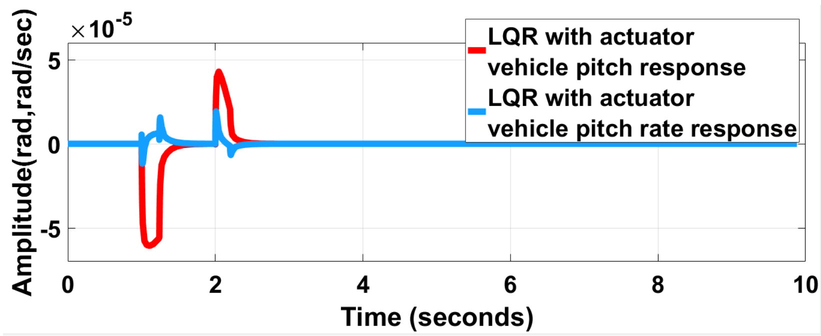
actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 91.3%,90.6% and 8.7% ,9.4% LQR excluding actuator dynamics respectively.The rise time are 1.5sec,2.5sec,and 1.5sec,2.5sec, for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 84.6%,100%,and 15.4%,0% in case of an active suspension system with actuator dynamics and excluding actuator dynamics rear wheels and front wheels respectively.



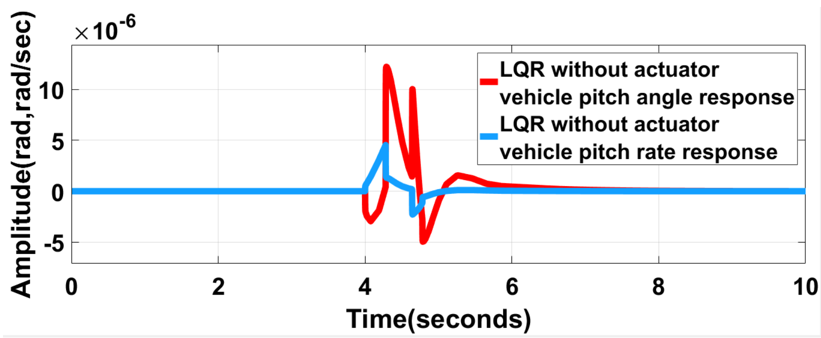
(a) Simulation result of vehicle bouncing dynamics with actuator body displacement using single bump for vehicle dynamics with actuator



(b) Simulation result of vehicle bouncing dynamics body displacement using single downward bump for vehicle dynamics without actuator



(a) Simulation result of vehicle pitch dynamics with actuator body displacement using single downward bump for vehicle dynamics with actuator



(b) Simulation result of vehicle pitch dynamics body displacement using single downward bump for vehicle dynamics without actuator

(a) Comparison of LQR without and with actuator dynamics for Roll angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$2.8*10^{-6}$	$2*10^{-6}$	40%
Settling time (sec)	$6second$	$5second$	20%
rise time (sec)	4	4	0%

(b) Comparison of LQR without and with actuator dynamics for Roll rate

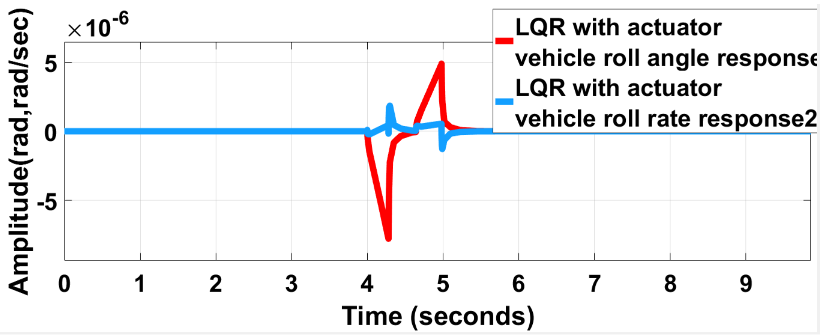
Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/sec)	$2*10^{-6}$	$1.5*10^{-6}$	33.33%
Settling time (sec)	$5.05second$	$5second$	1%
rise time (sec)	4.8	4.6	4.35%

(a) Comparison of LQR without and with actuator dynamics for front wheels

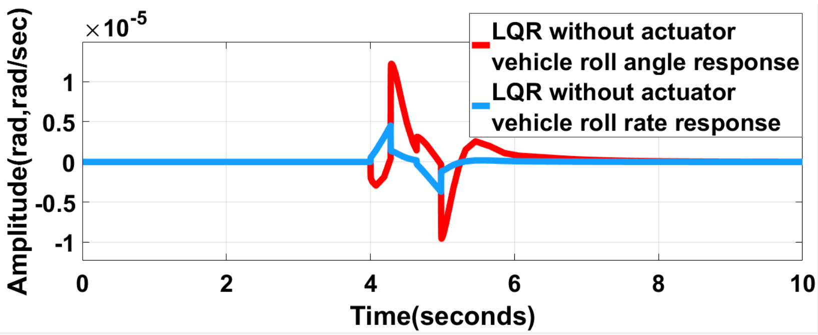
Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	40	15	62.5%
Settling time (sec)	5.3*second*	5*second*	6%
rise time (sec)	5	4	25%

(b) Comparison of LQR without and with actuator dynamics for Rear wheels

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	60	40	50%
Settling time (sec)	5.8*second*	5*second*	16%
rise time (sec)	4	4	0%



(a) Simulation result of vehicle roll dynamics with actuator body displacement using double bump for vehicle dynamics with actuator



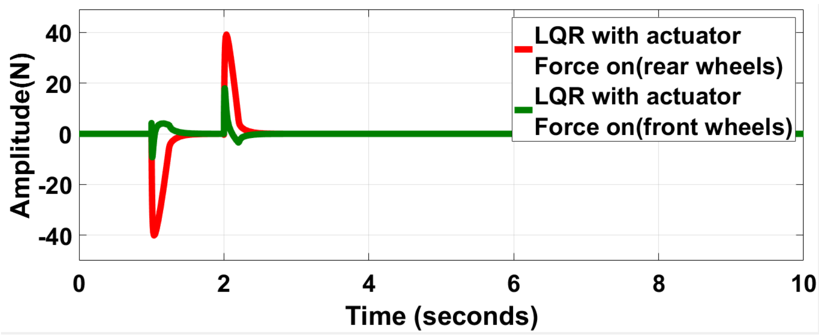
(b) Simulation result of vehicle roll dynamics body displacement using double bump for vehicle dynamics without actuator

(a) Comparison of LQR without and with actuator dynamics for bouncing position

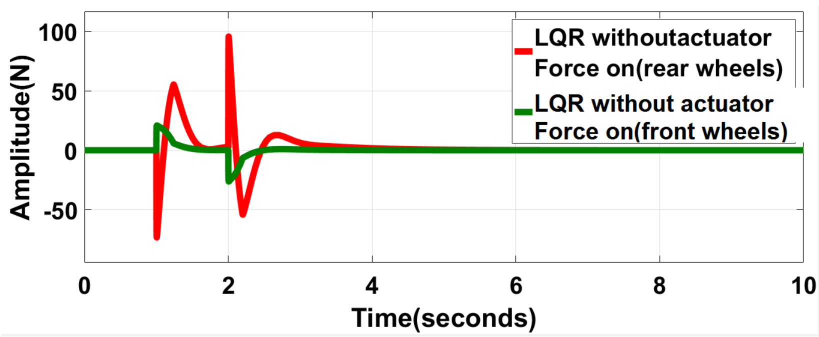
Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m)	$5*10^{-5}$	$2.8*10^{-5}$	78.6%
Settling time (sec)	$5.4 second$	$4.5 second$	20%
rise time (sec)	1.5	1.35	11.11%

(b) Comparison of LQR without and with actuator dynamics for bouncing velocity

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m/s)	$2*10^{-5}$	$1.5*10^{-5}$	33.33%
Settling time (sec)	$4.5 second$	$4.4 second$	2.27%
rise time (sec)	3	1.6	87.5%



(a) Simulation result of force generated from front and rear wheels with actuator using downward double bump



(b) Simulation result of force generated from front and rear wheels without actuator using downward double bump

Table 4.7: Comparison of LQR without and with actuator dynamics for pitch angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$4 * 10^{-5}$	$1.8 * 10^{-5}$	55%
Settling time (sec)	$5.5 second$	$4.98 second$	10.44%
rise time (sec)	3.5	1.4	60%

(a) Comparison of LQR without and with actuator dynamics for pitch rate

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/s)	$1.96 * 10^{-5}$	$0.99 * 10^{-5}$	96%
Settling time (sec)	$4.5 second$	$4.4 second$	2.27%
rise time (sec)	3.6	3.5	2.85%

(a) Comparison of LQR without and with actuator dynamics for Roll angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$2 * 10^{-5}$	$1.98 * 10^{-5}$	1.01%
Settling time (sec)	$5.6 second$	$4.2 second$	33.33%
rise time (sec)	3.5	1.8	94.44%

(b) Comparison of LQR without and with actuator dynamics for roll rate

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/s)	$1 * 10^{-5}$	$0.7 * 10^{-5}$	42.8%
Settling time (sec)	$4.3 second$	$4.1 second$	4.9%
rise time (sec)	-	-	-

(a) Comparison of LQR without and with actuator dynamics for Front wheels

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	45	20	55.55%
Settling time (sec)	4.4*second*	4.1*second*	7.32%
rise time (sec)	3.5	2	75%

(b) Comparison of LQR without and with actuator dynamics for Rear wheels

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	100	50	50%
Settling time (sec)	4.6*second*	4.5*second*	2.22%
rise time (sec)	2.1	2	5%

(a) Comparison of LQR without and with actuator dynamics for Bouncing position

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m)	$1.5 * 10^{-5}$	$5 * 10^{-6}$	66.67%
Settling time (sec)	$6 second$	$5 second$	20%
rise time (sec)	4.7	4.5	4.44%

(b) Comparison of LQR without and with actuator dynamics for Bouncing velocity

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m/s)	$0.48 * 10^{-5}$	$0.5 * 10^{-6}$	89.58%
Settling time (sec)	$5.2 second$	$5 second$	4%
rise time (sec)	4.5	4.05	11.11%

(a) Comparison of LQR without and with actuator dynamics for pitch angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$10.5*10^{-6}$	$4.9*10^{-5}$	78.57%
Settling time (sec)	$5.89 second$	$2.8 second$	52.46%
rise time (sec)	4.5	2	55.55%

(b) Comparison of LQR without and with actuator dynamics for pitch rate

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/s)	$0.5*10^{-6}$	$0.04*10^{-6}$	25%
Settling time (sec)	$5 second$	$2.5 second$	50%
rise time (sec)	4.5	1.6	64.44%

(a) Comparison of LQR without and with actuator dynamics for roll angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$1.5*10^{-5}$	$5*10^{-6}$	96.67%
Settling time (sec)	$6second$	$5.2second$	15.3%
rise time (sec)	4.5	4.44	1.35%

(b) Comparison of LQR without and with actuator dynamics for roll rate

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/s)	$0.5*10^{-5}$	$2.5*10^{-6}$	95%
Settling time (sec)	$5.2second$	$5second$	4%
rise time (sec)	4.5	4.4	2.27%

(a) Comparison of LQR without and with actuator dynamics for front wheels

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	30	19	36.67%
Settling time (sec)	2.5*second*	2.3*second*	8.7%
rise time (sec)	1.5	1.3	15.4%

(b) Comparison of LQR without and with actuator dynamics for Rear wheels

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	100	40	60%
Settling time (sec)	3.5*second*	3.2*second*	9.4%
rise time (sec)	2.5	2.5	0%

For the double bump downward road profile inputs

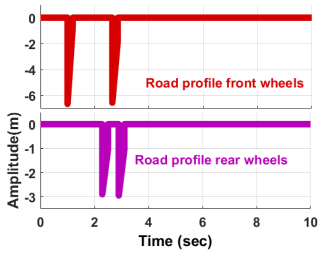
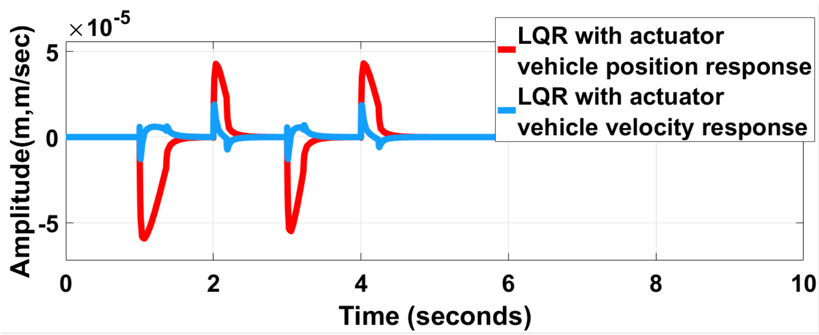
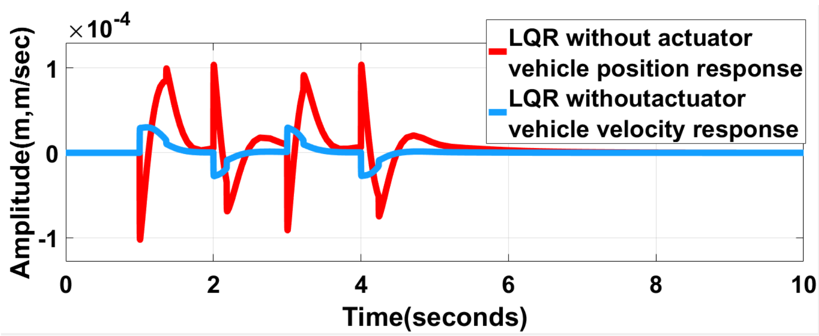


Fig. 4.18: Road input disturbance of a double down ward bump

The road bump profile in Fig 4.18 is appear for $1sec \leq t \leq 1.35sec$, and $2.5 \leq t \leq 2.75sec$ for front right and left wheels and $1.5sec \leq t \leq 1.75sec$ and $2.8sec \leq t \leq 3sec$ for rear right and left wheels. The width of each bump '(t)' in this case 0.35 sec ,0.25sec ,0.2sec,these indicates the duration of the road bump at each wheel.



(a) Simulation result of vehicle bouncing dynamics with actuator body displacement using double bump for vehicle dynamics with actuator



(b) Simulation result of vehicle bouncing dynamics body displacement using double bump for vehicle dynamics without actuator

From the Fig4.19a,and 4.19b its clear that, the chassis displacement (peak value) is reduced by 51%,96% and 49% ,4% in case of an active suspension system with actuator dynamics and excluding actuator dynamics respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.19a ,and 4.19b are 5sec ,5sec, and 4.5sec,4.5sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 88.89%and 11.11% LQR excluding actuator dynamics respectively.The rise time are 2.5sec,1.5sec,and 2.5sec,1.1sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 100%,63.64%,and 0%,36.36% in case of an active suspension system with actuator dynamics and excluding actuator dynamics respectively.

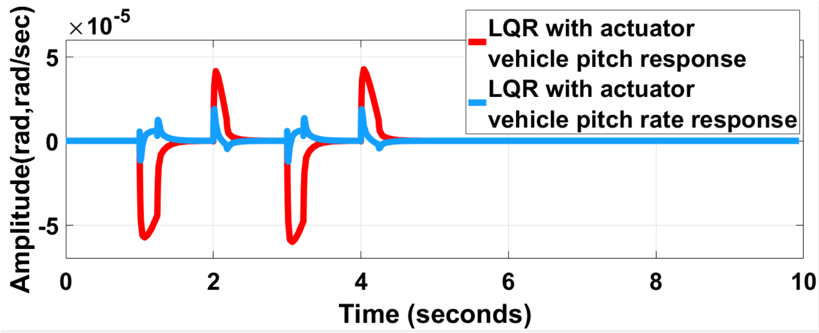
(a) Comparison of LQR without and with actuator dynamics for bouncing position

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m)	$1*10^{-4}$	$4.9*10^{-5}$	51%
Settling time (sec)	$5second$	$4.5second$	11.11%
rise time (sec)	2.5	2.5	0%

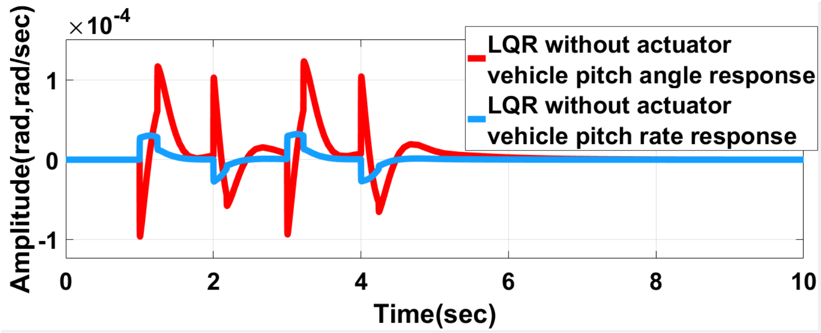
(b) Comparison of LQR without and with actuator dynamics for bouncing velocity

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (m/s)	$0.5*10^{-4}$	$2*10^{-5}$	96%
Settling time (sec)	$5second$	$4.5second$	11.11%
rise time (sec)	1.5	1.1	36.36%

From the Fig4.20a,and 4.20b its clear that, the chassis displacement (peak value) is reduced by 67.33%,85% and 32.67% ,15% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels and front wheels respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.20a ,and 4.20b are 5.6sec ,4.5sec, and 4.5sec,4.5sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 80.36%,100% and 19.64% ,0% LQR excluding actuator dynamics respectively.The rise time are 2.5sec,2.5sec,and 2.5sec,2.5sec for active sus-



(a) Simulation result of vehicle pitch dynamics with actuator body displacement using single downward bump for vehicle dynamics with actuator



(b) Simulation result of vehicle pitch dynamics body displacement using double downward bump for vehicle dynamics without actuator

pension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 100%,and 0%, in case of an active suspension system with actuator dynamics and excluding actuator dynamics pitch angle and pitch rate respectively.

From the Fig4.21a,and 4.21b its clear that, the chassis displacement (peak value) is reduced by 67.33%,60% and 32.67% ,40% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels and front wheels respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as we can observe from Fig 4.21a ,and 4.21b are 5sec ,1.5sec, and 4.5sec,1.4sec for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 88.89%,98.88% and 11.11% ,1.12% LQR excluding actuator dynamics respectively.The rise time are 0sec,and 1.6sec,4.5sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 100%,92.86%,and 0%,7.14% in case of an active suspension system with actuator dynamics and excluding actuator dynamics roll angle and roll rate respectively. From the Fig4.22a,and 4.22b its clear that, the chassis displacement (peak value) is reduced by 66.67%,63.33% and 33.33% ,36.67% in case of an active suspension system with actuator dynamics and excluding actuator dynamics force required the rear wheels and front wheels respectively, which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over active suspension system without actuator dynamics.The settling time, as

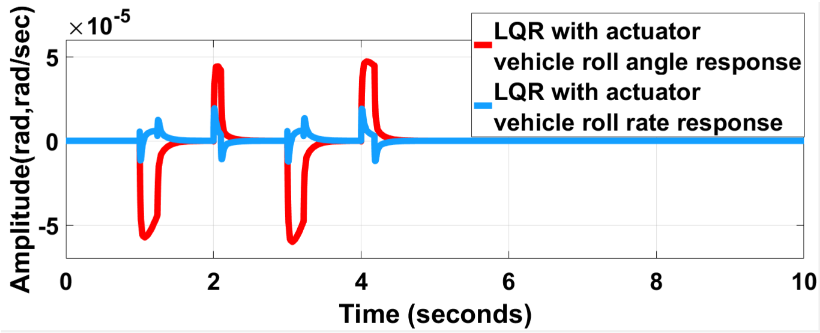
(a) Comparison of LQR without and with actuator dynamics for pitch angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$1.5*10^{-4}$	$4.9*10^{-5}$	67.33%
Settling time (sec)	$5.6second$	$4.5second$	19.64%
rise time (sec)	2.5	2.5	0%

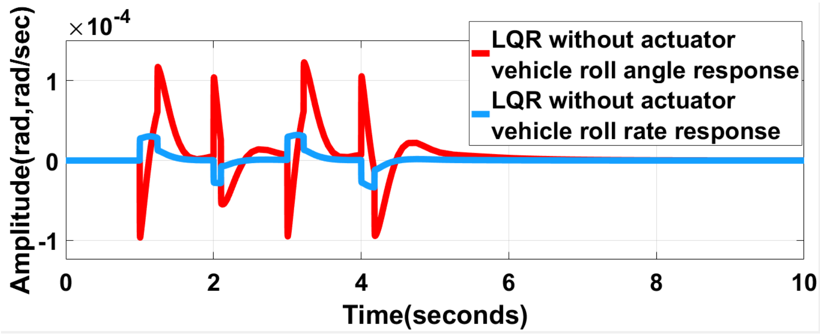
(b) Comparison of LQR without and with actuator dynamics for pitch rate

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/s)	$0.2*10^{-4}$	$3*10^{-5}$	85%
Settling time (sec)	$4.5second$	$4.5second$	0%
rise time (sec)	2.5	2.5	0%

we can observe from Fig 4.22a ,and 4.22b are 4.5sec ,5sec, and 4.5sec,4.5sec for rear wheels and front wheels for active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 100%,88.89% and 0% ,11.11% LQR excluding actuator dynamics respectively.The rise time are 2.1sec,2.5sec,and 2.45sec,2sec,for active suspension system without actuator dynamics and with actuator dynamics respectively.Thus reduction(improvement) in rise time for the active suspension including actuator dynamics are 95%,97.96%,and 5%,2.04% in case of an active suspension system with actuator dynamics and excluding actuator dynamics rear wheels and front wheels respectively. It is also important to simulate the system model for various road profile scenarios to check whether the system gives the expected result or not. The system should behave as required for all possible road profile scenarios as shown 4.5a,4.5b,4.6a,and 4.7a.



(a) Simulation result of vehicle roll dynamics with actuator body displacement using double bump for vehicle dynamics with actuator



(b) Simulation result of vehicle roll dynamics body displacement using double bump for vehicle dynamics without actuator

4.3 Comparisons of LQR with Fuzzy and PID Control

Under this section we would be compared the previous work with the current work and to look the over come of the LQR and the other controller such as PID and Fuzzy which was the previous work.

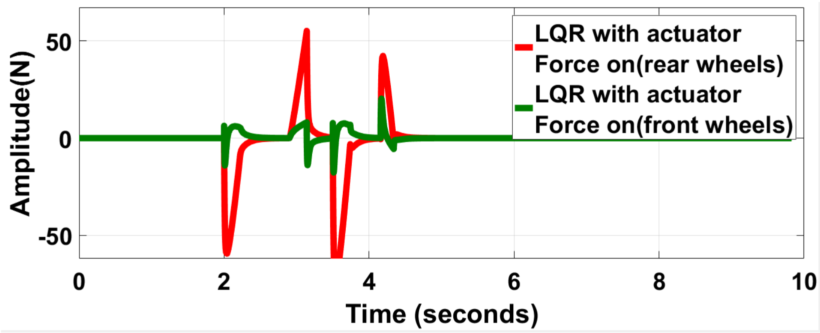
$$\overline{Ts_{iPID}} = \frac{\sum_{i=0}^{N-i} Ts_{iPID}}{N} \tag{4.2}$$

$$\overline{Ts_{iFuzzy}} = \frac{\sum_{i=0}^{N-i} Ts_{iFuzzy}}{N} \tag{4.3}$$

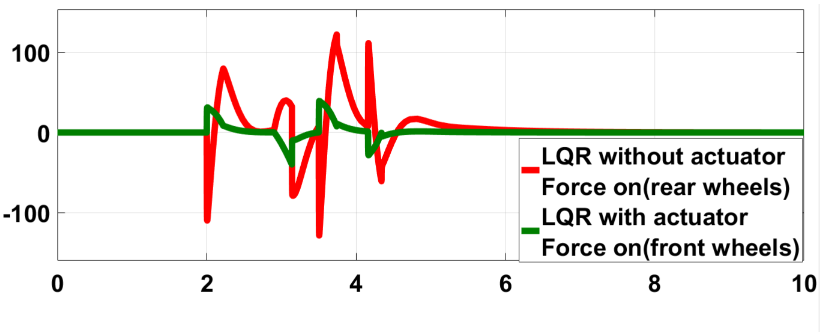
$$Hc_{RMS} = \sqrt{\frac{\sum_{i=0}^{N-i} (\overline{Ts_{iPID}} - TS_{iPID})^2}{N}} \tag{4.4}$$

Where
$\overline{Ts_{iPID}}$ is the mean of settling time in PID controller.
Ts is settling time for PID controller
N is number of settling time

$$Hc_{RMS} = \sqrt{\frac{\sum_{i=0}^{N-i} (\overline{Ts_{iFuzzy}} - TS_{iFuzzy})^2}{N}} \tag{4.5}$$



(a) Simulation result of force generated from front and rear wheels with actuator using downward double bump



(b) Simulation result of force generated from front and rear wheels without actuator using downward double bump

Where
$\overline{Ts_{iFuzzy}}$ is the mean of settling time in fuzzy controller.
Ts is settling time for Fuzzy controller
N is number of settling time
RMS root mean square

$$\%Re = \frac{LQR_{RMSvalue} - PID_{RMSvalue}}{PID_{RMSvalue}} \quad (4.6)$$

Where
$PID_{RMSvalue}$ is peak values(body displacement) attained due to pid controller.
$LQR_{RMSvalue}$ is peak values(body displacement) attained due to LQR controller
Re is reduction in error.

$$\%Re = \frac{LQR_{RMSvalue} - Fuzzy_{RMSvalue}}{Fuzzy_{RMSvalue}} \quad (4.7)$$

Where
$Fuzzy_{RMSvalue}$ is peak values(body displacement) attained due to fuzzy controller.
$LQR_{RMSvalue}$ is peak values(body displacement) attained due to LQR controller
Re is reduction in error.The road profile for individual wheel of full car model depicted in equations below. The unsprung mass will have be small alter in height of road Zu1 and Zu2 in order to compare the(PID,Fuzzy) and LQR's suspension system parameter of full

car model.

$$zu1 = \begin{cases} 0.15(1 - cos(2 * \pi * t)) & 1 \le t \le 2 \\ 0 & Otherwise \end{cases}$$

$$zu2 = \begin{cases} 0.1(1 - cos(2 * \pi * t)) & 1 \le t \le 2 \\ 0 & Otherwise \end{cases}$$

$$zu3, zu4 = \begin{cases} 0.1(1 - cos(2 * \pi * t)) & 6 \le t \le 7 \\ 0 & Otherwise \end{cases}$$

Table 4.18 lists the root mean squared values of suspension parameters used for simulation of full car model with current work. The active suspension of corner 1 has displacement of 2.4974cm and long settling in the PID controller time compare to LQR controller which has wheel displacement of 0.04cm and less settling time.Thus the reduction of LQR over PID controller previous work is 98.4%.This is direct indication of the superiority of active suspension system using LQR over PID controller.Similar case for the corner 2,3,and 4,and body displacement,body pitch angle,and body roll angle.

Table 4.19 lists the root mean squared values of suspension parameters used for simulation of full car model LQR. The active suspension of corner 1 has displacement of 2.4452cm and long settling in the Fuzzy controller time compare to LQR controller which has wheel displacement of 0.04cm and less settling time.Thus the reduction of LQR over Fuzzy controller is 98.4%.This is direct indication of the superiority of active suspension system using LQR with actuator dynamics over Fuzzy controller.Similar case for the corner 2,3,and 4,and body displacement,body pitch angle,and body roll angle.

Generally

The 7DOF(seven-degree-of-freedom) full car model has been developed based on linear characteristics as a result of its electro-hydraulic components has decreased the difficulty of creating mathematical model for active suspension system model also decreased. As we observe from table 4.18,and 4.19, the PID and fuzzy based controller do not give better result compared to LQR controller. The proposed LQR controller results have demonstrated that the magnitudes of the body displacement is decreased as well as the resonance peak value due to vehicle body is eliminated significantly compare to PID and fuzzy based controller. The simulation of proposed LQR controller for active suspension system has confirmed improvement of the ride comfort in vehicles.

(a) Comparison of LQR without and with actuator dynamics for roll angle

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad)	$1.5 * 10^{-4}$	$4.9 * 10^{-5}$	67.33%
Settling time (sec)	$5 second$	$4.5 second$	11.11%
rise time (sec)	-	-	-

(b) Comparison of LQR without and with actuator dynamics for roll rate

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (rad/s)	$0.5 * 10^{-4}$	$2 * 10^{-5}$	60%
Settling time (sec)	$1.5 second$	$1.4 second$	1.12%
rise time (sec)	4.5	1.6	7.14%

(a) Comparison of LQR without and with actuator dynamics for forces on front wheels

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	60	20	66.67%
Settling time (sec)	4.5*second*	4.5*second*	0%
rise time (sec)	2.1	2	5%

(b) Comparison of LQR without and with actuator dynamics for force on rear wheels

Performance specifications	LQR with out actuator dynamics	LQR with actuator dynamics	Reduction (improvement)
Peak response (amplitude) (N)	150	55	63.33%
Settling time (sec)	5*second*	4.5*second*	11.11%
rise time (sec)	2.5	2.45	2.04%

Table 4.18: Comparison of PID vs LQR

position	parameter	Root Mean Squared Value		
		PID	LQR	%Improvment(LQR over PID)
Corner1 (Font right wheel)	Displacement (cm)	2.4974	0.04	98.4↓
	Suspension Travel (cm)	0.2626	0.036	86.3↓
	Acceleration(m/s^2)	0.6583	0.017	97.4↓
Corner2 (Front left wheel)	Displacement (cm)	1.9673	0.07	96.44↓
	Suspension Travel (cm)	0.3512	0.028	92↓
	Acceleration(m/s^2)	0.5336	0.026	95↓
Corner3 (Rear right wheel)	Displacement (cm)	1.6943	0.042	97.5↓
	Suspension Travel (cm)	0.2965	0.005	98↓
	Acceleration(m/s^2)	0.4349	0.0063	98.5↓
Corner4 (Rear left wheel)	Displacement (cm)	1.6971	0.045	97↓
	Suspension Travel (cm)	0.3080	0.017	94.45↓
	Acceleration(m/s^2)	0.4363	0.004	99↓
Body	Displacement (cm)	1.4792	0.04	97.3↓
	Acceleration(m/s^2)	0.3917	0.0012	99.6↓
	Pitch angle (Deg)	0.5722	0.04	93↓
	Roll Angle (Deg)	0.3037	0.053	82.5↓

Table 4.19: Comparison of Fuzzy vs LQR

position	parameter	Root Mean Squared Value		
		Fuzzy	LQR	%Improvment(LQR over Fuzzy)
Corner1 (Front Right wheel)	Displacement (cm)	2.4452	0.04	98.4↓
	Suspension Travel (cm)	0.1745	0.036	79.4↓
	Acceleration(m/s^2)	0.7252	0.017	97.65↓
Corner2 (Front left wheel)	Displacement (cm)	1.9349	0.07	96.4↓
	Suspension Travel (cm)	0.2478	0.028	88.7↓
	Acceleration(m/s^2)	0.5063	0.026	95↓
Corner3 (Rear right wheel)	Displacement (cm)	1.666	0.042	97↓
	Suspension Travel (cm)	0.2407	0.005	98↓
	Acceleration(m/s^2)	0.4786	0.0063	98.7↓
Corner4 (Rear left wheel)	Displacement (cm)	1.674	0.045	99.3↓
	Suspension Travel (cm)	0.2874	0.017	94↓
	Acceleration(m/s^2)	0.48654	0.004	99.2↓
Body	Displacement (cm)	1.442	0.04	97.2↓
	Acceleration(m/s^2)	0.4288	0.0012	99.7↓
	Pitch angle (Deg)	0.5667	0.04	93↓
	Roll Angle (Deg)	0.2927	0.053	81.9↓

Chapter 5

Conclusions and Scope for Future Work

5.1 Conclusions

Active vehicle suspension system is one part of the essential mechatronic system. In this research paper,we did with some assumptions, the model of the active suspension system was developed and the state feedback controller (LQR) was designed.The lyapunov energy principle proved. By comparing the performance of the LQR without actuator dynamics and LQR with the actuator dynamics based on different road profile, the simulation results clearly indicated that LQR with actuator dynamics gave lower amplitude and faster settling time. The reduced value of peak response resulted in less sprung-mass travel and hence, the reduced vibrations felt by the passenger. The less settling time quickly suspended the oscillations induced in the car body which ensured better comfort to the passenger. Therefore, the proposed LQR controller with actuator dynamics which included in the system model was more effective in the vibration isolation of the car body than the LQR without actuator dynamics.This is because the vehicle model in minimal form and the number of sensors required to install in the vehicle dynamics were reduced and also the cost of the sensor was minimized. The simulation of proposed LQR controller for active suspension system has confirmed improvement of the ride comfort in vehicles. So, the proposed LQR controller with the selected weighting matrices Q and R is acceptable.

5.2 Scope for Future Work

Though the LQR controller in this thesis is recommended to control the oscillation of the car body, there are different techniques to select the weighting matrices Q and R to obtain the best optimal performance. The weighting matrices is tuned manually which needs experience otherwise it is something tedious. For future, we suggested to select the weighting matrices Q and R to design the LQR controller by using a technique that directly addresses the best optimal performance despite the presence of multi-objective function in the system. These are some evolutionary optimization techniques, like Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) will be used to optimize the multi-objective functions which are beyond the study.

This work has proposed some foundation of Linear quadratic (LQR) design strategies necessary to describe some characteristics of an vehicle dynamics, so its advisable to use for any vehicle dynamics. Although the linear quadratic regulator algorithm and its simulation presented here in can prove an useful intuitive understanding of generalized performance and control characteristics of the vehicle dynamics. Understanding the performance of what has been done here is essential in any future development of on-line and advanced control algorithm , even if an off line controlling mechanism has been proposed . Having said this, we can now consider some of the natural important of this research that must be considered on-line control algorithm(model predictive control ,adaptive model predictive control,and more advanced controls) at the same principle in any future work.

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Appendex A

Before minimal realization
state space matrix

$$A11 = 0_{7X7}$$

$$A12 = (eye)_{7X7}$$

$$A21 = \left(\begin{pmatrix} -76.72 & 0 & 0 & -0.0025 & -0.0025 & -0.0025 & -0.0025 \\ 0 & -84.5 & -18.43 & -0.00042 & -0.00042 & -0.00042 & -0.00042 \\ 0 & -18.43 & -98.44 & -0.00088 & -0.00088 & -0.00088 & -0.00088 \\ 290.95 & 662.56 & 576.14 & -576.16 & 0 & 0 & 0 \\ -290.95 & 662.56 & 576.14 & 0 & -0.016 & 0 & 0 \\ 179.85 & -539.55 & 327 & 0 & 0 & -0.016 & 0 \\ 179.85 & -539.55 & 327 & 0 & 0 & 0 & -0.016 \end{pmatrix}\right)$$

$$A22 = \left(\begin{pmatrix} -9.45 & 0 & 0 & -0.0025 & -0.0025 & -0.0025 & -0.0025 \\ 0 & 14.53 & 0.286 & -0.00042 & -0.00042 & -0.00042 & -0.00042 \\ 0 & 0.286 & -12.1 & -0.00088 & -0.00088 & -0.00088 & -0.00088 \\ 31.45 & 71.63 & 62.3 & -0.0158 & 0 & 0 & 0 \\ -31.45 & 71.63 & 62.3 & 0 & -0.0158 & 0 & 0 \\ 26.98 & -80.93 & -49.05 & 0 & 0 & -0.0158 & 0 \\ -26.98 & -80.93 & -49.05 & 0 & 0 & 0 & -0.0158 \end{pmatrix}\right)$$

Active suspension system matrix

$$Bu1(t) = 0_{7X4}$$

$$Bu2(t) = \left(\begin{pmatrix} 0.0012625 & 0.0012625 & 0.0012625 & 0.0012625 \\ 0.00048 & 0.00048 & 0.00048 & 0.00048 \\ -0.00088 & -0.00088 & -0.00088 & -0.00088 \\ -0.0158 & 0 & 0 & 0 \\ 0 & -0.0158 & 0 & 0 \\ 0 & 0 & -0.016 & 0 \\ 0 & 0 & 0 & -0.016 \end{pmatrix}\right)$$

passive suspension system matrix

$$F1(t) = 0_{10X4}$$

$$F2(t) = \left(\begin{pmatrix} 2896.34 & 0 & 0 & 0 \\ 0 & 2896.34 & 0 & 0 \\ 0 & 0 & 3041.16 & 0 \\ 0 & 0 & 0 & 3041.16 \end{pmatrix}\right)$$

Where

$$A = 14X14 = \begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix}, BU(t) = 14X4 = \begin{bmatrix} Bu1(t) \\ Bu2(t) \end{bmatrix}, F(t) = 14X4 = \begin{bmatrix} F1(t) \\ F1(t) \end{bmatrix}$$

$$C = eye_{2X14}$$
$$D = 0_{2X4}$$

Appendex B

After minimal realization

$$A1 = \left(\begin{bmatrix} 27.32 & 12.11 & 0.1663 & 2.727 & 2.9 \\ -275.1 & -121.4 & -3.54 & -31.31 & -42.68 \\ -35.37 & -33.52 & -18.11 & -19.93 & -39.73 \\ 49.79 & 77.02 & 0.7689 & 9.864 & 12.6 \\ 418.4 & 166.1 & 12.27 & 51.02 & 72.38 \\ 105.9 & 42.05 & 2.669 & 10.18 & 18.45 \\ -19.99 & -22.66 & -2.911 & -5.184 & -8.844 \\ -231.7 & -88.77 & -2.152 & -23.41 & -31.65 \\ 57.6 & 21.44 & 0.5102 & 5.629 & 7.659 \\ -2.021 & 0.8847 & 0.3647 & 0.09912 & 0.4104 \end{bmatrix}\right)$$

$$A2 = \left(\begin{bmatrix} 4.626 & 6.591 & -14.13 & -51.7 & -9.392 \\ -41.13 & -73.93 & 129 & 458.9 & 140 \\ 50.74 & -58.53 & -94.78 & -333.1 & 306.7 \\ 95.3 & 9.446 & -130.4 & -442.9 & 258.8 \\ 8.509 & 128.9 & -102.6 & -366.4 & -444.4 \\ 9.121 & 25.04 & -54.22 & -220.8 & -115.3 \\ -20.15 & -3.873 & 25.54 & 107.7 & 29.35 \\ -17.86 & -62.17 & 107.8 & 383.3 & 102.8 \\ 3.666 & 15.42 & -27.14 & -96.71 & -29.4 \\ 1.621 & -0.2813 & 1.607 & 5.955 & 6.55 \end{bmatrix}\right)$$

$$B1 = \left(\begin{bmatrix} -0.0007472 & -0.0002523 & -0.0002556 & -0.0002556 \\ 0.007674 & 0.001742 & 0.001764 & 0.001764 \\ 0.001594 & -0.005549 & -0.005627 & -0.005627 \\ -0.002254 & -0.006931 & -0.007009 & -0.007009 \\ -0.01206 & 0.00148 & 0.001504 & 0.001504 \\ -0.002615 & 0.0002514 & 0.0002505 & 0.0002505 \\ 0.001085 & 0.001254 & 0.001263 & 0.001263 \\ 0.005489 & -6.253*10^{-5} & -5.143*10^{-5} & -5.143*10^{-5} \\ -0.001284 & 0.0001962 & 0.0001947 & 0.0001947 \\ -1.332*10^{-5} & -0.0001266 & -0.0001273 & -0.0001273 \end{bmatrix}\right)$$

$$B2 = \left(\begin{bmatrix} 139.4 & 48.66 & 51.09 & 51.09 \\ -1401 & -313.8 & -329.5 & -329.5 \\ -181.7 & 1128 & 1184 & 1184 \\ 262.5 & 1120 & 1176 & 1176 \\ 2128 & -353.5 & -371.2 & -371.2 \\ 538.9 & 13.39 & 14.06 & 14.06 \\ -103.9 & -134.9 & -141.6 & -141.6 \\ -1178 & -160.7 & -168.8 & -168.8 \\ 292.8 & 21.45 & 22.52 & 22.52 \\ -10.03 & 10.74 & 11.28 & 11.28 \end{bmatrix}\right)$$

$$C1 = \begin{bmatrix} -0.02191 & 0.7149 & 0.07886 & 0.273 & 0.3606 \\ -2.197*10^{-5} & -2.618*10^{-5} & -0.01429 & 0.02454 & 0.006495 \end{bmatrix}$$

$$C2 = \begin{bmatrix} 0.3872 & 0.3572 & -0.008193 & -0.003523 & -4.74*10^{-5} \\ -0.04203 & 0.01898 & -0.06527 & -0.2894 & -0.9534 \end{bmatrix}$$

Where

$$A = 10X10 = \begin{bmatrix} A1 & A2 \end{bmatrix}, B = 10X8 = \begin{bmatrix} B1 & B2 \end{bmatrix}, C = 2X10 = \begin{bmatrix} C1 & C2 \end{bmatrix}$$

$$D = 0_{2X8}$$

$$P1 = 10^4 \begin{bmatrix} 3.6890 & 1.3378 & 0.2769 & 0.7375 & 1.4563 \\ 1.3378 & 3.3673 & 0.4879 & 1.6806 & 2.7824 \\ 0.2769 & 0.4879 & 0.1433 & 0.1769 & 0.4182 \\ 0.7375 & 1.6806 & 0.1769 & 0.9070 & 1.3896 \\ 1.4563 & 2.7824 & 0.4182 & 1.3896 & 2.3504 \\ 1.1437 & 2.5238 & 0.4114 & 1.2154 & 2.1644 \\ 1.4370 & 1.7835 & 0.2369 & 0.9555 & 1.4628 \\ 0.4491 & 0.0733 & 0.0398 & 0.0225 & 0.1612 \\ -0.4208 & -0.1688 & -0.0375 & -0.0870 & -0.1920 \\ 1.3826 & 0.5181 & 0.1011 & 0.2898 & 0.5557 \end{bmatrix}$$

$$P2 = 10^4 \begin{bmatrix} 1.1437 & 1.4370 & 0.4491 & -0.4208 & 1.3826 \\ 2.5238 & 1.7835 & 0.0733 & -0.1688 & 0.5181 \\ 0.4114 & 0.2369 & 0.0398 & -0.0375 & 0.1011 \\ 1.2154 & 0.9555 & 0.0225 & -0.0870 & 0.2898 \\ 2.1644 & 1.4628 & 0.1612 & -0.1920 & 0.5557 \\ 2.2289 & 0.8923 & 0.3525 & -0.2103 & 0.4235 \\ 0.8923 & 1.8069 & -0.2715 & -0.0715 & 0.5705 \\ 0.3525 & -0.2715 & 0.2971 & -0.1094 & 0.1539 \\ -0.2103 & -0.0715 & -0.1094 & 0.0625 & -0.1545 \\ 0.4235 & 0.5705 & 0.1539 & -0.1545 & 0.5197 \end{bmatrix}$$

$$K1 = \begin{bmatrix} 0.6010 & 0.0811 & -3.6798 & -2.1836 & -5.7238 \\ 5.4896 & -0.9549 & 0.1252 & 1.3610 & 1.3018 \\ 2.9927 & 1.4817 & -0.2809 & -0.2262 & -0.8365 \\ -5.8715 & -5.3659 & -0.4158 & -1.1364 & -2.7709 \\ -0.0879 & -6.4711 & 0.5973 & 1.5745 & 4.7167 \\ -4.0576 & 5.0335 & 5.3202 & -0.2962 & 2.5462 \\ 3.1802 & -2.9587 & 6.0423 & -6.3595 & -2.2771 \\ 0.6114 & -1.1297 & 4.5103 & 7.0167 & -5.2768 \end{bmatrix}$$

$$k2 = \begin{bmatrix} 4.6376 & 1.9967 & -4.7259 & 1.5213 & 1.7559 \\ 1.9167 & 7.2708 & 4.4583 & 6.7242 & 0.0071 \\ -8.0108 & 2.4204 & -4.7064 & 6.8166 & 6.4984 \\ -4.2448 & 5.4250 & 0.1999 & -1.3728 & 6.9121 \\ 1.9517 & -0.8910 & -5.1890 & 1.3844 & 0.1441 \\ 3.1149 & 3.2162 & -2.3402 & 0.8659 & -1.5685 \\ -0.3099 & -1.2070 & 0.3063 & -0.2327 & 0.8796 \\ 0.1302 & -0.8016 & -0.6000 & 0.0903 & 0.5837 \end{bmatrix}$$

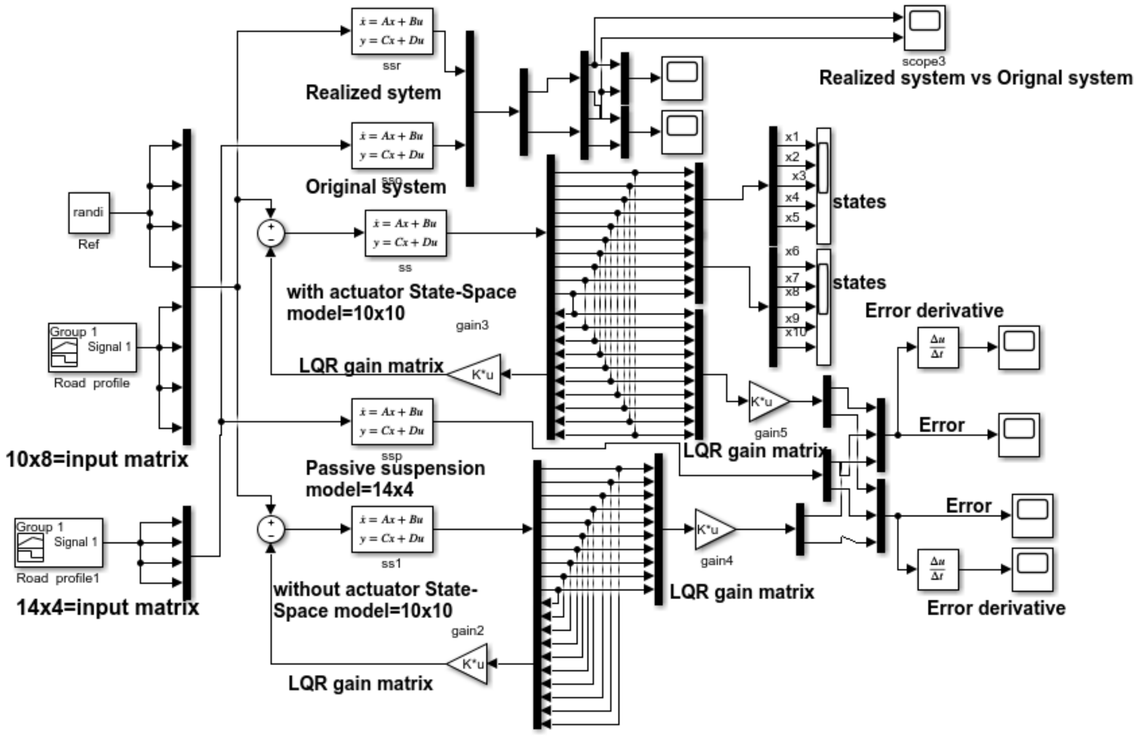
where

$$P = 10X10 = \begin{bmatrix} P1 & P2 \end{bmatrix}, K = 10X10 = \begin{bmatrix} k1 & k2 \end{bmatrix}$$

P is aljeberic raccit equation solution(ARE),k is state full feed back gain.

$$Q = 10X10 = 10000 * eye(10X10)$$
$$R = 10X8 = 0.01 * eye(10X8)$$

Where
Q is state weighting matrix
R is input weighting matrix



(a) The SIMULINK diagram of LQR based ASS and PSS

Appendix D

Table 5.1: vehicle suspension parameters

parameters for full car model	values	units
ms	1136	kg
muf	63	kg
mur	60	kg
Ip	2400	kgm^2
Ir	400	kgm^2
kf	36297	N/m
kr	19620	N/m
Tf	0.505	m
Tr	0.55	m
$ktf = ktr$	182470	N/m
bf	3924	N/m
br	2943	N/m
a	1.15	m
b	1.65	m

Table 5.2: Actuator parameters

Hydraulic actuuator parameters	values	units
Mb_1	40	kg
Mb_2	2	kg
Mb_3	2	kg
Jb_1	100	kgm^2
Jb_2	100	kgm^2
Kb_1	120	N/m
Kb_2	120	N/m
Cb_1	110	Nm/s
Cb_2	110	Nm/s
Kb_3	125	N/m
Cb_3	115	Nm/s
$\triangle_1$	0.035	m
$\triangle_2$	0.035	m
$Q(t)$	0.667	$\frac{1}{s}$
F_{frl2}	68.7	kN
F_{frl4}	68.7	kN
S	0.00746	m^2
$V0$	4440	m^3
M_b	503	Nm
R_a	0.15	m
f	0.99	-
k(p)	100	pa
$\dot{p}$	80	pa/s