Region Tracking Control for Robot Manipulators

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Abstract—Research of robotics aims to realize some aspects of human functions into a mechanical system. It is interesting to observe from most human reaching movements that the desired targets are regions rather than points. In this paper, a region tracking controller is proposed for robot manipulator. In the proposed controller, the desired objective of the robot end effector can be specified as a moving region. Lyapunov-like function is presented for the convergence analysis. Simulation results are presented to verify the theory.

I. INTRODUCTION

In many applications of robots, the desired path for the end-effector is specified in task space such as Cartesian space. In order for the robot to follow the desired path, one way is to solve the inverse kinematic problem to generate the desired angle in joint space [1]-[5]. When the control problem is formulated directly in task space [1], [6], [7], the need to solve the inverse kinematics problem is eliminated. When robots pick up some objects with unknown masses, the problem of dynamic uncertainty arises. To deal with trajectory tracking control problems in presence of dynamic uncertainty, several adaptive tracking controllers have been proposed [2], [11], [13], [14].

The results in [1]-[5] are focusing on setpoint control where the desired target is specified as a point. However it is interesting to observe in our daily activities that the desired target for our reaching movements is a region rather than a point or trajectory. For example, placing an object into a cup, throwing a dart to a dart board, picking up an object from a conveying belt, driving a car along a road etc. Observing these aspects, region reaching controllers have been proposed recently in [15], [16]. However, the region reaching controllers in [15], [16] are focusing on reaching for a stationary region. In some applications such as driving a car along a road, placing an object on a moving conveying belt, shooting a moving target etc. the desired region is not a static region but a moving region.

In this paper, adaptive region tracking controller is proposed for robot with dynamic uncertainty. In the proposed adaptive controller, the desired target can be specified as a moving region. Lyapunov-like functions are presented for the convergence analysis. Simulation results are presented to verify the theory.

II. ROBOT KINEMATICS AND DYNAMICS

In order to describe a task for the robot manipulator, the desired path for the end-effector is usually specified in task space. Let \( x \in \mathbb{R}^n \) represents the position vector of the manipulator in task space defined by [2], [8]:

\[
x = h(q),
\]

where \( q \in \mathbb{R}^n \) is a vector of generalized joint coordinates, \( h(\cdot) \in \mathbb{R}^n \rightarrow \mathbb{R}^n \) is generally a nonlinear transformation describing the relation between the joint and task space. The velocity vector \( \dot{x} \) is therefore related to \( \dot{q} \) as:

\[
\dot{x} = J(q)\dot{q},
\]

where \( J(q) \in \mathbb{R}^{n \times n} \) is the Jacobian matrix of mapping from joint space to task space. The equations of motion of the robotic manipulator with \( n \) degrees of freedom is given in joint space as [2], [9], [10]:

\[
M(q)\ddot{q} + \frac{1}{2}M(q) + S(q, \dot{q})\dot{q} + g(q) = \tau,
\]

where \( M(q) \in \mathbb{R}^{n \times n} \) is an inertia matrix, \( g(q) \in \mathbb{R}^n \) denotes a gravitational force vector, \( \tau \in \mathbb{R}^n \) denotes the control inputs, and,

\[
S(q, \dot{q})\dot{q} = \frac{1}{2}M(q)\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}q^TM(q)\dot{q}^T
\]
Property 1: The inertia matrix $M(q)$ is symmetric and positive definite for all $q \in \mathbb{R}^n$.

Property 2: $S(q, \dot{q})$ is skew-symmetric for all $q, \dot{q} \in \mathbb{R}^n$.

Property 3: The dynamic model as described by equation (3) is linear in a set of physical parameters $\theta = (\theta_d, \cdots, \theta_{dp})^T$ as

$$M(q)\ddot{q} + \frac{1}{2}M(q) + S(q, \dot{q})\dot{q} + g(q) = Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\theta_d = \tau.$$ (18)

where $Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\theta_d = M(q)\ddot{q}_r + \frac{1}{2}\dot{M}(q)\dot{q}_r + g(q)$. (19)

III. ADAPTIVE REGION TRACKING CONTROL OF ROBOTS

In this section, Adaptive Region Tracking controllers with single and multiple objective functions are presented. Stability analysis of the closed-loop system is presented with the aid of Lyapunov-like function.

A. Adaptive Region Tracking Controller with Single Objective Function

We first consider a region reaching control problem where the desired moving region is specified by an inequality function as follows:

$$f(\Delta x) \leq 0, \quad \text{where} \quad \Delta x = x - x_o, \quad x_o(t) \text{ is a reference point inside the desired region} \quad f(\Delta x) \in \mathbb{R} \text{ is a scalar function that satisfy} \quad f(\Delta x) \to \infty \text{ as} \quad ||\Delta x|| \to \infty \text{ and the desired region is moving at a speed of} \quad \dot{x}_o. \quad \text{An illustration of the moving desired region is shown in figure 2. Let} \quad f(\Delta x) \text{ be chosen in such a way that the boundedness of} \quad f(\Delta x) \text{ ensures the boundedness of} \quad \Delta x. \quad \text{In addition,} \quad f(\Delta x) \text{ is chosen to be a continuous function with continuous partial derivatives. For example, if} \quad x = [x_1, x_2]^T \in \mathbb{R}^2, \text{ a desired region can be specified as a circle as:}$$

$$f(\Delta x) = (x_1 - x_{o1})^2 + (x_2 - x_{o2})^2 - r^2 \leq 0, \quad \text{where} \quad r \text{ is a constant radius of the circle,} \quad (x_{o1}(t), x_{o2}(t)) \quad \text{is the center of a circle. As seen from the above example,} \quad \Delta x \quad \text{is bounded if} \quad f(\Delta x) \quad \text{is bounded.}$$

The potential energy function for the region tracking controller in task space is defined as follow:

$$P_a(\Delta x) = \frac{k_p}{2}[max(0, f(\Delta x))]^2. \quad \text{(7)}$$

That is,

$$P_a(\Delta x) = \begin{cases} 0, & f(\Delta x) \leq 0, \\ \frac{k_p}{2}f^2(\Delta x), & f(\Delta x) > 0, \end{cases} \quad \text{where} \quad k_p \text{ is a positive constant. Partial differentiating the potential energy (8) with respect to} \quad \Delta x, \quad \text{we have,}$$

$$\frac{\partial P_a(\Delta x)}{\partial \Delta x}^T = \begin{cases} 0, & f(\Delta x) \leq 0, \\ k_p f(\Delta x)(\frac{\partial f(\Delta x)}{\partial \Delta x})^T, & f(\Delta x) > 0, \end{cases} \quad \text{(9)}$$

Next, we let a region error

$$\delta \xi = max(0, f(\Delta x))(\frac{\partial f(\Delta x)}{\partial \Delta x})^T \quad \text{(11)}$$

and define a vector $\dot{x}_r \in \mathbb{R}^n$ as

$$\dot{x}_r = \ddot{x}_o - \alpha \delta \xi \quad \text{(12)}$$

where $\alpha \text{ is a positive constant. Differentiating equation (12) with respect to time we get}$$

$$\ddot{x}_r = \dot{x}_o - \alpha \ddot{\xi} \quad \text{(13)}$$

where $\dot{x}_o \in \mathbb{R}^n \text{ is the acceleration of the desired region in task space. Next, let}$

$$\dot{q}_r = J^{-1}(q)\dot{x}_r \quad \text{(14)}$$

Differentiating equation 14 with respect to time yields

$$\ddot{q}_r = J^{-1}(q)\ddot{x}_r + J^{-1}(q)\dot{\dot{x}}_r \quad \text{(15)}$$

A sliding vector is then defined in joint space as

$$s = \dot{q} - \dot{q}_r = J^{-1}(q)(\ddot{x} - \dot{x}_r) = J^{-1}(q)(\Delta \ddot{x} + \alpha \delta \xi) \quad \text{(16)}$$

where $\Delta \ddot{x} = \ddot{x} - \dot{x}_o$. Differentiating equation (16) with respect to time, we have

$$\dot{s} = \dddot{q} - \dddot{q}_r = J^{-1}(q)(\Delta \dddot{x} + \alpha \delta \dddot{\xi}) + J^{-1}(q)(\Delta \ddot{x} + \alpha \delta \dddot{\xi}) \quad \text{(17)}$$

where $\Delta \ddot{x} = \ddot{x} - \dot{x}_o$. Substituting equations (16) and (17) into equation (3) and using Property 3, we have

$$M(q)\ddot{s} + \frac{1}{2}M(q) + S(q, \dot{q})s + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\theta_d = \tau \quad \text{(18)}$$

where

$$Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\theta_d = M(q)\ddot{q}_r + \frac{1}{2}M(q) + S(q, \dot{q})\dot{q}_r + g(q) \quad \text{(19)}$$
In this paper, we propose an adaptive region tracking controller as
\[ \tau = -J^T(q)(K_v \Delta \dot{x} + K_p \delta \xi) + Y_d(q, \dot{q}, \dot{q}_r, \ldots) \] (20)
where \( K_v = k_p I \) and \( K_p = k_v I \), \( k_v \) is a positive constant and \( I \) is an identity matrix.
The estimated dynamic parameters \( \hat{\theta}_d \) are updated by
\[ \hat{\theta}_d = -L_d Y_d^T(q, \dot{q}, \dot{q}_r, \dot{q}_r) \] (21)
where \( L_d \) is a positive definite matrix.
The closed-loop dynamic equation is obtained by substituting equation (20) into equation (18):
\[ M(q)s + \left( \frac{1}{2} \dot{M}(q) + S(q, \dot{q}) \right) s + Y_d(q, \dot{q}, \dot{q}_r, \dot{q}_r) \Delta \dot{\theta}_d + J^T(q) (K_v \Delta \dot{x} + K_p \delta \xi) = 0 \] (22)
where \( \Delta \dot{\theta}_d = \dot{\theta}_d - \dot{\theta}_d \). Let us define a Lyapunov-like function as
\[ V = \frac{1}{2}s^T M(q)s + \frac{1}{2} \Delta \dot{\theta}_d^T L_d^{-1} \Delta \dot{\theta}_d + \frac{1}{2}(k_p + \alpha k_v) \max(0, f(\Delta x))^2 \] (23)
Differentiating equation (23) with respect to time and using property 1 and property 2 we have
\[ \dot{V} = s^T M(q)s + \frac{1}{2} \Delta \dot{\theta}_d^T L_d^{-1} \Delta \dot{\theta}_d + \Delta \dot{x}^T (k_p + \alpha k_v) \max(0, f(\Delta x)) \left( \frac{\partial f(\Delta x)}{\partial \Delta x} \right)^T \] (24)
Substituting \( M(q)s \) from equation (22) and \( \hat{\theta}_d \) from equation (21) into equation (24) we have
\[ \dot{V} = -s^T J^T(q)(k_v \Delta \dot{x} + k_p \delta \xi) + \Delta \dot{x}^T (k_p + \alpha k_v) \delta \xi \] (25)
Substituting equation (16) into equation (25) we get
\[ \dot{V} = -(\Delta \dot{x} + \alpha \delta \xi)^T (k_v \Delta \dot{x} + k_p \delta \xi) + \Delta \dot{x}^T (k_p + \alpha k_v) \delta \xi = -\Delta \dot{x}^T k_v \Delta \dot{x} - \alpha \delta \xi^T k_p \delta \xi \] (26)

We are now in a position to state the following theorem:

**Theorem 1**: For a finite work space such that the Jacobian matrix is non-singular, the adaptive control law (20) and the parameter update law (21) for the robot system (3) guarantees the convergence of the region and velocity tracking errors. That is, \( x \) converges to the moving desired region \( f(\Delta x) \leq 0 \) and \( \dot{x} \to \dot{x}_o \) as \( t \to \infty \).

**Proof**: Since \( M(q) \) is uniformly positive definite, \( V \) in equation (23) is positive definite in \( s, \Delta \dot{\theta}_d \), and \( \max(0, f(\Delta x)) \). Hence, \( s, \Delta \dot{\theta}_d \) and \( f(\Delta x) \) are bounded. The boundness of \( f(\Delta x) \) ensures the boundness of \( \Delta x \).

Hence, \( \frac{\partial f(\Delta x)}{\partial \Delta x} \) and \( \frac{\partial^2 f(\Delta)}{\partial (\Delta x)^2} \) are also bounded. Therefore, \( \delta \xi \) is also bounded. Next, \( \dot{x} \) is bounded if \( \dot{x}_o \) is bounded as can be seen from equation (12). The boundness of \( \dot{x}_e \) ensures the boundness of \( \dot{q}_r \) if \( J(q) \) is nonsingular. Therefore, \( \dot{q} \) in equation (16) is also bounded since \( s \) is bounded. The boundness of \( \dot{q} \) guarantees the boundness of \( \dot{x} \) since \( J(q) \) is a trigonometric function of \( q \). Hence, \( \Delta x \) is bounded if \( \dot{x}_o \) is bounded. Differentiating equation (11) with respect to time yields
\[ \delta \xi = \begin{cases} \dot{f}(\Delta x)(\frac{\partial f(\Delta x)}{\partial \Delta x})^T, & f(\Delta x) \leq 0 \\ +f(\Delta x)(\frac{\partial^2 f(\Delta x)}{\partial (\Delta x)^2}) \dot{\Delta x}, & f(\Delta x) > 0 \end{cases} \] (27)
where \( \dot{f}(\Delta x) = (\frac{\partial f(\Delta x)}{\partial \Delta x}) \dot{\Delta x} \). Since \( \frac{\partial f(\Delta x)}{\partial \Delta x} \) and \( \frac{\partial^2 f(\Delta x)}{\partial (\Delta x)^2} \) are bounded, \( \delta \xi \) is therefore bounded. From equation (13), \( \dot{x} \) is bounded if \( \dot{x}_o \) is bounded. Therefore, \( \dot{q} \), in equation (15) is bounded if \( J(q) \) is nonsingular. From the closed-loop equation (22), we can conclude that \( s \) is bounded. Therefore, \( \dot{q} \) is bounded as seen from equation (17). From equation (17), \( \Delta x \) is also bounded since \( \Delta \dot{x}, \delta \xi, \delta \xi \) are bounded. Differentiating equation (26) with respect to time we get
\[ \dot{V} = -2(\Delta \dot{x}^T k_v \Delta \dot{x} + \alpha \delta \xi^T k_p \delta \xi) \] (28)
Hence, \( \dot{V} \) is bounded since \( \Delta \dot{x}, \Delta \dot{x}, \delta \xi, \delta \xi \) are bounded. Therefore, \( \dot{V} \) is uniformly continuous. Applying Barbalat’s lemma [12], we have \( \Delta \dot{x} \to 0 \) and \( \delta \xi \to 0 \). The convergence of \( \delta \xi \) to zero implies that \( f(\Delta x) \to 0 \). Therefore, \( x \) converges to moving desired region \( f(\Delta x) \leq 0 \) and \( \dot{x} \) converges to \( \dot{x}_o \) as \( t \to \infty \).

B. Adaptive Region Tracking Controller with Multiple Objective Functions

In the previous subsection, the region tracking controller with single objective function was presented. This section will discuss the extension of the result in the previous subsection to the case of multiple objective functions. That is, the desired region is specified as intersection of multiple function as follows:
\[ f(\Delta x) = [f_1(\Delta x_1), f_2(\Delta x_2), ..., f_N(\Delta x_N)]^T \leq 0, \] (29)
where \( \Delta x_i = x - x_{oi}, x_{oi}(t) \) is the reference point within the \( i \)th desired region, \( i = 1, 2, ..., N \), \( N \) is the total number of objective functions, \( f_i(\Delta x_i) \) are continuous scalar functions with continuous partial derivatives that satisfy \( f_i(\Delta x_i) \to \infty \) as \( ||\Delta x_i|| \to \infty \). Each region is chosen to have the same speed \( \dot{x}_o \). For example, a desired region can be specified as an intersection of two circles shown in figure 3. In this case, the objective functions are specified as:
\[ f_1(\Delta x_1) = (x_1 - x_{o1})^2 + (x_2 - x_{o2})^2 - r_1^2 \leq 0, \]
\[ f_2(\Delta x_2) = (x_1 - x_{o2})^2 + (x_2 - x_{o3})^2 - r_2^2 \leq 0, \] (30)
where \( r_1, r_2 \) are the constant radii of the two circles, \( (x_{o1}(t), x_{o2}(t)) \) and \( (x_{o21}(t), x_{o22}(t)) \) represent the centers of the two circles. In this example, the two circles move at the same speed if the centers of the two circles are chosen as:
\[ x_{o1}(t) = x_{o21}(t) + c_1, \quad x_{o12}(t) = x_{o22}(t) + c_2 \]
where \( c_1 \) and \( c_2 \) are constant offsets.
Fig. 3. Trajectory of Desired Region as an Intersection of two Circles

Figure 4 shows another example where the desired region is a cube specified by,

\[ f_1(\Delta x_1) = (x_1 - x_{o11})^2 - a_1^2 \leq 0, \]

\[ f_2(\Delta x_2) = (x_2 - x_{o22})^2 - a_2^2 \leq 0, \]

\[ f_3(\Delta x_3) = (x_3 - x_{o33})^2 - a_3^2 \leq 0, \]

(31)

where \( a \) represents half the length of the cube and \( (x_{o11}, x_{o22}, x_{o33}) \) is the centroid of the cube. The speed of each region in this case is independent of each other and the speed of the centroid of the cube is therefore \( \dot{x}_o = [\dot{x}_{o11}, \dot{x}_{o22}, \dot{x}_{o33}]^T. \)

Fig. 4. Trajectory of Desired Region as a Cube

The potential energy function for the region tracking controller with multiple functions is defined as follows:

\[ P_b(\Delta x) = \sum_{i=1}^{N} P_{b_i}(\Delta x_i), \]

(32)

where

\[ P_{b_i}(\Delta x_i) = \frac{k_{pi}}{2} \max(0, f_i(\Delta x_i))^2 \]

and \( k_{pi} \) are positive constants. Note that \( P_b(\Delta x) = 0 \) only if all the objective functions in (29) are satisfied.

The potential function is chosen in such a way that it has a unique minimum region as the desired region. Partial differentiating the potential energy function described by equation (32) and equation (33) with respect to \( \Delta x, \) we have,

\[ \frac{\partial P_b(\Delta x)}{\partial \Delta x} = \sum_{i=1}^{N} k_{pi} \max(0, f_i(\Delta x_i)) \frac{\partial f_i(\Delta x_i)}{\partial \Delta x_i} \]

(34)

The adaptive region tracking controller with multiple objective functions is proposed as:

\[ \tau = -J^T(q)(K_p \Delta \dot{x} + K_p \delta \dot{\xi}) + Y_d(q, \dot{q}, \ddot{q}, \dddot{q}, \theta) \]

(35)

where \( \dot{q}, \ddot{q}, \theta \) are bounded. Hence, \( \dot{V} \) is uniformly continuous. Applying Barbalat’s lemma [12], we have \( \dot{\delta \dot{\xi}} \rightarrow 0. \) The convergence of \( \delta \dot{\xi} \) to zero implies that \( f_i(\Delta x_i) \leq 0 \)
for \( i = 1, 2, 3 \ldots N \) since potential function is chosen to have a unique minimum region at the desired region. Therefore, \( x \) converges to the moving desired region \( f(\Delta x) = [f_1(\Delta x_1), f_2(\Delta x_2), \ldots, f_N(\Delta x_N)]^T \leq 0 \) and \( \dot{x} \) converges to \( \dot{x}_o \) as \( t \to \infty \).

IV. SIMULATION

This section presents some simulation results to verify the theory. The simulation is based on a 2-link robot as illustrated in figure 5. The robot’s end effector is required to follow a desired circular region with a radius of 0.05 m. The center of the desired region is moving in a circular trajectory in Cartesian space with a radius of 0.2m and center at (0.35, 0.35) (see figure 6).

The desired circular region is specified as:

\[
f(\Delta x) = (x_1 - x_{o1})^2 + (x_2 - x_{o2})^2 - 0.05^2 \leq 0 \tag{44}
\]

and the center of circular region is moving and is specified by the following trajectory:

\[
x_{o1}(t) = 0.35 + 0.2 \sin(3t) \\
x_{o2}(t) = 0.35 + 0.2 \cos(3t) \tag{45}
\]

The equation of motion of the 2-link robot without object can be derived as:

\[
M(q)\ddot{q} + \left(\frac{1}{2} \dot{M}(q) + S(q, \dot{q})\right)\dot{q} + g(q) = \tau
\tag{46}
\]

where

\[
M(q) = \begin{bmatrix}
\theta_1 + \theta_2 + 2\theta_3 c_2 & \theta_2 + \theta_3 c_2 \\
\theta_2 + \theta_3 c_2 & \theta_3 c_2
\end{bmatrix},
\]

\[
\frac{1}{2} \dot{M}(q) + S(q, \dot{q}) = \begin{bmatrix}
-\theta_3 s_2 \dot{q}_2 & -\theta_3 s_2 \dot{q}_1 + \dot{q}_2 \\
\theta_3 s_2 \dot{q}_1 & 0
\end{bmatrix},
\]

and

\[
g(q) = \begin{bmatrix}
\theta_3 c_1 + \theta_3 c_{12} \\
\theta_3 c_{12}
\end{bmatrix},
\]

and \( \theta_1 = m_1 l_2^2 c_1 + m_2 l_2^2 + l_1, \theta_2 = m_2 l_2^2 + l_2, \theta_3 = m_1 l_1 c_2, \theta_4 = (m_1 l_1 + m_2 l_2) g \) and \( \theta_5 = m_2 g l_2 c_2 \) are the unknown parameters, \( s_1 = \sin(q_1), c_1 = \cos(q_1), s_{12} = \sin(q_1 + q_2), c_{12} = \cos(q_1 + q_2) \).

In the above dynamic equation, \( m_1, l_1, l_2, l_1 \) and \( m_2, l_2, l_2, l_2, l_2 \) are the mass, link length, center of gravity and inertia of link 1 and link 2 respectively. The masses \( m_1 \) and \( m_2 \) approximately equal to 17.4kg and 4.8kg respectively. The initial position of joint 1 and joint 2 are unknown. The length of first and second links are equal to \( l_1 = 0.43m \) and \( l_2 = 0.43m \) respectively. Hence, from equation (46), we obtain

\[
M(q)\ddot{q}_r + \left(\frac{1}{2} \dot{M}(q) + S(q, \dot{q})\right)\dot{q}_r + g(q) = Y_d(q, \dot{q}, \ddot{q}_r, \dddot{q}_r)\theta_d,
\tag{47}
\]

where

\[
Y_d(q, \dot{q}, \ddot{q}_r, \dddot{q}_r) = \begin{bmatrix}
y_{11} & y_{12} & y_{13} & y_{14} & y_{15} \\
0 & y_{22} & y_{23} & 0 & y_{25}
\end{bmatrix},
\tag{48}
\]

and \( y_{11} = \dddot{q}_r, y_{12} = \dddot{q}_r + \dddot{q}_r, y_{13} = 2c_2 \dddot{q}_r + c_2 \dddot{q}_r - s_2 \dddot{q}_2 \dddot{q}_r + s_2 (\dddot{q}_1 + \dddot{q}_2) \dddot{q}_r, y_{14} = c_1, y_{15} = c_{12}, y_{22} = \dddot{q}_r + \dddot{q}_r, y_{23} = c_2 \dddot{q}_r + s_2 \dddot{q}_2 \dddot{q}_r, y_{25} = c_{12} \) and \( \theta_d = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5] \). The proposed controller in theorem 1 is used in the simulation with \( k_v = 30, k_p = 50, L_d = \text{diag}\{200, 200, 200, 200, 200\} \) and \( \alpha = 20 \). The initial position of robot end-effector was specified as \( (x_1(0), x_2(0)) = (0.3042, 0.6846) \).
For the multiple objective functions case, the robot end-effector is required to track the desired region which is an intersection of two circles specified by:

\[ f_1(\Delta x_1) = (x_1 - x_{o11})^2 + (x_2 - x_{o12})^2 - 0.05^2 \leq 0, \]
\[ f_2(\Delta x_2) = (x_1 - x_{o21})^2 + (x_2 - x_{o22})^2 - 0.06^2 \leq 0 \]

The trajectory of the centers of the two circles are specified as:

\[ x_{o11}(t) = 0.35 + 0.2 \sin(3t) \]
\[ x_{o12}(t) = 0.35 + 0.2 \cos(3t) \]
\[ x_{o21}(t) = 0.35 + 0.2 \sin(3t) \]
\[ x_{o22}(t) = 0.41 + 0.2 \cos(3t) \]

Fig. 8. Trajectory of a Desired Moving Region as an Intersection of two Circle

The proposed controller in theorem 2 is used in the simulation with \( k_v = 30, k_{p1} = 75, k_{p2} = 250, L_d = \text{diag}(200, 200, 200, 200, 200) \), \( \alpha_1 = 1.5, \alpha_2 = 5 \) and \( \alpha = 10 \). As can be seen from the graph in figure 9, the tracking errors converge with the online updating of and dynamics parameters.

![Region Error for Multiple Objective Functions](image)

Fig. 9. Region Error for Multiple Objective Functions

V. CONCLUSION

In this paper, task-space region tracking controllers with both single and multiple objective functions have been proposed. It has been shown that the robot end-effector is able to track a moving desired region with the dynamic parameters being updated online. Lyapunov-like functions have been proposed for the stability analysis of the region tracking controller. Simulation results have been presented to verify the theory.

REFERENCES